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EITHER EPISTEMICISM OR LOGIC

Abstract. Epistemicism seems to be the most dominating approach to vagueness in the recent twenty years. In the logical and philosophical tradition, e.g. Peirce, vagueness does not depend on human knowledge. Epistemicists deny this fact and contend that vagueness is merely the result of our imperfect mind, our dearth of knowledge, sort of phantom, finally, that it simply does not exist. In my opinion, such a stance not only excludes vagueness comprehended in terms of human knowledge, but which is worse, stems from spurious logical arguments. The part of arguments called Sorensen's Arguments or even Proofs were the subject of my analysis in the book *Paradoksy* (2006; in Polish) and in the paper "Epistemicism and Roy Sorensen Arguments" published in the *Bulletin of the Section of Logic* (2007). Here I shall only briefly refer to these works and focus mainly on the arguments launched by Tymothy Williamson. One of them is to uncover why we are not able to recognize the alleged sharp boundary between positive and negative extensions of any vague predicates. Williamson's reasoning is based on his *margin for error principle*. Another argumentation of Williamson aims at the refutation of the principle *I know that I know*. It should be emphasized that all the aforementioned arguments are fundamental for epistemicism and all of them are fallacious because of either formal or false-premise fallacy. There is the circumstance that we cannot deem epistemicism logical. Finally, we show that within the epistemic frame the following thesis is valid: if what epistemicism states is the case, then what epistemicism states is not the case. This immediately implies (by '$(p \to \neg p) \to \neg p$') that it is not the case what epistemicism states. So, either epistemicism or logic.

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1. Introduction: notorious *petitio principi* in Sorensen's arguments

In [2] I have presented the logical analysis of the four most famous Sorensen's arguments for the non-existence of vagueness. Let us shortly recapitulate the main points.

One of the eminent Sorensen's arguments, the argument of **Grey Sphere**, is to show that a grey sphere (vague by assumption as grey fades gradually into white) has sharp boundaries:

> 'My thesis is that all other finite, material objects must have boundaries. Moreover, those boundaries must be *sharp*. The basis of my claim is a topological thought experiment. Visualize a "vague" grey sphere that fades into a white background. A spherical white cavity begins to grow from the centre. Gradually, the cavity grows so large that it destroys the grey sphere. The destruction of the sphere seems gradual—as one would expect for a "vague object". Yet the sphere's boundary disappears suddenly. Perhaps the sphere's outer *layer* erodes bit by bit, but a boundary has no width, and therefore, no relevant parts to lose. The boundary depends on the sphere and so cannot outlast it. Therefore, the sphere must be destroyed as instantaneously as its boundary. Since the sphere is sensitive to arbitrarily small changes, it must have a sharp boundary. [7, p. 275]

It is a noteworthy feature of the presented reasoning that it is not exempt from a tacit assumption which, when made explicit, says that ... the boundaries of the grey sphere are sharp. Indeed, since there is only one ideal white colour[1], and everything that differs is not-white, "officially" a vague grey sphere has sharp boundaries; consequently, the clearly delineated white area has to possess a sharp complement, which is not a matter of observation but of topology. Plainly, the assumption (that grey sphere is vague) is merely an illusion, while the demonstrable premise is exactly the opposite. Everyone who accepts:

1. white area has sharp boundaries, and
2. everything beyond the white area is a grey sphere,

has to accept

3. the grey sphere has sharp boundaries.

[1] Let us assume that there are different hues of white color in the mental experiment. Then, no one could know in which moment the grey sphere disappears—gray would fade gradually into different shades of white. It means that the grey sphere will disappear instantaneously only when there is one shade of white.

Evidently, the grey sphere is clear-cut independently from its graphic filling and from our imagination: It does not matter whether it is something grey fading gradually into white or something composed of stripes of different hues of grey, or anything else. In fact, it is sharp thanks to 1 and 2. Basing on both assumptions and showing that the grey sphere is sharp is essentially the error of *petitio principi*. *Petitio principi* is an integral element of the Sorensen's reasoning because of the topological character of this mental experiment. Therefore, the Grey Sphere argument shows nothing even if white and grey colours are replaced by red, green (or any other) colour. In all possible versions of the reasoning, the names of colours used in the narration will not represent any real colours but they will have mathematical meaning given by the topological assumption. In mathematical reasoning the material shape of the word has nothing to do with its meaning. The meaning of words is defined by accepted premises.

The mathematical idea standing behind the Grey Sphere argumentation is also employed for the **Sharp Boundaries For Blobs**. On the ground of topology, but using words of the material world, Sorensen argues that the blob (e.g. of water) is a sharp (not vague) material object:

1. The blob must have a boundary.
2. If a spherical cavity grows from the centre of the blob, the blob's outer boundary is completely unaffected as long as some of the blob remains.
3. As soon as nothing remains of the blob, the blob's boundary goes out of existence all at once.
4. Lemma: The blob's boundary goes out of existence instantaneously.
5. Conclusion: The blob goes out of existence instantaneously.

[7, p. 276]

Obviously, the material blob is a topological object comprehended as all but its sharply defined complement. It is difficult to say why a spherical cavity (growing from the centre of the blob) has been chosen for consideration and not the following:

a. point coming from the centre of the blob in one moment passes the sharp boundary of the blob; or

b. the blob has a sharp boundary.

