Jiří Raclavský
Petr Kuchyňka

CONCEPTUAL AND DERIVATION SYSTEMS

Abstract. Pavel Materna proposed valuable explications of concept and conceptual system. After their introduction, we contrast conceptual systems with (a novel notion of) derivation systems. Derivation systems differ from conceptual systems especially in including derivation rules. This enables us to show close connections among the realms of objects, their concepts, and reasoning with concepts.

Keywords: concepts; conceptual systems; deduction; derivation systems.

1. Introduction

The present study adheres to the tradition according to which *concepts* are not linguistic expressions or 'mental images' but abstract (objective) entities which *determine*, or pick out *objects*. Since there are various relations between concepts we can consider various *conceptual systems*.

Though it is certainly true that we have concepts collected in conceptual systems, it is also hardly deniable that we *ratiocinate* with (or with the help of) these concepts. Framed within a conceptual system, we perform operations (including inferences) with concepts, exploiting various *rules*. We will call the entities involving both concepts and rules *derivation systems*.

Rigorous work with our notions necessitates their proper *explication*. There are more logical explications of the notion of concept at hand. The present authors prefer the explication offered by Pavel Materna (Materna 1998, 2004) because we consider it well defended and best suitable for our goals. The exclusive feature of Materna's proposal consists in that it

captures the structured and procedural (algorithmic) nature of concepts. For that purpose, Materna profited from the logical theory developed by Pavel Tichý, i.e., *Transparent intensional logic*, TIL (Tichý 2004, 1988). Tichý also elaborated a noteworthy explication of the notion of (linguistic) *meaning* as a structured and procedural entity. Note that one thus can obtain a valuable connection of the explication of concepts with the explication of meanings. Materna also offered an explication of the notion of conceptual system (esp. Materna 2004) which gives us another reason for the continuation in his research. On the other hand, we met the need for an extension of his views, mainly by introducing the notion of derivation system. This enables us to enrich his investigations concerning concepts by means of Tichý's works on *deduction* (*cf.* the papers Tichý 1982, Oddie and Tichý 1982, Tichý 1986, which are collected in Tichý 2004).

The paper is organized as follows. We begin — "From extensional to hyperintensional explication of concepts" — with a selective overview of explications of concept, motivating thus the choice of Materna's proposal. In "Tichý's transparent intensional logic and meaning" we explain essential notions of Tichý's logic, showing also how he explicated meanings of linguistic expressions by means of TIL. The section "Materna's concepts" presents main theses of Materna's theory of concepts. Materna's theory of conceptual systems is explained in "Materna's conceptual systems". The essentials of the system of deduction we prefer are explained in "Tichý's system of deduction". Finally, we expose our main notion "Derivation systems".

It will be apparent from the paper that we subscribe to *objectual conception of logic*. On this view, the subject matter of logic is not a set of linguistic expressions of some artificial language but a range of abstract entities coded by certain expressions. The overall aim of such logic is to *contribute to the explication of our whole conceptual scheme* which concerns with concepts rather than expressions.

2. From extensional to hyperintensional explication of concepts

There seem to be two traditions in the explication of concepts. According to the tradition A, concept is something general. HORSE is thus construed as a concept, while THE HIGHEST MOUNTAIN is not. A typical representative of this tradition, if we do not mention the traditional

conception of concepts, is Gottlob Frege (esp. Frege 1891) who fittingly claims that concept has a predicative nature. It is in the spirit of this conception to view concept as a set of things or an entity determining such set.

According to the tradition B, however, concept is any (or nearly any) constituent of a thought ('proposition'). Thus also THE HIGHEST MOUNTAIN is construed as a concept. A prominent figure of this tradition is Alonzo Church who wittingly expanded Frege's theory which was restricted to predicates. Church considered any sense of a word to be a concept (Church 1956, 1985). Thus *concept* is something possibly *expressed by an expression* of this or that language, whereas the concept *determines an object* (a class or non-class), which is the *denotatum of that expression*. Let us note that Materna was inspired by Church's conception; that is why we also adhere to the tradition B.

