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ON A NON-REFERENTIAL THEORY OF MEANING FOR SIMPLE NAMES BASED ON AJDUKIEWICZ'S THEORY OF MEANING

Abstract. In 1931–1934 Kazimierz Ajdukiewicz formulated two versions of the theory of meaning (A1 and A2). Tarski showed that A2 allows synonymous names to exist with different denotations. Tarski and Ajdukiewicz found that this feature disparages the theory. The force of Tarski's argument rests on the assumption that none of adequate theories of meaning allow synonymous names to exist with different denotations. In the first part of this paper we present an appropriate fragment of A2 and Tarski's argument. In the second part we consider an elementary interpreted language in which individual constants occur, but not functional symbols. For such a language we define semantically a relation of synonymity for simple names and show that it fulfills syntactical conditions formulated by Ajdukiewicz in A2 and allows synonymous names to exist with different denotations.

Keywords: Ajdukiewicz, Tarski, theory of meaning, simple names, synonymity, meaning directives.

1. Historical remarks

In 1931–1934 Kazimierz Ajdukiewicz formulated two versions of the theory of meaning ([1, 2]). It is assumed that it was the first attempt to define this term using the methods taken from formal logic. Ajdukiewicz's theories of meaning have been studied by a number of researches (see e.g. [6, 7, 9, 10, 11, 13, 14]). Despite the great precision of the language of Kazimierz Ajdukiewicz, his texts leave considerable room for interpretation. This is how this dissertation should be considered.

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The Ajdukiewicz's idea was to bind an intuitive concept of synonymity with a syntactic-pragmatic relation of mutual interchangeability of expressions over a set of so-called meaning directives of language. In these two theories, the definition of the mutual interchangeability of expressions was formulated differently. In [9] I tried to compare both theories exactly for the sake of Ajdukiewicz's modification of the definition. Let $A1$ represent the theory from 1931 and $A2$ — the theory from 1934. It is a well-known fact that the theory from 1934 allows synonymous names to exist with different denotations. It was Alfred Tarski who noticed this fact first and then conveyed his observation to Ajdukiewicz during their conversation.

The fact that the theory of meaning allows synonymous names to exist with different denotations was considered by Ajdukiewicz and Tarski to be an evidence that the theory was incorrect. In fact, the problem of interpreting Tarski's argument in relation to the theory $A2$ is more complicated. I discussed it in [9]. I also showed in this work that contrary to popular opinion, the theory $A1$ also allows synonymous names to exist with different denotations.[1] I expressed the opinion backed up with arguments that in certain very special cases theories of meaning should allow such names to exist with different denotations. I also framed an open problem of semantic defining a relation of synonymity of names, for which Ajdukiewicz's theory would provide a syntactic sine qua non and which could be fundamental for a non-referential theory of meaning of names. The following dissertation is a partial solution of this problem.

2. About meaning directives

We are going to deliberate here on very special cases of applying the $A2$ theory. However, we will present its general principles, in order to situate these special cases even better. One of the fundamental concepts of both Kazimierz Ajdukiewicz's theories of meaning is a concept of a meaning directive. Let L be any interpreted language, formal or ethnic. A possible meaning directive of L is an ordered pair $\langle W_\alpha, \alpha\rangle$, where α is a sentence in the language L, and W_α is a possible condition (a set of conditions) for acceptance of α by users of L, regardless of what is the character of the

[1] There are two definitions of synonymity in [1], on pp. 132 and 134. In the second one Ajdukiewicz uses the notion of an essential directive and I assume that A1 is based on this definition.

condition and how it is defined. Let W^L represent the set of all possible conditions for acceptance of sentences in the language L, S^L – the set of all sentences in the language L and Exp^L – the set of all expressions of L. The concept of a possible meaning directive for L becomes a formal concept only when both L and W^L are formally defined.

Let $\langle W_\alpha, \alpha \rangle$ be a possible meaning directive of L. We say that $\langle W_\alpha, \alpha \rangle$ is a meaning directive for L, if the following condition is met: if W_α is satisfied, a member of community using the language L cannot reject the sentence α, he has to accept it. Ajdukiewicz understands an act of acceptance in a purely pragmatic way and it does not have to be connected with a belief or knowledge of a language user. It is an expression of community's language habit and it show how the community uses the language. Rejecting the sentence α, when the condition W_α is fulfilled and when the pair $\langle W_\alpha, \alpha \rangle$ is the meaning directive of the language L, means that a given person does not belong to the commuity using the language L and assigning the same meanings to its expressions. Ajdukiewicz assumes that, for any expression of L, there are meaning directives for L in which this expression occurs.

The condition W_α, which is the component of the meaning directive $\langle W_\alpha, \alpha \rangle$ in a special case can consist in accepting some sentences (it can be the empty set). We will say then, that it has a linguistic character and we will identify it with the set of those sentences.

Depending on the type of condition W_α, Ajdukiewicz distinguishes three kinds of meaning directives. Axiomatic directives are characterized by an empty set of conditions of accepting sentences. We identify such directives with the sentences. In deductive directives the condition of sentences' acceptance is to accept other sentences; in empirical directives it is a user's appropriate extralinguistic experience (e.g. sensory, metaphysical). In the last case, we identify W_α with the type of that experience. The above characteristic of the meaning directives differs a bit from the original perspective of Kazimierz Ajdukiewicz. However, I think that it properly conveys the nature of his conception; the differences are technical, and the omitted details are of secondary importance.[2]

Ajdukiewicz assumes that expressions of any interpreted language L, used by a language community for communicative purposes, have meanings that allow users to recognize them and make users understand each other. For every such language there exists a relation of synonymity

[2] I discussed this matter in more detail in [9, pp. 128–129].

between expressions, which is intuitively recognized by users and put into language practice. Let $\overset{\text{int}}{\sim}$ represent this relation when it is known which language is meant.