Both sentences express the same thought about spherical cavity growing from the centre of the blob. The assumption and the conclusion of the reasoning are the same: the blob has a sharp boundary. Thus, there is an error of *petito principi*, ... again.

The reasoning is methodologically mysterious and provokes the questions: how is it possible to state an empirical fact by topological thinking? Is it really possible to discover some physical properties of water by the semantic analysis of the word 'water'? Any argumentation concerning the character of the material blob should be based on molecular physics rather than purely mathematical speculation.

A **Thousand Clones** is also a very well-known Sorensen's argument. Let us fancy that there are two men: Mr. Original and his clone Mr. Copy both growing at exactly the same rate, and both undergoing the same stages of development according to the *Principle "Earlier is First"*: If an item undergoes some finite process of change, then had it started earlier and changed at just the same rate, it would have finished sooner. Mr. Copy is an ideal copy of Mr. Original, and also his process of growing is a copy of the process of growing of Mr. Original.[2] Thus Sorensen assumes both: that one is an ideal copy of the other and that the development of one is the ideal copy of the development of the other. The latter assumption is dubious. The case of these two men makes Sorensen state plainly that since Mr. Original must stop growing as first, he will stop growing at some moment t becoming a tall man. But at the same moment t Mr. Copy still grows so he is not a tall man yet. Hence the line separating the tall from not tall lies somewhere in between Mr. Original and Mr. Copy's heights at t. Since Mr. Copy could have been copied (cloned) at any moment after the birth of Mr. Original, it could be a second, the height of Mr. Copy could be very close to that of Mr. Original. This means that the boundary between tall and not tall is sharp, as it lies within an arbitrarily small numerical interval.

Naturally, we encounter here the *petitio principi* fallacy, known to us from earlier Sorensen's arguments, but not only this. At first Sorensen assumes that growing of both men stops at once (!) at some moment t. Surely, growing is understood topologically and thus is measurable in micrometers as everything that is not growth. Moreover, Sorensen treats the end of the process of growing as precisely determined, neglecting the fact that the actual height depends on many factors including time of the day, mood, etc. Nevertheless, for epistemicicts the world is sharp and so all these factors

[2] The second assumption seems to be false in the light of modern biology.

must be taken into consideration. (And all this in spite of the fact that Sorensen speaks about "arbitrarily small differences" in the height of both men.) So the height of both men is to be determined with mathematical and not empirical precision, and in the exactly same conditions. Such a strictly mathematical approach to the reality, in fact neglecting its character, is erroneous , since the reality, contrary to the epistemicits' dreams, cannot be described with arbitrarily small differences. But it is neither the least nor the last error here.

The next consists in identifying the two completely different expressions: 'the final height of Mr. Original achieved at moment t' and 'the height of a tall man'. By assumption, at t, Mr. Copy is shorter than Mr. Original, even if the difference between them is minimal. But in compliance with the meaning of 'a tall man' at moment t, both men can be tall, the smaller the difference between them, the more probable it is. If Mr. Copy were cloned twenty years after Mr. Original birth, he could be really short at moment t, while Mr. Original could be (already) tall. But in case of an arbitrarily small span of age, one cannot tell that at the same moment one man is tall and the other is not. All these trivial remarks concerning the meaning of the expression 'a tall man' are shaken by Sorensen who has adopted a logically illegitimate assumption that the above two expressions may be treated as identical. Since if we deem true that a tall man is someone whose height corresponds exactly to the height of Mr. Original at the moment t, or is taller, then we render the expression 'a tall man' sharp (not vague) and, in fact, we replace it by another. Assume that Mr. Original's height at the moment t equals a real positive number r,[3] then the expression 'a tall man' is identified with 'a man whose height is a number from the closed interval $[r, \infty)$'. This is in apparent contradiction with the meaning of the first term. Here Sorensen commits equivocation and no wonder his conclusions are so bizarre. If a tall man is a man whose height at t amounts to, or exceeds, r, then whatever is the height of Mr. Copy, being shorter than r at t, is never judged to be tall. For example, if Mr. Original is 189.76824113 cm tall and Mr. Copy is "merely" 181.76824112 cm tall, then the former is a tall man while the latter is not! Such an obvious violation of the commonplace meaning of 'a tall man' has its origin with the aforesaid error of equivocation. This fallacy immediately generates another one because a vague expression 'a tall man' is identified

[3] Obviously even such an assumption is refutable as no real number corresponds to human height; only some approximation around which the real height oscillates depending on the factors like mood, condition, etc.

with a non-vague 'Mr. Original at moment *t*'. Apparently, Sorensen takes for granted the non-vagueness of the latter expression. And thus, in the proof of vagueness of this expression ('a tall man') he presupposes that the expression ('a tall man') is vague: The obvious case of *petitio principi*. It is also clear that the Clone Argument is "a clone" of Grey Sphere Argument. Observe the similarities: *the sphere loses gradually its colour to become white and Mr. Copy loses its shortness to become tall—both processes, maybe even treated as continuous, have upper, sharp boundary, and thus whiteness and tallness are achieved at one instance*. In both, Sorensen commits the same fallacy. In case of a sphere he assumes that there is one white colour, and even the least shade of grey may be recognized as grey. In case of a clone, he assumes that a tall-man height is tantamount to the height of Mr. Original at moment *t*, and anyone shorter even by micrometer is not tall. That Sorensen presupposes the existence of a sharp boundary in both cases is evident. The Sorensen "proofs", we have just discussed, are in fact one and the same argumentation presented for various cases in the disguise of different words.

