Logic and Logical Philosophy
Volume 22 (2013), 63–73
DOI: 10.12775/LLP.2013.004

Piotr Łukowski

IS HUMAN REASONING REALLY NONMONOTONIC?

Abstract. It seems that nonmonotonicity of our reasoning is an obvious truth. Almost every logician not even believes, but simply knows very well that a human being thinks in a nonmonotonic way. Moreover, a nonmonotonicity of thinking seems to be a phenomenon parallel to the existence of human beings.[1] Examples allegedly illustrating this phenomenon are not even analyzed today. They are simply quoted. Nowadays, this is a standard approach to nonmonotonicity. However, even simple analysis of those "obvious" examples shows that they illustrate various problems of our thinking, among which none concerns nonmonotonicity.

Keywords: entimematic premise, error of generality, human thinking, nonmonotonic reasoning, nonmonotonicity, non-nonmonotonicity, Tweety the ostrich, reasoning increasing preciseness, reasoning from open and hidden premises, stereotypical thinking, thinking by the most frequent cases.

Introduction

Reasoning as a human act bases on some previously accepted inference relations. It seems that, in the case of a human being the inference is not classical. However, some classical rules are used by us everyday. Such rules like *Modus Ponens*, *Modus Tollens*, *Disjunctive Syllogism*, *Hypothetical Syllogism*, *Simplification* seem to be quite natural for our thinking. Can we really think nonmonotonically using all these classical rules and some more? Thus, there is a question, if the inference used in

[1] "Of course, humans have been reasoning nonmonotonically for as long as they have been reasoning at all", Makinson [1994], p. 36.

Received March 9, 2012; *Revised* July 31, 2012
 ISSN: 1425-3305

human thinking is monotonic or not. An inference $\vdash$ is *monotonic* if and only if, for any $s, t \in L$ and $A \subseteq L$: if $A \vdash s$, then $A \cup \{t\} \vdash s$, where L is a language. Thus, $\vdash$ is *nonmonotonic* if and only if, it is not monotonic, i.e.: for some $s, t \in L$ and $A \subseteq L$: $A \vdash s$ and $A \cup \{t\} \nvdash s$. In other words, an inference is nonmonotonic, if sometimes some formula follows from the given set of premises and the same formula does not follow from the set being a superset of the given one.

Directly from the definition, it follows that nonmonotonicity cannot appear in one separate step of reasoning. For recognition of nonmonotonicity we need at least two-step reasoning. Moreover, if the inference will be monotonic (it does not matter, whether classical or not), the reasoning basing on this inference will be monotonic too. It is not difficult to notice, that if the first step of reasoning $A \vdash s$ uses e.g. *Modus Ponens* (also *Modus Tollens*, *Disjunctive Syllogism*, and so on) for inferring s from A, then in the second step, s must be also inferred from $A \cup \{t\}$, for any t. Rules of inference that we use every day make our thinking monotonic: if the set A, sentence s and rules inferring s from A will not change, then s will be inferred from every superset of A thanks to exactly the same set of rules, as in the first step of reasoning.

From the logical point of view, nonmonotonicity is totally unintuitive. Let us assume that among all formulas from $Z = \{p_1, \ldots, p_n\}$, only p_2, p_3, p_5 suffice for inferring of p. Thus, thanks to premises p_2, p_3, p_5, and rules from the set R, we have $Z \vdash p$. In other words, when we have p_2, p_3, p_5, and R, we always have p also. It is a matter of premises and rules, only. Now, let us assume that the set Z is enlarged by $q \notin Z$. Is it possible that having still the same set of premises and the same rules we can be in the situation that p cannot be inferred from $Z \cup \{q\}$? Yes, but only when q makes invalid at least one premise from p_2, p_3, p_5. Let us suppose that p_2 is invalidated by q. In such a situation, possessing Z and q, we do not have p, because we cannot use p_2. However, it means that we still can infer p from $Z \cup \{q\}$, but using q "suggests" us not to use p_2. Why? Probably because $Z \cup \{q\}$ would be an inconsistent set of believes. In such a case, when we have q, we "should" always reject p_2. Thus, the correct diagnosis of the situation is as following:

1. $\{p2, p3, p5\} \vdash p$,
2. $Z \vdash p$,
3. $Z \cup \{q\} \vdash p$,
4. $(Z \setminus \{p2\}) \cup \{q\} \nvdash p$.

