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TWO TYPES OF ONTOLOGICAL STRUCTURE: Concepts Structures and Lattices of Elementary Situations*

Abstract. In 1982, Wolniewicz proposed a formal ontology of situations based on the lattice of elementary situations (cf. [7, 8]). In [3], I constructed some types of formal structure — Porphyrian Tree Structures (PTS), Concepts Structures (CS) and the Structures of Individuals (U) — that formally represent ontologically fundamental categories: species and genera (PTS), concepts (CS) and individual beings (U) (cf. [3, 4]). From an ontological perspective, situations and concepts belong to different categories. But, unexpectedly, as I shall show, some variants of CS and Wolniewicz's lattice are similar. The main theorem states that a subset of a modified concepts structure (called CS^+) based on CS fulfils the axioms of Wolniewicz' lattice. Finally, I shall draw some philosophical conclusions and state some formal facts.

Keywords: formal (formalized) ontology, ontology of situations, concepts structure, lattice.

1. Preliminaries

Following [3] and [4], let us remind ourselves of some essential definitions.

Let Q be a set of cardinality $\aleph_0$. Then for any subset X of Q, $\{0,1\}^X$ is the set of all functions from X into $\{0,1\}$. We put:

$$\mathrm{CS} := \bigcup\{\{0,1\}^X : X \text{ is a finite subset of } Q\}. \qquad \text{(def CS)}$$

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All functions from CS will be called *formal concepts* (henceforth: *concepts*). The function on $\varnothing$ (equal to $\varnothing$) will be denoted by $c_\varnothing$ and called the *root of* CS.

Example. If we consider the qualities (according to their familiar definitions): of being a number (q_1), being a natural (q_2), being divisible by 2 (q_3), then the concepts of (a): even number and of (b): odd number, can be defined by functions from $X = \{q_1, q_2, q_3\}$ into $\{0, 1\}$ in the following way:

even number	odd number
$q_1 \to 1$	$q_1 \to 1$
$q_2 \to 1$	$q_2 \to 1$
$q_3 \to 1$	$q_3 \to 0$

Definition 1. A set CS with the relation $\leq_{\mathrm{CS}}$ of inclusion on CS (in short, $\langle \mathrm{CS}, \leq_{\mathrm{CS}} \rangle$) will be called a *concepts system.*

Remark. For all sets $X, Y \subseteq Q$ and all functions $a\colon X \to \{0,1\}$ and $b\colon Y \to \{0,1\}$ (i.e. $a, b \in \mathrm{CS}$): $a \leq_{\mathrm{CS}} b$ iff $a \subseteq b$ iff $X \subseteq Y$ and $\forall_{q \in X}\ a(q) = b(q)$.

We obtain an expected result:

Fact 1. $\langle \mathrm{CS}, \leq_{\mathrm{CS}} \rangle$ *is a partially ordered set.*

Let us extend the set CS to a set CS^+. Our aim is to obtain a structure that will be parallel to Wolniewicz's structure.

Definition 2. $\lambda_{\mathrm{CS}} := Q \times \{0, 1\}$.

Definition 3. $\mathrm{CS}^+ := \bigcup \{\{0, 1\}^X : X \subseteq Q\} \cup \{\lambda_{\mathrm{CS}}\}$.

Definition 4. Let $\leq_+$ denote the inclusion on CS^+. Then the pair $\langle \mathrm{CS}^+, \leq_+ \rangle$ is an *extended concepts system.* Of course, for any $c \in \mathrm{CS}_+$, $c \leq_+ \lambda_{\mathrm{CS}}$.

Remark. CS^+ includes all functions defined on any subset (not only finite) of Q. We have designated elements of CS^+ by c_1, c_2, c_3, … or more simply by a, b, c. Any function c defined on the finite set $\{q_1, q_2, \ldots, q_n\}$ into $\{0, 1\}$ we depict as the set $\{q_1^*, q_2^*, \ldots, q_n^*\}$, where $^* \in \{0, 1\}$; hence, q_j^* means that the given function has value * on q_j; functions defined on Q are denoted by c_∞, c'_∞, c''_∞ etc. and called *maximal concepts*; instead of $\leq_+$ we will write: $\leq$.