In his theories of meaning, Ajdukiewicz assumes that the fact that the expressions of the language L have meanings is equivalent to the existence of the set of meaning directives for L, and that the synonymity relation for L depends on the form of that set. Therefore, by referring to meaning directives, we can formulate conditions that describe the intuitive relation $\overset{\text{int}}{\sim}$ of synonymity.

3. Basic definitions

Let L be an interpreted language and D a set of meaning directives for L.

The syntactic definition of the relation of a mutual interchangeability of expressions of L over the set D of meaning directives for that language is one of the crucial definitions of the theory $A2$ (and also $A1$). Ajdukiewicz connected this relation with the intuitive synonymity relation $\overset{\text{int}}{\sim}$ for L. In his definition he used syntactic operations on expressions, which can be defined in the following way:

DEFINITION 1. Let L be any language and α, β, λ be any members of Exp^L. We define the operation

$$\alpha \circlearrowleft \beta : Exp^L \mapsto Exp^L$$

by putting that $\lambda^{\alpha \circlearrowleft \beta}$ is the expression resulting from λ by replacing all occurrences of α with β (and vice versa).

It turns out that the transformations defined in Definition 1 have many merits in comparison to analogical transformations defined in [1]. However, they have a serious defect as well, which was not noticed by Kazimierz Ajdukiewicz. They are not always possible to execute. If some expressions α and β are not disjoint, that is to say, α is a part of β or vice versa, then for some expressions γ the operation of the mutual interchange is not possible to execute.[3] So Definition 1 defines only partial operations in such cases. Nothing stands in the way of using Definition 1, provided that the parameters of operations $\alpha \circlearrowleft \beta$ are going through the set of simple expressions of the given language.

[3] That was noticed by professor W. Buszkowski in [6].

We always obtain, in such a case, the total operations. If we want to apply Definition 1 without limitation, in the same way as Ajdukiewicz did it, we have to modify the definition, so that it would consider the fact, that the transformation $\alpha \circlearrowright \beta$ is not always possible. Otherwise, the next definition is not correct. In this article we will only consider simple names, so we do not have to worry about this problem. It follows from Definition 1 that if $\alpha \notin \lambda$ and $\beta \notin \lambda$, then $\lambda^{\alpha \circlearrowright \beta} = \lambda$. The notation $\alpha \notin \lambda$, in the context of expressions, stands for α does not occur in λ.

The syntactic operations defined above allow us to define analogical operations on conditions W_α and, in a consequence, on meaning directives. We assume that if W_α is a component of an axiomatic or empirical meaning directive, then $W_\alpha^{\gamma \circlearrowright \beta} = W_\alpha$. Let's now assume that $W_\alpha = \{\delta_1, \ldots, \delta_n\}$, where $\delta_1, \ldots, \delta_n$ are sentences. Then we set:

$$W_\alpha^{\gamma \circlearrowright \beta} = \{\delta_1^{\gamma \circlearrowright \beta}, \ldots, \delta_n^{\gamma \circlearrowright \beta}\}.$$

Let Δ be a meaning directive of the form $\langle W_\alpha, \alpha \rangle$. We set that,

$$\Delta^{\gamma \circlearrowright \beta} = \langle W_\alpha^{\gamma \circlearrowright \beta}, \alpha^{\gamma \circlearrowright \beta} \rangle.$$

Now we can give a definition of a mutual interchangeability of expressions over a set D of meaning directives of a language L.

DEFINITION 2. Let L be any language, D a set of meaning directives of L, $\alpha, \beta \in Exp^L$. We say that α and β are mutual interchangeable over D, written $\alpha \overset{\mathrm{D}}{\approx} \beta$, if and only if for any directive $\Delta \in D$, $\Delta^{\alpha \circlearrowright \beta} \in D$. In other words: $\alpha \overset{\mathrm{D}}{\approx} \beta$ if and only if the set of directives D is closed under the operation $\alpha \circlearrowright \beta$.

We noticed, that Tarski showed that the theory $A2$ allows synonymous names to exist with different denotations. Now, we can present his reasoning.

Example 1 (Tarski). Let L be an elementary language, in which the individual constants a and b are the only specific symbols. We assume that a model $\mathfrak{M}$ is an interpretation of the language L and its universe consists of two elements $a^{\mathfrak{M}}$ and $b^{\mathfrak{M}}$, for which we have $a^{\mathfrak{M}} \neq b^{\mathfrak{M}}$. Let a set of meaning directives D be an elementary theory generated by two sentences : $a \neq b$ and $b \neq a$.[4] As all the tautologies expressible in the

[4] Tarski in his example assumed that the set D of meaning directives consists

language L belong to the set D, we can assume that the meaning of the symbols of equality and negation which occur in these two sentences, is the meaning which is assigned to classical logic. It is hard to believe that a user of the language L, accepting the set D as the set of the sentences defining the meanings of that language's symbols, could use the symbols of equality and negation in different meanings from the classical. Therefore, the names a and b have different denotations and, moreover, regardless of that fact, the sentences $a \neq b$ and $b \neq a$ should be interpreted as the sentences saying that a and b have different denotations.[5] However, if we take any sentence $\Delta \in D$, it is easy to notice that the sentence $\Delta^{a \circlearrowleft b}$ belongs to D as well. And so, according to Definition 2.2, the result is that $a \stackrel{\mathrm{D}}{\approx} b$. ⊣

