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UNGROUNDED CAUSAL CHAINS AND BEGINNINGLESS TIME

Abstract. We use two logical resources, namely, the notion of recursively defined function and the Benardete-Yablo paradox, together with some inherent features of causality and time, as usually conceived, to derive two results: that no ungrounded causal chain exists and that time has a beginning.

Keywords: recursively defined function, Benardete's paradox, Yablo's paradox, causality, time, irreversibility.

1. Introduction

The logico-mathematical theory of recursive functions relies heavily on the notion of *recursively defined function*. I will argue that this notion proves useful to approach some classical ontological problems by providing some of the terms in which a precise formulation of those problems is possible. One such problem is whether an ungrounded causal chain is possible; another is whether beginningless time is possible. My claim here will be that the notion of recursively defined function, together with some ontological features of causation and time, should make it possible to prove two propositions, namely, that there is no ungrounded causal chain and that there is no beginningless time, at least if causality and time are *normal* in a sense to be defined. The fundamental idea behind the second result stems from the shared structure of Benardete's and Yablo's paradoxes [2, 8].

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The general enterprise might seem an odd one, since we do not usually prove *ontological* theorems. But consider that just like we prove theorems in Geometry from axioms asserting evident features of space, it should be possible to prove theorems reasoning from evident traits of causality and time.

Of causality, as usually conceived, the following holds:

(C1) *All the items in a causal chain, except the first one, if it exists, get completely determined on the basis of other items that are anterior in the causal chain.*

By a *chain* we understand here not just a *totally ordered set* but a *sequence*; as a sequence, it can be finite, singly-infinite or doubly-infinite, i.e. it can have the order type of an initial segment of N, the order type of N or the order type of Z.

So, causality is usually conceived of as a step-by-step process in which the *nth* item (i.e. the *nth causing or caused event*), except the first if it exists, is a function of what the previous items are, this function being an expression of the causal laws that rule the chain.

Hereafter a *normal causal chain* is a causal chain to which (C1) applies. Proving assertions about normal causal chains requires proofs from axioms expressing ontological traits of the normal causal relation. (C1) is such an axiom. (C1) will allow us to prove that no normal causal chain is ungrounded, that is, that every normal causal chain contains a first cause.

Of time, as usually conceived, the following is true:

(T1) *At any time t, the events in the past of t are irreversibly determinate.*

That is, the irreversible determinateness of the past is essential to our usual conception of time.

Certainly, the determinateness of the past is relative, because *past* and *future* are always understood as relative to some instant or event. Any lapse of time T is fixed but only relative to any time in the future of T. T is never past in an absolute, atemporal way; hence the irreversible determinateness of the past affects a time stretch only as seen from those temporal points in the past of which it lies. Usually, in mathematics, when something is definite or determinate, it is so regardless of the temporal location considered. Here we have the novelty that we must consider a kind of definiteness or determinateness relative to some temporal instant or instants. However, this mathematically unusual relativity of determinateness should not be a hindrance when it comes to prove facts about time; on the contrary, since it is inherent to the very nature of time, it should be part of the premises.

A proof concerning the nature of time must be a proof from axioms capturing ontological features of time, in exactly the same way as geometrical proofs rely on axioms describing the nature of space. (T1) captures a feature of time out of which the following axiom can be asserted:

(T2) *At any time t the past of t is irreversibly determinate in such a way that, if the state of the world at t depends only on the states of the world prior to t, then the state of the world at t is determinate.*

That is, if the shape of the present is a function of past events, then, as the past is irreversibly determinate in the present according to (T1), then the shape of the present must be also determinate.

The reason why (T2) follows from (T1) is this: *when* the determinateness of the state of the world at all times anterior to x is required for the determinateness of the state of the world at x, i.e. precisely at x, the state of the world at all times anterior to x is *already* determinate as required. So, the conditions necessary for the state of the world **at** x to be determinate are all given in due time. The determinateness of the past of x at x implies the determinateness of the past of x **to the effect of** the determination of x. To see this, we can imagine the determination of the state of the world at x as the action of an agent who has been given precise instructions to shape the world at x as a function of the states of the world in times previous to x. Assume the agency is instantaneous and the agent has the required knowledge of the past. So the agent must act at x and he can do so in fact, because at x it is fixed what the state of the world was at any time anterior to x; it is fixed because the past of x is irreversibly determinate at x. This agent is the personification of a causal function ruling the becoming of the world.

