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Jan Woleński

AN ABSTRACT APPROACH TO BIVALENCE*

Abstract. This paper outlines an approach to the principle of bivalence based on very general, but still elementary, semantic considerations. The principle of bivalence states that (a) "every sentence is either true or false". Clearly, some logics are bivalent while others are not. A more general formulation of (a) uses the concept of designated and non-designated logical values and is captured by (b) "every sentence is either designated or non-designated". Yet this formulation seems trivial, because the concept of non-designated value is negative. In order to refine the analysis, the class of anti-designated values has been distinguished. The non-trivial version of the principle of bivalence is expressed by (c) "every sentence is either designated or anti-designated". The last part of the paper mentions some extralogical reasons for considering the principle of bivalence with truth being a designated value as intimately connected to human thinking and behavior.

Keywords: logic; logical value; truth; falsehood; designated; non-designated; anti-designated.

The principle of bivalence is usually expressed as the following statement ("either or" expresses the exclusive disjunction)

(1) every sentence is either true or false.

Any logic based on (1) is called two-valued or classical. Its traditional importance as the very foundation of formal logic consists in its relation to the so-called highest (the most fundamental) laws of thought. In the framework of classical metalogic, (1) is equivalent to the conjunction of

* This paper is a continuation of my earlier writings, namely Woleński 1998, Woleński 2004. In fact, my approach is not the most abstract as compared with account offered in Shramko and Wansing 2011.

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two other basic rules of thought, namely the metalogical principle of excluded middle

(2) every sentence is true or false;

and the metalogical principle of non-contradiction

(3) no sentence is both true and false.

The adjectives "true" and "false" in (1), (2) and (3) are just metalogical (i. e. they belong to metalanguage and as such must be sharply distinguished from theorems $A \vee \neg A$, $\neg(A \wedge \neg A)$ and $(A \vee \neg A) \wedge \neg(A \wedge \neg A)$ that is, theorems of propositional calculus formulated in the object-language of logic. Jan Łukasiewicz was perhaps the first logician who observed that (1) is metalogical (see Łukasiewicz 1930).[1] It is important to see that the equivalence

(4) $(1) \Leftrightarrow (2) \wedge (3)$

holds for classical logic only.[2]

A very general semantic approach to logic consists in adopting some logical values as designated and some as non-designated. Intuitively speaking, if a formula A always has a designated value as its logical value it is a theorem of a given logic. Further, I will refer to sentences having designated values as designated sentences and to sentences having non-designated values as undesignated sentences. Assume that a function *val* (a valuation function) from $\mathbf{L}$ being a set of sentences to a set $\mathbf{V}$ of logical values is defined. There are three case ("card" is the abbreviation for "cardinality"):

(5) $\mathrm{card}(\mathbf{V}) = 0$; no A is a theorem;
(6) $\mathrm{card}(\mathbf{V}) = 1$; all A's are theorems (a given logic is inconsistent);
(7) $\mathrm{card}(\mathbf{V}) \geqslant 2$; some A's are theorems, some A's are not theorems.

Let $\mathbf{D}$ be a set of designated values. A logic $\mathbf{Lg}$ is consistent if $\mathrm{card}(\mathbf{V}) > \mathrm{card}(\mathbf{D})$ (a necessary condition). Moreover, if A is not a theorem, $\neg A$ is contradictory (absolutely or relatively, depending whether

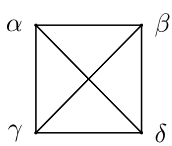
[1] See Betti 2001 for a discussion of Łukasiewicz's views on (1).

[2] See Béziau 2004 for a discussion of relations of various logics to (1), (2) and (3). I will discuss his suggestions in a special digression a the end of this paper. Russell and Cohn 2012 (a selection of papers from Wikipedia) provide a useful survey of problems related to the principle of bivalence.

the issue concerns pure logic or extralogical theories). In the case of a complete logic, A is contradictory iff A is always non-designated.

Clearly, (1) displays the minimal situation where we have exactly two values. It is normally assumed that truth is designated and falsity is non-designated. On these grounds, A is a tautology iff A is true for all valuations, and a counter-tautology iff A is counted as false by all valuations. On the other hand, dual logic (and other similar constructions) designates falsehood. This shows that designating truth is based on some pragmatic assumptions even in the most popular case of two-valued logic with truth as the designated value. Anyway, the sets of designated and non-designated values are mutually exclusive in any valuation. Moreover, both sets exhaust $\mathbf{V}$ (for simplicity sake, I consider truth-value gaps as logical values in this context). And the division of logical values into designated and non-designated forms the logical division in the traditional understanding (see Greniewski 1970; I skip the difference between the bivalentists and the pseudo-bivalentists introduced by this author). The problem of stability of having definite logical values (the question whether sentences change their values, for instance, in the course of time or dependently on the region of space in which facts described by the sentences in question occur) is more complex and I will omit it here.

