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SOME REMARKS ON FORMAL DESCRIPTION OF GOD'S OMNIPOTENCE

Abstract. There are proposed two simple formal descriptions of the notion of God's omnipotence which are inspired by formalizations of C. Christian and E. Nieznański. Our first proposal is expressed in a modal sentential language with quantifires. The second one is formulated in first order predicate language. In frame of the second aproach we admit using self-referential expressions. In effect we link our considerations with so called *paradox of God's omnipotence* and reconstruct some argumentation against the possibility of reference God's omnipotence to a lack of itself.

Keywords: omnipotence, self-reference, formalization, theodicy.

1. Introduction

In the present paper we are going to construct two descriptions of the notion of God's omnipotence which are inspired by formalizations of C. Christian [1] and E. Nieznański [4]. Formalisms of these authors may be based on some ideas of Thomas Aquinas and G. W. Leibniz and for this reason our approach is also linked with at least some fragments of classical theodicy. Following Christian and Nieznański we will depend the notion of omnipotence on the notion of *will* of the creator excluding from the range of it *a contradiction*. Our first proposal we will express in a modal sentential language with quantifires. The second description is formulated in predicate language and here we admit using self-referential

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expressions. In this language we will answer a question about consequences of referring the omnipotence of God to a lack of itself — the question which may give the occasion to formulate so-called *paradox of omnipotence*.

2. Omnipotence in frame of God's will and noncontradition

An idea of linking omnipotence with will of God is already known from Aquinas' theology. Thomas underlined the role of will in undertaking any action:

> Even in us the cause of one and the same effect is knowledge as directing it, whereby the form of the work is conceived, and will as commanding it, since the form as it is in the intellect only is not determined to exist or not to exist in the effect, except by the will. [...] But the power is cause, as executing the effect, since it denotes the immediate principle of operation. [5, I, 19, 4]

and in case of Omnipotent Creator he claimed that:

> [...] the will of God is the universal cause of all things, it is impossible that the divine will should not produce its effect. [5, I, 19, 6]

If we say, that a creator has a power to create any given situation s if and only if the fact that situation s is subject of his will is a sufficient condition of the actual existence of s, the omnipotence is some kind of generalization of such creative power: the creator would be omnipotent only if he would have the creative power to all subjects of his will. There is however no point to discuss that the range of will of omnipotential creator must be limited to consistent objects:

> Therefore, everything that does not imply a contradiction in terms, is numbered amongst those possible things, in respect of which God is called omnipotent: whereas whatever implies contradiction does not come within the scope of divine omnipotence [...]. [5, I, 25, 3–4]

The same idea is repeated also by Leibniz in his *Theodicy*:

> [God's power] extends ad maximum, ad omnia, to all that implies no contradiction [...]. [3, 227]

As Thomas Aquinas argues, the limitation of God's omnipotence to objects which are consistent in any sense does not downgrade His power. Let us continue the quoted text of *The Summa*:

> whereas whatever implies contradiction does not come within the scope of divine omnipotence, because it cannot have the aspect of possibility. Hence it is better to say that such things cannot be done, than that God cannot do them. Nor is this contrary to the word of the angel, saying: "No word shall be impossible with God." For whatever implies a contradiction cannot be a word, because no intellect can possibly conceive such a thing. [5, I, 25, 3–4]

Both mentioned limits of the omnipotence: the will of the creator and the consistency of what may be its subject, are also considered by modern formalizations — in particular these proposed by Christian and Nieznański. However the attempts of these authors lead to complications which we refer just to avoid them in our proposal.

Following Christian and Nieznański we use individual variables: x, y, z, . . . , which are meant to represent *persons*; sentential variables: p, q, r, . . . , for *situations*; logical symbols: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\forall$, $\exists$ (quantifries are applied both to individual and sentential variables) and constant symbol W. The formula $W(\ldots,\ldots)$ we read as: ". . . wants, that . . . ". Both authors don't explicitly define well-formed expressions of their languages showing the way of constructing them in concrete contexts. Actually this fact leads to some misunderstandings. First let us notice that the constant W is called by Christian a predicate and used in context: $W(x, p)$ that should be read: "(being) x wants, that p". This expression occurs in his definition of omnipotence:

$$\mathrm{OM}\, x \leftrightarrow \forall_p (W(x, p) \rightarrow p) \qquad (\mathrm{def\,OM})$$

i.e., x is omnipotent iff for every situation p: if x wants that p, then p.

