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NO SUCCESSFUL INFINITE REGRESS

Abstract. We model infinite regress structures — not arguments — by means of ungrounded recursively defined functions in order to show that no such structure can perform the task of providing determination to the items composing it, that is, that no determination process containing an infinite regress structure is successful.

Keywords: infinite regress; order; non well-foundedness; recursion; ungroundedness; recursively defined function; determination system

1. Introduction and prefatory characterization

We are concerned here with modeling and assessing with respect to their possibility *infinite regress structures*, not *infinite regress arguments*.[1] The latter are arguments that use the derivation of the existence of an infinite regress structure as some kind of argumentative resource, often as a form of *reductio ad absurdum*. We are not directly concerned with these arguments here. However, as we will use our modeling to argue that no infinite regress structure can successfully perform a determination process, our result may lend support to a vast class of infinite regress arguments.

We will first propose a preliminary definition of infinite regress structures and try then to turn it into a precise mathematical modeling.

[1] For the relation between infinite regress and infinite regress arguments, see [2, Part 5], [3, Chapter 1, pp. 1–56] and [4, pp. 421–438].

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We have an infinite regress if and only if:

i. we have a set D that is ordered by some relation $<_{\text{o}}$, and the transmission of some sort of determination P from some members of D to others, in such a way that,
ii. $<_{\text{o}}$ is a non well-founded relation, and
iii. each member of D receives P *only* from some $<_{\text{o}}$-prior members of D.[2]

An order $<_{\text{o}}$ is non well-founded if it contains a backward infinite chain of the form $\ldots x_2 <_{\text{o}} x_1 <_{\text{o}} x_0$. In an infinite regress structure, $<_{\text{o}}$ turns D into a channel of transmission of P, which we will call a *determination channel*. If D is finite, then the determination channel contains a loop.

Determination P may be almost anything: existence brought about by other existing members of D put into existence by still others; explanation provided by other members of D that are in turn explained by others; definition of some members of D in terms of others, which are defined in terms of yet others, or any other kind of condition or determination.

Although most times we visualize an infinite regress structure as a set with order type ω_0^* — the order type of the negative integers — or ω_0 the order type of the naturals (e.g. [3, Chapter 1, pp. 1–56]) such structures need not be non-dense or even discontinuous: we can set up an infinite regress structure on the real line, for example, by the following stipulation inspired by the Benardete and Yablo paradoxes [1, p. 259] and [10]:

$$\forall x \in \mathbb{R} \qquad f(x) = \begin{cases} 1 & \text{if } \max\{f(y) \mid y \in \mathbb{R} \,\&\, y < x\} = 0 \\ 0 & \text{otherwise.} \end{cases}$$

where $\mathbb{R}$ is the set of all real numbers. This is why graphs, which are discrete structures, may not be adequate to model infinite regress structures with full generality.

The particular order type of D will not make any difference, provided it is non well-founded. The point of an infinite regress structure is non

[2] The existence of such a dependence relation is sometimes used to distinguish *vicious* from *benign* infinite regress; see, for instance, [4, Section 3], [6] and [7]. Making or avoiding that distinction only amounts to a difference in terminology: 'infinite regress' means here what other authors would call 'vicious infinite regress'.

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well-foundedness or *ungroundedness*: the fact that the determination channel contains *recursion* — that is, determination of each item based on determination of previous items — but no initial or base case. In addition, one essential condition must be fulfilled by any infinite regress structure: there must be a genuine dependence relation between the items in D as regards the acquisition of P, that is, each item in D must in fact depend on prior items in order to get P; the items in D are determined as regards P exclusively by other items in D; this is what clause iii. makes explicit. This condition we call *exclusiveness*. Recursion plus ungroundedness plus exclusiveness characterize infinite regress structures.

This is our preliminary and intuitive approach to infinite regress and we will assume it as a definition. It is against this prefatory definition that we will painstakingly assess our definitive modeling of infinite regress structures in the appendix.