From such a point of view, nonmonotonicity is just a trick preserving an ordinary monotonic reasoning from inconsistency. Some of currently available premises become forbidden because of some new information. That is all. But since they are forbidden, no-one can say that the old set of premises is still kept. The forbidden premise is never again a premise. It means that the premise is just rejected from the set of believes. Of course, such a trick is against the definition of nonmonotonic reasoning. Obviously, a two-step reasoning (expressed in points 1 and 4) based on inference $\vdash$ satisfying the conditions 1–4 is monotonic (although maybe not according to the classical logic). There is neither reason nor sense to suppose that the inference $\vdash$ satisfying 1–4 is nonmonotonic. If, in the second step of reasoning, we have to remove at least one premise from the set of believes crucial for the reasoning, we cannot say that the reasoning is nonmonotonic. Instead of nonmonotonicity we have here *contraction* plus *expansion*. This general case shows that instead of looking for nonmonotonicity, we should be more careful and strict analyzing our thinking, which probably uses monotonic inferences.

Now, let us analyze some examples which are fundamental for nonmonotonicity. For all of them one necessary from the point of view of the definition of nonmonotonic inference condition is satisfied: *the set of rules of the inference and meaning of premises are exactly the same for both steps of successively considered reasoning*. Satisfaction of this condition is necessary for proper recognition of nonmonotonicity of the inference in such a sense that if the condition is not satisfied, an inference cannot be recognized as nonmonotonic (and so monotonic).

1. Diagnosis for medical treatment

One of the most standard examples for nonmonotonicity of human thinking is medical diagnosis. At first view it seems that physicians think in a nonmonotonic way.

Knowing the results $p_1, \ldots, p_n$ of several medical tests and researches doctors decide: in this case it is the illness z_1. Rarely, first medical tests are complete. That is why, during next days it is possible to make another tests. When physicians receive another information p_{n+1}, they think: in this new situation (but still in the same case as before) we think that the patient suffers from the illness z_2. Of course, later this diagnosis can be change again. Thus allegedly, the structure of reasoning here has the following form:

$\{p_1, \ldots, p_n\} \vdash z_1$
$\{p_1, \ldots, p_n, p_{n+1}\} \vdash z_2$ and $\{p_1, \ldots, p_n, p_{n+1}\} \nvdash z_1$
$\{p_1, \ldots, p_n, p_{n+1}, p_{n+2}\} \vdash z_3$, but $\{p_1, \ldots, p_n, p_{n+1}, p_{n+2}\} \nvdash z_1$ and $\{p_1, \ldots, p_n, p_{n+1}, p_{n+2}\} \nvdash z_2$.

Unfortunately, the truth is quite different. In many real medical cases, no well educated physician believes that results p_1, ..., p_n indicate only one illness z_1. Every doctor knows very well that one set of symptoms can indicate a group of various illnesses: $\{p_1, \ldots, p_n\} \vdash \{z_1, \ldots, z_k\}$. Indeed, thanks to medical studies, a physician remembers that the sentence accepted on the ground of medicine is of the form $(p_1 \wedge \cdots \wedge p_n) \rightarrow (z_1 \vee \cdots \vee z_k)$, and not $(p_1 \wedge \cdots \wedge p_n) \rightarrow z_1$. Maybe a doctor does not remember all z_1, ..., z_k. In such a case they will believe that the shorter disjunction is a successor of the implication. Of course, an acceptance of the implication is a matter of medicine only, and not logic, neither classical nor non-classical.[2] Finally, a physician can use an extremely intuitive, classical rule of *Modus Ponens*. Thus, they infer $z_1 \vee \cdots \vee z_k$ from the set $\{p_1, \ldots, p_n\}$. It means, that