The determinateness of the past is essentially a consequence of the irreversibility of time. An asymmetry is to be found in the lapse of time, as we usually conceive it: since the past flows into the future but not vice versa, the past influences the future but not vice versa. The direction of time settles the direction of causation. This irreversibility prevents circularity: if not only the past acted upon the future but also the future influenced the past, a vicious circle could ensue. Note in particular that, even if the future of x is deterministically decided from all time, it can be indeterminate for an agent acting at x because the future of x can be dependent upon this agent's agency, so that if the future were determinate for that agent, that agent could decide his own behavior on the basis of the future, which in turn could depend on his agency, so possibly bringing forth again a vicious circle.

Unlike the future, the past of x is objectively fixed for any agent at x, provided that the flow of time is irreversible, and this is why any well-defined causal law determining the future on the basis of the past, must succeed if time flows only in the usual direction. However, we will find that, if time has no beginning, there can be causal laws determining any instant on the basis of its past that fail; this is what Benardete's and Yablo's paradoxes contribute. From the possibility of such causal laws, given a beginningless time, the conclusion can be drawn that an irreversible time must have a beginning.

Hereafter a *normal time* is a time for which (T2) holds.

Let me add a conceptual precision. It is evident that time cannot begin in time: the existence of a point in time at which time begins makes no sense; hence, the claim that time has a beginning must be understood as meaning that time has a first instant or event.

2. Impossibility of ungrounded normal causal chains

A relation R is said to be well-founded if there is no backward infinite chain xRy, zRx, vRz, wRv, ... We say that a causal chain is ungrounded iff the relation x *causes* y restricted to the items in the chain is not well-founded; that is, the chain is ungrounded iff the chain contains no first cause.

Causal laws in normal causality have the form of *recursively defined* functions, that is, they can be conceived of as functions assigning values to the terms in a sequence, each of them on the basis of the values assigned to anterior terms, except perhaps for some base cases that are independently defined. If base cases exist in a recursively defined function, the function is said to be *grounded*. The classical example of a grounded recursively defined function is the function generating the Fibonacci series, which can be described as follows:

The number 1
The number 1
The sum of the two preceding numbers
The sum of the two preceding numbers
The sum of the two preceding numbers
⋮

It is clear that ungrounded recursively defined functions fail to determine. Consider the case—proposed by Goldstein [3, p. 872]—of an ungrounded Fibonacci series:

⋮
The sum of the two preceding numbers
The sum of the two preceding numbers
The sum of the two preceding numbers
⋮

The chain of phrases above has the order type of the integers; if it were to define a function f, f would be from the set Z of the integers to the empty set, which means that the function would be defined for no argument. Since no ungrounded recursively defined function succeeds in defining or determining values for its arguments and any normal causal chain is a recursively defined function that succeeds in determining a chain of events, no normal causal chain is ungrounded.

To frame this all in a reasonably formal way, we define *recursively defined function.*

DEFINITION 1. The member $f\colon S \to V$ of a quadruple $\langle f, S, V, R\rangle$ is a recursively defined function iff:

1. S is a set of items and V is a set of values for the members of S.
2. S is ordered by the relation R (the *anteriority relation*) into a (possibly infinite, either singly or doubly) sequence.
3. For all x in S either:

   i. $f(x)$ is one of the base cases that ground the function, i.e. there is a finite initial chain G in $\langle S, R\rangle$ such that

   $$x \in G \;\&\; \forall y\,(y \in G \to f(y) \text{ is independently defined})\,,$$

   or

ii. $f(x)$ is defined on the basis of the value of R-anterior arguments, i.e. there is a function g and a subset T of S such that:

$$\forall y\,(y \in T \to yRx)\ \&\ f(x) = g(f\lceil T),$$

where $f\lceil T$ is the function f restricted to T, that is, the set of ordered pairs $\langle u, v\rangle$ such that $\langle u, v\rangle \in f$ and $u \in T$.

From this definition and the normality axiom (C1) we can prove:

THEOREM 1. *No ungrounded recursively defined function is total.*

PROOF. In the absence of initial arguments whose values are independently defined, the definition of a recursively defined function only permits establishing conditional statements of the form:

$$f(x') = y'\ \ \&\ \ f(x'') = y''\ \ \&\ \ \cdots \to f(x) = y\,,$$

where $x'Rx$, $x''Rx$, ...