However (def OM) can not be constructed in well formulated (first order) predicate language since in definiens of (def OM) the sentential variable p stands in two different categories: first as a name and second as a sentence.[1] The way of treating symbol W by Nieznański seems not to lead to such difficulties. Although he copies the definition of omnipotence by Christian it may be that W is taken as the index modal sentential operator and so the formalization of Nieznański could be considered as some kind of multimodal aproach. This suggestion seems to be plausible in view of semantics which is sketched by the author of [4]. The meaning of symbol W in context: Wbp that should be read: "God wants that p" is relativisied to a possible world w and the truth condition

[1] The same problem occurs in case of Christian's definition of omniscience: "x is omniscient $\leftrightarrow \forall_p(p \rightarrow (x$ knows, that $p))$". By this occasion we also remark that although definitions of omnipotence and omniscience proposed by P. Weingartner in [6] are more complex that these of Chrstian, as far as they are intended to be expressed in first order predicate language they are burdened with the same gramatical trouble as Christian's formulation.

for the expression Wbp depends on accessibility relation which is reflexive and transitive. The author claims that Wb has S4-properties. From the other side in standard Kripke semantics for multimodal languages there are considered many accessiblitiy relations which are linked with different *agents* represented in syntax by indexes and this construction is passed over in [4].

Independently of listed problems (or lacks of clarity) we should however notice that the notion of God's will may have at least two possible formal representations — as a modal operator and as a predicate. Just because of this two possibilites we are going to propose two descriptions of God's omnipotence.

3. Omnipotence of God in sentence and predicate terms. Two descriptions

We will base our proposals on classical sentential logic with quantifires, expressed in the language with the following vocabulary: (i) sentential variables: p, q, r, ... (the set SV); (ii) logical symbols: $\to$, $\forall$; (iii) brackets.

The set of formulas, FOR, is defined inductively. It is the smallest set satisfying the following conditions:

(1f) $\mathrm{SV} \subseteq \mathrm{FOR}$,
(2f) if $v \in \mathrm{SV}$ and $A, B \in \mathrm{FOR}$, then $(A \to B) \in \mathrm{FOR}$ and $\forall_v A \in \mathrm{FOR}$.

In the metalanguage we take the following usual notation:

- by $\mathrm{fv}(A)$ we denote the set of all free variables in A;
- by $\mathrm{Sb}(A)$ we understand a formula which is obtained from A by substitution in A some free variable by any formula with the restriction that: (i) all other free variables in A remains also free in $\mathrm{Sb}(A)$ and (ii) all free variables of the substituted formula remain free in $\mathrm{Sb}(A)$.

We define assumed logic in the way of Tarski and Bernays. The calculus of classical sentential logic with quantifiers, called PL$\forall$, is characterised by the following axioms:

$$(p \to q) \to ((q \to r) \to (p \to r)) \tag{A1}$$

$$q \to (p \to q) \tag{A2}$$

$$((p \to q) \to p) \to p \tag{A3}$$

and rules:

$$A, A \to B \ / \ B \tag{MP}$$

$$A \ / \ \mathrm{Sb}(A) \tag{RSb}$$

$$A \to B \ / \ \forall_v A \to B \tag{RGen}$$

$$A \to B \ / \ A \to \forall_v B, \quad \text{where } v \notin \mathrm{fv}(A) \tag{RGen$'$}$$

Notice that, by (A2), (RSb), (RGen′) and (MP), the logic PL∀ is closed under the following rule of generalization:

$$A \;/\; \forall_v\, A \tag{RG}$$

Moreover, in the logic PL∀ we have the following theses:

$$p \to p \tag{1}$$
$$(p \to (q \to r)) \to (q \to (p \to r)) \tag{2}$$
$$p \to ((p \to q) \to q) \tag{3}$$

To extend the language of PL∀ by ⊥ (*falsum*) and ¬ (*negation*) there are assumed following metadefinitions:

$$\bot := \forall_p\, p \tag{def$\bot$}$$
$$\neg A := A \to \bot \tag{def$\neg$}$$

Notice that the logic PL∀ has the following theses:

$$p \to \neg\neg p \tag{4}$$
$$(p \to \neg q) \to (q \to \neg p) \tag{5}$$

Indeed, we use (def¬) and moreover for (4): (3); for (5): (2).