Since it is difficult to treat seriously such arguments as:

"language learning ends at one moment (!) when it is already known";

"climbing the mountain ends at one moment (!) when one stands on its top" and

"the process of development of a tadpole ends at one moment (!) when it becomes a frog"

let us pass to another, namely, **Vagueness Cannot Be Tolerant**. It is a sorites reasoning where the concept of tolerance is applied to a name, 'a short man'. A name is tolerant if there are changes that are too small to affect its applicability; of course, an appropriate multiplication of such changes affects the applicability. Sorensen accepts vagueness of 'a short man', and uses the sorites reasoning against its tolerance. Namely, he argues that vague expressions cannot be tolerant.

Traditionally, the sorites argument has been treated as a test for vagueness which is positive when the result is contradiction. Thus, if some expression (name or predicate) generates contradiction in sorites, it is proved that the expression is vague. Sorensen's conclusion is quite different. Let us recall his reasoning [4, p. 249]:

(1) A Sorites argument concerning 'a short man' has a false induction step if the step's increment equals or exceeds ten thousand millimetres.

(2) If a Sorites argument concerning 'a short man' has a false induction step if the step's increment is n millimetres, it also has a false induction step if the step's increment is $n - 1$.

Thus,

(3) All Sorites arguments concerning 'a short man' having induction steps with increments convertible to millimetres have false induction steps.

The conclusion is straightforward: the most significant change turns out to be insignificant. In other words, there is no sharp boundary between a significant change and an insignificant change for tolerant names. Thus 'tolerance' is vague, and 'significant (insignificant) change', likewise.

At the same time, according to Sorensen, the reasoning proves that there are no names that could be vague and tolerant. Not only has he separated vagueness from tolerance, but also placed them in opposition. It is a completely unusual understanding of vagueness, since traditionally, tolerance is a fundamental feature of vague expressions—they are vague because they are tolerant.

Let us analyse the reasoning. According to the "starting" assumption, difference of ten thousand millimetres brings about a significant change of the terms of application of a name 'a short man'. (This is obvious.) The second, inductive assumption (also obvious) says: if n millimetres is a significant change, then $n - 1$ millimetres is also a significant change. The conclusion from those assumptions is straightforward: any difference in human height is significant. The conclusion of the appropriate, analogous argumentation is just the opposite: any difference in human height is insignificant. The first (Sorensen's) reasoning shows that the name: 'significant change of the terms of application of a name: 'a short man'' is vague. Parallel reasoning shows that the name: 'insignificant change of the terms of application of a name: 'a short man'' is vague. In both cases the vagueness of 'tolerance' is proved, since both '(in)significant change of the terms of application of a name: 'a short man'' are sensitive to sorites reasoning. Thus, to say that reasoning in question proves that a vague expression cannot be tolerant is absurd. In order to justify the use of the word 'absurd', we shall consider a twin reasoning, reconstructed for the class of Sorensen's predicates 'to be n-small', for $n \in \mathbb{N}$. Let us recall that a natural number k is n-small if and only if k is a small number or $k < n$. Directly from the definition it follows that 'to be n-small' is a vague predicate if $n \in \{1, 2, 3, 4, 5\}$ and 'to be 10^{23}-small' is already sharp.[4] In his reasoning Sorensen uses the name 'a short man' to show that it cannot be tolerant. We use the name 'n-small number' to show that it cannot be sharp:

(1) A 'n-small number' is a sharp name for $n = 10^{23}$.

(2) If a 'n-small number' is a sharp name for n, then a 'n-small number' is a sharp name also for $n - 1$.

Thus,

(3) A 'n-small number' is a sharp name for all natural numbers n.

Traditional and ordinary conclusion from the above reasoning is that 'sharpness' is vague. But following Sorensen's understanding of the sorites argumentation we must say that the conclusion is quite different, namely: since sorites employed for the sharpness of the sharp name leads to contradiction, it means that sharp name cannot be ... sharp. So, let sharpness be counter-set to ... sharpness.

2. More precisely about epistemicist's sharp boundary of vague predicate's extensions

Let us recall that every vague predicate P bears by definition a penumbra, i.e. the batch of cases in which neither P nor *non-P* can be asserted (the word 'set' is here purposefully avoided). The problem resides in the fact that the penumbra of an ordinary vague predicate has a fuzzy boundary or in other words has no (sharp) boundary. It means that in some cases it is unclear what belongs to extension (positive or negative) and what to penumbra. These "extraordinary" vague predicates, whose penumbra is a set are interesting intersections of a vague and not vague predicates, e.g. "to be a small natural number or to be natural number different from 100".[5] Their role in the course of discussion on vagueness cannot be overestimated. Of course, epistemicists do not state that a penumbra is sharply delineated

[4] In the reasoning, number 1023 is used because in physics it is a diagonal of the universe given in centimeters. It means that they are not used to working with numbers bigger than 10^{23}, and so, it is really a big number.

[5] The penumbra of the predicate is a set, if number 100 is a borderline case of the predicate 'to be a small number'. Then, the penumbra is a one-element set $\{100\}$. Everybody who thinks that 100 is not a borderline case of the predicate 'to be a small number', should replace 100 by a better (in his/her opinion) number.

but go further proclaiming its non-existence. In their opinion there is a sharp boundary between positive and negative extensions.