No categorical statement of the form $f(x) = y$ follows from such definitions. Hence the function is defined for no argument. ⊣

THEOREM 2. *No normal causal chain is ungrounded.*

PROOF. Any causal chain to which (C1) applies realizes a *causal law* equivalent to a recursively defined total function: just take S to be the set of items in the causal chain, take R to be the causal order in S, let c_0 be the first item in S, if there is such, and set $V = S - \{c_0\}$; clearly, this makes the function total; besides, (C1) renders it recursively defined. Now, by Theorem 2, no such function is ungrounded. ⊣

Note that Theorem 2 states that any normal causal law must act upon some initial and independently given items in order to effectively determine the events in a causal chain. Indeed, any normal causal law is on its own an ungrounded recursively defined function.

I wish to finish this section with a reference to the thesis Rowe called the *Hume-Edwards Principle* (HEP, hereafter). In Rowe's words [6, p. 153] HEP states:

> *If the existence of every member of a set is explained, the existence of that set is thereby explained.*

This parallels the famous passage of Hume's *Dialogue* [4, p. 59] where Cleanthes argues for the possibility of an ungrounded causal chain:

> *In such a chain, too, or succession of objects, each part is caused by that which preceded it, and causes that which succeeds it. Where then is the difficulty? But the WHOLE, you say, wants a cause. I answer, that the uniting of these parts into a whole* [. . .] *is performed merely by an arbitrary act of the mind and has no influence on the nature of things.*

HEP, in Rowe's formulation, seems rather intuitive, at least whenever a totality is ontologically exhausted by its individual components. But, as I see it, this principle need not conflict with our result here, because it does not entail the possibility of an ungrounded normal causal chain: the principle may well apply to any possible causal chain but, as Theorem 1 shows, an ungrounded normal causal chain is impossible.

3. Benardete's paradox and beginningless time

Aquinas, though he believed that the existence of a first cause could be proved, rejected the possibility of proving the existence of a beginning of the universe. In [1], *Summa Theologica* I, q46, a2, he wrote:

> *Respondeo dicendum quod mundum non semper fuisse, sola fide tenetur, et demonstrative probari non potest* [. . .].

In my translation:

> *I answer that the fact that the world has not always existed can be known only by faith, and can be proved by no demonstration.*

I will try to show that the theory of recursively defined functions and (T2) prove Aquinas wrong.

Nowadays, most physicists believe that empirical evidence and our best physical theories combine to show that time had a beginning some 13.700 billion years ago. So, the result that time had a beginning is in itself perhaps not as interesting as the possibility to obtain it by non empirical means. If I am right, Benardete's and Yablo's paradoxes offer this possibility.

Shackel [7] has shown that Benardete's paradox and Yablo's paradox have a common structure. In [5] I have used that structure to argue for the paradoxicality of beginningless time. Consider the following version of Benardete's paradox.

Assume the existence of a beginningless time. Assume that on each day of that time a gong peal sounds. Assume that each day a hearer is present, the same all along, and the gong peal is always so loud that it would deafen the hearer if and only if the hearer has not yet been deafened by some previous gong peal; this implies that the hearer can only be deaf as an effect of a heard gong peal. We can show that on any day D the hearer is already deaf, for suppose he weren't; then he would also be not deaf the day before D and then he would have been deafened by the gong peal sound of the day before D so that he would be deaf on D, which is a contradiction. Now, if the hearer is already deaf on any day, he is deafened by the gong peal of no day. So, he is deaf, he can only be deaf as a consequence of his hearing a gong peal, and he has heard no gong peal.

This contradiction shows that the whole situation is impossible. But, on the one hand, it seems obvious that if a beginningless time is possible, then the whole situation is and, on the other hand, the contradiction disappears as soon as we turn to a time with a first day; this should render the impossibility of a beginningless time obvious.

Indeed, there seems to be no impossibility in a gong peal sounding on every day of any existing succession of days, no impossibility in a hearer being present on every day. And no impossibility in the following law determining whether any particular gong peal deafens the hearer:

(L) *For any gong peal x, x deafens the hearer if and only if no anterior gong peal has deafened him.*

Certainly, (L) seems able to determine, for any gong peal x, whether x deafens the hearer or not, because when x sounds it is irreversibly determinate whether some previous peal has deafened the hearer or none has; since this is so due to the very nature of time, as depicted in (T2), this version of Benardete's paradox is a paradox of normal beginningless time.