Frege's penetrating idea was to explicate entities in terms of functions. Unfortunately, he seems to oscillate between the old and modern construal of function.[1] This led to various obscurities in his theory (*cf.* Tichý 1988). Anyway, Frege claimed that concept is a certain function and he in fact identified concept with a characteristic function, i.e., *class*. The view that concept is a class is still espoused by a wide range of contemporary theoreticians.

There is, however, a serious objection to this *extensional conception of concept* (as we call it). This conception cannot distinguish between *non-empirical* (e.g., PRIME) and *empirical* (e.g., HORSE) *concepts*, which is originated in ignorance of *modal* and *temporal variability*. While the concept PRIME has one and the same extension in all possible worlds and moments of times, the concept HORSE has various extensions across the logical space and time-scale. Although the concept HORSE is still the same, its extensions (i.e., classes of objects *falling under* this concept) vary.

We have just sketched the core reason for *intensional conception of concept*. According to this conception, empirical concept is a function from possible worlds W and moments of times T (i.e., an *intension*) to its extensions (in those W and T). (In the case of the tradition B, the values in W's and T's need not to be classes.) Non-empirical concept

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[1] According to the modern logical tradition, function is a mere correspondence of arguments with values, a flat mapping. According to the older tradition, function is a way or a rule how to get from arguments to values, i.e., a structured recipe.

can be then identified either with the extension or with the respective constant function from worlds and times.[2]

Though the intensional conception of concept just sketched seems to be plausible enough, it still has drawbacks. The crucial objection was already formulated by Bernard Bolzano (Bolzano 1837) many decades before the modal and temporal variability (intensionality) was appreciated. He attacked the traditional conception of concepts by means of examples such as UNLEARNED SON OF A LEARNED FATHER / LEARNED SON OF AN UNLEARNED FATHER. Bolzano pointed out that these concepts cannot be understood merely as a sum of their parts, i.e., UN-, LEARNED, SON, FATHER. The two concepts are apparently different, thus there must be something over this sum which makes them different. Obviously, it is structuredness, which binds component subconcepts together and gives rise to the *structured complex whole*, which is thus a feature peculiar to (compound) concepts.

Bolzano's lesson can be easily generalized to other examples (*cf.* Materna's 1998, 2004). For instance, concepts NUMBER DIVISIBLE BY THE SQUARE ROOT OF TWENTY FIVE and NUMBER DIVISIBLE BY FIVE are distinct. Yet no set-theoretical conception (extensional or intensional one) can plausibly explain the difference between the two.

Two (other) kinds of conceptions can. One of their symptomatic examples was developed by Church. An inspection of his work reveals that his intensions were primitive entities of his system. Individuals are members of ι_0, the type of such primitive objects; concepts of individuals (i.e., certain Church's intensions) were collected in another atomic type, namely ι_1 (analogously up to ι_n). The trouble is that no function from ι_0 to ι_1 (etc.) explains *why* something is a concept of some entity. Within the system, we know only that a member of ι_1 is a concept of certain member of ι_0. To make the example more concrete, we can only know that FIVE is a concept of the number 5 and that TEN_MINUS_FIVE is a concept of 5 as well. We do not know, however, why this is so and how the two concepts are related to each other. We do not properly know that the second concept is compound, having the first concept as its own subconcept. For the only thing we know about a member of an atomic type is that it is self-identical and numerically distinct from any other

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[2] Let us remark that intensions of, e.g., Church are not functions from possible worlds. Consequently, Church could not distinguish between non-empirical and empirical concepts.

member of that type. Being primitive objects of the system, Church's intensions display no proper structuredness.

However, we need a conception which acknowledges that concept is a *structured entity*; we would like to call such conception *hyperintensional conception*. Concept would then be an entity composed, but not in a set-theoretic fashion, from its parts which are organized into a unique complex. This structured whole determines (in an algorithmic way) an object, which is a class or non-class (e.g., an intension in the possible-world sense). The features of concepts just discussed are incorporated in Materna's proposal; this is why we choose it.[3] Materna's explication of (the notion of) concept is formulated in the frame of Tichý's logical system, thus we have to expose it first.