On the other hand, there is no a priori reason to restrict $\mathbf{V}$ to two items. Consider the logical square (**S1**) with truth as modality ($\mathbf{T}$ – is true):



where particular points have the following interpretation: α – $\mathbf{T}A$, β – $\mathbf{T}(\neg A)$, γ – $\neg T(\neg A)$, $\neg\mathbf{T}(A)$. We have the following dependencies (analogical to the relations between modalities or categorical sentences):

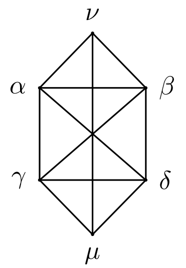
(8) $\alpha \Rightarrow \gamma$ (truth of A implies truth of non-truth of $\neg A$ is true);
(9) $\beta \Rightarrow \delta$ (truth of $\neg A$ implies non-truth of A);
(10) $\neg(\alpha \wedge \beta)$ (truth of A and truth of $\neg A$ are contraries);
(11) $\gamma \vee \delta$ (non-truth of $\neg A$ and non-truth of A are complementarities);
(12) $\alpha \Leftrightarrow \neg\delta$ (truth of A and non-truth of A are contradictories);
(13) $\beta \Leftrightarrow \neg\gamma$ (truth of $\neg A$ and non-truth of $\neg A$ are contradictories).

There is, however, a question where falsehood should be located in the diagram (**S1**) (the formula $\mathbf{F}A$ abbreviates "it is false that A" or "A is false"; I regard the operator and the predicative formulation as expressing the same proposition). There are two possibilities, namely to place $\mathbf{F}A$ either at the point β or at the point δ. The first choice equates $\mathbf{F}A$ with $\mathbf{T}(\neg A)$, while the second decision identifies $\mathbf{F}A$ with $\neg\mathbf{T}A$. In fact, (1) assumes that $\mathbf{F}A$, $\mathbf{T}\neg A$ and $\neg\mathbf{T}A$ are equivalent. To put it formally we have the following equivalences

(14) (a) $\mathbf{T}\neg A \Leftrightarrow \neg\mathbf{T}A$;
(b) $\mathbf{F}A \Leftrightarrow \mathbf{T}\neg A$ (or $\neg\mathbf{T}A$).

This a tacit assumption of the traditional principle of bivalence.

However, (**S1**), unless (14) is accepted, does not preclude that there are A's which are neither true nor false. It is illustrated by means of the logical hexagon (**S2**):



Additional points ν and μ have the reading as respectively "A is either true or false" and "A is neither true not false" provided that falsehood is located at the point β; this choice seems the most intuitive from the general point of view. If (14) is accepted, (**S2**) becomes reducible to the triangle determined by the points $\alpha\beta\nu$ and we can universally generalize ν. This move gives

(15) $\forall A(\mathbf{T}(A) \vee \mathbf{F}(A))$,

that is (1). Clearly, the principle of bivalence in this construction is not a necessary (analytic, metalogically tautological) statement, because it must be justified by stipulations recorded in (14); the mentioned reduction fails, if (14) is rejected. This observation immediately supports Łukasiewicz's view that the principle of bivalence goes beyond purely logical reasons. And what about the point μ, if (14) is rejected? There are several interpretative possibilities, for instance, (a) to admit the existence of truth-value gaps (sentences devoid of logical values; some logicians point out that the Liar sentence is gappy); (b) to allow other logical values than **T** and **F**, or (c) to introduce fuzziness. Such solutions lead to various reforms of classical logic, for instance, by introducing many-valued logic, logic with truth-value gaps or systems formalizing fuzzy (inexact, rough) concepts. The details of such constructions are fairly complicated. For example, fuzzy logic is frequently considered as a special kind of many-valued logic. We can also regard the gappiness as a separate logical value (this pragmatic solution was adopted above for simplicity. My further considerations are restricted to (b), except for some illustrations. It is sufficient to consider the three-valued system with **T** as the designated value.[3]

It is possible to interpret (**S1**) and (**S2**) by replacing **T** by **D**. In this context **D** refers to a designated value, not to the set of designated values. Thus, the formula $\mathbf{D}A$ means "it is designated that A", "A is designate" or "A is a designated sentence". We can formulate the principle of bivalence as:

(16) $\forall A(\mathbf{D}(A) \vee \neg\mathbf{D}(A)$,

which expresses the proposition that (every sentences is designated or non-designated.[4] For instance, every sentence is true or (false or neutral), every sentence is either true or (false or gappy). However, (16) seems trivial, although it is universally valid, according to the logic of generalized logical squares (see Woleński 2008 for a more extensive examination of such diagrams). In particular, the category of non-designated items is ambiguous or heterogenic, because it covers falsehoods and neutralities (or gappy sentences). In fact, this solution is too easy, because if we have a given non-void universe $\boldsymbol{U}$ consisting of three (or more) mutually exclusive sets X, Y, Z, that is, $\boldsymbol{U} = X \cup Y \cup Z$, it is possible to form the set$X' = Y \cup Z$ and transform the initial equality into $\boldsymbol{U} = X \cup X'$; the symbol X' refers to the complement of X.

In order to refine the picture, we can divide non-designated sentences into anti-designated (Rescher 1969, Malinowski 1993, Gottwald 2001, Shramko, Wansing 2011), $\mathbf{D}^{\mathbf{A}}$ and simply non-designated $\mathbf{D}^{\mathbf{S}}$. The rule is that if $\mathbf{D}^{\mathbf{A}}(A)$, then $\mathbf{D}^{\mathbf{S}}(A)$, but not conversely. Moreover, $\mathbf{D}(A)$ and $\mathbf{D}^{\mathbf{A}}(A)$ are contraries, but $\mathbf{D}(A)$ and $\mathbf{D}^{\mathbf{S}}(A)$ — contradictories. Simply

[3] Special problems are connected with paraconsistent logic. See Digression at the end of this paper.

[4] This version occurs in Kotarbiński 1913.

non-designated sentences can have designated values in some models and non-designated values in other models. Having these dependencies we can formulate a non-trivial generalized version of the principle of bivalence as:

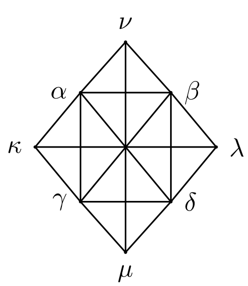
(17) $\forall A(\mathbf{D}(A) \vee \mathbf{D}(\neg A))$,

which says that every sentence is either designated or anti-designated. Clearly, (15) (and, *a fortiori*, (1)) becomes a special case of (17), if **T** and **F** are admitted as the only logical values and truth is designated, but falsehood functions as anti-designated. Moreover, there is no difference between being anti-designated and non-designated for the latter. On the other hand, (17) also holds if **F** serves as the designated value (dual logic). Clearly, the principle (17) is not-tautological, because it is false in some metalogics.

If **T** is designated, its behavior is governed by the **T**-scheme

(18) $\mathbf{T}A \Leftrightarrow A$,

but we cannot replace **T** by **D** in (18). To see it, consider the diagram (**S3**)



The points κ and λ refer to A and $\neg A$, respectively. Although

(19) $\mathbf{D}(A) \Rightarrow A$,

holds for any designated value, because if it is true that A is designated, A represents its designated value, its converse generally fails. In fact, if a sentence A represents a value v, it does not mean that it is designated. For instance, if *val* ascribes the neutrality value n to A in the three-valued system of Łukasiewicz with **T** as designated, we still have that $\mathbf{D}A \Leftrightarrow A$, but $n(A)$ does not imply $\mathbf{D}A$.

The above analysis suggests that Tarski had a good intuition when he characterized the concept of truth by applying **T**-equivalences. Propositional calculus is the simplest case of logic. We can introduce truth and falsehood as unary connectives by equations (1, 0 refer to values of the function *val*)

(20) $v(\mathbf{T}A) = 1$ iff $v(A) = 1$; otherwise $v(A) = 0$.