The interpretation of Tarski's argument is clear, provided that in the theory $A2$ Ajdukiewicz equated the relation of synonymity with the relation $\stackrel{\mathrm{D}}{\approx}$. However, it is not true. Regardless of how Tarski's argument concerns the theory $A2$, in my view, he attracts attention to an interesting problem: whether there are any permissible cases in an accurate theory of meaning when synonymous names have different denotations. It seems that Tarski, as well as Ajdukiewicz, rejected such possibility.[6]

If we limit the field of $\stackrel{\mathrm{D}}{\approx}$ to simple expressions, it will turn out that it is an equivalence relation. Let δ be any expression of L, α, β, γ any simple expressions of that language. Then

$$((\delta^{\alpha \circlearrowleft \beta})^{\beta \circlearrowleft \gamma})^{\alpha \circlearrowleft \beta} = \delta^{\alpha \circlearrowleft \gamma}.$$

Checking whether the relation $\stackrel{\mathrm{D}}{\approx}$ is reflexive and symmetrical is not a problem. We assume that $\alpha \stackrel{\mathrm{D}}{\approx} \beta$ and $\beta \stackrel{\mathrm{D}}{\approx} \gamma$. Let $\Delta \in D$. Then, by Defi-

of all logical tautologies expressible in the language L and of the sentences $a \neq b$ and $b \neq a$. We assumed a slightly different solution. However, it does not influence the interpretation of the example discussed. The above example can be also easily supplemented by a set of logical deductive directives, because the interchange of all names a and b (and vice versa) transforms any substitution instance of modus ponens into a substitution instance of modus ponens and the similar case concerns the rule of generalization.

[5] If the set D of directives consisted only of the sentences $a \neq b$ and $b \neq a$, nothing would entitle us to claim that these sentences express the fact of different denotations of names a and b. In that case, these sentences would express the same as e.g. the sentences $R(a, b)$ and $R(b, a)$.

[6] Ajdukiewicz stated that clearly in [4].

nition 2, $\Delta^{\alpha\circlearrowleft\beta}$, $(\Delta^{\alpha\circlearrowleft\beta})^{\beta\circlearrowleft\gamma}$ also belong to D. And so $((\Delta^{\alpha\circlearrowleft\beta})^{\beta\circlearrowleft\gamma})^{\alpha\circlearrowleft\beta} = \Delta^{\alpha\circlearrowleft\gamma}$. Hence $\alpha \overset{\mathrm{D}}{\approx} \gamma$.[7]

The way in which a relation of expression's interchangeability is connected with a synonymity relation was presented by Ajdukiewicz in the form of the following thesis.

Thesis (Ajdukiewicz) *Let L be any interpreted language, D a set of meaning directives of that language, α and β its any expressions. If $\alpha \overset{\mathrm{int}}{\sim} \beta$, then $\alpha \overset{\mathrm{D}}{\approx} \beta$.*

4. About theories of meaning of names

As a matter of fact, the theories $A2$ and $A1$ are universal and refer to any expressions of any language — ethnic or interpreted formal language.[8] There are also no limiting conditions imposed on sets of meaning directives. As far as kinds of the languages in question, a shape of the meaning directives' set and the range of the expressions are concerned, we will further assume far-reaching limitations. As we have mentioned, Ajdukiewicz in his theory does not impose any essential conditions on a language. A set of meaning directives depends on linguistic habits of language users, which are not limited by any conditions. Therefore we can assert that our findings are not exactly modifying Ajdukiewicz's theory of meaning but rather focus attention on some special applications. However, it does not mean that Ajdukiewicz's theory of meaning can be reduced to these particular cases. We assume the following:

1. We limit the class of considered languages to interpreted, elementary languages based on classical logic. In order to specify our deliberations, we choose the specific system of logic. Let it be the system of elementary logic with the identity defined in [12]. In this system, the modus ponens and the rule of generalization are rules of inference (we have only the general quantifier in the language of this logic).

[7] I am copying here the reasoning of W. Buszkowski from [6].

[8] In the second part of [2] Ajdukiewicz restricted considered languages to coherent and closed languages. However, this assumption was connected with the definition of a translation, thus with pondering the synonymy of expressions belonging to different languages. If we limit ourselves to pondering the synonymy of expressions belonging to one language, that assumption is not needed.

2. We limit members of sets of meaning directives to axiomatic directives. Furthermore, we assume that a set of axiomatic directives is an elementary theory, true in a distinguished model, being an interpretation of a given language.
3. We limit the range of considered expressions to simple names, i.e. individual constants of elementary languages.

Thereby, we assume that not all true sentences of a given language L have to be accepted by users of that language on the strength of the meaning of expressions (the knowledge does not have to be analytical), and that users of L have a full ability to deduce.

Point 2 requires a justification. Let L be any interpreted elementary language and D a set of meaning directives for L. The assumption that there are no empirical directives in D seems natural. If we assume that the set of deductive directives for L consists of all substitution instances of modus ponens and the rule of generalization, it will turn out that it does not influence the form of the relation $\overset{\mathrm{D}}{\approx}$.

