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## ONCE MORE ON ANALYTIC VS. SYNTHETIC\*

**Abstract.** The boundary between analytic and synthetic sentences is well definable. Quine’s attempt to make it vague is based on a misunderstanding: instead of freeing semantics from shortcomings found, e.g. in Carnap’s work, Quine actually rejects semantics of natural language and replaces it by behavioristically articulated pragmatics. Semantics of natural language as a *logical analysis* is however possible and it can justify hard and fast lines between analyticity and syntheticity.

*Keywords:* Analytic, synthetic, intensions, constructions, concepts, pragmatics.

### 1. What does “unrevisability in the light of experience” mean?

Let S be an analytically true sentence, i.e., true in virtue of its meaning only. In which sense can it be said to be unrevisable in the light of experience?

Since S is true in virtue of its meaning only it is immune as for its truth-value from any influence of changes in reality, i.e., from experience. The seeming counterexamples are produced by a fatal misunderstanding that essentially consists in mixing up meaning with assertion. A paradigmatic example:

(S) *All bachelors are men.*

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Which kind of changed experience could cause a change in the semantic status of the analytic(ally true) sentence (S)? Let us helpfully construct such a case. Imagine the following situation: Practically all adult men get married while there are many adult spinsters. The word ‘bachelor’ begins to be used in the ‘everybody embracing’ sense—bachelor is now whoever an unmarried adult is. Now the sentence (S) seems to be false.

If examples of this kind should be considered as arguments for the claim defended by Quine in his “Two dogmas of empiricism”, namely that “no statement is immune to revision”, then, first of all, we would have to wonder: how come that the overall popularity of the mentioned claim could be supported by examples that are evidently heavy-laden by an elementary mistake?

Well, we could be accused of simplifying Quine’s view: to this point we will return below. Now we will recapitulate the mistake committed by whoever would use the argument above; true, the mistake is an elementary one, but it is symptomatic for the way some pragmatically oriented philosophers think.

Let us ask *What is a sentence?* Given a language  $\mathbf{L}$ , a sequence of signs of  $\mathbf{L}$  is a sentence iff it possesses a meaning due to which we can arrive to a truth-value (if any) or to a proposition in the sense of a function from possible worlds (and, as the case may be, time moments) to a truth-value. Clearly, we will say that two *sentences are distinct* if either the respective sequences of signs (of the given  $\mathbf{L}$ ) differ or *if their meanings differ*.

Up to now it is not necessary to specify the notion of meaning; let it suffice to say that meaning of an expression  $E$  is a procedure that results in the object denoted by  $E$ . From this characteristic of a possible explication and from our story about bachelors in the changed situation it obviously follows that there are two distinct sentences here: (S) and

(S') *All bachelors' are men,*

where *bachelor'* is an expression which has got another meaning according to our story. Thus the analytic sentence (S) is still true, and it will always remain true: any thinkable counterexample can be formulated only because the meanings of some subexpressions of (S) change so that *another sentence* arises.

Summarizing, the cases such as our ‘bachelor example’ are not cases of a genuine revision in virtue of a change in the extra-linguistic reality. The latter can be exemplified by principally different examples, e.g. by the case

when a claim concerns a state of the world at a time moment  $t$  and the state changes at some other moment  $t'$ . For example, the statement that some birds of the kind *Dodo ineptus* live in the island called Mauritius was true at some times but is no more true as soon as the kind *Dodo ineptus* died out.

Nevertheless, we can easily see that Quine has understood 'revisability in the light of experience' in another sense: for him such Kuhnian 'incommensurable' cases as transition from Newton to Einstein, i. e. cases where it was not reality what changed but our conceptual tools (under the pressure of reality but not in parallel with its changes), were the typical reasons for revision of our claims. Yet all such cases share the fact that statements are revised in the sense that reality is described by means of other concepts. Analytic sentences are stable, 'unrevisable'. (A good example is connected with the extreme cases of revisability concerning *logical* claims. Quine adduces the case of quantum mechanics, which has been allegedly simplified due to revising the law of the excluded middle. We can adduce the similar case with 'paraconsistent logic'. Is the truth of the matter really in admitting that, e.g. the law of (forbidden) contradiction can be revised? Not at all: you can introduce axioms that will block the 'undesirable' consequences of this law, but only seemingly: actually, you have redefined the logical objects connected with the symbols used for formulating the law of contradiction. Thus the law of contradiction, as well as the law of the excluded middle continues to hold; the other formulations are no more 'revised' laws of contradiction or of the excluded middle.)