Traditionally, a chain of determination containing an infinite regress was deemed impossible or doomed to failure for (broadly taken) logical reasons. And in our view this opinion rests indeed on an imperious intuition; if, for instance, z explains y but y requires further explanation to be found in x, and so on, it is intuitive that no explanation is actually provided. Let us recall, however, that not everyone is of the same opinion. Here is the famous argument by Cleanthes in Hume's *Dialogue Concerning Natural Religion*:

> In such a chain, too, or succession of objects, each part is caused by that which preceded it, and causes that which succeeds it. Where then is the difficulty? But the WHOLE, you say, wants a cause. I answer, that the uniting of these parts into a whole [...] is performed merely by an arbitrary act of the mind and has no influence on the nature of things. [5, p. 59]

It is natural to assume that, regarding our description above, Cleanthes would claim that there is no reason for the chain of *P-receivers* and *P-givers* to fail, since each term in it receives P from some prior term, so that, for any member x of D, the possession of P by x is fully explained. But this is far from obvious since one can reasonably suspect that *actual explanation* requires *ultimate explanation* — explanation that is not indefinitely postponed — and that in such a structure the possession of P is actually explained for no member of D. We will put forward two examples to support this intuition.

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Consider a set of lenders and borrowers formed by John, Mary and Peter, and let C be a coin. Assume that John claims he is legitimately in possession of C and is entitled to lend it out because Mary has lent it to him; assume Mary claims she had the right to lend C to John on the grounds that she was previously lent C by Peter, and that Peter claims the right to lend C to Mary, at the time he did, based on the fact that he was previously lent C by John. This depicts a loop that would have John, Mary and Peter lending C to one another from eternity. The right claimed by each of them seems justified by the loan made by some other. However, there can be no such rights because there is no *owner* of C who would ground on ownership rights the rights claimed by John, Mary and Peter.

My second case involves no loop. Imagine I am holding between my open hands the following structure: on top, a plank of wood one centimeter broad lying on another plank of wood broad half the first but equally shaped for the rest, and so on, with each of the infinitely many planks having half the breadth of the one just above it. The whole structure is two centimeter broad. Of course, this need not be physically possible: the example is a thought experiment and should be granted the benevolence usually bestowed upon the inessential details of such experiments. Anyway, imagine I drop the structure: will it fall? According to Cleanthes, it should not: the plank on top should not fall because it leans on the one immediately below and the same is true for any other. Since no plank should fall, the whole structure should keep floating on air. However, we know that it will fall as soon as no force counterpoises gravity.

2. Mathematical modeling and examples

We could claim that all possible cases of infinite regress are ultimately the same as one of the two examples above and that this makes an intuitive case against the possibility of successful determination through infinite regress. But for most of our contemporary philosophers showing that something is intuitive or counterintuitive is not enough to make the case for or against it. So, we will try to turn the intuitive idea that infinite regress must always fail into a formal argument by modeling infinite regress with the help of extremely simple mathematical tools. As there is little doubt that circularity entails failure in any determination channel,

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we will focus on noncircular cases of infinite regress. By 'infinite regress' we will understand hereafter 'noncircular infinite regress'.[3]

We aim to model infinite regress in terms that may permit us reasoning about it with due rigor.[4] So, we have to model non well-founded relations of exclusive dependence. Functions are the kind of mathematical objects that might seem to serve the end of capturing dependence relations and, particularly, functions defined by recursion, at least if they are ungrounded (i.e. they are defined for any argument in terms of their own values for prior arguments), may seem adequate to model infinite regress. Yet, functions, as they are understood in modern mathematics, are much too formal to actually embody relations of genuine and exclusive dependence: a set of ordered pairs, as the purely extensional entity it is, can hardly encapsulate real links of dependence, let alone exclusive dependence. This will be the main hurdle in our way in what follows, for we will model infinite regress structures by means of functions defined through ungrounded recursion and will have to manage to accommodate the *exclusiveness* requisite.

As said above, a function f defined by ungrounded recursion has no *base case*, that is, no value of f is given independently of values of f for prior arguments. Such functions are sometimes called *ungrounded*. Strictly speaking, what is ungrounded is not the defined function — ultimately, because sometimes no function is actually defined — but the definition of the function by recursion. Still, for simplicity, our talk will be of functions rather than of their (sometimes failed) definitions.