Definition 5. For any $a, b \in \mathrm{CS}^+$ concepts a and b are *inconsistent* iff there exist pairs $\langle q, n\rangle \in a$ and $\langle q, m\rangle \in b$ such that $\langle q, n\rangle \neq \langle q, m\rangle$. Otherwise, the concepts a and b are *consistent*.

Fact 2. *For any $a \in \mathrm{CS}^+ \setminus \{c_\varnothing\}$: a and λ_{CS} are inconsistent.*

Proof. Let $a \in \mathrm{CS}^+ \setminus \{c_\varnothing\}$. Of course, there exists $\langle q, k\rangle \in a$. But, if $k \neq n$ and $k, n \in \{0, 1\}$, then $\langle q, n\rangle \in \lambda_{\mathrm{CS}}$. Hence, by Definition 5, a and λ_{CS} are inconsistent. ⊣

Fact 3. *For any $a \in \mathrm{CS}^+$: the concepts a and $\varnothing$ are consistent.*

Fact 4. *Let $a, b \in \mathrm{CS}^+$ and a and b be consistent. Then for any $x \leq a$, x and b are consistent.*

Proof. Indeed, by *absurdum*, if x and b are inconsistent, then there exist pairs $\langle q, n\rangle \in x$ and $\langle q, m\rangle \in b$, for $n \neq m$ and $n, m \in \{0, 1\}$. But $x \leq a$, so $\langle q, n\rangle \in a$. In consequence: a and b are inconsistent; contradiction. ⊣

Fact 5. *$\langle \mathrm{CS}^+, \leq\rangle$ is a lattice, i.e. $\langle \mathrm{CS}^+, \leq\rangle$ is a poset and for any $a, b \in \mathrm{CS}^+$ there is a supremum and infimum of $\{a, b\}$.*

Proof. It is easy to remark that the infimum of $\{a, b\}$ is $a \cap b$ and $a \cap b \in \mathrm{CS}^+$. Next, the supremum of $\{a, b\}$ is $a \cup b$, if a and b are consistent, and is λ_{CS}, if a and b are inconsistent. But $\lambda_{\mathrm{CS}} \in \mathrm{CS}^+$ and in the first case $a \cup b \in \mathrm{CS}^+$, hence $(\mathrm{CS}^+, \leq)$ is a lattice. ⊣

Now, I introduce two operations on CS^+: a) an operation of *consistent join of concepts* (&) and b) operation of *common content of concepts* (#).

Definition 6. Let # denote the set-theoretical intersection, and for $a, b \in \mathrm{CS}^+$, let

$$a \mathbin{\&} b = \begin{cases} a \cup b, & \text{if } a \text{ and } b \text{ are consistent} \\ \lambda_{\mathrm{CS}}, & \text{otherwise.} \end{cases}$$

By Fact 5, we have that $\&, \#\colon \mathrm{CS}^+ \times \mathrm{CS}^+ \to \mathrm{CS}^+$.

Remark. A counterpart of & is Wolniewicz's operation ; (semicolon) on sets of elementary situations and a counterpart of operation # is his operation ! (exclamation mark). The former is the supremum in the lattice, the later is the infimum (cf. [8]). In what follows, instead of ';' and '!' the signs '∨' and '∧' will be used as in [9].

Fact 6. *For any* $a \in \mathrm{CS}^+$*:*

(&1) $a \,\&\, \lambda_{\mathrm{CS}} = \lambda_{\mathrm{CS}} \,\&\, a = \lambda_{\mathrm{CS}} \,\&\, \lambda_{\mathrm{CS}} = \lambda_{\mathrm{CS}}$
(#1) $a \,\#\, \lambda_{\mathrm{CS}} = \lambda_{\mathrm{CS}} \,\#\, a = a$
(#2) $\lambda_{\mathrm{CS}} \,\#\, \lambda_{\mathrm{CS}} = \lambda_{\mathrm{CS}}$

Proof. The facts are evident by Definitions 2, 5 and 6. ⊣

Fact 7. $\langle \mathrm{CS}^+, \&, \# \rangle$ *is a lattice.*

Proof. Commutativity and associativity for & and # are established by Definition 6, the commutativity and associativity of ∪, ∩, and finally, by Fact 6. Let us prove absorption laws, i.e.:

(1) $a \,\&(a \,\#\, b) = a$,
(2) $a \,\#(a \,\&\, b) = a$.