So do we not do Quine an injustice? After all a logician as he surely was must have seen that there was something wrong with possible examples like ours. So what would Quine probably object to our claim that such examples make his 'universal revisability thesis' dubious?

Quine's answer is to be found in his "more thorough pragmatism" (Quine 1953, p. 46), which on the one hand refuses to accept a semantic notion of meaning and on the other hand (closely connected with the first one) defends the holistic thesis that the distinction between the linguistic and the empirical factor (that makes it possible to distinguish between analytic and synthetic sentences) "is not significantly traceable into the statements of science taken one by one" (Quine 1953, p. 42). Our criticism is from this viewpoint irrelevant: first, it presupposes the 'atomistic' thesis, which connects truth-values and truth conditions (propositions) with particular sentences "one by one"—so a thesis not accepted by Quine—and, second, uses a semantic notion of meaning (so an "obscure" notion) instead of talking about "symbol in use".

We could say, exaggerating a little, that Quine's philosophy has been defined once for all by what has been suggested *in nuce* just in "Two Dogmas". One cannot try to explicate the distinction analytic vs. synthetic without showing that as far as Quine's philosophy contains criticism of purely semantic analyses it is wrong.<sup>1</sup>

In what follows such an attempt is made. To achieve the desired aim it will suffice to show positively that

- (a) a logical, i.e., non-empirical, not behaviorist analysis of scientific and natural language can be defined and usefully exploited,
- (b) a purely semantic notion of meaning is definable so that more pretentious commitments than those ones connected with mathematics and/or logic are not needed,
- (c) the difference between analytic and synthetic (empirical) sentences is definable on the basis of the points (a) and (b) so that the positive core of Quine's theory is inapplicable as a criticism of the resulting theory.

## 2. Semi-expressions and expressions (ad a)

Pavel Tichý in (2004, p. 55, first published in Czech 1966) defining the task of *logical analysis of natural language* (he speaks about *logical semantics*) writes:

We assume, of course, a normal linguistic situation, in which communication proceeds between two people, both of whom understand the language. *Logical semantics does not deal with other linguistic situations.* (Emphasis ours.)

(Here to be referred to as LANL PRINCIPLE.)

This is only a small footnote written 1966. When reading some contemporary considerations concerning analyzes of natural language one gets an impression that some analysts are not aware of the simple fact mentioned in this footnote. It is as if doing an analysis of natural language one would have to describe factual functioning of language instead of being after the logical structures underlying the expressions of a language. It was Quine, who started the trend of 'pragmatization of semantics', of replacing an analysis by empirical generalization. Therefore, among others, he cannot accept

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<sup>1</sup>In my opinion, it is well possible to separate a 'positive core' of Quine's theory: this would be a kind of behaviorist analysis of natural language, probably best developed in his (1960).

that there is a qualitative distinction between empirical and mathematical claims/problems.<sup>2</sup>

Commenting the fact that Galileo, as a result of his alleged dropping pebbles from the Tower of Pisa, has put together a table showing the course of values of a function Tichý writes:

while plotting the values of the function against its arguments Galileo was not doing mathematics. He was just taking down what was dictated to him by nature.

[...] But Galileo not only *identified* the free-fall function. He also noted that there is quite a straightforward way of *calculating* the values of the function from its arguments. Given an argument, all one needs to do is multiply it by two, divide the result by 9.7, and then take a square root. *It was when he made this second discovery, a discovery concerning a complex involving functions and numbers, that he was doing mathematics.* (Emphasis ours.)