Let $\Delta \in D$ be any substitution instance of modus ponens, α and β any disjoint expressions of L belonging to the same syntactic category. Then $\Delta^{\alpha \circlearrowright \beta}$ is also a substitution instance of modus ponens, regardless of whether the expressions α and β are terms, formulas or variables, provided that the operation $\alpha \circlearrowright \beta$ is possible to execute. In such a case for any formula γ and δ we have

$$(\gamma \to \delta)^{\alpha \circlearrowright \beta} = \gamma^{\alpha \circlearrowright \beta} \to \delta^{\alpha \circlearrowright \beta}.$$

Hence $\Delta^{\alpha \circlearrowright \beta} \in D$. Therefore, these directives do not influence the relation $\overset{\mathrm{D}}{\approx}$ on the set of simple expressions of these categories. They can influence this relation on the set of logical symbols. However, it is easy to show that even only the axiomatic directives are the reason for which the relation of mutual interchangeability coincides with the identity relation on this set. It would be enough to consider some tautologies in the language L, in order to check that $\overset{\mathrm{D}}{\approx}$ is always an identity relation on the set of logical symbols. For example: if α is any axiomatic directive, then $\neg\alpha \to \neg\alpha$ and $\alpha \vee \neg\alpha$ are also axiomatic directives, but $(\neg\alpha \to \neg\alpha)^{\to \circlearrowright \wedge}$, $(\neg\alpha \to \neg\alpha)^{\to \circlearrowright \vee}$ and $(\alpha \vee \neg\alpha)^{\wedge \circlearrowright \vee}$ are not. The similar situation is for the rule of generalization. Therefore, provided the assumptions, deductive directives do not influence the relation $\overset{\mathrm{D}}{\approx}$ on the set of logical symbols too, and can be omitted.

The fact that deductive directives do not influence the relation of mutual interchangeability for non-logical expressions is the advantage of the theory $A2$. I suppose it was one of the causes for changing the definition by Ajdukiewicz. Thus, if some meaning directives cause that on the set of the logical symbols the relation of mutual interchangeability coincides with the identity relation, then all the remaining deductive directives can be omitted, without changing the form of the relation on the set of all expressions. Therefore we can assume that sets of meaning directives consist only of axiomatic directives. The sentences which are elements of sets of axiomatic meaning directives we shall simply call the meaning directives.

Let us sum up our settlements. We consider interpreted elementary languages. Every set of meaning directives consists of true sentences and is closed under the logical consequence.

The topic of our interest is the possibility of formal defining for any interpreted, elementary language L a semantic relation (on the set of simple names of L), which could be identified with intuitively understood relation $\overset{\text{int}}{\sim}$ occurring in Ajdukiewicz's Thesis. However, that is not all. We would like the definition of a synonymity relation to be the base of the non-referential theory of meaning for simple names, i.e. the theory allowing names to exist with different meanings and the same denotation as well as names with different denotations and the same meaning.

We shall define the above postulates in more detail for a bit more general case, in which we take into account any elementary language and all constant terms, not only simple names. Let $\mathfrak{Tr}^L$ denote the algebra of constant terms of L. We define it in standard manner. $\mathfrak{Tr}^L$ has as its universe the set $\mathfrak{Tr}^L$ of all constant terms of L. The signature of the algebra consists of all individual constants and all function symbols of L. They are interpreted as follows: if c is an individual constant, then $c^{\mathfrak{Tr}^L} = c$; if F is an n-argument function symbol and $t_1, \ldots, t_n$ are any elements of the universe, then

$$F^{\mathfrak{Tr}^L}(t_1, \ldots, t_n) = F(t_1, \ldots, t_n).$$

Let us denote the family of all consistent theories in L by $Th(L)$. Any consistent theory in L can serve as the set of axiomatic directives for L, provided that the interpretation of L is a model of that theory. In every such case, our desired theory of meaning should allow to define with semantic means the relation $\mathcal{R} \subseteq \mathfrak{Tr}^L \times \mathfrak{Tr}^L \times Th(L)$ satisfying for any $T \in Th(L)$ the following conditions:

1. $\overset{\mathrm{T}}{\sim} = \{\langle t_1, t_2\rangle : \langle t_1, t_2, T\rangle \in \mathcal{R}\}$ is a congruence relation on $\mathfrak{Tr}^L$,
2. if $t_1 \overset{\mathrm{T}}{\sim} t_2$, then $t_1 \overset{\mathrm{T}}{\approx} t_2$.

It follows from the above conditions that the synonymity relation $\overset{\mathrm{T}}{\sim}$ over the set T of meaning directives determined on the set of constant terms of L should be a congruence on $\mathfrak{Tr}^L$ and it should meet Ajdukiewicz's Thesis. Certainly, when there are no function symbols in L, every equivalence relation on the universe of $\mathfrak{Tr}^L$ is a congruence.

If $\mathfrak{M}$ is an interpretation of L, then there exists a distinct congruence relation on $\mathfrak{Tr}^L$, connected with that interpretation. This is the congruence relation $\overset{\mathfrak{M}}{\asymp}$ defined as follows :

$$t_1 \overset{\mathfrak{M}}{\asymp} t_2 \text{ if and only if } t_1^{\mathfrak{M}} = t_2^{\mathfrak{M}}.$$

Let us denote the congruence lattice of $\mathfrak{Tr}^L$ by $\mathbf{Con}(\mathfrak{Tr}^{\mathfrak{L}})$. We can divide theories of meaning of names into three kinds, depending on an area of the universe of $\mathbf{Con}(\mathfrak{Tr}^{\mathfrak{L}})$ in which synonymity relations determined by a given theory, relative to various sets $T \subseteq Th(\mathfrak{M})$ of meaning directives, can occur.

1. The theories in which synonymity relations always come out below the relation $\overset{\mathfrak{M}}{\asymp}$. We will call them connotative theories.

2. The theories in which synonymity relations always come out above the relation $\overset{\mathfrak{M}}{\asymp}$. We do not have a good name for these theories, besides, they do not occur in practice.

3. The theories in which synonymity relations can occur incomparable with relation $\overset{\mathfrak{M}}{\asymp}$. We will call them non-referential theories.[9]

5. Languages, in which only simple names occur

In this chapter we will consider elementary languages in which individual constants occur, but not functional symbols. We say that such languages are functionless.