It will be convenient to use also other logical symbols introduced by (for any $A, B \in \mathrm{FOR}$ and $v \in \mathrm{SV}$):

$$(A \wedge B) := \neg(A \to \neg B) \tag{def$\wedge$}$$
$$(A \vee B) := \neg(\neg A \wedge \neg B) \tag{def$\vee$}$$
$$(A \leftrightarrow B) := ((\neg A \vee B) \wedge (A \vee \neg B)) \tag{def $\leftrightarrow$}$$
$$\exists_v\, A := \neg\forall_v\, \neg A \tag{def$\exists$}$$

Notice that, by (5), (2) and (MP), we obtain a thesis $(A \to \neg T) \to \neg A$, for any thesis T. Hence, by (def∧) and (4), we have:

$$(p \to (q \wedge \neg q)) \to \neg p \tag{6}$$

Hence, by (def ↔), (def∧), (def∨) and (4), we also obtain:

$$(p \to (q \leftrightarrow \neg q)) \to \neg p \tag{7}$$

A. Characteristics with modal operator of God's will. We limit our interest to God's will by itself and in particular we will not consider a problem of its relation to will of other persons (*agents*). For this reason we don't proceed in style of already mentioned multimodal formalization. So we enrich the vocabulary of PL∀ only by one modal operator **W** read: "God wants that". We add to the set of formulas new expressions by adding to the definition of FOR the following condition:

(3f) if $A \in \mathrm{FOR}$, then $\mathbf{W}A \in \mathrm{FOR}$.

Following suggestions of Nieznański we accept that:

$$\exists_p \, \mathbf{W}p \qquad (\textit{God wants something}) \qquad (\mathrm{A}\exists)$$

$$\mathbf{W}p \to p \qquad (\mathtt{T})$$

As it is solved in [4] we also accept that:

$$\mathbf{W}(p \to q) \to (\mathbf{W}p \to \mathbf{W}p) \qquad (\mathtt{K})$$

$$\mathbf{W}p \to \mathbf{W}\mathbf{W}p \qquad (\mathtt{4})$$

and we add a rule of introducing **W**:

$$A \;/\; \mathbf{W}A \qquad (\mathrm{R}\mathbf{W})$$

The theory based on PL∀ extended by (A∃), (T), (K), (4) and (R**W**), called T**W**, characterises the operator **W** as S4-modality. (However for our considerations we need only a fragment of T**W**.)

Le us introduce a new sentential constant which describes a situational counterpart of God's omnipotence:

$$\mathbf{s} := \forall_p (\mathbf{W}p \to p) \qquad (\mathrm{def}\mathbf{s})$$

In consequence we immediately obtain:

(t1)	$\mathbf{s}$	(*God is omnipotent*)	(T), (RG), (def**s**)
(t2)	$\neg\mathbf{W}\bot$	(*God doesn't want a contradiction*)	(T), (RSb), (def¬)
(t3)	$\neg\mathbf{W}\neg\mathbf{s}$	(*God doesn't want to be not omnipotent*)	(T), (RSb), (5), (t1)

B. God's will expressed by predicate. Let us take now the extension of PL∀ language obtained by adding to its vocabulary:

- the one place constant predicate $\mathbf{W}'$; a formula of the form $\mathbf{W}'(\ldots)$ we read: "… is wanted by God",
- the naming operator: $\ulcorner\ \urcorner$.[2]

The set of terms Γ and the set of formulas FOR′) are defined as follows. They are the smallest sets satisfying the following conditions:

(1f′) $\mathrm{SV} \subseteq \mathrm{FOR}'$,
(2f′) if $v \in \mathrm{SV}$ and $A, B \in \mathrm{FOR}'$, then $(A \to B) \in \mathrm{FOR}'$ and $\forall_v A \in \mathrm{FOR}'$,

[2] In non-substantative stylistics the operator $\ulcorner\ \urcorner$ might be called *reificator*.

(3f′) if $A \in \mathrm{FOR}'$ then $\ulcorner A \urcorner \in \Gamma$,
(4f′) if $\tau \in \Gamma$ then $\mathbf{W}'(\tau) \in \mathrm{FOR}'$.