We have to consider four cases:

1° $a = \lambda_{\mathrm{CS}}$ and $b = \lambda_{\mathrm{CS}}$,
2° $a = \lambda_{\mathrm{CS}}$ and $b \in \mathrm{CS}^+ \backslash \{\lambda_{\mathrm{CS}}\}$,
3° $a \in \mathrm{CS}^+ \backslash \{\lambda_{\mathrm{CS}}\}$ and $b = \lambda_{\mathrm{CS}}$,
4° $a, b \in \mathrm{CS}^+ \backslash \{\lambda_{\mathrm{CS}}\}$.

Let us consider the law (1). In case of 1° and 2° (1) is fulfilled by (&1), (#1), (#2). In case 3° by (&1), (#1), (#2). To prove (1) for 4°, i.e. if a and b are inconsistent, (1) is true by the absorption law for ∪ and ∩. But if a and b are inconsistent, then $a \,\#\, b \subseteq a$ and then $a \,\&(a \,\#\, b) = a$.

Similarly, by (&1), (#1), (#2) and the absorption law for ∩ and ∪, we obtain (2). ⊣

2. Wolniewicz's axioms for lattices of elementary situations

The axioms for the lattice of situations were given in [7] and [8]. Yet both sets of axioms are different. The axiomatics presented below follows [8].

S.1. $\mathrm{SE} = \mathrm{SEC} \cup \{o, \lambda\}$, where SEC is a (empty or non-empty) set of the contingent situations, o is called an empty situation and $\lambda (\neq o)$ the impossible one. SE is a universe of *elementary situations.*

S.2. $\leq$ is a partial order on SE such that o is its zero and λ is its unit. Hence, for any $x \in \mathrm{SE}$: $o \leq x \leq \lambda$.

S.3. For any $A \subseteq \mathrm{SE}$ there exists $x \in \mathrm{SE}$ such that: $x = \sup A$.

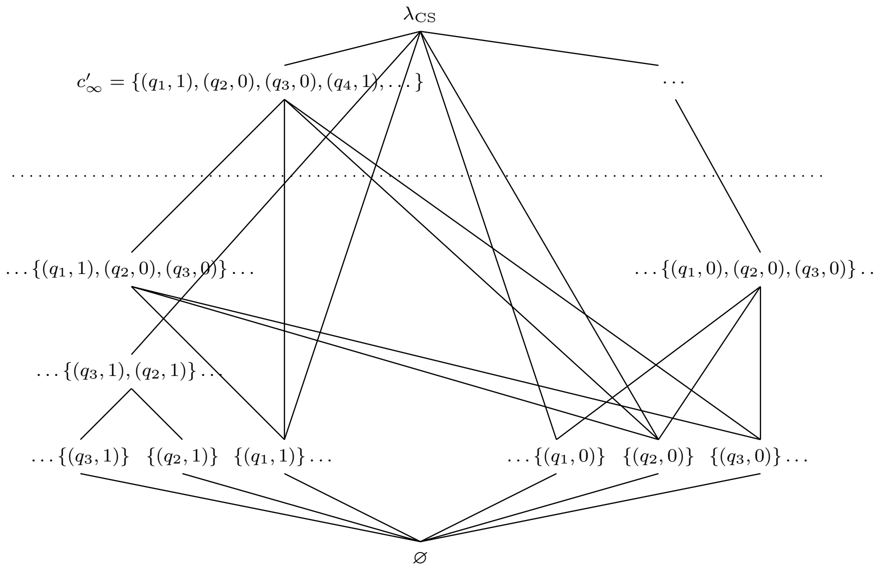

Figure 1. Diagram of fragment of the lattice CS^+.