This reveals that the paradoxical issue about Benardete's example is *determination*: the question is whether (L) determines for any particular gong peal x whether x deafens the hearer or it doesn't.

On the one hand, for any gong peal x, (L) determines whether x deafens the hearer or it doesn't, because, according to (L), this depends only upon whether some prior gong peal has done it and this is in turn determinate when x sounds. On the other hand, for no gong peal x is it determinate on the basis of (L) whether x deafens the hearer or not. For take an arbitrary x and assume it deafens the hearer; then the hearer was not yet deaf the day before; then the hearer was deafened by the gong peal of the day before; hence, x does not deafen the hearer and we've got a contradiction. Since x was arbitrary, no gong peal deafens the hearer. Assume now that x does not deafen the hearer; then he was deafened by some previous gong peal, which is impossible since we have established that no gong peal deafens him.

It is important to note that the paradox would not even show up if (L) were ill-defined. Suppose, for instance, that instead of (L) we had:

(L′) *For any gong peal x, x deafens the hearer if and only if he is handsome and no anterior gong peal has deafened him.*

Since being handsome is vague enough, we would reject (L′) as simply ill-defined. Or suppose that the days are of two alternating kinds, even and odd, and consider:

(L″) *For any gong peal x, if x is even, then x deafens the hearer if and only if the immediately anterior gong peal has not deafened him, and if x is odd, then x deafens the hearer if and only if the immediately posterior gong peal will not deafen him.*

We would simply reject (L″) as circular. Or consider:

(L‴) *For any gong peal x, x deafens the hearer if and only if no posterior gong peal will deafen him.*

(L‴) is unable to determine whether x deafens the hearer because it leads to an infinite regress, and that's all.

The problem with (L) is that it is necessarily well-defined, because when x sounds, it is already determinate whether some prior gong peal has deafened the hearer. So (L) necessarily determines and at the same time it cannot determine. If it were not the case that the facts involved in (L) are determinate in such a way that (L) must be able to determine in each case, we would simply dismiss (L) as contradictory.

The ultimate root of the paradox is the fact that (L) must at the same time be and not be able to determine. This contradiction should serve as a *reductio* for the possibility of a beginningless time. In the next section we put it in a more formal way.

4. The impossibility of a beginningless normal time

DEFINITION 2. A recursively defined function (see Definition 1) f is *time-like* iff for each item x in S, the value of any item in S that is R-anterior to x is determinate to the effect of the determination of the value of x:

$$\forall x \Big(\exists T \exists g (T \subseteq S \ \& \ \forall y (y \in T \to yRx) \ \& \ f(x) = g(f \lceil T)) \\ \to f(x) \text{ is defined} \Big).$$

THEOREM 3. *Time-like recursively defined functions are total.*

PROOF. It follows from Definitions 1 and 2. Take an arbitrary x from S; according to Definition 1, either there is a finite initial chain G of $\langle S, R\rangle$ such that $x \in G$ and $f(x)$ is independently defined or

$$\exists T \, \exists g \ \ (f(x) = g(f \lceil T))$$

and $f(x)$ is defined, according to Definition 2. ⊣

THEOREM 4. *No time-like recursively defined function f is ungrounded.*

PROOF. f is total (Theorem 3); since it is also recursively defined, it follows from Theorem 1 that it is not ungrounded. ⊣

THEOREM 5. *No normal time is beginningless.*

PROOF. If a normal time were beginningless, there would be an ungrounded time-like recursively defined function. Assume time is normal and beginningless and consider the following quadruple $\langle f, S, V, R\rangle$. Let $\langle S, R\rangle$ be the chain of instants in time ordered by the chronological anteriority relation R, let $V = \{0, 1\}$ and let f assign 1 to x, for all x, if and only if all instants in S that are anterior to x (if some exist) are assigned 0 by f:

$$\forall x \in S \, (f(x) = 1 \longleftrightarrow \forall y \in S (yRx \to f(y) = 0))$$

It is evident that f is recursively defined (in a Benardete-Yabloesque way, in fact). It follows from (T2) and Definition 2 that f is time-like. Since time has no first instant, f is ungrounded. As there is no ungrounded time-like recursively defined function (Theorem 4), time, if normal, is not beginningless. ⊣

If time and causality are *normal*, as I contend we have intuitive evidence to think, theorems 2 and 5 are about time and causality as they frame the world, that is, they are genuine ontological theorems provable from fundamental ontological principles.

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