[9] The term was borrowed from [11]. Of course non-referential theories are not extensional, but the question whether a meaning theory have to be extensional seems to be open. I discussed this problem in [9].

Definition 3. Let L be a functionless elementary language, a and b individual constants of L, $\mathfrak{M}$ a model for L.

1. We define the function $f^{a\circlearrowright b}\colon M \to M$ by
$$f^{a\circlearrowright b}(x) = \begin{cases} a^{\mathfrak{M}} & \text{if } x = b^{\mathfrak{M}}, \\ b^{\mathfrak{M}} & \text{if } x = a^{\mathfrak{M}}, \\ x & \text{otherwise.} \end{cases}$$

2. We define the model $\mathfrak{M}^{a\circlearrowright b}$ of the same type as $\mathfrak{M}$ putting that $\mathfrak{M}^{a\circlearrowright b}$ and $\mathfrak{M}$ have the same universe and
$$a^{\mathfrak{M}^{a\circlearrowright b}} = (b^{\mathfrak{M}}),$$
$$b^{\mathfrak{M}^{a\circlearrowright b}} = (a^{\mathfrak{M}}).$$

   We assume that interpretations of the remaining symbols of L are in $\mathfrak{M}^{a\circlearrowright b}$ the same as in $\mathfrak{M}$.

Definition 4. Let L be a functionless elementary language, $\mathfrak{M}$ a model of L. We define the relation $\overset{\mathfrak{M}}{\sim}$ on the set of the individual constants of L, assuming for any constants a and b that $a \overset{\mathfrak{M}}{\sim} b$ if and only if the function
$$f^{a\circlearrowright b}\colon M \longmapsto M^{a\circlearrowright b}$$
is an isomorphism between $\mathfrak{M}$ and $\mathfrak{M}^{a\circlearrowright b}$.

Lemat 1. *Let a, b and c be individual constants of a functionless elementary language L and let $\mathfrak{M}$ be a model of L. If $a^{\mathfrak{M}} = b^{\mathfrak{M}}$ and $c^{\mathfrak{M}} \neq b^{\mathfrak{M}}$ then both $c \overset{\mathfrak{M}}{\nsim} a$ and $c \overset{\mathfrak{M}}{\nsim} b$.*

Proof. We assume the appropriate premises and for the indirect proof let us suppose that $c \overset{\mathfrak{M}}{\sim} a$. Then $f^{c\circlearrowright a}$ is an isomorphism between $\mathfrak{M}$ and $\mathfrak{M}^{c\circlearrowright a}$. Thus
$$f^{c\circlearrowright a}(b^{\mathfrak{M}}) = b^{\mathfrak{M}^{c\circlearrowright a}} = b^{\mathfrak{M}} = f^{c\circlearrowright a}(a^{\mathfrak{M}}) = a^{\mathfrak{M}^{c\circlearrowright a}} = c^{\mathfrak{M}}.$$
Thereby, $b^{\mathfrak{M}} = c^{\mathfrak{M}}$, which is contradictory to the assumption. We show similarly that $c \overset{\mathfrak{M}}{\nsim} b$. ⊣

Lemat 2. *Let L be a functionless elementary language and $\mathfrak{M}$ a model of L. The relation $\overset{\mathfrak{M}}{\sim}$ defined on the set of the constants of L, according to Definition 4, is a congruence on the algebra $\mathfrak{Tr}^{L}$.*

Proof. If there are not function symbols in L, then every equivalence relation on the set of the constant terms is a congruence relation on the algebra $\mathfrak{Tr}^L$. It shall be enough to show that $\overset{\mathfrak{M}}{\sim}$ is an equivalence relation. The condition of reflexivity and symmetry is easy to check. We shall show that it is also a transitive relation. Let us suppose that for some individual constants a, b, and c of L we have $a \overset{\mathfrak{M}}{\sim} b$ and $b \overset{\mathfrak{M}}{\sim} c$. It means that there exist isomorphisms $f^{a\circlearrowright b}\colon \mathfrak{M} \longrightarrow \mathfrak{M}^{a\circlearrowright b}$ and $f^{b\circlearrowright c}\colon \mathfrak{M} \longrightarrow \mathfrak{M}^{b\circlearrowright c}$.

If $b^{\mathfrak{M}} = c^{\mathfrak{M}}$ or $b^{\mathfrak{M}} = a^{\mathfrak{M}}$, then appropriately $f^{a\circlearrowright c} = f^{b\circlearrowright c}$ or $f^{a\circlearrowright c} = f^{a\circlearrowright b}$.

Thus we can assume that $b^{\mathfrak{M}} \neq c^{\mathfrak{M}}$ and $b^{\mathfrak{M}} \neq a^{\mathfrak{M}}$. We know from Lemma 1 that in such case $a^{\mathfrak{M}} \neq c^{\mathfrak{M}}$ as well. In case $a^{\mathfrak{M}} = c^{\mathfrak{M}}$, then because $b^{\mathfrak{M}} \neq c^{\mathfrak{M}}$, we would obtain $b \overset{\mathfrak{M}}{\nsim} c$, which is contrary to our assumption. So, the elements $a^{\mathfrak{M}}$, $b^{\mathfrak{M}}$ and $c^{\mathfrak{M}}$ are different in pairs.