(Tichý 1995, 183–184; 2004, 883–884)

Thus while we register real phenomena, writing down particular facts and generalizing we do an empirical work. Using complexes involving functions and various kinds of abstract objects we do mathematics. All results of the former work are in principle revisable. Once a mathematical result is true it cannot be ever revised. (Indeed, we can wrongly apply some mathematical tool to a particular empirical problem, but this is no case of revisability of a *mathematical truth*.)

This difference can be characterized in other words as the difference between logically contingent and logically necessary claims.<sup>3</sup> And this is surely not an only quantitative (“of degree”) difference. To refute this statement Quine needs holism in the sense that only knowledge system as a whole (notably science) can be semantically evaluated.

We would like to show that the non-holistic (“atomistic”) approach to analysis of natural/scientific language can be justified as soon as we intend to do a *logical* analysis, i.e., a search of the logical structures that underlie expressions of the language. The main assumption that legitimizes this justification can be formulated as Tichý did it in his footnote quoted above. Logical analysis of the given language—as done, e.g. by Montague’s IL or Tichý’s TIL—takes the language (its given stage of development) as a definite system of phonetic, syntactic, and semantic rules, which are given by a

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<sup>2</sup>See his claim that the difference between issues concerning classes and issues concerning centaurs or brick houses on Elm Street “is only one of degree” (1961, p. 46).

<sup>3</sup>As for the relation between logical and analytic necessity see Section 4.

partly anonymous linguistic convention. In other words, no empirical study of the facts concerning the language is a part of the *logical analysis*: from the vantage point of the latter the meanings of the expressions are given, their connection with the expressions is *a priori* from this standpoint. This kind of abstraction is similar to the abstraction done by theoretical physics: admitting only a slight simplification we can imagine a following dialogue between a naïve observer and a physicist:

Obs.: *I dropped a stone to this well and then I did the same with a much lighter piece of wood. I have measured the time of falling of both bodies. You make us believe that the time should be the same, but this is not the case, the wooden piece fell much slower.*

Ph.: *Indeed, you must see that the time would be the same if the bodies had fallen in vacuum.*

Obs.: *But no vacuum is here. Why do you physicists work out such theories that are not able to predict phenomena under normal circumstances?*

This simile is not precise, of course, physics as a whole is—unlike logical analysis of language—an empirical science. But an essential similarity can be stated. Under ‘normal circumstances’ even the speakers of their native language often behave in an unpredictable way. This empirical fact has obviously fascinated Wittgenstein when he ceased doing tractarian analyses and began to swim in the attractive waters of ‘language games’. Let him do it, it is a most interesting job, but as soon as it is interpreted as a criticism of *logically* analyzing language a useless misunderstanding starts a nonsensical war against logical analysis of (natural) language, LANL.

Interestingly enough, if the physicist took into account the factors he had abstracted from, he would be able to predict future cases of, say, free-fall on the basis of the abstract law + the added factors. Not vice versa: mere empirical data cannot be simply generalized to result in a mathematically presented abstract law. Mere registration of the way the jungle people speak cannot unambiguously reproduce the jungle language as a definite system of phonetic, syntactic, and semantic rules. Quine’s “indeterminacy of translation” is no contribution to semantics: it is an unintentional confirmation of the asymmetry of the relation between empirical data and mathematically formulated abstract law.

A Slovak philosopher Pavel Cmorej (2005) explained the position of LANL in terms of distinguishing what he calls ‘semi-expression’ from the (linguistic) expression. The core of his conception can be described as follows:

Any system called *language* contains (virtually) infinitely many finite sequences of signs. No such sequence stands in an intrinsic relation to *meaning*. Any of them can, however, be associated to a meaning due to a linguistic convention. Consider a pair  $\langle S, M \rangle$ , where  $S$  is such a sequence and  $M$  the meaning associated with it. As already stated, there is no intrinsic relation between  $S$  and  $M$ , so  $M$  is only contingently associated with  $S$ . The  $S$ s in such pairs can be called *semi-expressions*. As for the (genuine) *expressions* of the given language, they correspond to such *pairs*. Yet whereas the  $M$ s belong to  $S$ s (i.e., the meanings to the semi-expressions) only contingently, they belong to the pairs necessarily, since what arises due to a contingent association of meaning to a semi-expression is just an expression, and the meaning belongs to the expression (unlike to the semi-expression) necessarily—so two distinct expressions arise not only when  $S$ s are distinct but also if  $M$ s are distinct (even with  $S$  the same—the phenomenon of homonymy).<sup>4</sup>