Let $f\colon D \to R$ be a nonempty function with domain D and range R. Let $<_{\mathrm{o}}$ be an order on D. Then for any $x \in D$ and $C \subseteq D$, we say that *$f(x)$ is essentially defined in terms of the values of f for the members of C* iff the following conditions are fulfilled:

i′. $f(x)$ is defined by means of some function $g\colon K \times J \to R$, where K is a subset of R^{κ}, for some cardinal $\kappa \neq 0$, and each κ-tuple $\langle f(y), f(z), \ldots\rangle$ in K is such that all the κ objects: y, z, $\ldots$ are in C, and J is a nonempty set of χ-tuples of parameters a, b, $\ldots$, for some cardinal $\chi \neq 0$, together with a clause of the form

$$f(x) = g(\langle\langle f(y), f(z), \ldots\rangle, \langle a, b, \ldots\rangle\rangle);$$

[3] However, the fact that the order relation $<_{\mathrm{o}}$ introduced below is not required to be strict allows one type of circularity, the shortest possible: $f(x) = g(\langle f(x)\rangle)$.

[4] Related attempts have been undertaken before (see [9]).

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if no parameters are required, J can be thought of as containing just one member, so that it contributes a constant value; we call function g the *auxiliary function*; we will dispense with angle brackets when not needed to make g have only one argument.

ii′. $f(x)$ cannot be defined otherwise.

Now, we will say that f is *recursively defined* iff the value of f for any of its arguments is essentially defined in terms of the value of f for some $<_{\text{o}}$-prior arguments; i.e., iff:

$$(\forall x \in D)(\exists C \subseteq D)[C \neq \emptyset \ \& \ (\forall y \in C)(y <_{\text{o}} x) \ \& \ f(x) \text{ is essentially defined in terms of the values of } f \text{ for the members of } C].$$ [5]

Function f gives determinations from R (or conveys P) to the members of D. The auxiliary function g is charged with the task of defining the values of f out of values of f for $<_{\text{o}}$-prior arguments. So, f, $<_{\text{o}}$, and g turn D into a determination channel. The definition of g obeys the following requirements. First of all, in defining f(x), g must act upon values of f for arguments $<_{\text{o}}$-prior to x, namely, those in C; there is no need to set a limit to the number of such values and this is why κ is any cardinal other than 0. But there is no reason either to prohibit g from acting on an arbitrary number of arbitrary parameters, provided this does not permit defining $f(x)$ otherwise than by recursion; the former is made possible by J and χ, which can be any cardinal other than 0; the latter is taken care of by ii′.

Note that we are trying not to lose generality in our modeling of infinite regress, so that we can claim that our model is able to model *any* infinite regress structure.

Clause ii′ is meant to capture the exclusiveness condition introduced above and it is necessary to guarantee that any recursively defined function will give rise to an infinite regress structure, because it is necessary to preserve ungrounded recursion as the only form in which f(x) can be defined.

An example of a recursively defined function is

$$\forall x \in \mathbb{Z}: \quad f_1(x) = g(f_1(x-1)) = 2f_1(x-1),$$

[5] The recursive clause in a recursively defined function is sometimes called a *recurrence equation*.

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where $\mathbb{Z}$ is the set of all integers. For any $x \in \mathbb{Z}$, $f_1(x)$ is essentially defined in terms of $f_1(x-1)$; obviously, trying to compute $f_1(x)$, for whatever x, leads to infinite regress.