These equations as well as earlier observations, however simple and perhaps even trivial, are interesting for at least four reasons. Firstly, they show why (1) is frequently confused with the law of excluded middle. In fact, (20) justifies the reduction of $\mathbf{T}A \vee \neg\mathbf{T}A$ (or $\mathbf{T}A \vee \mathbf{T}\neg A$ or $\mathbf{T}A \vee \mathbf{F}A$) to $A \vee \neg A$. Yet this is only a partial success, because if we move to predicate calculus, it turns out that introducing "it is true" as a monadic connective does not suffice as the foundation of semantics and the full-blooded truth-definition based on the concept of satisfaction must be employed. Secondly, (20) additionally explains why (**S2**) and (**S3**) cannot be reduced to the triangle $\alpha\beta\nu$ in the case of admitting neutralities or truth-value gaps. Thirdly, considerations about the **T**-scheme show that **T**-equivalences are not longer logical tautologies beyond propositional calculus (see Woleński 2008a for further arguments). Fourthly, (16), (17) and (20) contribute to an analysis of the Suszko Thesis (see Suszko 1977, see also Woleński 2009 and Caleiro, Carnielli, Coniglio and Marcos 2007; while I am skeptical about Suszko's thesis, the Brazillian authors defend it), according to which every logic is bivalent. More specifically, the Suszko Thesis distinguishes logical values $(\mathbf{T}, \mathbf{F})$ and algebraic values (for instance, neutralities in three-valued logic). The simplest argument saying that this distinction is conventional consists in pointing out that every logic can be made bivalent by (16), but not by (17). In other words and on purely logical grounds, the Suszko Thesis is problematic even if we replace **T** by **D**. In fact, it is trivial for **D** and $\mathbf{D}^{\mathbf{S}}$, but not for **D** and $\mathbf{D}^{\mathbf{A}}$, although some abstract algebraic constructions somehow "de-trivialize" the problem. Yet it is dubious whether they remain inside logic.

The formula (17) holds for any two-valued case (the paraconsistent case will be considered separately below), for instance, when **F** serves as the designated value. Yet logical investigations much favor (15), particularly in metalogic. Moreover, metalogical studies are based on classical logic even if they pertain to non-classical systems. Although there are various attempts to execute non-classical metalogic, the results are fragmentary and limited in their applications. For instance, important metatheorems concerning intuitionistic logic have no proofs by which would use methods accepted by the intuitionsts; the completeness theorem for intuitionistic predicate calculus is a very spectacular example in this respect. In fact, no general metatheory of logic based on non-classical logic is available. Although this conclusion has the status of an empirical generalization, it suggests that non-classical logics arise when some refinements of classical logic help to shed some light on specific problems (constructive proofs, fuzziness, gaps, future contingents, paradoxes, quantum mechanics, etc.). Anyway, the truth-talk has a privileged status in metalogic and there is nothing to indicate that the situation will change. For example, logicians speak about truth-functions although the label "falsehood-functions" appears as fully legitimate from the theoretical point of view.

What are the reasons for seeing bivalence as basic and truth as designated? Apart from pragmatic motives pointed out in the last paragraph, we can list the following circumstances (I restrict myself to a stock of some factors without entering into details; the list below is nothing more than a body of speculative suggestions):

(a) Truth represents facts, but falsehood indicates the lack of facts. People are more interested in facts than in the absence of facts (this point was made by Jerzy Perzanowski in one of our discussions about the concept of truth;
(b) Spatial and temporal oppositions (symmetric or not): left–right, bottom–top, earlier–later, back–front, outer–inner;
(c) Biological oppositions: life–death;
(d) Motorical oppositions: active–passive;
(e) Other distinctions: having something–having nothing, having all (in some collection)–having not-all; modal contrasts and dualities differences in skills, utilities, duties, values, attitudes, etc., for instance, duty–right or useful–non-useful. Observe that some oppositions involve positive and negative contrasts, while other are based on the distinction between the positive and the privative;
(f) Binary rhythms;
(g) Sensual contrasts: dark–bright, loud–silent, colorful–colorless;
(h) The structure of double helix;
(i) Binary character of genetic codes;
(j) 1-0 nature of information;
(k) the belief that preserving truth help protecting information, but preserving falsehood is a "measure" of information–dispersion;
(l) The dual behavior of many quantifiers in the natural language.

For these and presumably many other reasons, logic acting as the framework of human thinking in a notoriously contrastive environment seems to require a minimal world as its ontological counterpart and this is the world with two items, i.e. logical values. For pragmatic reasons, one component was called Truth and the other was dubbed Falsehood. Let me finish with an anecdote. Several years ago, perhaps at the beginning of the 1970s, Roman Suszko delivered a talk on Non-Fregean logic. He argued that the Frege axiom asserting that all true propositions refer to one object, namely the True, and all false propositions denote the False as their very reference, is not intuitive and should be rejected. According to Suszko, propositions refer to situations and we know that there are many situations. In the discussion held after the talk, Andrzej Wroński remarked that the logical world should be as limited as possible, that is, to two items. Clearly, (7) shows the minimal furniture of the logical reality. Although the principle of bivalence with $\mathbf{T}$ as the designated value bears no stamp of logical necessity, we can give several empirical reasons to support it.