This conception can develop and become more precise, of course. At least one point should be mentioned: Let  $A_1, A_2$  be semi-expressions with meanings  $B_1, B_2$ , respectively. Let  $\mathbf{F}$  be a syntactic function whose application to  $A_1, A_2$  results in grammatically well-defined linguistic entity, and let  $\mathbf{G}$  be a semantic function whose application to  $B_1, B_2$  results in a new meaning so that the pair  $\langle \mathbf{F}(A_1, A_2), \mathbf{G}(B_1, B_2) \rangle$  corresponds to a new expression in harmony with the principle of compositionality. Would we be able to agree that  $\mathbf{F}(A_1, A_2)$  is a semi-expression?

It is probably clear in which way Cmorej's conception can be successfully developed. What is important in the present context is that it makes explicit the reason why LANL not only should but also *can* presuppose that expressions of a well-defined stage of a language are related to their meanings necessarily.

Another argument consists in stating that in the normal situation mentioned in Tichý's footnote quoted in the outset of the present section the expressions of the given language are *understood*; if the meaning of the expression  $E$  were not connected *a priori* with  $E$  then understanding would be dependent on some experience, but then understanding would be a product of *regressus infinitus*, which is impossible, of course.

Doing LANL we cannot but admit that we know the given language and are thus able, for example, connect various kinds of expression with types

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<sup>4</sup>Independently of Cmorej Robert May in his "The invariance of sense", *Journal of Philosophy* 103 (2006), pp. 111–144 has argued that Frege's conception of natural language presupposes that language is a system of *signs*, where "a sign is a pairing of a symbol and a sense".

(this stage of analysis is in general shared by Montague and Tichý), apply the principle of compositionality (knowing the grammar of the language) etc. But exploiting our knowledge of the language for analysis we contribute to our knowledge, being able to explain some *logical problems* connected with language. Some of those problems are not trivial, and knowing the language in the sense of being simply a user of the language is not sufficient for finding solution of them. Not finding solution to some such problems may cause some communication disorders so that not only empirical study of language but also the *a priori* LANL can prove its usefulness in this direction. (After all, who wants to claim that being *a priori* means being useless?)

Thus we can state that there are two classes of theories scientifically concerned with language. To one of them such activities belong that consist in studying language as a natural phenomenon. These are *empirical studies*, and all linguistic disciplines are members thereof.<sup>5</sup> When Quine applied his philosophy to more specific problems he gave us his *Word and Object* (1960), where a theory of language is presented that obviously belongs to the class in question. To the second class belong in particular works by Montague (see (1974)) and his school, and works based on Tichý's *The Foundations of Frege's Logic* (1988) and his numerous articles collected in *Pavel Tichý's Collected Papers in Logic and Philosophy* (2004); the system is known as Transparent Intensional Logic, TIL.

Hopefully, the point a) has been fulfilled. Going to the point b) we will exploit TIL.

### 3. Meaning (ad b)

#### 3.1. Criticism of the prevailing conception of the Fregean schema sense-reference

It follows from what has been said in Section 1 that any attempt to pragmatize semantics (like to replace the notion of meaning by the Quinean notion of 'stimulus meaning') is incompatible with pursuing our aim.

We can see that the term 'meaning', as used in semantics, corresponds rather to Frege's term 'sense' (*Sinn*) than to his term *Bedeutung*. The latter has been translated as *denotation* by Church (1956) and more often as *reference* (see, e.g. Kirkham (1997)). Later we will show that a fundamen-

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<sup>5</sup>'Empirical' is here antonym to '*a priori*' rather than to 'theoretical'. Clearly, Chomsky's are very theoretical works but they remain being empirical.



tal difference between *denotation* and *reference* can be defined. Now we will concentrate on *meaning* as what Frege probably had in his mind when speaking about sense.