3. Tichý's transparent intensional logic and meaning

Tichý presented basic ideas of TIL already in 1971 (*cf.* in Tichý 2004) and he soon started to write an extensive monograph on TIL (Tichý 1976) which has remained unpublished. There are several important differences of this early TIL from the rivalling (and well-known) system developed by Montague. Firstly, functions from possible worlds are represented by λ-terms abstracting over possible worlds. The treatment of modal (and temporal) parameter is quite explicit, which has many advantages. The second major difference consists in that the explication of meaning is not an intension (or simply an extension), but an objectual correlate, called *construction*, of the λ-term representing that intension.

As any other object, an intension can be arrived at, *constructed by* infinitely many constructions. Although such constructions are *equivalent* (for they construct one and the same object), they are *not identical*. This means that constructions have more *fine-grained criteria* of their individuation than intensions (or extensions).

In other words, Tichý proposed a *hyperintensional model of meaning*. Its need was found inevitable mainly in the 1980s and 1990s. There were proposed many arguments in its favour (*cf.*, e.g., Cresswell 1985), yet we are going to formulate only one of the most persuasive ones. According to intensional semantics, all true mathematical sentences have one and the same proposition (namely the analytically true proposition) as their

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[3] Arguably, George Bealer (Bealer 1982) proposed another viable proposal.

meaning. Nevertheless, an agent A can believe that $2 + 3 = 5$ without believing that $\sqrt{25} = 5$, thus the argument 'A believes that $2 + 3 = 5$', '$2 + 3 = \sqrt{25}$', 'Therefore, A believes that $\sqrt{25} = 5$' is incorrect. In spite of that, intensional semantics renders it as valid (for A is supposed to believe one and the same proposition). It follows that mere propositions are too course-grained to be objects of beliefs / meanings of sentences.[4] According to Tichý, objects of beliefs are constructions of propositions, i.e., abstract structured entities of an algorithmic nature which yield propositions. For a detailed argumentation in favour of this proposal, see mainly (Tichý 1988).

Now let us say more about TIL, though the lack of space leads us to a very simplified exposition (see thus Tichý 1988 for details).

TIL makes use of (Tichý's) *type theory*. It is quite general because it has an unspecified *objectual basis OB* (OB is a class of mutually non-overlapping collections of objects). In its simple type-theoretic part, common set-theoretical objects over OB are classified:

   i. Any member of *OB* is a type over *OB*;
  ii. If ζ, ξ_1, ..., ξ_m are (not necessarily distinct) types over *OB*, then $(\zeta\xi_1 \ldots \xi_m)$, which is a collection of all total and partial functions from ξ_1, ..., ξ_m into ζ, is a type over *OB*;
 iii. Nothing is a type over *OB* unless it so follows from i.–ii.

Objectual basis of TIL comprises ι (individuals), o (truth-values T and F), ω (possible worlds), and τ (time-moments/real numbers). *Intensions* are functions from the members of ω to total or partial chronologies of objects of type ξ; a chronology is a function of type $(\xi\tau)$. Briefly speaking, *intensions* are functions from ⟨possible world, time-moment⟩ couples; '$((\xi\tau)\omega)$' will be abbreviated to '$\xi_{\tau\omega}$'. *Propositions* are of type $o_{\tau\omega}$; *properties of individuals* are of type $(o\iota)_{\tau\omega}$; *individual offices* are of type $\iota_{\tau\omega}$; etc. Objects which are not intensions may be called *extensions*. For instance, classical unary ($\neg$) or binary ($\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$) truth-functions

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[4] Various attempts to solve the puzzles of hyperintensional contexts are easily criticisable by Church-like arguments raised originally against Carnap's notion of intensional isomorphism because the proposals expose the idea that an agent is related (or at least partly related) to the very expressions of natural or artificial language which expresses the object of belief. (A remarkable 'non-linguistic' approach has been developed by Cresswell; for its criticism *cf.* Tichý 2004.)

are of types (oo) and (ooo), respectively; quantifiers $(\forall^\xi, \exists^\xi)$ are of type $(o(o\xi))$; $=^\xi$ is of type $(o\xi\xi)$.[5]