S.4. For any $x, y, z \in \mathrm{SE}$: $x \leq y \leq z \Rightarrow \exists_{y' \in \mathrm{SE}}(x = y \wedge y'$ and $z = y \vee y')$.

S.5. For any $x, y, z \in \mathrm{SE}$:
(a) $(x \vee y \neq \lambda$ and $x \vee z \neq \lambda) \Rightarrow (x \vee y) \wedge (x \vee z) \leq x \vee (y \wedge z)$,
(b) $y \vee z \neq \lambda \Rightarrow x \wedge (y \vee z) \leq (x \wedge y) \vee (x \wedge z)$.[1]

S.6. Let $\mathrm{SE}' = \mathrm{SE} \setminus \{\lambda\}$. Then: $\forall_{x \in \mathrm{SE}'} \exists_{w \in \mathrm{Max}(\mathrm{SE}')} x \leq w$.

The set $\mathrm{Max}(\mathrm{SE}') = \{x \in \mathrm{SE} : \lambda$ covers $x\}$ of maximal possible situations, where b *covers* a, for $a \neq b$, means that $\{x : a \leq x \leq b\} = \{a, b\}$, is called the *logical space* (SP) and its elements *logical points* or *possible worlds*.

S.7. $x, y \in \mathrm{SE}$: $x \neq y \Rightarrow \exists_{w \in \mathrm{SP}}((x \leq w$ and $\sim y \leq w)$ or $(\sim x \leq w$ and $y \leq w))$.

S.8. Let $\mathrm{SA} = \{x \in \mathrm{SE} : x$ covers $o\}$. Then: $\forall_{x \in \mathrm{SE}} \exists_{A \subseteq \mathrm{SA}}(x = \sup A)$ (so, the lattice SE is *atomistic*).

S.9. For any $x, y \in \mathrm{SE}$: $x \vee y = \lambda \Rightarrow \exists_{a, a' \in \mathrm{SA}}(a \leq x$ and $a' \leq y$ and $a \vee a' = \lambda)$.

S.10. $\forall_{x,y,z \in \mathrm{SA}}((x \vee z = \lambda \wedge y \vee z = \lambda) \Rightarrow (x = y \vee x \vee y = \lambda))$.

[1] The conditions (a) and (b) are equivalent in each lattice with the unit element (see [1, 5]).

Now, following Wolniewicz, to define logical dimensions of the space SP, we redefine equivalence relation $\underset{d}{=}$ on SA.

DEFINITION 7. For any $x, y \in$ SA: $(x \underset{d}{=} y$ iff $(x = y$ or $x \vee y = \lambda$. The classes of the partition SA$/_{\underset{d}{=}}$ are the *logical dimensions* of SP, provided SEC is not empty. Otherwise, SP $= \{o\}$, SA $= \{\lambda\}$, SA$/_{\underset{d}{=}} = \{\{\lambda\}\}$.

S.11. dim SP $= n$, where dim SP is the number of logical dimensions.

3. Concepts Structure CS$^+$ and Wolniewicz's structure

The axioms **S.1**–**S.11** can be rewritten in the following way:

(REW) We take CS$^+$ for SE with $\varnothing$ and λ_{CS} for o and λ. Partial order in CS$^+$ is the order given by Definition 4, operations & and # correspond to Wolniewicz's $\vee$ and $\wedge$. Next, as maximal possible situations we will consider the functions c_∞, c'_∞, c''_∞ from Q into $\{0, 1\}$ and the set of atoms CS$^+$At in CS$^+ = Q \times \{0, 1\}$. Finally, I define the relation $\underset{D}{\approx}$ on CS$^+$At by the condition:

DEFINITION 8. For any $a, b \in$ CS$^+$At: $a \underset{D}{\approx} b$ iff $(a = b$ or $a \,\&\, b = \lambda_{\mathrm{CS}})$.

The equivalence relation $\underset{D}{\approx}$ is to be a counterpart of $\underset{d}{=}$ defined by Wolniewicz.

3.1. CS$^+$ fulfils S.1–S.10 and does not fulfil S.11

THEOREM 1. *The Axioms* **S.1**–**S.10** *hold for the structure* $\langle$CS$^+, \&, \#\rangle$.