Let meaning and denotation be whatever, one point is intuitively clear: if two such notions are introduced then *meaning is more fine-grained than denotation*. To illustrate this claim consider the expressions ‘2+3’ and ‘1+4’. We would say that both denote the same object, but they somehow differ: therefore the identity ‘(2 + 3) = (1 + 4)’ is interesting unlike the identity ‘5 = 5’. Another example, this time from the area of empirical expressions, the expressions ‘a brother of a parent of XY’ and ‘an uncle of XY’ both denote the same property but they again somehow differ<sup>6</sup>. Frege’s idea has been that two expressions may denote one and the same object but their ‘modes of presentation’ can be different; this ‘mode of presentation’ (*die Art des Gegebenseins*) has been never well-defined by Frege, who called it ‘sense’.<sup>7</sup>

From the time of Frege’s 1892 semantics of natural language felt to be obliged to react in some way to his idea of distinguishing between ‘sense’ and ‘denotation’. It is not the purpose of the present study to reflect all the attempts to make this idea exact or at least clear. Instead we will try to distinguish between general approaches to this problem.

First of all, let us consider a general schema broadly accepted as an explication of Frege’s semantics:

$$\text{expression} \rightarrow \text{sense} = \text{intension} \rightarrow \text{denotation} = \text{reference} = \text{extension}$$

(see, e.g. (Kirkham 1997)).

In what follows this schema will be criticized from the viewpoint of TIL.

### (a) denotation = reference?

We will show that at least in the case of empirical expressions this identity is incompatible with the *a priori* character of LANL.<sup>8</sup>

We will first analyze three examples of empirical expressions; our claim will be easily generalized.

<sup>6</sup>The ‘classical’ examples can be found in Frege (1892) (intersection point of medians, morning star vs. evening star).

<sup>7</sup>In Bealer’s 1982 this distinction is defined as the difference between two kinds of ‘intension’, one being ‘coarse-grained’, the second ‘fine-grained’.

<sup>8</sup>As for terminology, it developed gradually in TIL. Tichý, for example, did not use the terms ‘denotation’ and ‘reference’ as semantically distinct but his theory made it possible to explicate these terms in the way we present here.

### First example: a definite description

Let it be

(D) *the head of the Catholic Church*

As explained in Section 2, given that meaning is associated with an expression of the respective language necessarily then the object “presented” by the meaning of an expression  $E$  should be given just by the meaning and be so entirely independent of empirical facts, i.e., of any events in reality. This is obvious because if the meaning (sense) of  $E$  has been associated with the respective semi-expression, then the way to the denotation must be an unambiguous way, independent of whatever happens in the world: the convention connected with meaning could not foresee which events will come. Thus not only meaning (sense) but also denotation is associated with the expression necessarily, i.e., independently of empirical facts. If so, then the definite description (D) should denote one and the same object (unless the language itself changes) independently of who just occupies the mentioned office. (In other words, the person to which we *refer* by (D) is not what is *denoted* by (D).) Indeed, it would be very strange if what the given expression denotes should depend on the instantaneous state of the world. Due to the meaning of (D)—whose independence of the state of the world is evident—we can understand (D) without taking into account who is just now the head of the Catholic Church. Thus the sentence “The head of the Catholic Church is visiting Africa” is fully intelligible and can be true or false independently of whether the sentence “John Paul II. is visiting Africa” is true or false.

So what is the denotation of (D)?

The *seeming* variability of the denotation can be illustrated as follows:

*If the world at the time moment  $T_1$  looks as follows . . . then the head of the Catholic Church is Pius XII.*

⋮

*If the world at the time moment  $T_k$  looks as follows . . . then the head of the Catholic Church is John Paul II.*

⋮

In general we can describe this situation as follows:

*The entities (here: descriptions) can be  $X$  depending on  $Y$ ,*

where  $X$  is an individual and  $Y$  are some empirical facts (conditions). Now a good general principle (formulated by Janssen (1986, p. 29) in a slightly

different wording) says:

*If an entity  $E$  can be  $X$  depending on  $Y$ , then the most natural (Jans) way of construing  $E$  is to say that  $E$  is a **function** (a **mapping**) the arguments of which are members of  $Y$  and the values are  $X$ s.*

This principle is, by the way, more than welcome by TIL, since the latter takes the notion *function* as the most important primitive notion. In our example  $X$ s will be individuals; what will be the arguments of the respective function? Verbally they will be the states of the world at various times. But logic can handle these abstract objects, if, of course, it is an intensional logic. An important feature of TIL is that it heeds the principle of extensionality with all its consequences. Transparent Intensional Logic flouts none of the principles of extensional logic and is, in this respect, an extensional logic.