We define the function $h\colon M \longrightarrow M$ assuming that $h = f^{a\circlearrowright b} \circ f^{b\circlearrowright c} \circ f^{a\circlearrowright b}$. Of course, h is a bijection. Now, we shall only show that this is the function $f^{a\circlearrowright c}$, and that h is a homomorphism sending the model $\mathfrak{M}$ into the model $\mathfrak{M}^{a\circlearrowright c}$. Let's first consider the constants: a, b and c.

$$\begin{aligned}
h(a^{\mathfrak{M}}) &= f^{a\circlearrowright b} f^{b\circlearrowright c} f^{a\circlearrowright b}(a^{\mathfrak{M}}) \\
&= f^{a\circlearrowright b} f^{b\circlearrowright c}(a^{\mathfrak{M}^{a\circlearrowright b}}) \\
&= f^{a\circlearrowright b} f^{b\circlearrowright c}(b^{\mathfrak{M}}) \\
&= f^{a\circlearrowright b}(b^{\mathfrak{M}^{b\circlearrowright c}}) \\
&= f^{a\circlearrowright b}(c^{\mathfrak{M}}) \\
&= c^{\mathfrak{M}^{a\circlearrowright b}} \\
&= c^{\mathfrak{M}} \\
&= a^{\mathfrak{M}^{a\circlearrowright c}}.
\end{aligned}$$

$$\begin{aligned}
h(b^{\mathfrak{M}}) &= f^{a\circlearrowright b} f^{b\circlearrowright c} f^{a\circlearrowright b}(b^{\mathfrak{M}}) \\
&= f^{a\circlearrowright b} f^{b\circlearrowright c}(b^{\mathfrak{M}^{a\circlearrowright b}}) \\
&= f^{a\circlearrowright b} f^{b\circlearrowright c}(a^{\mathfrak{M}}) \\
&= f^{a\circlearrowright b}(a^{\mathfrak{M}^{b\circlearrowright c}}) \\
&= f^{a\circlearrowright b}(a^{\mathfrak{M}}) \\
&= a^{\mathfrak{M}^{a\circlearrowright b}} \\
&= b^{\mathfrak{M}} \\
&= b^{\mathfrak{M}^{a\circlearrowright c}}.
\end{aligned}$$

$$
\begin{aligned}
h(c^{\mathfrak{M}}) &= f^{a \circlearrowright b} f^{b \circlearrowright c} f^{a \circlearrowright b}(c^{\mathfrak{M}}) \\
&= f^{a \circlearrowright b} f^{b \circlearrowright c}(c^{\mathfrak{M}^{a \circlearrowright b}}) \\
&= f^{a \circlearrowright b} f^{b \circlearrowright c}(c^{\mathfrak{M}}) \\
&= f^{a \circlearrowright b}(c^{\mathfrak{M}^{b \circlearrowright c}}) \\
&= f^{a \circlearrowright b}(b^{\mathfrak{M}}) \\
&= b^{\mathfrak{M}^{a \circlearrowright b}} \\
&= a^{\mathfrak{M}} \\
&= c^{\mathfrak{M}^{a \circlearrowright c}}.
\end{aligned}
$$

If $e \in M$and $e \notin \{a^{\mathfrak{M}}, b^{\mathfrak{M}}, c^{\mathfrak{M}}\}$, then

$$
\begin{aligned}
h(e) &= f^{a \circlearrowright b} f^{b \circlearrowright c} f^{a \circlearrowright b}(e) \\
&= f^{a \circlearrowright b} f^{b \circlearrowright c}(e) \\
&= f^{a \circlearrowright b}(e) \\
&= e.
\end{aligned}
$$

We have shown that $h = f^{a \circlearrowright c}$, and that the condition of homomorphism for the constants a, b, c is met. Now, let d be an individual constant of L different from a, b and c. We know, from Lemma 1, that in this case $d^{\mathfrak{M}}$ is different from $a^{\mathfrak{M}}$, $b^{\mathfrak{M}}$ and $c^{\mathfrak{M}}$. Then $d^{\mathfrak{M}^{a \circlearrowright c}} = d^{\mathfrak{M}}$ and

$$
h(d^{\mathfrak{M}}) = d^{\mathfrak{M}} = d^{\mathfrak{M}^{a \circlearrowright c}}.
$$

Thus, the homomorphic condition on the individual constants is met. Let P be an n-argument predicate symbol and $n_1, \ldots, n_k \in M$. Then

$$
\begin{aligned}
P^{\mathfrak{M}}(n_1, \ldots, n_k) &\Leftrightarrow P^{\mathfrak{M}^{a \circlearrowright b}}(f^{a \circlearrowright b}(n_1), \ldots, f^{a \circlearrowright b}(n_k)) \\
&\Leftrightarrow P^{\mathfrak{M}}(f^{a \circlearrowright b}(n_1), \ldots, f^{a \circlearrowright b}(n_k)) \\
&\Leftrightarrow P^{\mathfrak{M}^{b \circlearrowright c}}(f^{b \circlearrowright c} f^{a \circlearrowright b}(n_1), \ldots, f^{b \circlearrowright c} f^{a \circlearrowright b}(n_k)) \\
&\Leftrightarrow P^{\mathfrak{M}}(f^{b \circlearrowright c} f^{a \circlearrowright b}(n_1), \ldots, f^{b \circlearrowright c} f^{a \circlearrowright b}(n_k)) \\
&\Leftrightarrow P^{\mathfrak{M}^{a \circlearrowright b}}(f^{a \circlearrowright b} f^{b \circlearrowright c} f^{a \circlearrowright b}(n_1), \ldots, f^{a \circlearrowright b} f^{b \circlearrowright c} f^{a \circlearrowright b}(n_k)) \\
&\Leftrightarrow P^{\mathfrak{M}}(f^{a \circlearrowright b} f^{b \circlearrowright c} f^{a \circlearrowright b}(n_1), \ldots, f^{a \circlearrowright b} f^{b \circlearrowright c} f^{a \circlearrowright b}(n_k)) \\
&\Leftrightarrow P^{\mathfrak{M}}(h(n_1), \ldots, h(n_k)) \\
&\Leftrightarrow P^{\mathfrak{M}^{a \circlearrowright c}}(h(n_1), \ldots, h(n_k)).
\end{aligned}
$$