PROOF. The truth of **S.1**–**S.3** is evident (by (REW)).

To prove **S.4** we show that:

for any $a, b, c \in$ CS$^+ : a \leq b \leq c \Rightarrow \exists b' \in$ CS$^+ (a = b \,\#\, b'$ and $c = b \,\&\, b')$.

Obviously, if $a = b$, we put $b' = c$, but if $b = c$, we take $b' = a$. The case when $a = b = c$ is trivial for then $b' = b$.

Now, let us assume that $a \leq b \leq c$ and $a \neq b \neq c$. If $a = \varnothing$, then we put $b' = c \setminus b \neq \varnothing$. Then $a = \varnothing = b \,\#\, b'$ and $c = b \,\&\, b'$. If $a \neq \varnothing$, we take $b' = (c \setminus b) \cup a$. So,

$$b \,\#\, b' = b \cap ((c \setminus b) \cup a) = (b \cap (c \setminus b)) \cup (b \cap a) = \varnothing \cup a = a \text{ and}$$
$$b \,\&\, b' = b \cup ((c \setminus b) \cup a) = c \cup a = c.$$

Ad **S.5**. For $a, b, c \in \mathrm{CS}^+$:

(a) $(a \,\&\, b \neq \lambda_{\mathrm{CS}} \wedge a \,\&\, c \neq \lambda_{\mathrm{CS}}) \Rightarrow (a \,\&\, b) \,\#(a \,\&\, c) \leq a \,\&(b \,\#\, c)$,
(b) $b \,\&\, c \neq \lambda_{\mathrm{CS}} \Rightarrow a \,\#(b \,\&\, c) \leq (a \,\#\, b) \,\&(a \,\#\, c)$.

In the case of (a) we assume the predecessor. Then a, b, c are concepts and by the definitions of & and #:

$$\begin{aligned}(a \,\&\, b) \,\#(a \,\&\, c) &= (a \cup b) \cap (a \cup c) = ((a \cup b) \cap c) \cup ((a \cup b) \cap c) = \\ &= a \cup ((a \cap c) \cup (b \cap c)) = (a \cup (a \cap c)) \cup (b \cap c) = \\ &= a \cup (b \cap c) = a \,\&(b \,\#\, c).\end{aligned}$$

The last equation follows from Fact 4. Hence, if $(a \,\&\, b) \,\#(a \,\&\, c) = a \,\&(b \,\#\, c)$, then, by reflexivity of $\leq$: $(a \,\&\, b) \,\#(a \,\&\, c) \leq a \,\&(b \,\#\, c)$.

To prove (b), we assume: $b \,\&\, c \neq \lambda_{\mathrm{CS}}$. But then: $b \neq \lambda_{\mathrm{CS}}$, $c \neq \lambda_{\mathrm{CS}}$, $b \cup c$ (i.e. $b \,\&\, c$) $\in \mathrm{CS}^+ \backslash \{\lambda_{\mathrm{CS}}\}$ and b and c are consistent. So, we have:

$$a \,\#(b \,\&\, c) = a \cap (b \cup c) = (a \cap b) \cup (a \cap c) = (a \,\#\, b) \,\&(a \,\#\, c),$$

The last equality holds because the concepts: $(a \cap b)$ and $(a \cap c)$ are consistent. Hence,

$$a \,\#(b \,\&\, c) \leq (a \,\#\, b) \,\&(a \,\#\, c).$$

Axiom **S.6** is evident. If we take any concept c defined on a proper subset Q' of Q, then there exists a function c_∞ on Q, such that $c_\infty|_{\mathrm{DOM}(c)} = c$. Then, of course, $c \leq c_\infty$. But if $c = c_\infty$, then $c_\infty \leq c_\infty$.

Ad **S.7**. We will show that: $a, b \in \mathrm{CS}^+$: $a \neq b \Rightarrow \exists c_\infty \in \mathrm{SP}$: $((x \leq c_\infty$ and $\sim y \leq c_\infty)$ or $(\sim x \leq c_\infty$ and $y \leq c_\infty))$.