The theory T**W** may be easily translated in the described language. We would use for that aim the function $\sharp$ that assigns to every formula from FOR a formula from FOR′ in the following way:

$$\begin{aligned} (v)^\sharp &= v, \quad \text{for any } v \in \mathrm{SV}, \\ (A \to B)^\sharp &= ((A)^\sharp \to (B)^\sharp), \\ (\forall_v\, A)^\sharp &= \forall_v\, (A)^\sharp, \\ (\mathbf{W}A)^\sharp &= \mathbf{W}'(\ulcorner (A)^\sharp \urcorner). \end{aligned}$$

Let us consider only the translation of (T):

$$\mathbf{W}'(\ulcorner p \urcorner) \to p \tag{T*}$$

and follow the idea accepted in T**W** to define the situation of God's omnipotence:

$$\mathbf{s}^* := \forall_p(\mathbf{W}'(\ulcorner p \urcorner) \to p) \tag{s*}$$

Just by of PL∀ , (T*) and (s*) we get also $\sharp$-translations of (t1), (t2) and (t3) of T**W**:

(t1*) $\mathbf{s}^*$
(t2*) $\neg\mathbf{W}'(\ulcorner \bot \urcorner)$
(t3*) $\neg\mathbf{W}'(\ulcorner \neg\mathbf{s}^* \urcorner)$

However we may use the proposed predicate language to consider also a problem which is not expressible in the language of T**W**, since it is linked with the possibility of using self-referential expressions. To analyse at least some formulation of so called *paradox of omnipotence* we assume that (cf. Feferman [2]):

(*Self-Reference*) For every formula A with a therm $\ulcorner v \urcorner$ it can be constructed a formula B such that $\mathrm{fv}(B) = \emptyset$ and $B := A(v/B)$, where $A(v/B)$ is the formula that results from A by replacing all occurrences of v, in A, by B (also in all occurrences of $\ulcorner v \urcorner$).

To consider a question what would happen if the omnipotence of God would refer to a lack of itself we define a *range of* omnipotence as:

$$\mathbf{Z}(\ulcorner p \urcorner) := \mathbf{W}'(\ulcorner p \urcorner) \wedge p \tag{defZ}$$

and so we get:

(t4*) $\mathbf{W}'(\ulcorner p \urcorner) \to (\mathbf{Z}(\ulcorner p \urcorner) \leftrightarrow p)$

Because of (*Self-Reference*) we can consider now the following definition:

$$\mathbf{s}^\bullet := \neg\mathbf{Z}(\ulcorner\mathbf{s}^\bullet\urcorner) \qquad (\text{def}\mathbf{s}^\bullet)$$

The sentence $\mathbf{s}^\bullet$ is self-referential — in $\mathbf{s}^\bullet$ there is said that $\mathbf{s}^\bullet$ does not belong to the range of God's omnipotence.

From (t4*), (def$\mathbf{s}^\bullet$) and (RSb) we obtain:

(t5•) $\mathbf{W}'(\ulcorner\mathbf{s}^\bullet\urcorner) \to (\mathbf{Z}(\ulcorner\mathbf{s}^\bullet\urcorner) \leftrightarrow \neg\mathbf{Z}(\ulcorner\mathbf{s}^\bullet\urcorner))$

nad so, by (7), we also obtain:

(t6•) $\neg\mathbf{W}'(\ulcorner\mathbf{s}^\bullet\urcorner)$

As we may notice in the proof of (t5•) there are invented essentially the same steps as these formulated by Tarski in the Liar paradox.[3] Anyway we do not obtain *paradox of omnipotence* — the contradiction is blocked by the predecessor $\mathbf{W}'(\ulcorner\mathbf{s}^\bullet\urcorner)$. After all, (t5•) could be regarded as an answer for the formulated question about the consequences of referring God's omnipotence, or its range, to a lack of itself. Intuitively speaking we would say that it is even not possible that God would wish this lack since this would lead to a contradiction — the range of His omnipotence couldn't be defined in the proposed way just by the same argumentation which we know from Tarski. And perhaps this remark might be treated as an explanation of a classical conviction that God couldn't be the cause of His weakness since it would stand in contradiction to His *nature*.

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