In practice, it means that sweet tea differs from not-sweet tea by exactly one grain of sugar. For a cup of tea given, there exists natural number k_0 such that k_0 number of grains of sugar in it changes nothing, tea remaining not sweet, while k_0+1 grains changes everything and the tea becomes sweet.

Figure 1.

Moreover, all this does not suffice to obtain a sharp boundary of predicate 'to be sweet tea', since we should also know when the substance added is sugar and when it is not. Sweetness of sugar varies as it depends on the percentage of saccharose ($C_{12}H_{22}O_{11}$) and thus t_0 indicates the sharp boundary between sugar and not-sugar:

Figure 2.

Naturally, for two different t_1, and t_2, greater than t_0, the sudden switch from not-sweet to sweet tea, postulated by epistemicists, cannot be determined by the same number k_0 of sugar grains, but by two appropriate numbers, k_{t_1}, and k_{t_2}. It is straightforward that the above diagram illustrating the difference of sweet and not-sweet tea should use two dimensions and be as on Fig. 3.

The shaded area implies that sweetness of sugar cannot be infinitely increased—the content of saccharose cannot exceed 100%—and that tea can absorb limited number of sugar grains. Let us add that the boundaries need not be straight lines.

Naturally, it is not sufficient to consider only those two parameters as we should also have the size of sugar grains determined in order to refer to it as a grain at all (and not a lump or dust). The question is not trivial because the size being doubled, the amount (number of pieces) of sugar poured in must be instantly halved thus transplanting the borderline (the

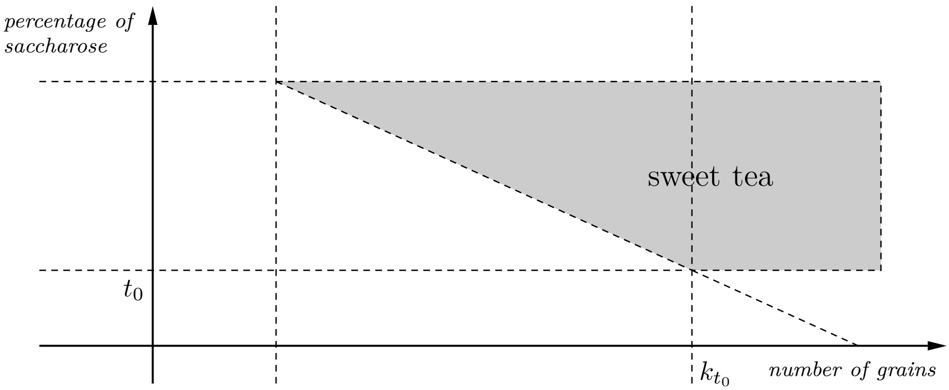

Figure 3.

sharp boundary). Similarly not every infusion may be called "tea": some cannot be told from water and some from essence. Introducing three parameters involves the third dimension, and four, the fourth, etc. In general, if in case of a given predicate we have to consider n factors, the diagram representing sharp boundaries for both, positive and negative, extensions must be n-dimensional.

The resulting complexity of the diagram, remote from the simple linear representations traditionally used by epistemicists, remains however, a minor problem. Much worse is the fact that their considerations seem to be entirely deprived of any sense as there can be found no scale allowing to judge if tea is sweet in a manner that would be objective (and not relative). Being objective here implies that everybody who tastes the tea estimates its taste in the exactly same way, which is obviously impossible. Every tea drinker has his own impression which may be the result of his drinking habits, or of what he has just eaten, etc. The criteria of "being sweet" reside in humans and not in cups of tea. Tea contains sugar and we have the impression. The transition of typically human categories onto the reality is untenable and may reveal some tacit adherence to the na?ve, primitive anthropological thinking.

Let us now analyse the line of reasoning of Williamson leading to the fundamental thesis of epistemicism:

> We do not know the sharp boundary of vague predicate and we shall never know it for some principal reason.

The assumption of existence of any sharp boundary between positive an negative extensions of a vague predicate—in fact, as we have shown, the assumption of existence of sharply delineated area corresponding to the boundary in question—apparently excludes the above mentioned fact of "human" origin of the category of sweetness. For we may ask where so great variety of tastes comes from if such a boundary does exist. Hence the epistemicists are compelled to exercise their acumen to solve the crucial problem of some inability of a man to recognise this allegedly existing boundary.

Timothy Williamson' s argumentation was to provide the expected solution of this problem. The gist of it is the *margin for error principle* introduced by him. The principle takes the form:

(MEP) A is true in all cases similar to cases in which 'It is known that A' is true.

Let us incidentally notice that once the principle holds, the conviction of infallibility of knowledge should be maintained, as well. Since if I know that A, then from this, A follows. Obviously, knowledge may be so defined as to guarantee the truth of all statements. However, in such a case, no scientific output can be referred to as knowledge.

Coming back to the problem considered, let us assume, as the epistemicists do, that there exists such a natural number k_0 that:

- tea with k sugar grains dissolved is not sweet if only $k < k_0$;
- tea with k sugar grains dissolved is sweet if $k \geqslant k_0$.