Objects are constructible by *four kinds of constructions* (Tichý 1988, where also two other kinds are specified). They can be viewed as objectual correlates of λ-terms, namely of constants, variables (as letters), applications, and λ-abstractions. Let X be any object (a construction or non-construction) and C any construction; let v be any valuation (it is a field consisting of sequences of objects of given types):

1. *Trivialization* ^0X directly v-constructs X (i.e., ^0X takes X and leave it as it is).[6]
2. *Variable* x_k v-constructs the k-th member of the sequence of objects of a given type.
3. *Composition* $[CC_1 \dots C_m]$ v-constructs the value of the function constructed by C at the string of objects (i.e., the argument for that function) which are constructed by C_1, ..., C_m. If C or C_1 (etc.) does not v-construct such object(s) or the function is undefined for that argument, $[CC_1 \dots C_m]$ is v-*improper*, it does not v-construct anything at all.
4. *Closure* $\lambda x[\dots x \dots]$ v-constructs, in a nutshell, the function which takes particular values of x to the objects v-constructed by $[\dots x \dots]$ on the respective valuations like v.[7]

Realize clearly that constructions are not expressions — they are language-independent entities. For instance, the term '$\lambda n[[n\,^0\!\times n]^0\!-\!^03]$' is used to denote the construction $\lambda n[[n\,^0\!\times n]^0\!-\!^03]$. Note also that the term '$\lambda n[[n\,^0\!\times n]^0\!-\!^03]$' denotes the procedure-construction as such, not the function (which maps 1 to -2, 2 to 1, 3 to 6, etc.) constructed by $\lambda n[[n\,^0\!\times n]^0\!-\!^03]$; analogously, '$[^08^0\!\div^02]$' denotes the construction $[^08^0\!\div^02]$, not its result (the number 4).

For non-circularity constraints, Tichý introduced a sophisticated theory of types which combines in fact certain simple and ramified theory of types. Its definition (see Tichý 1988) has three parts: a. types of ('classical') set-theoretic objects (*cf.* the simple-type theoretic part above),

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[5] In our examples the proper type indications will be usually omitted.

[6] Trivializations of well-known mathematical or logical binary functions will be written in the infix manner.

[7] For the definition of *subconstructions* and *open/closed constructions*, see, e.g., (Tichý 1988).

b. types of constructions (some constructions are *first-order construc-tions*, belonging to the type $*_1$, other constructions are second-, third-, ..., n-order constructions, belonging thus to the type $*_2$, $*_3$, ..., $*_n$, respectively), c. types of functions from/to constructions.

As mentioned above, Tichý proposed that constructions are explica-tions of linguistic meanings. The *semantic scheme* of his semantic theory is this:

an expression E

     expresses (means) in L

a construction, which is the *meaning* (logical analysis) of E in L

     constructs

an intension / non-intension / nothing (*cf.* '3 ÷ 0'), which is the *denotatum* of E in L.

The value (if any) of an intension in a possible world W, moment of time T is the *referent* (in L) of an empirical expression E (such as 'dog', 'the king of France', 'It rains in London'); the denotatum and referent of a non-empirical expression are understood as identical.

Here is an example. Let w and t be variables v-constructing possible worlds and moments of times, respectively; 'C_{wt}' abbreviates '$[[Cw]t]$'. The sentence 'The king of France is bald' expresses (in English) the construction $\lambda w \lambda t [^0\mathrm{Bald}]_{wt} {}^0\mathrm{KF}_{wt}]$ (of course, $^0\mathrm{KF}$ is a simplification). The construction constructs the proposition "The king of France is bald" which is denoted by that sentence.