A digression

Jean-Yves Béziau (see Béziau 2004) derives the principle of bivalence from the following statements (I use my terminology):

(i) The set of logical values is limited to two values;
(ii) These truth-values are truth and falsehood;
(iii) *Val* is a function;
(iv) *Val* is a total function.

The points (i)–(iv) are summarized in one statement (Béziau 2004, p. 74):

The evaluation relation is a total function whose domain is the set of propositions and the codomain is a set of truth-values, true and false.

Clearly, (i) excludes many-valueness, (ii) identifies truth-values, (iii) states the standard property of *val* and (iv) excludes truth-value gaps.

Béziau's main intention is to show that (1) should not be identifies with the conjunction of (2) and (3), because, on one hand, we have a bivalent semantics for logic without the principle of excluded middle, and, on the other hand, there is a bivalent semantics, for logic without the principle of non-contradiction. Béziau's makes several observations, for instance, he says (p. 76) that bivalence "reappears at another level" in many-valued logic, namely as the principle expressed by (16). However, as I pointed out, this is quite trivial version of (1) as compared with (17). Using Béziau's terminology, (16) obeys the principle of bivaluation, but not the principle of bivalence. Thus, I exclude bivaluational semantic constructions as necessarily bivalent (in my sense).[5] Moreover, Béziau toys with the principle of excluded middle (EM, for brevity). First of all, it is not always sufficiently clear whether he appeals to its logical or metalogical version, as both versions appear in his considerations. Furthermore, Béziau offers a non-standard formulation of the principle in question, namely

(EM) A sentence A and its negation cannot be both false.

Béziau argues that (1) does not entails (EM), because there is a bivaluation ascribing 0 to A as well as to $\neg A$. An answer to this argument points out that (i)–(iv) do not impose truth-conditions for connectives. Yet the classical bivalent semantics as contrasted with various bivaluations defines the functor of negation in the manner used in (1).

The paraconsistent case is perhaps particularly interesting. Doubtless, paraconsistent semantics obeys (i)–(iv). The third condition allows that both A and $\neg A$ can have the value 1 in some paraconsistent systems or the value 0 in other systems. The first case concerns so-called dialetheias (contradictory statements which are both true), but the second case tolerates anti-dialetheias (the name is *ad hoc*), that is, ascribes falsehood to both mutual contradictories.[6] Moreover, we can designate $\mathbf{T}$ (or $\mathbf{F}$, if one likes), define a non-explosive consequence operation and determine a sound "bivalent" logic (the quotes here will be explained in a moment). However, there are some survey problems to be tackled. First of all, dialetheias (and anti-dialetheias as well) cannot be generally generated, but arise only from very special cases. These are, for instance, the Russell sentence about sets which are not their own elements, the Liar sentences, etc. One can ask what is a difference between such sentences and "normal" contradictories. Secondly, I did not find any natural matrix semantics for paraconsistent logic.[7] Hence, properties of connectives are somehow vague or even mysterious. For

[5] See Caleiro, Carnielli, Coniglio and Marcos 2007 for a very sophisticated account of dyadic (bivaluational) semantics. However, constructions presented in this paper seem to be based on the distinction of designted and non-designated values.

[6] Dialetheieas and anti-dialetheias are someitems called truth-value gluts.

[7] Compare for instance Béziau, Carnielli, and Gabbay 2007 or Priest and Tanaka 2009.

instance, the negation in discussive logic is classical, but the negation in one of Béziau's examples is not. Thirdly, the relation of paraconsistent logic to many-valued logic is not clear. Some logicians (see Priest and Tanaka 2009 for further information) regard many-valueness as the best way to explain paraconsistency, but other tend to search for different solutions. Fourthly and more specifically, it is very unclear how the **T**-scheme should be understood if we insert dialetheias (or anti-dialetheias) in it. In the former case, it is very problematic whether paraconsistency obeys the principles of bivalent semantics, but if the latter direction is taken, the issue appears to be open. These arguments justify the hypothesis that paraconsistency is bivalent if and only if it is bivaluational. In fact, although bivalence leads to a bivaluation, the reverse dependency does not generally hold. Let me add that my critical remarks about paraconsistent logic are not intended as offering arguments against formal studies on consistency, its sources, scope and limitations. All I wanted was to show why the standard concept of bivalence should not be extended beyond classical logic.

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Jan Woleński
Institute of Philosophy
Jagiellonian University
31-044 Kraków, Poland
wolenski@if.uj.edu.pl