Thus, by assumption, it is always true that:

$Z_0 =$ 'tea with $k_0 - 1$ sugar grains is not sweet and tea with k_0 sugar. grains is sweet'

Nevertheless, quite simple argumentation makes us aware of the fact that we can never grasp the truth of his sentence. According to Williamson, this fact is caused by *the margin for error principle.* In this point, let us reproduce the reasoning of Williamson:

The fact that I know the value of k_0 is tantamount to the truth of the sentence 'I know that Z_0', Z_0 being a conjunction, is true when both conjuncts are true.

'I know that tea with $k_0 - 1$ sugar grains is not sweet' and

‘I know that tea with k_0 sugar grains is sweet’

Suppose that the former sentence is true. Then by the margin for error principle, the sentence ‘Tea with $k_0 - 1$ sugar grains is not sweet’ is true, and the sentence ‘Tea with k_0 sugar grains is sweet’ is false. Applying the *margin for error principle*, we receive falsity of the conjunction Z_0 from the truth of ‘I know that tea with $k_0 - 1$ sugar grains is not sweet’.

Now, let us assume that true is the sentence ‘I know that tea with k_0 sugar grains is sweet’. The same principle yields the truth of ‘Tea with k_0 sugar grains is sweet’ and ‘Tea with $k_0 - 1$ sugar grains is not sweet’ is false. Again we may conclude that the truth of ‘Tea with k_0 sugar grains is sweet’ implies that the conjunction is false.

To sum up, if we assume that we know as true at least one conjunct of Z_0, then we obtain falsity of the whole conjunction, which shows that it cannot be known as true. In this way, the conjunction Z_0 treated as true on the ground of epistemicism, totally escapes our knowledge (knowing).

The above reasoning, so essential for epistemicism, involves two logical problems. Witness the first. The reasoning of Williamson should prove that the sentence ‘I know that Z_0’ is false. However, this is not so, since the reasoning brings us to the conclusion that Z_0 is false. And as it comprises a predicate ‘I know’, the effect desired can be only produced by the conclusion: ‘I know that Z_0’ is false (and not that Z_0 is false). In fact, we may be never aware of Z_0 being false and thus the epistemic conflict Williamson is eager to accentuate simply does not exist. In view of this, how are we to understand the fact that the truth of the assumption (‘I know that (tea with $k_0 - 1$ grains of sugar is not sweet)’) implies merely that Z_0 is false? Williamson would rather need the following implication: if ‘I know that (tea with $k_0 - 1$ grains of sugar is not sweet)’ is true, then ‘I know that (Z_0)’ is false. Only then I know why I do not know—the state of my not-knowing represents the fact that what I know is false, and by no means the fact that some statement independent from my state of knowledge is false. Perhaps the reasoning presented should be concluded with the use of some auxiliary assumption that we may call the *margin of error principle bis*.

(MEP-*bis*) The sentence ‘I know that A’ is false in all cases similar to the cases where sentence A is false.

Now, we shall unravel the missing part of Williamson’s reasoning. Having obtained the falsity of Z_0, we may, by MEP-*bis*, draw the conclusion that also 'I know that Z_0' is false. At last I know something! Falsity of the sentence 'I know that Z_0' renders the state of my knowing (the dearth of knowledge, in fact) while the falsity of Z_0 does not. Alas, it seems that the MEP-*bis* cannot be derived from the MEP. This clearly means that we cannot suppose that Williamson was aware of the reconstructed part of the reasoning, and that he has only omitted it for the sake of brevity (as too evident).

The second problem is slightly different. We may think—when the question just considered is simply forgotten—that Williamson's reasoning is simple, logical and necessarily ends with a true conclusion. Notwithstanding, the truth of conclusion relies not only upon the impeccability of the very reasoning (inference) but on the truth of the premises, as well. In case premises are false, even sound reasoning is no guarantee of the truth of the conclusion. And the viability of the premise (assumption) that Z_0 is true is dubious. Most logicians, especially those who do not adhere to epistemicism, consider Z_0 as obvious falsehood. Furthermore, also MEP opens the gate to some rudimentary doubts situating the truth of a sentence on a par with the knowing-the-truth of this sentence.

What does Williamson's argument prove if Z_0 is false? To estimate the value of his argument, let us apply it to the case wherein the falsity of a premise should be evident even for epistemicists.

To this aim, we adopt LOEGP (*Life-on-Earth general principle*[6]) which resembles MEP in such a respect that it renders impossible conceding a pair of sentences:

- *If I know that some species lives deep in the ocean then it does not live on deserts.*
- *If I know that some species lives on deserts then it does not live deep in the ocean.*

Additionally, let us presuppose that the author of this paper intends to defend the truth of:

$$Z_1 = \text{`Bison lives deep in the ocean and bison lives on deserts'}.$$

As it is extremely difficult to believe that the above sentence is true, the author is forced to demonstrate that there exists some principal (deeply seated) inability (obstacle) that does not allow us to accept this sentence

[6]Obviously, pompous language is not accidental here.

as true. To this aim LOEGP is really indispensable. Beneath we copy the Williamson's reasoning for the "bison" case.

Since Z_1 is a conjunction, truth of 'I know that Z_1' is tantamount to the truth of the two following sentences: 'I know that bison lives deep in the ocean' and 'I know that bison lives on deserts'.