$$\{p_1, \ldots, p_n\} \vdash z_1 \vee \cdots \vee z_k \,.$$

Of course a disjunction $z_1 \vee \cdots \vee z_k$ is not efficient conclusion. That is why, basing on experience a physician makes a choice: z_1. Of course, they are influence e.g. by the knowledge about frequency of illnesses z_1, ..., z_k in this area, the probability of appearance of these illnesses, etc. One way or another, the physician chooses z_1 knowing that also another illnesses could be associated with these symptoms. Moreover, it is not excluded that another physician would diagnose another illness in this case. This way of reasoning can be repeated for new information. Then, new illness can be selected. During the process, the list of illnesses becomes shorter with every step. The list can also be supplemented by illnesses previously not considered.

[2] Unfortunately, from time to time, there appears an opinion that a physician cannot classically infer z_1 from $\{p_1, \ldots, p_n\}$, but infer it using some non-classical inference. In fact, this inference is possible thanks to *Modus Ponens* applied to p_1, ..., p_n and implication $(p_1 \wedge \cdots \wedge p_n) \rightarrow z_1$. An acceptance of $(p_1 \wedge \cdots \wedge p_n) \rightarrow z_1$ is possible (or not) thanks to inductive reasoning applied to a big amount of sentences expressing many observed and recorded individual facts.

Thus, in fact, the structure of the reasoning is the following:

$$\{p_1, \ldots, p_n\} \vdash z_1 \vee z_2 \vee z_3 \vee \cdots \vee z_k$$
$$\{p_1, \ldots, p_n, p_{n+1}\} \vdash z_2 \vee z_3 \vee \cdots \vee z_k$$
$$\{p_1, \ldots, p_n, p_{n+1}, p_{n+2}\} \vdash z_3 \vee \cdots \vee z_k.$$

The inference $\vdash$ has nothing in common with nonmonotonicity. It is just monotonic. Indeed, we still have:

$$\{p_1, \ldots, p_n, p_{n+1}\} \vdash z_1 \vee z_2 \vee z_3 \vee \cdots \vee z_k$$
$$\{p1, \ldots, p_n, p_{n+1}, p_{n+2}\} \vdash z_1 \vee z_2 \vee z_3 \vee \cdots \vee z_k.$$

This schema is general. So, it is necessary to mention that in some cases it is possible that one step of the trial of diagnosis can be the last one. No-one can exclude the situation that some test or other research will give an unambiguous result.

A recognition of the medical diagnosis as an illustration of nonmonotonic reasoning obscures an interesting problem represented by this kind of examples: a **reasoning increasing preciseness**. This important type of human thinking is an essence of all diagnosis: medical and any other.[3] Thanks to such reasoning, step by step we limit a scope of possible solutions. The less solutions, the more precise and so better knowledge.

The next case concerns the so called "unless" problem.

2. Meeting in the pub

Let us assume that John has an appointment with Mark: they plan to spend the Saturday night in the pub "Ten Bells". This fact entails some consequences. At proper time, John has to call a taxi, leave home and go to the pub. Just before that time John's phone starts ringing. Anna informs John that Mark has had a car accident. Of course, John will not go to the pub. Maybe he will go to the hospital or just stay at home. Thus, the new information invalidates earlier premises of John's reasoning.

It allegedly seems that the schemata of the reasoning represented by this standard for *defaults* example is the following:

$$\{p_1, \ldots, p_n\} \vdash z$$
$$\{p_1, \ldots, p_n, p_{n+1}\} \nvdash z.$$

[3] We also use a *reasoning increasing preciseness* when we look for something. Then, step by step we eliminate checked places.