Tichý's semantic theory is easily capable to dissolve various puzzles created by 'intensional' and 'hyperintensional contexts'. To illustrate, when an agent A seeks the king of France, she is related to the individual office "the king of France" as such, not to its actual holder (if any); the respective sentence is analyzed as expressing $\lambda w \lambda t [^0\mathrm{Seek}_{wt} {}^0A {}^0\mathrm{KF}]$ (the inference to 'There is a individual which is the king of France' is blocked, which is right). When A calculates $2+3$ (or $3 \div 0$), A is not related to the result (if any) of that calculation, but to the calculation-construction as such ($\lambda w \lambda t [^0\mathrm{Calculate}_{wt} {}^0A {}^0[^03 {}^0\div {}^00]]$; note that $[^03 {}^0\div {}^00]$ is trivialized, i.e., taken as it is, the constructing of $[^03 {}^0\div {}^00]$ is 'stopped'). When A believes that the king of France is bald and $2 + 3 = 5$, A is related exclusively to what is expressed by 'The king of France is bald and $2 + 3 = 5$' (the analysis of the respective *de dicto* belief sentence is thus $\lambda w \lambda t [^0\mathrm{Believe}_{wt} {}^0A {}^0\lambda w \lambda t [[^0\mathrm{Bald}_{wt} {}^0\mathrm{KF}_{wt}]^0 \wedge [[^02 {}^0+ {}^03] {}^0 = {}^05]]])$, A is not related to the proposition which is denoted also by 'The king of France is bald and $\sqrt{25} = 5$' (the 'paradox of omniscience' is thus prevented).

For many applications of TIL, see works by Tichý or his followers (e.g., Duží, Jespersen, Materna 2010, Kuchyňka 2011, Raclavský 2009).

4. Materna's concepts

As concluded above, a plausible explication of concept should be a hyperintensional one. We have just presented elements of Tichý's explication of meaning which is hyperintensional. According to the line of explication of concepts followed by us, it is natural to maintain that every (or nearly every) meaning of a linguistic expression is a concept. On the basis of also other preferences formulated above, Tichý's logical framework and his explication of meaning is straightforwardly available for the explication of (the notion of) concept. This task has been performed by Materna; we can state here only key theses of his proposal (for more see Materna 2004, for the earlier version see his 1998).

The crucial claim of his theory is this. *Concept* is a *construction* which is (a) *closed*, (b) *α-normalized*, and (c) *η-normalized*. All three specifications (a)–(c) are supported by Materna's arguments for preferred intuitions.

Firstly consider:

(a) having $\lambda n[n \, ^0> \, ^07]$ and $[n \, ^0> \, ^07]$ as an example. Whereas the first construction corresponds to (and thus explicates) the intuitive concept NUMBER HIGHER THAN SEVEN, there seems to be no intuitive concept involving a free variable. Note also that what is constructed by $[n \, ^0> \, ^07]$ varies dependently on valuation and if $[n \, ^0> \, ^07]$ was a concept, it would be a concept of no particular object, which is also counterintuitive.

Now consider:

(b) It seems counterintuitive to maintain that the two non-identical constructions $\lambda n[n \, ^0> \, ^07]$ and $\lambda m[m \, ^0> \, ^07]$ correspond to two distinct intuitive concepts. Rather, they both seem to explicate one and the same concept. In other words, α-convertibility of constructions (i.e., collisionless replacement of λ-bound variables) is a feature peculiar to constructions, not to concepts. It seems thus adequate to choose α-normalized construction as the explication of concept.[8]

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[8] Similarly to other such conversions, α-normalization leads to a unique construction.

Very similarly:

(c) gives up η-convertibility of constructions. Thus not $\lambda nm[n \ ^0> m]$, but $^0>$ is chosen as the explication of the intuitive concept HIGHER THAN.

Materna treated many topics disputed in theories of concepts (such as emptiness of concepts, mentioning / using of concepts).

5. Materna's conceptual systems

Conceptual systems have been studied by Materna already in his book (Materna 1998). Nevertheless, he made some substantial changes in his (Materna 2004). When presenting his main theses, we start with (a version of) his earlier views, going then to his final proposal.

Conceptual system CS is a *class of concepts*. Materna suggests that each conceptual system has some (first-order) concepts as basic, thus we can identify SC_{CS}, the class of *simple concepts* (Materna: 'primitive') of that CS. The other concepts of CS, called by us *compound concepts*, are made exclusively from the members of SC_{CS} and variables by means of modes of forming constructions (*cf.* the kinds of constructions in Section 3). Here is an example: $SC_{CS} = \{^0Bald, {}^0Tom\}$, thus CS contains also $\lambda w \lambda t [^0Bald_{wt} {}^0Tom]$ (which is made from ^0Bald, ^0Tom, w and t by means of composing and closing) and even the concept ^{00}Tom (a concept of the concept ^0Tom; it is made from ^0Tom by trivializing). Since they are no principal restrictions on building of concepts from members of SC_{CS}, each CS is infinite. It also follows that every CS is uniquely determined by its SC_{CS}.