1. Let us assume that the former sentence is true, and thus I know that bison lives deep in the ocean. Then, by LOEGP, the sentence 'Bison does not live on deserts' is true, and consequently, 'Bison lives on deserts' is false. Applying LOEGP we have inferred, from the truth of the sentence 'I know that bison live deep in the ocean', that the conjunction Z_1 is false.

2. Now let us assume that the sentence 'I know that bison lives on deserts' is true. In such a case, by LOEGP, 'Bison does not live deep in the ocean' is a true sentence, and subsequently, 'Bison lives deep in the ocean' must be false. As in the former case, by LOEGP and having assumed the truth of 'I know that bison lives on deserts', we infer the conclusion that the conjunction Z_1 is false.

It is not hard to notice that in both cases our reasoning ends at the stage where Williamson terminates his. This implies that, against what is admissible, we accept the falsity of Z_1 as a satisfactory conclusion. Summing up, if we assume that we know that at least one conjunct of Z_1 is true, then instantaneously, our conjunction is false. Now, we may further urge that Z_1 is true and if anyone thinks otherwise (since it is absurd), we may say that what they think is an illusion caused by the fact that to know the truth of this sentence is basically and objectively impossible due to the character of our cognition. Consequently, we may state that the sentence 'Bison lives deep in the ocean and bison lives on desert' is true but its truth is simply inaccessible for a human being.

At this moment, it would be not amiss to question the soundness of the whole reasoning of Williamson. First, let us notice that it is not a branching proof, not being an alternative. Having the conjunction: 'I know that tea with $k_0 - 1$ sugar grains is not sweet' and 'I know that tea with k_0 sugar grains is sweet' as a starting point, we follow two parallel lines of argumentation to demonstrate, finally, its falsity. Naturally, in order to show that a conjunction is false it suffices to prove the falsity of one of its conjuncts. Williamson wants to prove that both are false, therefore he presupposes that the false (even in his opinion) premises: 'I know that tea with $k_0 - 1$ sugar grains is not sweet' and 'I know that tea with k_0 sugar grains is sweet' are true. Such a procedure is legitimate when they lead to contradiction, i.e. the falsity of each premise is proved independently, as the truth of each premise yields contradiction. As everyone is quick to point out, it is not the case here. Truth of each premise amounts to the same consistent conclusion, i.e. to the falsity of Z_0 = 'tea with $k_0 - 1$ sugar grains is not sweet and tea with k_0 sugar grains is sweet'. Those who do not believe in epistemicism may be satisfied with this conclusion, for them it is false. So why, according to Williamson, the conclusion plays any logical role? The only explanation I can think of is that for Williamson obtaining the falsity of Z_0 is on a par with arriving at contradiction. But when is it true? Clearly, only when the truth of Z_0 is formerly presupposed. Here, however, we encounter some inherent difficulty, as for most logicians and philosophers this sentence is false and thus the whole reasoning of Williamson is based upon a false premise, i.e. Z_0.

Another question concerns the sense of the very MEP. Let us recall it:

> A margin for error principle is a principle of the form: 'A' is true in all cases similar to cases in which 'It is known that A' is true.
>
> [11, p. 227]

Apparently the principle is the declaration of our belief in the infallibility of our propositions (knowledge). Since, if it is true that we know that A, then under this principle, A itself must be true. Moreover, it must be true in all cases similar to the one in which it is true that we know that A. On account of contemporary epistemology and methodology of science it seems to be a na?ve view. For the sentence 'I know that A' expresses solely a belief in truth of A, which means that 'I know that A' is interpreted as 'I believe that A', rather than a statement of the fact that we have just grasped some truth about reality—the truth that is to last for ever. Furthermore, the examples Williamson provides elicit the fact, that for him 'I know that A' means 'I believe that A' or 'I am convinced that A' and not at all that he understands the sentence as referring to the state of our acquisition of any final truths about the world. This may be easily settled when we remember that he applies the schemata 'I know that A' to the sentence A that speaks about the amount of people in the crowd [11, pp. 218–220].

To recapitulate, the principle MEP makes us commit ourselves to a claim that the truth of A in cases similar to P depends on the truth of 'I know that A' in case P. Without too much attention paid to the complex problem of similarity, the objective truth value of A is to depend on the subjective truth of the sentence 'I know that A'! Such a stance is so bizarre that I can hardly believe that it looks utterly natural to anyone.

Coming back to the amount of people in a crowd, let us switch to the next reasoning proposed by Williamson which is an articulation of symbolic logic in defence of the thesis of epistemicism.

3. "Some serious problem stems from the fact that we may know something and at the same time we may not know that we know it"

The thesis, that one can know something and at the same time he does not know that he knows, seems to be connected with psychology of thinking process rather than with logical calculus and its tautologies. Nonetheless, Williamson attempts to prove it employing the tools of symbolic logic. In fact, whether one is aware of the knowledge imbibed or not, seems to be individual and depends on

1. the person just thinking;
2. the problem just considered.

Specifically, it is interlaced with particular intellectual potency, education, complexity of the problem, etc. For all those reasons, it is not susceptible to global solutions by means of propositional calculus: it should concern every person in exactly the same way.