This form suggests that there is a nonmonotonicity here. Obviously, it is not true.

Let us see that a car accident is not the only case which can make the meeting impossible. There are plenty of events which may invalidate the meeting: John or Mark could forget about the meeting, the pub could be suddenly closed, John or Mark could be ill, John's mother (father, brother, sister, close friend, etc.) could need his help, and so on. It means that z does not follow only from the set $\{p_1, \ldots, p_n\}$. The conclusion z follows from the much larger set of premises $\{p_1, \ldots, p_n\} \cup \{q_1, \ldots, q_s\}$, where $q_1, \ldots, q_s$ are negations of sentences stating all those cases which, if came true, would make the meeting impossible. Usually, nobody realizes all $q_1, \ldots, q_s$. We can think only about some of them. For example, if I feel not excellent, I would say to Mark: "Yes, I will meet with you in the pub, only if my sore throat passes". Although most of $q_1, \ldots, q_s$ are not recognized by us, we know very well that such big set of "additional" conditions exists. Moreover, we understand well that every condition is for z necessary, but not sufficient.

Thus, in fact, the schema of the reasoning in such cases is the following:

$$\{p_1, \ldots, p_n\} \cup \{q_1, \ldots, q_s\} \vdash z$$
$$\{p_1, \ldots, p_n\} \cup \{q_1, \ldots, q_{i-1}, \neg q_i, q_{i+1}, \ldots, q_s\} \nvdash z, \text{ for any } i \in \{1, \ldots, s\}.$$

This schemata represents a **reasoning from open and hidden premises**, which is not nonmonotonic. Premises $p_1, \ldots, p_n$ are *open* (*explicit*, *direct*) and $q_1, \ldots, q_s$ are *hidden* (*implicit*, *indirect*). All premises are necessary for the conclusion. However, we treat them in a different way. We used to speak loudly only about all open premises, and usually do not say at all about any of the hidden ones. Thus, $q_1, \ldots, q_s$ are *entimematic* premises. Usually, we even does not think about any hidden premise.

The schema above can be simplified:

$$\{p_1, \ldots, p_n\} \cup \{q\} \vdash z$$
$$\{p_1, \ldots, p_n\} \cup \{\neg q\} \vdash z,$$

where q = "it is not true that something happened that invalidates z". This schema seems to be closer to reality, because we actually know that our plans will be realized, if everything will goes well, without any troubles. Monotonicity of $\vdash$ is clear, and moreover we realize all premises which are necessary for z.

An interesting comment which is suitable for this and previous set of examples was formulated by Poole:

> In nonmonotonic reasoning we want to reach conclusions that we may not reach if we had more information. There seem to be two ways to handle this; we could change logic to be defeasible, or we could allow some premises of the logical argument that may not be allowed when new information is received. Default logic is a formalization of the latter; it provides rules that add premises to logical arguments.
>
> Poole [1994], p. 189.

Thus, Poole clearly said that in nonmonotonic reasoning there is some premise in the first step, which disappears in the second step. It is a contradiction: *the nonmonotonic reasoning is not nonmonotonic.*

3. Tweety the ostrich

The case of Tweety the ostrich is probably the most popular example for nonmonotonic reasoning. Knowing that Tweety is a bird, we think "Tweety can fly". Later, receiving a new information that Tweety is an ostrich, we know that "Tweety cannot fly".

Allegedly, the schemata of the reasoning is the following:

$$\{p\} \vdash z$$
$$\{p, q\} \nvdash z,$$

with $p =$ "Tweety is a bird", $q =$ "Tweety is an ostrich", $z =$ "Tweety can fly". At first view it seems that it is a case of nonmonotonic reasoning. For the proper, logical recognition of the problem let us recall some facts basic for semiotics.