Note that f_1 is essentially the same as a dynamical system of the form

$$x_{n+1} = f(x_n),$$

where $x_n = x(n)$, so that, substituting f for x and g for f, we can write

$$f(n+1) = g(f(n)).$$

In dynamical systems, a base case is usually supplied by the choice of an initial value or *seed* x_0. We could also provide f_1 with a base case, for instance:

$$f_1(0) = 1$$

But then the function could also be defined without recursion by

$$f_1(x) = 2^x$$

and it would no longer be a recursively defined function, for it would not comply with clause ii'. Note also that ungroundedness is not a sufficient condition for a function defined by recursion to be recursively defined: not all ungrounded functions defined by recursion are recursively defined functions. Consider the expression:

$$\forall x \in \mathbb{Z}\colon h(x+1) = g(h(x)) = 0 \cdot h(x).$$

This sole clause is enough to let us know that

$$\forall x \in \mathbb{Z}\colon h(x) = 0.$$

Thus, h does not satisfy clause ii$'$. Or consider $j\colon \mathbb{Z} \to \mathbb{Z}$, defined as injective, and $g = j$ such that

$$\forall x \in \mathbb{Z}\colon j(x) = g(j(x-1)) = j^2(x-1) \Rightarrow j(x-1) = x \Rightarrow j(x) = x+1.$$

h and j fail to model the dependence relation required by any infinite regress structure because neither $h(x+1)$ really depends on $h(x)$ nor $j(x)$ on $j(x-1)$, since in the former case all possible values of the function have collapsed into one and in the latter '$j(x)$' can be re-written as '$x+1$', where '$j(x-1)$' does not occur. We will call ungrounded functions that are defined by recursion but are not recursively defined functions *degenerate cases*. So, h and j are degenerate cases.

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The exclusion of degenerate cases guarantees that recursively defined functions involve infinite regress. That they can model all cases of infinite regress follows from the fact that we have proceeded without loss of generality, that is to say, imposing no unnecessary restrictions on the objects involved. So, if our preliminary characterization of infinite regress structures is adequate, as it seems to be, recursively defined functions have been defined in as general a way as possible, given the class of structures they are devised to model. So, it is sensible to claim that any infinite regress structure can be modeled by some recursively defined function. In any event, the reader can find a more detailed assessment of these issues in the appendix.

Elaborating on an idea we have used before [8], we can collect the main elements constituting a recursively defined function into a *determination system*. A determination system is then a sextuple $\Sigma = \langle D, R, <_\text{o}, f, \Delta, \Gamma\rangle$, where D and R are nonempty sets; $<_\text{o}$ is an order on D; $f\colon D \to R$ is a function; Γ is a set of auxiliary functions g, as defined above; Δ is a set of definitions d in which f is defined, by recursion with the help of some member g of Γ. Note that we are not requiring so far that f be recursively defined.

Determination systems, when successful, can be viewed as devices taking arguments from D and assigning them values or determinations from R in a recursive way by means of functions f's and their auxiliary functions g's, in accordance with the stipulations in the d's. So, recursion in the d's, always based on $<_\text{o}$, turns D into a determination channel.

A couple of examples may be in order here. If what we wish to model by some Σ is the temporal course of a system ruled by a causal law, D will be a set of time units ordered by the usual temporal order and R a set of events. Then f will give us the content of a time unit in terms of the content of preceding time units by means of some g in Γ, representing the incumbent causal law. For a very simple case, if we want to model the law that, for all $n \in \mathbb{Z}$, it rains at day t_{n+1} iff it didn't rain at t_n, we can represent presence of rain by 1 and absence of rain by 0, and then d will be as follows:

$$f(t_{n+1}) = g(f(t_n)) = 1 - f(t_n)$$

If we want to model a chain of definitions of concepts by means of other concepts, D will be a set of concepts ordered by some epistemological dependence relation and R will be a set of definitions of concepts. Then f will give us, with the help of the auxiliary functions, the definition of

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a concept in terms of the definitions of some other concepts. This is how the concept of addition (+) is usually defined in terms of the concept of successor (s), the concept of multiplication ($*$) is defined in terms of the concept of addition, and the concept of exponentiation (ˆ) in terms of the concept of multiplication:

$$\begin{aligned} \mathrm{o}_0 &= \mathrm{s}, \\ \mathrm{o}_1 &= +, \\ \mathrm{o}_2 &= *, \\ \mathrm{o}_3 &= \hat{} \end{aligned}$$

which are such that:

$$\begin{aligned} \forall x, y \in \mathbb{N}\colon x + 0 = x \;\&\; x + y + 1 = g_1(x + y) = \mathrm{s}(x + y) \\ \forall x, y \in \mathbb{N}\colon x * 0 = 0 \;\&\; x * (y + 1) = g_2(x * y) = x * y + x \\ \forall x, y \in \mathbb{N}\colon x\hat{}0 = 1 \;\&\; x\hat{}(y + 1) = g_3(x\hat{}y) = (x\hat{}y) * x \end{aligned}$$