To prove it, let us consider two different non-empty concepts a and b. Then, there exists $\langle q, k\rangle$ such that $\langle q, k\rangle \in a$ and $\langle q, k\rangle \notin b$ (or vice versa). Let us take into account a c_∞ such that $a \leq c_\infty$ and consider two cases: (a) if $b \leq a$, we can point to c'_∞ such that $\langle q, m\rangle \in c'_\infty$ for $m \neq k$; and then $b \leq c'_\infty$ and $\sim(a \leq c'_\infty)$; if $\sim(b \leq a)$, then there exists a c_∞ such that $a \leq c_\infty$; but $\langle q, k\rangle \in c_\infty$ and $\langle q, k\rangle \notin b$, so $\sim(b \leq c_\infty)$. This means that the successor is true.

S.8. is trivial by the definition of concept and the definition of $\mathrm{CS}^+\mathrm{At}$.

S.9. follows from the definition of concepts that are not inconsistent.

Ad **S.10**. We have to prove that for any a, b, $c \in \mathrm{CS}^+\mathrm{At}((a \,\&\, c = \lambda_{\mathrm{CS}}$ and $b \,\&\, c = \lambda_{\mathrm{CS}}) \Rightarrow (a = b$ or $a \,\&\, b = \lambda_{\mathrm{CS}}))$.

We remark that for an atom $\{\langle q,^{*}\rangle\}$ there exists only one atom inconsistent with it. It is an atom of the form $\{\langle q,-^{*}\rangle\}$, where $-^{*}=1(0)$, if $^{*}=0(1)$. So, if $x\,\&\,y=\lambda_{\mathrm{CS}}$ and $y\,\&\,z=\lambda_{\mathrm{CS}}$, then x and y are inconsistent and $z=x$. ⊣

3.2. A fragment of CS^{+} fulfils S.1–S.11

THEOREM 2. $\langle\mathrm{CS}^{+},\&,\#\rangle$ *has an infinite number of logical dimensions* $(\dim\mathrm{SP}=\aleph_0)$.

PROOF. It is easy to notice that any dimension has two elements (of the form: $\{q_i^1,q_i^0\}$). Hence, $\dim\mathrm{SP}=\aleph_0$, because the cardinal number of Q is $\aleph_0$. ⊣

It appears that a fragment of CS^{+} fulfils the axiom **S.11**. Namely, the following theorem is true.

THEOREM 3. *Let* $X\subsetneq Q$, $\mathrm{card}(X)<\aleph_0$ *and* $\mathrm{At}_{\mathrm{CS(FIN)}}=X\times\{0,1\}$. *The set* $\mathrm{CS}^{+}_{\mathrm{FIN}}$ *(with operations* $\&$, $\#$*) such that:*

(1) $\mathrm{At}_{\mathrm{CS(FIN)}}\subseteq\mathrm{CS}^{+}_{\mathrm{FIN}}$,
(2) $\mathrm{CS}^{+}_{\mathrm{FIN}}$ *is closed on* $\&$ *and* $\#$,

is a lattice fulfilling **S.1–S.11**.

Remark. Functions defined on the set X are maximal elements of $\mathrm{CS}^{+}_{\mathrm{FIN}}$.

PROOF OF THEOREM 3. Let us show that **S.11** holds. Indeed, considering the equivalence relation we obtain $\mathrm{card}(X)$ dimensions, $\mathrm{card}(X)<\aleph_0$. In turn, axioms **S.1–S.10** can be considered as particular cases of Theorem 1. ⊣

4. Conclusions and perspectives

1. The investigations were presented in the simplest form possible. The case of CS^{+}, where the partial functions on $X\subseteq Q$ into $\{0,1\}$ are considered, corresponds to Wolniewicz's lattice for Wittgenstein's atomism (each dimension has then 2 elements). It is evident, however, that CS structures can be extended to the case where functions on X have values from the set $\{1,\dots,k\}$, for $k\in\omega$ (or even from some infinite set). At the present time we have k inconsistent elements in each dimension. If we, additionally, reject — in spite of Wolniewicz's suggestion — axiom

S.11, then we can speak of lattices with infinite width and length. A CS structure is suitable for this kind of lattice.