Therefore, worth witnessing is the argumentation of Williamson. Let us assume that we observe a crowd of people. Naturally, we cannot know exactly how many people there are. The assumption implies that there is a number m for which I do not know that there are exactly m people there. For example, I do not know that there are 1000 people, neither do I know that there are 800 people, etc. These numbers constitute a subset T of the set of natural numbers. Every subset of the set of natural numbers has the property of having the least number and so has the set T. Assume that the number equals n. Hence,

(1) *I do not know that there are not exactly n people there.*

and

(2) *I know that there are not exactly $n-1$ people there.*

If 582 is the least number, for which (1) is true, then the greatest number I know that it is not tantamount to the number of people in the crowd is 581. Now, basing on his experience Williamson immediately adds:

(3) *I know that if there are exactly n people there, then I do not know that there are not exactly* $n-1$ *people there.*

Williamson claims that (1), (2), (3) appear to be mutually inconsistent unless we reject the principle *I know that I know.* Only then, two propositions (2) and (3) imply the negation of the first. Let us apply simple propositional calculus with operator 'K' and where 'N' denotes proposition: 'there are exactly n people there', and $N-1$ – 'there are exactly $n-1$ people there'. Hence we receive:

(1) $\neg\,\mathsf{K}(\neg N)$,

(2) $\mathsf{K}(\neg(N-1))$,

(3) $\mathsf{K}(N \to \neg\,\mathsf{K}(\neg(N-1)))$

the *detachment rule* for implication within the range of operator 'K', should be of the form: if $\mathsf{K}(a \to b)$ and $\mathsf{K}(\neg b)$, then $\mathsf{K}(\neg a)$. From the set of propositions (2) and (3) we may infer a proposition inconsistent with (1), when we additionally admit:

(4) $\mathsf{K}(\neg(N-1)) \to \mathsf{K}\,\mathsf{K}(\neg(N-1))$.

Next, from (2) and (4) we obtain:

(5) $\mathsf{K}\,\mathsf{K}(\neg(N-1))$, i.e. $\mathsf{K}\neg\neg\,\mathsf{K}(\neg(N-1))$.

At last (5) and (3) yield $\mathsf{K}(\neg N)$, inconsistent with (1). In this way Williamson proves the failure of the KK-principle, expressed by premise (4): *If I know something, then I know that I know it.* In the opinion of Williamson, this principle leads to inconsistency of otherwise consistent set of premises (1), (2), and (3).

Formal aspect concerning inconsistency inferred from the three premises and KK-principle seems cogent but the entire reasoning is simply not. The problem resides in the three premises Williamson keeps to, and more precisely in the second one. It is obvious that while observing a crowd of people I can point out a sequence of numbers m such that would satisfy the first proposition. However, by no means, the set thus determined has sharp boundaries. So I cannot assume that this is a standard sub-set of the set of natural numbers $\mathbb{N}$, and consequently, that it contains a least number. Mere observation is against such a conclusion. It is trivial to notice that there are numbers that we seriously doubt if they satisfy the first proposition. Furthermore, this bulk of numbers is also deprived of sharp boundaries, which elicits the fact that the estimation of the number of people gathered as a crowd is likewise approximated. Here Williamson fails on formal side: to infer legitimately from (1), (2), (3), only one number n may be considered—the same for each premise. (1) and (2) imply that n is a least number among sharply determined numbers, of which I do not know that they do not correspond to the amount of people in the crowd observed. Clearly, $n - 1$ is the greatest of these numbers less than numbers constituting the clearly determined set of numbers of which I know that they do not describe the amount of people in the crowd. This is, as we have shown, a strong assumption of vagueness absence, i.e. the assumption that a vague object (phenomenon) is not vague—the error of *petitio principi*. As I have already mentioned, epistemicism is not exempt from this error. It alone suffices to reject the argument so gravely disproved. However, spelling out the reasoning in terms of some concrete data would elicit its bizarreness. If a number n is to represent the amount of people in the crowd, it must be big enough (10 is not a crowd). Moreover, n must be a number that we cannot verify (count) at one simple glance. Let us arbitrarily choose $n = 2375$ and substitute it for n in (1) and (2):

1* *I do not know that there are not exactly 2375 people in the crowd*,

2* *I know that there are not exactly 2374 people in the crowd.*

The result of the substitution is quite bizarre. Such a set of premises is usually the basis of erroneous reasoning—one of the premises is false. Indeed the premises could be treated as true if the interval between them were greater. To illustrate this, we alter the second premise:

2** *I know that there are not exactly 10 people in the crowd.*

It is absolutely sure that I know that 10 people cannot be a crowd. Unfortunately, the number of the first premise must be an immediate successor of the second, which warrants the truth of at least one premise. The result is the falsity-of-premise fallacy. I wonder if there exists an epistemicists who, seeing a crowd of people, would be able to tell (and not guess) the two succeeding natural numbers n_0 and $n_0 - 1$ satisfying (1) and (2)?[7]

We may overtly refute the argument with the false presupposition that every man knows with utmost precision that there are not n number of people in the crowd. Here the error of false premise merges with the *petitio principi* fallacy (the assumption of a sharp boundary separating our knowing from not-knowing of the exact amount of people in the crowd). Such is the result of the simple analysis of only two premises. After all else, let us consider the third. The addition of the third proposition makes the set of premises an exemplary case of the assumption of sharp boundaries—n is a number of people in the crowd. This is clearly implied by Williamson's reasoning where n is a unique number considered. This assumption although present behind (1) and (2) is considerably strengthened by (3).