If f in Σ is a recursively defined function, then Σ is a *recursively defined* determination system (RD-Σ, hereafter).[6] RD-Σ's always involve infinite regress because recursively defined functions do. Conversely, all infinite regress structures, as defined, can be modeled by some RD-Σ, since all of them can be modeled by recursively defined functions, as shown above and an RD-Σ is only the assembling of the objects constituting a recursively defined function.

3. Conclusion: No successful infinite regress

Now we want to show that no RD-Σ succeeds in determining values for the members of D because recursively defined functions fail to assign values to their arguments. Then we will show that, if a determination process running into an infinite regress could succeed, there could also be a *successful* RD-Σ: this implication encapsulates the entire merit of our modeling for our present purposes.

That recursively defined functions must fail to provide values for their arguments can be shown as follows. Let $f\colon D \to R$ be one such function

[6] In [8] I used a related construction; unfortunately, in that paper I forgot to explicitly exclude degenerate cases and cases in which $C = \emptyset$. I owe Casper Hansen acknowledgement for spotting the latter.

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and let's assume for simplicity that $<_{\mathrm{o}}$ is such that each member x of D has a $<_{\mathrm{o}}$-successor $\mathrm{s_o}(x)$ and that, for each $x \in D$, $f(\mathrm{s_o}(x))$ is a function of just $f(x)$ — generalization to other cases is straightforward. Then, if f is recursively defined, all we can get from its definition is a set of chains of conditionals of the form

$$\begin{aligned}
&\ldots\\
&\text{if } f(\mathrm{s_o^{-1}}(x)) = k \text{ then } f(x) = g(k),\\
&\text{if } f(x) = g(k) \text{ then } f(\mathrm{s_o}(x)) = g(g(k)),\\
&\text{if } f(\mathrm{s_o}(x)) = g(g(k)) \text{ then } f(\mathrm{s_o}(\mathrm{s_o}(x))) = g(g(g(k))),\\
&\ldots
\end{aligned}$$

and so for each pair $\langle \mathrm{x}, \mathrm{s_o}(\mathrm{x})\rangle$ of successive members of D. Each possible value of $f(x)$ would induce one such series of concatenated conditionals.

But it is a logical truth that no categorical sentence of the form '$f(x) = k$', for some member x of D, follows from such a chain of conditionals and only a categorical sentence of that form would actually determine a value for a member of D. This fact and clause ii. entail that f, as defined by d in Σ, is undefined for all members of D. Indeed, the recursive definition of the values of f leaves us with only a doubly infinite chain of hypothetical sentences concatenated as in this general form

$$\begin{aligned}
&\ldots\\
&\text{if } p_{n-1} \text{ then } p_n,\\
&\text{if } p_n \text{ then } p_{n+1},\\
&\text{if } p_{n+1} \text{ then } p_{n+2},\\
&\ldots
\end{aligned}$$

and, in general, from these no categorical sentence can be derived. In our case only a categorical sentence would deliver the determination of a value of f. Furthermore, by clause ii$'$, this necessarily failing way is the only way in which values of f could possibly be determined. These two facts ensure that no value is actually assigned by f to any x in D. So, *the fact that a set of concatenated conditionals entails no categorical statement turns out to be decisive for the question whether a process of determination involving an infinite regress can be successful.*

It follows that RD-Σ's fail to assign members of R to the members of D: there is no *successful RD-Σ*. But it follows from the fact that RD-Σ's can model any infinite regress structure that, if some items were

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actually produced, explained, entailed, qualified or anyhow else determined through infinite regress — shortly: if a *successful infinite regress* existed — a successful RD-Σ would also exist, namely, the one modeling the successful infinite regress structure at issue. Now it only takes a humble *Modus Tollens* to infer that there is no successful infinite regress.