We should remember that from Definition 3 follows $P^{\mathfrak{M}} = P^{\mathfrak{M}^{b\circlearrowleft c}} = P^{\mathfrak{M}^{a\circlearrowleft b}} = P^{\mathfrak{M}^{a\circlearrowleft c}}$. The function h is the isomorphism we were looking for and the relation $\overset{\mathfrak{M}}{\sim}$ is transitive. $\dashv$

Definition 5. Let L be a given functionless elementary language and D a set of its meaning directives. We define the synonymity relation $\overset{\mathrm{D}}{\sim}$ on the set of individual constants of L, assuming for any constants a and b of L that $a \overset{\mathrm{D}}{\sim} b$ if and only if for every model $\mathfrak{N}$ of L, the following condition is met: if $\mathfrak{N} \models D$, then $a \overset{\mathfrak{N}}{\sim} b$.

Therefore, for any language L and a set D of meanings directives for L we have :

$$\overset{\mathrm{D}}{\sim} = \bigcap\{ \overset{\mathfrak{N}}{\sim} : \mathfrak{N} \models D\}.$$

Since a product of congruences is also a congruence relation, let us make a note of the following trivial fact:

Fact 3. *If L is a functionless elementary language, D a set of meaning directives for L, then the synonymity relation $\overset{\mathrm{D}}{\sim}$ on the set of individual constants of L is a congruence relation on the algebra $\mathfrak{Tr}^L$.*

Theorem 4. *Let L be a functionless elementary language, D a set of meaning directives for L. For any individual constants a and b of L, the following condition is met: if $a \overset{\mathrm{D}}{\sim} b$, then $a \overset{\mathrm{D}}{\approx} b$.*

Proof. We assume that $a \overset{\mathrm{D}}{\sim} b$ and $\varphi \in D$, where φ is a sentence in the language L. Let $\mathfrak{M}$ be a model of L and $\mathfrak{M} \models D$. If in the sentence φ the constants a and b do not occur, then $\varphi = \varphi^{a\circlearrowleft b}$ and the thesis is self-evident. We assume that a and b occur in φ and maybe some other constants $c_1, \ldots, c_k$ also, and the sentence φ takes the form $\varphi(\overline{c}, a, b)$. To make the calculation simpler, we assume that in φ we have one occurrence of a and the other one of b. In case of more occurrences, we follow in a similar way. We know that if $a \overset{\mathrm{D}}{\sim} b$, then $a \overset{\mathfrak{M}}{\sim} b$. Let us consider two cases.

$a^{\mathfrak{M}} = b^{\mathfrak{M}}$: We have from logic that $\mathfrak{M} \models \varphi \leftrightarrow \varphi^{a\circlearrowleft b}$ and hence $\mathfrak{M} \models \varphi^{a\circlearrowleft b}$.

$a^{\mathfrak{M}} \neq b^{\mathfrak{M}}$: In this case we know from Lemma 1 that for any $1 \leqslant i \leqslant k$, $a^{\mathfrak{M}} \neq c_i^{\mathfrak{M}}$ and $b^{\mathfrak{M}} \neq c_i^{\mathfrak{M}}$. Then

$$\begin{aligned}\mathfrak{M} \models \varphi(\overline{c}, a, b) &\Rightarrow \mathfrak{M} \models \varphi(c_1^{\mathfrak{M}}, \ldots, c_k^{\mathfrak{M}}, a^{\mathfrak{M}}, b^{\mathfrak{M}}) \\ &\Rightarrow \mathfrak{M}^{a\circlearrowleft b} \models \varphi(f^{a\circlearrowleft b}(\overline{c}^{\mathfrak{M}}), f^{a\circlearrowleft b}(a^{\mathfrak{M}}), f^{a\circlearrowleft b}(b^{\mathfrak{M}}))\end{aligned}$$

$$\begin{aligned}&\Rightarrow \mathfrak{M} \models \varphi(f^{a\circlearrowright b}(\overline{c}^{\mathfrak{M}}), f^{a\circlearrowright b}(a^{\mathfrak{M}}), f^{a\circlearrowright b}(b^{\mathfrak{M}}))\\ &\Rightarrow \mathfrak{M} \models \varphi(c_1^{\mathfrak{M}}, \ldots, c_k^{\mathfrak{M}}, b^{\mathfrak{M}}, a^{\mathfrak{M}})\\ &\Rightarrow \mathfrak{M} \models \varphi(c_1, \ldots, c_k, b, a)\\ &\Rightarrow \mathfrak{M} \models \varphi(\overline{c}, a, b)^{a\circlearrowright b}.\end{aligned}$$

We follow by analogy when the constant a occurs in φ and there are no occurrences of the constant b or vice versa. And so in every case we obtain $\mathfrak{M} \models \varphi^{a\circlearrowright b}$, which means that $\varphi^{a\circlearrowright b} \in D$ because of the fact that D is closed under the logical consequence. Hence $a \overset{\mathrm{D}}{\approx} b$. $\dashv$