Before proceeding further, let us spell out the (quite outstanding) conclusions of our knowledge when 2375 stands for n.

(1) there are exactly 2375 people in the crowd (fact).

(2) I do not know that there are not exactly 2375 people in the crowd; (1).

(3) I know that there are not exactly 2374 people in the crowd; (2).

(4) I know that if there are exactly 2375 people in the crowd, then I do not know that there are not exactly 2374 people in the crowd; (3).

(5) If I know that that there are not exactly 2374 people in the crowd, then I know, that I know that if there are not exactly 2374 people in the crowd; KK) applied to (2) .

(6) I know, that I know that there are not exactly 2374 people in the crowd; (3), (5).

(7) I know that it is not true that I do not know that there are not exactly 2374 people in the crowd; (6).

(8) I know that there are not exactly 2375 people in the crows; (4), (8).

Contradiction (2) and (8).

However, we cannot conjecture that the contradiction received is due to KK. To see this it suffices to remark that when we repeat the reasoning for

[7] In (1) and (2) there is a predicate 'I know that', and not 'I guess that'.

more plausible premises, we avoid contradiction, in spite of the fact that the same KK has been employed. So,

1^* there are exactly 2375 people in the crowd (fact).

2^* I do not know that there are not exactly 2375 people in the crowd.

3^* I know that there are not exactly 50 people in the crowd.

4^* I know that if there are exactly 2375 people in the crowd, then I do not know that there are not exactly 2374 people in the crowd.

5^* If I know that that there are not exactly 50 people in the crowd, then I know, that I know that if there are not exactly 50 people in the crowd.

6^* I know, that I know that there are not exactly 50 people in the crowd; 3^*, 5^*.

The set of premises like 1^*, 2^*, 3^*, 4^*, do not imply contradiction even when the KK is employed not only to 3^*, but "even" to 2^* and 4^*. The premises 1^*, 2^*, 3^*, 4^*, may be true—contrary to (1), (2), (3) and (4)—and, consequently, there may arise a situation described by them; while (1), (2), (3), and (4) refer to unreal, implausible situation and as such cannot be a starting point of any sensible proof. Especially they cannot be the reason of rejecting such a safe principle as *I know that I know*. Although we may think of situations where the usage of this principle may be futile, it should be countenanced as a well-formed tool of a thinking man.

Let us assume that what epistemicism claims is the case

Although our next move seems to be incompatible with what we have said about KK-*principle*, (for a while) we shall reject it for the sake of simple, indirect proof we wish to carry on.

Let us assume, after epistemisists, that KK does not hold and that any time it is applied, we run into the risk of misvaluating the propositions. According to epistemicists, our beliefs cannot be adequately trustworthy because the very self-reflection only augments the margin of error. But the genuine arguments of epistemicists are samples of thinking about thinking, considerations that may be notoriously treated as instances of the use of predicate 'I know that …'. Hence … the arguments of epistemicists, and especially conclusions they have drawn, are in the light of their own theory, examples of imprecise (inadequate) knowledge susceptible of erring, the greater deficiency the more we ponder about it (self-reflection). Without any prejudice in our attempt to mirror precisely the reasoning characteristic for epistemism, basing solely on their claim that vague predicates have sharp boundaries and that this knowledge is also vague (imprecise), and resorting to the theory of epistemicism (!), we have arrived at the conclusion that it is not precisely the case that vague predicates extensions have sharp boundaries. Have we not, basing only on epistemicism, reached the conclusion that vague predicates have no sharply determined extensions? Again we must admit that vagueness wins! It has become evident that vagueness may be relocated but not removed. The conclusion is philosophically pungent:

> *If what the epistemicism states is the case, then it is not the case what the epistemicism states.*

This fact imposes the necessity of rejecting epistemicism if we are not to give up logic: ‘$(p \to \neg p) \to \neg p$’ being among tautologies of the classical propositional calculus. Therefore, not to betray logic, we must conclude:

> *If (if what epistemicism states is the case, then it is not the case what epistemicism states) then it is not the case what epistemicism states.*

Thus, by *modus ponens* applied to both sentences in italic, we obtain:

> *What epistemicism states is not the case.*

4. Conclusions

According to the authors of epistemicism, Williamson and Sorensen, every predicate has sharp boundary, clearly dividing the universe into two, not vague sets: sets in mathematical terms. They constitute positive and negative extensions. Williamson has introduced MEP in order to explain why we do not see the boundary in predicates of the type: ‘to be a tall man’ or ‘to be red’. In consequence, he receives what he has intended to receive. But what is the value of such a reasoning? Obviously none! Only its appearance is healthy and thus it seems to bring some effect. Under the close inspection, however, it turns out to be a paralogism. The schemata of reasoning of Williamson when dressed in different (but analogous) propositions reveals its true character. Likewise does the argument, in which Williamson tries to refute 'I know that I know' principle, is based upon premises that cannot be both true; in other words the reasoning starts with false premises. No wonder, Williamson may obtain any conclusion he needs. Unfortunately, such reasoning has nothing in common with logic.

The main conclusion following the analysis of Wiliamson and Sorensen arguments should be that epistemicism has no logical foundations, but moreover, it resorts to ... logical fallacies: of *petitio principi* and the falsity of premise.

Therefore, no logical justification can be found for this philosophical current. Epistemicism is accepted simply because is believed in, with the faith not abided by any logical rules.

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