Appendix

Our definition of recursively defined function has been devised to meet these demands:

a) Ensure that any recursively defined function models an infinite regress structure.
b) Ensure that no other conditions than the required for a) are imposed.

If, and only if, (b) is actually satisfied, any infinite regress structure can be modeled by some recursively defined function. Let us make reasonably sure that a) and b) have actually been met. a) is very intuitive, since ungrounded recursion and clause ii$'$. clearly make infinite regress unavoidable. So, let us address (b), which is the requirement of greatest possible generality. To address b), we exposit our definition of recursively defined functions more compactly through conditions (α) and (β) below.

For any nonempty sets D and R, a function $f\colon D \to R$ is recursively defined iff f satisfies the following conditions:

(α) there is a relation $<_{\mathrm{o}}$ ordering D and for any $x \in D$ there are: a nonempty subset C of D, cardinal numbers κ and χ, a subset K of R^{κ}, a κ-tuple $\overbrace{\langle f(y), f(z), \ldots\rangle}^{\kappa} \in K$, a set J of χ-tuples of parameters, a χ-tuple $\overbrace{\langle a, b, \ldots\rangle}^{\chi} \in J$, and a function $g\colon K \times J \to R$ such that

$$(\forall w \in C)(w <_{\mathrm{o}} x) \;\&\; (\forall \overbrace{\langle f(y), f(z), \ldots\rangle}^{\kappa} \in K)(\overbrace{y, z, \ldots}^{\kappa} \in \mathrm{C}) \;\&\; f(x) = g(\langle \overbrace{\langle f(y), f(z), \ldots\rangle}^{\kappa}, \overbrace{\langle a, b, \ldots\rangle}^{\chi}\rangle).$$

(β) $\forall x\colon f(x)$ can only be defined as in (α).

The formula in (α), even if it is a rather cumbersome one, should facilitate supervising all objects and conditions involved in our characterization of recursively defined functions.

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Let us assess the definitions (α) and (β) for generality. Concerning α, we must show that f, $<_{\mathrm{o}}$, C and g are required for infinite regress and as general as possible. First of all, these four objects are required: f is necessary to assign determinations from R to members of D, which represents the transmission of P from items to items in the determination channel, while $<_{\mathrm{o}}$, C and g are necessary to express recursion. Let us now evaluate their generality.

As regards f: f depends only on D, R, g, and C; D and R are as general as possible, for they are only prohibited to be empty; we consider C and g below.

As regards $<_{\mathrm{o}}$: being any order, $<_{\mathrm{o}}$ is as general as possible.

Note that f, which involves D and R, and $<_{\mathrm{o}}$ are the only objects independent of x: all others may be different for different members of D, and this lends the definition greater generality.

As regards C: from C, we only ask that it be a nonempty subset of D and that its members be $<_{\mathrm{o}}$-prior to x; the latter is clearly required for recursion and the former is necessary for $f(x)$ to be defined in terms of the value of f for other members of D, which is required by our definition of infinite regress according to which P is received by members of D from members of D. So C is as general as possible.

As regards g: g depends only on K, J, R, C, κ, and χ; we have already seen that R and C are as general as possible; κ and χ are just any cardinals other than 0; this restriction is necessary for g to be nonempty and for C to be nonempty, since infinite regress demands that $f(x)$ be defined in terms of some members of D that are $<_{\mathrm{o}}$-prior to x, and these are the members of C; hence, κ and χ are as general as possible; J is just any set and its presence contributes to make g as general as possible, for it provides arbitrary parameters for g; and if no parameters are required, J contributes just a constant; K must be a nonempty subset of R^{κ} because g must act on values of f for members of C; thus, the arguments of the members of the κ-tuples in K must be members of C; we set no other restrictions on K; hence K is as general as possible; hence, so is g.

Once C and g have been shown to be as general as possible, we know that also f is so.

Finally, (β) is nothing but clause ii', which is necessary to fulfill the exclusiveness condition present in our provisional definition of infinite regress.

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