Before we expose Materna's last proposal, we briefly remark that conceptual systems can be compared or classified due to various criteria (see Materna's books for details). For example, CS is an *empirical conceptual system* if at least one of its concepts determines a (non-constant) intension; it is *non-empirical* otherwise. It is also clear that conceptual systems can be in various set-theoretical relations, most notably "being included in". Conceptual systems cannot be sensibly compared due to their cardinality (which is one and the same for all CSs), yet they can be unequal as regards the number of objects determined by them. The class of all objects determined by the members of CS is the *area* of that CS. Comparing two conceptual systems with respect to their areas leads

to important findings concerning their comprehensiveness. Of course, all these classifications have a number of fruitful applications not only in philosophy.

Materna's late proposal preserves most of his elaborations of the earlier one but it involves an important ramification of the very concept of conceptual system. It is motivated by examples of conceptual systems like the following two. Consider a conceptual system CS_1 within which mathematicians count numbers, more concretely real numbers. Now contrast it with a conceptual system CS_2 of arithmetic of natural numbers. Both systems operate over distinct systems of functions. The division function in the area of CS_1 is undefined for pairs $\langle n, 0 \rangle$ (where n is any real number). On the other hand, the division function in the area of CS_2 is not a partial function because the number 0 (and thus any pair with it) is missing in the area of CS_2. The phenomenon of two or more such dissimilar systems has occurred in the development of mathematics many times and even in the present times many such distinct conceptual systems are in use (they all coexist in the realm of abstract entities).

Materna's final explication of the notion of conceptual system characterizes the just discussed specificity of conceptual systems as dependent on specificity of objectual basis (and entities over it). According to Tichý, if a particular objectual basis OB is given, there is a system of functions over OB and then also a system of constructions (concepts among them) of all such objects over OB. Realize that each OB is individuated not only by types (such as ι) included in it, but also by the members of these types (thus whether Tom is, or is not, a member of ι of a given OB affects whether Tom can be a possible argument or value of some function over OB).

Conceptual system is then characterized as a two-dimensional entity. In its first dimension, all entities (including constructions) over OB are given. In its second dimension, only some constructions (namely some concepts) over OB are accepted. The second dimension amounts roughly to conceptual system in Materna's earlier sense; the first dimension adds the variability (or specificity) in the very domain of objects. *Conceptual system* CS is thus a quintuple (a slight adaptation of Materna 2004, 78):

$$\langle OB_{CS}, \text{defTT}, \text{defKC}, \text{defC}, SC_{CS} \rangle,$$

where OB_{CS} is a particular objectual basis, defTT is the definition of Tichý's type theory, defKC is the definition of kinds of constructions,

defC is Materna's definition of concept, and SC$_{\mathrm{CS}}$ is a particular class of simple concepts of that CS.[9]

6. Tichý's system of deduction

We can easily see that Materna's conceptual systems do not involve deduction, they are rather fields for deduction. On the other hand, there is a (general) system of deduction for TIL (and not only TIL) developed by Tichý in his (Tichý 1976) and then in his three papers from 1980s (see Tichý 2004). The aim of the present section is to introduce, in a bit simplifying manner, its key notions.

Match M is an ordered couple $X : C$, where C is a simple or compound construction and X is a trivialization of an object (of a given type ξ) or a variable for the type ξ.[10] A valuation v is said to *satisfy* M if C v-constructs one and the same object as X. *Sequent $\Phi \Rightarrow M$* consists of a finite set of matches Φ and a match M. $\Phi \Rightarrow M$ is *valid* if every valuation which satisfies all members of Φ satisfies also M.[11] *Rule of derivation (rule of inference)*, $\Phi_1 \Rightarrow M_1; \ldots; \Phi_n \Rightarrow M_n \models \Phi \Rightarrow M$, is a validity preserving operation on sequents — the sequent $\Phi \Rightarrow M$ is valid if all $\Phi_1 \Rightarrow M_1, \ldots, \Phi_n \Rightarrow M_n$ are valid. A sequent $\Phi \Rightarrow M$ is said to be *derivable* from a set of sequents *according* to a given *derivation rule*. A finite string of sequents is said to be a *derivation with respect to a set of derivation rules R* (writing it $\vdash_R \Phi \Rightarrow M$) if every item of this string, i.e., every *step* of the derivation, is derivable from the earlier steps according to some derivation rule from R.