2. Crossing the boundary between 2-element dimensions to k-element ones causes the change of paradigm connected with grasping given property (feature or quality). If $\mathrm{card}(D) = 2$, then we obtain the so-called Meinongian case (let us remind ourselves that Meinong proposed the concept of a complement property non-P besides the property P, for example: redness and non-redness). The so-called complete objects are characterized by the condition: for any property P, either the object has P or has non-P. In my proposal possessing P means that the value of property P is 1 and possessing non-P amounts to 0. The matter is discussed by Kaczmarek in [2, 3, 4]. In turn, the case when the functions from CS^+ have the value from $\{1, \ldots, k\}$, is present in information systems (cf. Pawlak [6]). Pawlak proposes to replace the concept of a property by the concept of an attribute (for example, age, growth, colour) and to bind with any attribute a a set of k_a values ($k_a \in \omega, k_a \geqslant 1$). I propose to call this approach an *attributive paradigm* and I investigate it elsewhere. It is interesting and fruitful that the set of information and partial information with an order on that set is isomorphic to a fragment of a set $\mathrm{CS}^+(k)$ containing all functions from subsets of Q into $\{1, \ldots, k\}$. So, I present the following facts:

Fact 8. (1) *Let* $k \geqslant 2$, $\lambda_{\mathrm{CS}} = Q \times \{1, \ldots, k\}$, $\mathrm{CS}^+(k) = \bigcup\{\{1, \ldots, k\}^X : X \subseteq Q\} \cup \{\lambda_{\mathrm{CS}}\}$ *and* $\leq_{\mathrm{CS}}$ *be the inclustion on the set* $\mathrm{CS}^+(k)$. *Then* $\langle \mathrm{CS}^+(k), \leq_{\mathrm{CS}} \rangle$ *is a lattice.*

(2) *Let* $S = \langle O, A, V, \rho \rangle$ *be an information system, where* O *is a finite set of objects,* A *is a finite set of attributes,* $V = \bigcup_{a \in A} V_a$, *where* $\{V_a\}_{a \in A}$ *is an indexed family of sets, and* ρ *is function on* $O \times A$ *into* V, *such that* $\rho(x, a) \in V_a$ *for any* $x \in O$ *and* $a \in A$. *Let us define a set* $\mathrm{Inf}^*(S) = \bigcup\{\{0, 1\}^B : B \subseteq A\}$ *of all information and partial information of* S, *the number* $k = \max(\mathrm{card}\, V_a)$ *for* $a \in A$ *and* $\leq_{\mathrm{inf}}$ *is the inclusion on the set* $\mathrm{Inf}^*(S)$. *Then there exists a set* $\mathrm{CS}^+_{\mathrm{FIN}}(k) \subseteq \mathrm{CS}^+(k)$ *such that* $\langle \mathrm{CS}^+_{\mathrm{FIN}}(k), \leq_{\mathrm{CS}} \rangle$ *and* $\langle \mathrm{Inf}^*(S), \leq_{\mathrm{inf}} \rangle$ *are isomorphic.*

Fact 9. *Consider* CS^+ *given in Definition 3 and the lattice* $\mathbf{CS}^+ = \langle \mathrm{CS}^+, \&, \#, \varnothing, \lambda_{\mathrm{CS}} \rangle$. *Let* $\mathbf{SE} = \langle \mathrm{SE}, \vee, \wedge, o, \lambda \rangle$ *be a lattice of elementary situations fulfilling* **S.1**–**S.6**, **S.8** *and* **S.11**, *such that for any dimension* *d*: card(d) = 2. *There exists a sublattice* $\mathbf{CS}_{\boldsymbol{A}}$ *of* $\mathbf{CS}^{+}$ *such that* $\mathbf{CS}_{\boldsymbol{A}}$ *and* **SE** *are isomorphic.*

In my opinion, by proving these facts we have shown that mutual relations between certain aspects of formal ontology and informatics do exist.

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