A name is *general* if it has more than one designate. "Bird", "ostrich", "animal", "man" are examples of general names. The set of all designates of a general name A is called a *range* of general name A. Since a range has more than one element, it is possible to divide it on disjoint subsets which sum is equal to the given range. In other words, it is possible to make a *logical division* of every range. For every subset of the logical division it is also possible to find/give a separate name for all designates of this subset. Recapitulating, for every general name A there are subordinate names $A_1, \ldots, A_n$ such that a range of A is a sum of ranges of $A_1, \ldots, A_n$. Thus, $\underline{A} = \underline{A}_1 \cup \cdots \cup \underline{A}_n$, where $\underline{X}$ is a range of X. In such configuration, A (also $\underline{A}$) is called *genus*, and every A_i (also

$\underline{A}_i$) *species*. It is easy to notice that logical (i.e. not biological) "genus" and "species" are relative notions. Every general name can be *genus*, can be also *species*. That is why, it is reasonable to use words "genus" and "species" for the pair of names A and B, saying "A is a species of the genus B" or "B is a species of the genus A".

Sometimes some species B dominates among all other species of the given genus A. Such a situation we have for two names: A = "bird", B = "flying bird". Of course, there are many birds which cannot fly: penguins, ostriches, kiwis, many small birds of tropical jungle. But much more birds can fly. So, flying has become for us a "typical" feature of bird. Thinking about every representative of a genus A as belonging to dominate species B is a logical mistake called the **error of generality**. Then, the dominate species plays a role of a "typical" representative of the genus. Such a thinking is called **stereotypical**.[4]

The ostrich Tweety is an example of the *stereotypical thinking*. Let us recall that such thinking bases on simple logical mistake. A schemata of the reasoning without the *error of generality* is the following:

$$\begin{array}{l} \{p\} \nvdash z \\ \{q_1\} \vdash z, \ldots, \{q_s\} \vdash z \\ \{r_1\} \nvdash z, \ldots, \{r_t\} \nvdash z, \end{array}$$

where p = "Tweety is a bird", z = "Tweety can fly", $q_1, \ldots, q_s$ – sentences stating that Tweety is a representative of the flying species (e.g. q_1 = "Tweety is a sparrow", q_2 = "Tweety is a duck", q_3 = "Tweety is a swan", ...), $r_1, \ldots, r_t$ – sentences stating that Tweety is a representative of the not-flying species (e.g. r_1 = "Tweety is an ostrich", r_2 = "Tweedy is a penguin", r_3 = "Tweedy is a kiwi", ...).

Sometimes, the ostrich Tweety is replaced in the example by a mammal which cannot lay eggs. In both cases we meet a stereotypical thinking, which is possible due to the error of generality. From the point of view of the thinking result both examples are identical. However, from the point of view of the origins of thinking they are different. In the case of the ostrich, stereotypical reasoning is a standard kind of a **sloppy thinking**. In the case of mammals which cannot lay eggs, stereotypical reasoning is a result of the **lack of knowledge**.

[4] Stereotypical kind of thinking is easy for recognition (especially) in another "nonmonotonic example" of pacifists-Kwarks and nonpacifist-Republicans, e.g. Ginsberg [1994], p. 12.

4. Car in front of the house

It seems that there is another version of Tweety the ostrich illustrating something different from stereotypical thinking. This another kind of reasoning can be called **thinking by the most frequent cases**, similar (but not identical) to the well known **authomatic thinking**. Let us assume that John's car is standing in front of his house. It can suggest that John is at home. However, John is not at home in the moment, because he is in the flat of his next-door neighbor (in the shop, etc.).