Let us bring out a very important feature of some derivation rules, namely the fact that they *exhibit properties of constructions and objects constructed by them*. This enlightens some deep connections among objects, concepts, conceptual systems, and derivation systems (we introduce below). First recall that every construction is specified by two things: i. the object constructed by it and ii. how it constructs that object (by means of which subconstructions). Now realize that deriva-

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[9] DefKC is in fact redundant here, constructions over OB are provided already by defTT.

[10] Two notes. Tichý viewed X simply as an object, not as the trivialization of that object. Tichý allows also matches whose first component is missing.

[11] Note that sequents concerning constructions of truth-values/propositions are only special cases; it follows that Tichý system rather extends the field for deduction.

tion rules of form $\models x : C_1 \Leftrightarrow x : C_2$ (i.e., $\models \{x : C_1\} \Rightarrow x : C_2$ and $\models \{x : C_2\} \Rightarrow x : C_1$) show which object is constructed by the construction C_1 (or C_2). Some such derivation rules have one of C_1 and C_2 very simple. Consider, for instance, $\models f : \lambda pq[p\,{}^0\!\vee q] \Leftrightarrow f : \lambda pq[[{}^0\neg p]\,{}^0\!\rightarrow q]$ where p and q range over truth-values and f ranges over binary truth-functions; the construction $\lambda pq[p\,{}^0\!\vee q]$ is η-reducible to ${}^0\vee$. We view *definitions* as rules of this sort.[12] The definition just mentioned says that $\lambda pq[p\,{}^0\!\vee q]$ is equivalent to $\lambda pq[[{}^0\neg p]^0 \rightarrow q]$, thus it enlightens exactly which object (which truth-function) is constructed by ${}^0\vee$. But there are also other derivation rules displaying properties of objects. For instance, one property of implication (viz. that it maps $\langle T, T\rangle$ to T) is exhibited by the rule $\Phi \cup \{{}^0T : p\} \Rightarrow {}^0T : q \models \Phi \Rightarrow {}^0T : [p\,{}^0\!\rightarrow q]$.

7. Derivation systems

Putting it simply, derivation system is a couple $\langle CC, R\rangle$ whose first component CC is a class of constructions and its second component R is a class of derivation rules operating on CC. Analogously to conceptual systems, a more accurate proposal is two-dimensional. Thus *derivation system* DS is a quintuple:

$$\langle OB_{DS}, \text{defTT}, PC_{DS}, Q_{DS}, R_{DS}\rangle,$$

where:

– OB_{DS} is a particular objectual basis;
– defTT is the definition of Tichý's theory of types;
– PC_{DS} is a particular class of trivializations, a subclass of the class AC of all constructions over OB_{DS};
– Q_{DS} is a particular class of qualities of constructions from AC (i.e., properties such as "having the order k", "having c as its subconstruction", "having the complexity-rank r"), the 'conjunction' of all these qualities characterizes the class CR_{DS}, which is that subclass of AC which contains all constructions occurring in members of R_{DS};
– R_{DS} is a particular class of derivation rules whereas sequents involved in these rules are made from matches of form $X : C$ where X is a variable or a member of PC_{DS} and C is a member of CR_{DS}.

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[12] It preserves many intuitions concerning definitions but we cannot discuss the topic here.

It is readily seen that, unlike conceptual systems, derivation systems are characterized especially by their class of derivation rules. To put it differently, there are various derivation systems for one and the same conceptual system (whereas the derivation systems differ in their Rs).