It follows from Fact 3 and Theorem 4 that in our simple case of a functionless elementary language L we can define the desired relation $\mathcal{R} \subseteq \mathfrak{Tr}^L \times \mathfrak{Tr}^L \times Th(L)$ putting

$$\langle a, b, T\rangle \in \mathcal{R} \iff a \overset{\mathrm{T}}{\sim} b.$$

It is worth noting that the above facts remain true also for languages, in which function symbols occur, provided that we limit our synonymity relations only to sets of individual constants of that languages. In such a case synonymity relations stop being congruences in algebras of constant terms of languages but remain equivalence relations, because the function $h(x)$, defined by us, remains an isomorphism. Let F be a k-argument function symbol and let $n_1, \ldots, n_k \in M$.

$$\begin{aligned}h(F^{\mathfrak{M}}(n_1, \ldots, n_k)) &= f^{a\circlearrowright b} f^{b\circlearrowright c} f^{a\circlearrowright b}(F^{\mathfrak{M}}(n_1, \ldots, n_k))\\ &= f^{a\circlearrowright b} f^{b\circlearrowright c}(F^{\mathfrak{M}^{a\circlearrowright b}}(f^{a\circlearrowright b}(n_1), \ldots, f^{a\circlearrowright b}(n_k)))\\ &= f^{a\circlearrowright b} f^{b\circlearrowright c}(F^{\mathfrak{M}}(f^{a\circlearrowright b}(n_1), \ldots, f^{a\circlearrowright b}(n_k)))\\ &= f^{a\circlearrowright b}(F^{\mathfrak{M}^{b\circlearrowright c}}(f^{b\circlearrowright c} f^{a\circlearrowright b}(n_1), \ldots, f^{b\circlearrowright c} f^{a\circlearrowright b}(n_k)))\\ &= f^{a\circlearrowright b}(F^{\mathfrak{M}}(f^{b\circlearrowright c} f^{a\circlearrowright b}(n_1), \ldots, f^{b\circlearrowright c} f^{a\circlearrowright b}(n_k)))\\ &= F^{\mathfrak{M}^{a\circlearrowright b}}(f^{a\circlearrowright b} f^{b\circlearrowright c} f^{a\circlearrowright b}(n_1), \ldots, f^{a\circlearrowright b} f^{b\circlearrowright c} f^{a\circlearrowright b}(n_k))\\ &= F^{\mathfrak{M}}(h(n_1), \ldots, h(n_k))\\ &= F^{\mathfrak{M}^{a\circlearrowright c}}(h(n_1), \ldots, h(n_k)).\end{aligned}$$

Synonymity relations for simple names defined above are very sensitive, even to small changes in the considered language. It is illustrated by the following examples.

Example 2. Let us consider the example given by Tarski, in which we only make a slight change in the language L. We add a new individual constant to the language, that is c. The set S of meaning directives remains the same. There exists a model $\mathfrak{M}_1$ for the set S, such that $c^{\mathfrak{M}_1} = a^{\mathfrak{M}_1}$. Thus, from the Lemma 1 we have $a \overset{\mathfrak{M}_1}{\not\approx} b$ and $b \overset{\mathfrak{M}_1}{\not\approx} c$. There exists also a model $\mathfrak{M}_2$ for the set S, in which we have $c^{\mathfrak{M}_2} = b^{\mathfrak{M}_2}$. Hence $a \overset{\mathfrak{M}_2}{\not\approx} c$. So, we obtain that $a \overset{\mathrm{S}}{\not\approx} c$, $a \overset{\mathrm{S}}{\not\approx} b$, $b \overset{\mathrm{S}}{\not\approx} c$, i.e. all the names are having different meanings. Therefore, adding one symbol to the language without changing the set of meaning directives causes the significant change of the synonymity relation. Since the set S of meaning directives remains the same, this example also shows that with our interpretation of the intuitive synonymity relation, the implication in Ajdukiewicz's Thesis cannot be reinforced to the equivalence. If we additionally change the set S of directives by adding the axiomatic directive $c = a$ to it, then we will obtain $a \overset{\mathrm{S}}{\sim} c$, $a \overset{\mathrm{S}}{\not\approx} b$, $b \overset{\mathrm{S}}{\not\approx} c$. ⊣

Example 3. Let L be a functionless elementary language containing individual constants a, b, c, d and a unary predicative symbol P. We assume that a model $\mathfrak{M}$ is an interpretation of the language L such that $a^{\mathfrak{M}} = b^{\mathfrak{M}}$, $c^{\mathfrak{M}} \neq d^{\mathfrak{M}}$, $c^{\mathfrak{M}} \neq a^{\mathfrak{M}}$, $d^{\mathfrak{M}} \neq a^{\mathfrak{M}}$, $P^{\mathfrak{M}} = \{a^{\mathfrak{M}}\}$. Let D be a set of meaning directives. We assume that D is an elementary theory generated by the following sentences: $c \neq d$, $c \neq a$, $c \neq b$, $d \neq a$, $d \neq b$, $(\forall x)\{P(x) \leftrightarrow x = a\}$. There exists a model $\mathfrak{M}_1$ such that $\mathfrak{M}_1 \models D$ and

$$\mathfrak{M}_1 \models a \neq b \,\wedge\, \neg P(b).$$

Therefore, $a \overset{\mathfrak{M}_1}{\not\approx} b$ and $a \overset{\mathrm{D}}{\not\approx} b$. On the other hand, for any model $\mathfrak{N}$ of L such that $\mathfrak{N} \models D$, we have $c \overset{\mathfrak{N}}{\sim} d$ so $c \overset{\mathrm{D}}{\sim} d$. Then we obtain $a \overset{\mathrm{D}}{\not\approx} b$ and $c \overset{\mathrm{D}}{\sim} d$. In the interpreted language L, the names a and b have the same denotation and different meanings, whilst the names c and d have different denotations and the same meaning. ⊣

The open problem: find a definition of synonymity relation of names, which will be an extension of the definition given in this article on any elementary languages.

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