Probably, it is one of the most illogical examples for nonmonotonicity. It seems to be reasonable to accept an implication $s \to t$, where $s =$ "John's car is standing in front of his house" and $t =$ "Now, John is at home". In the first step we infer t from $\{s \to t, s\}$. In the second step, we find that t is not true. By the way, the second step is not a case of reasoning. An unique illogicality of this example comes from the fact that defense of nonmonotonicity is here possible only thanks to acceptance of contradiction. Indeed, at the same moment we need to accept all previous premises i.e. $s \to t$, s, together with a new conclusion $\neg t$. However, in such a situation, we need to accept simultaneously t and $\neg t$. Probably, this acceptance has nothing in common with the classical logic. Extremely popular *Modus Ponens* is the only necessary tool for this inference. Is there any sense to pay such a high price for nonmonotonic interpretation of the situation, when a monotonic solution is simple and natural? First, we should simply know that $s \to t$ is obviously false. So, an implication $s \to t$ should be replaced by $s \to t'$, where e.g. $t' =$ "Now, John should be at home". Obviously, we have no contradiction now. Moreover, an inference leads to something obvious: $s \to t'$, s, t' and $\neg t$, i.e. "If John's car is standing in front of his house, John should be at home. John's car is standing in front of his house. John should be at home but John is not at home". The solution is logical, natural and shows the monotonicity of our thinking in this case. Of course, this thinking does not need to be classical, it only needs the rule of *Modus Ponens*.

Conclusion

Some believers in nonmonotonicity of human thinking defend their faith by underlying that usually nonmonotonic inferences are stronger than the classical inference in such a sense that for nonmonotonic inference $\vdash$ there are some A and s, such that $A \vdash s$ and $A \nvdash_{\mathrm{CL}} s$. At first, classical logic is not the only alternative option for nonmonotonic logic. Secondly, it is interesting that the mentioned property of nonmonotonicity is possessed by every logically correct, monotonic inference "extended" by logically incorrect rules. In fact, ordinary logical errors make every inference they pollute stronger than classical: *it is possible to infer what should not be (classically) inferred.* Examples for nonmonotonicity existing in literature reveal that to say "nonmonotonicity strengthens classical inference" is like to claim that "logical incorrectness as well as material errors strengthens classical inference".

The definition of nonmonotonic inference is clear and strict. In both expressions "$A\cup \vdash s$" and "$A\cup\{t\} \nvdash s$", appearing in the definition, there is exactly the same set A and exactly the same set of rules defining $\vdash$. That is why, nonmonotonicity bases on some manipulation with either A or $\vdash$: in the second step of reasoning it is already either "not exactly the same set A" or "not exactly the same set of rules of inference $\vdash$". It seems extremely difficult to show a reliable example for nonmonotonicity in our thinking. Four classes of examples considered above illustrate in various ways how our thinking has nothing in common with nonmonotonicity. Examples of the first class represent, used by us every day, **reasoning increasing preciseness**. Examples of the second class show also similarly common **reasoning from explicit and implicit premises**. The third class' examples illustrate another popular reasoning known as a **stereotypical thinking**, which is possible due to the logical error of generality. Finally, the last class of examples illustrate **thinking by the most frequent cases**.

Thanks to nonmonotonic interpretation, all these interesting from the logical point of view examples are crumpled into one case of nonmonotonic thinking, which probably is not a human phenomenon. Nobody can say that there are no examples for nonmonotonicity of reasoning of human being, even when such examples are still unknown. Obviously, such opinion would be methodologically wrong. However, it seems to be clear that in present situation, nonmonotonicity unjustifiably interprets all cases considered here, and limits interesting and important researches which could illuminate a mystery of human thinking to diver in its manifestations.

References

Gabbay, D. M., C. J. Hogger and J. A. Robinson (eds.), *Handbook of Logic in Artificial Intelligence and Logic Programming*, Volume 3, Clarendon Press, Oxford, 1994.

Ginsberg, M. L., "AI and nonmonotonic reasoning", pages 1–33 in Gabbay (1994).

Poole, D., "Default logic", pages 189–215 in Gabbay (1994).

Makinson, D., "General patterns in nonmonotonic reasoning", pages 35–110 in Gabbay (1994).

PIOTR ŁUKOWSKI
Department of Cognitive Science
Institute of Psychology
University of Łódź, Poland
lukowski@uni.lodz.pl