Let us provide at least one illustration how the concept of derivation system can be employed (the brevity of space does not enable us to offer more). Consider the derivation system DSF of the standard axiomatic system ZFC of the set theory. $\text{OB}_{\text{DSF}} = \{o, \sigma\}$, where σ is the collection of sets. PC_{DSF} contains trivializations of truth-values, logical functions, and the function $\in$ of the type $(o\sigma\sigma)$. The conjunction of Q_{DSF} admits only (first-order) constructions whose subconstructions are not variables ranging over types other than σ, $(o\sigma)$, and $(o\sigma\sigma)$. In R_{DSF} there are derivation rules corresponding to the axioms of ZFC. These rules specify the relation $\in$; they expresses, among others, that all sets are well-founded — and, consequently, that no set has the relation $\in$ to itself. Some authors (e.g., Aczel 1988) endeavour to propose variants of ZFC violating the rule of well-foundedness. The concept of derivation system makes clear (imagine the corresponding derivation rules) that what they really do is specify some other relation than $\in$. If we replace their symbols '$\in$', '$\{$', '$\}$' by '$\in^*$', '$\{^*$', '$^*\}$', we will readily see that '$\{^*\Omega_1, \Omega_2, \ldots {}^*\}$' means nothing other than '$\iota x(\forall y((y \in \{\Omega_1, \Omega_2, \ldots\}) \to (y\in^* x))$', i.e., 'the only set x such that all elements of $\{\Omega_1, \Omega_2, \ldots\}$ have the relation $\in^*$ to x'.

The concept of derivation system as a framework of reasoning with concepts seems to be intuitively known enough, thus it seems to need no special justification. Nevertheless, one may raise the following objection: a conceptual system collects some concepts of certain objects; since the very identity of those concepts is (partly) depending on those objects, their properties are 'supervening' on the properties of those objects; derivation systems only make these properties explicit (by means of derivation rules, *cf.* the second part of Section 6); thus the very concept of derivation system is dispensable (why should we understand derivation systems as criteria for distinguishing between various systems of concepts-constructions?).

Before we try to resist such objection, recall that any construction of the truth-value T (or, e.g., conjunction $\wedge$) has properties which are 'supervening' on the properties of T (or $\wedge$). The very reason of 'interpreted axiomatization', i.e., the formulation of a derivation system for (say) ZFC, is to make the investigation of such constructions quite systematic

and rigorous.[13] Without derivation rules, an equivalence (for example) of two constructions would be rather a matter of intuitive insights — which are often unclear and easily fallible.

Derivation systems are thus important for the study of concepts (and other constructions) especially from the *methodological point of view*. They enable us to *precisely specify conceptual systems* and to *prove claims about them*. Therefore, they yield rigorous and controllable results. Implicit relations between constructions (or objects) are made explicit, which is inevitable if we need to move from our intuitive use of concepts towards their conscious reflection.[14]

A final important comparison: Materna's conceptual systems can be seen as special cases of derivation systems. Their PCs comprise selected simple closed constructions (viz. first-order trivializations of objects), their Qs contain properties "having a member of PC or a variable as its subconstruction", "being closed and in α- and η-normal form", and their Rs are just $\varnothing$. Our concept of derivation system is thus demonstrably more general than Materna's concept of conceptual system. It is also more fruitful because it is more useful for the formulation of general laws.

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[13] Realize that *axiomatic system* is a linguistic entity which is amenable to this or that interpretation (nothing of this sort is true about any derivation system).

[14] An example. The very first introduction of the notion of derivation system (as a couple $\langle PC_{DS}, R_{DS} \rangle$) comes from the paper solving a notorious puzzle about verisimilitude (Raclavský 2007). According to that proposal, definitions (as derivation rules) show which simple concepts of some conceptual system are *primary* and which simple concepts are *derived*, i.e., depending on the primary concepts (which shows which intensions are primary and which are not). This distinction helps to solve the trouble revolving around one of the important notions of our conceptual scheme, that of likeness of scientific theories to truth.

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Jiří Raclavský and Petr Kuchyňka
Faculty of Arts, Masaryk University
Brno, the Czech Republic
raclavsky@phil.muni.cz, 7765@mail.muni.cz