In what follows some principles and notions of TIL are partly explained and partly presupposed. Everything necessary is contained in the monographs (Tichý 1988, Materna 1998, 2004) and in numerous articles, notably by Marie Duží, Bjørn Jespersen, see also <http://til.phil.muni.cz>.

To solve the problem of denotation of (D) we will need some definitions

DEFINITION (Types of order 1). (i)  $\iota, o, \tau, \omega$  are *types of order 1*.

(ii) Where  $\alpha, \beta_1, \dots, \beta_m$  are *types of order 1*,  $(\alpha\beta_1 \dots \beta_m)$ , i.e., the collection of all partial functions from  $\beta_1 \times \dots \times \beta$  to  $\alpha$ , is a *type of order 1*.

(iii) Whatever is a *type of order 1* is it only due to i) and ii). □

*Comments.* We will only briefly explain which general intuitions underlie the definition, more detailed comments can be found in the literature mentioned above.

$\iota$  is the universe, i.e., the collection of *individuals*. These are construed as *bare individuals*, so that the universe is shared by all possible worlds. Good arguments for this *anti-essentialism* can be found, e.g. in (Tichý 1983; 2004, pp. 505–523) or in (Jespersen, Materna 2002).

$o$  is the set  $\{\mathbf{T}, \mathbf{F}\}$  of *truth-values*. There are no further ‘truth-values’ for TIL, only partiality with *no value* on some arguments.

$\tau$  is the set of *real numbers*, which serves at the same time as the set of *time moments*.

$\omega$  is the logical space (relative to a given language), whose members are *possible worlds*, which are construed in the tractarian spirit and whose construal is perfectly justified in Chapter 11 (especially Section 38) of (Tichý 1988).

To illustrate the point ii), the types of following objects: *(an) even number, a class of even numbers, a class of individuals, smaller than, a binary relation between numbers, adding, a binary function of real numbers* are, respectively:

$$(o\tau), (o(o\tau)), (o(o\iota)), (o\tau\tau), (o(o\tau\tau)), (\tau\tau\tau), (o(\tau\tau\tau)).$$

Notice that any class/relation(-in-extension) is construed as the characteristic function thereof. The discourse in TIL is a *functional discourse*.

DEFINITION (Intensions). Let  $\alpha$  be any type<sup>9</sup>. Objects belonging to (or, equivalently: being of) the type  $(\alpha\omega)$ , very often  $((\alpha\tau)\omega)$ , are called *intensions*. Objects that are not *intensions* are called *extensions*.  $\square$

*Remark.* We abbreviate “ $((\alpha\tau)\omega)$ ” by “ $\alpha_{\tau\omega}$ ”.  $\square$

Some intensions frequently dealt with are:

*individual roles*, type  $\iota_{\tau\omega}$ : given a world W the respective individual role (called by Church *individual concept*) returns for any given time moment at most one individual.

*properties of individuals*, type  $(\iota\omega)_{\tau\omega}$ : For a world W and time T it returns a class (maybe empty) of individuals.

*propositions*, type  $o_{\tau\omega}$ : given a possible world W the function from times to truth-values (a *chronology*) in W gives at most one truth-value for any time moment.

We will exemplify these cases when solving the *denotation vs. reference* problem for the three examples announced above. So our present example is the case of definite description, and we are by now able to decide what the denotation of the description (D) is.

The parameters *possible world* and, more often than not, *time (moment)* are what can serve as the arguments of the function proposed by (1). From this, from our definitions and from the surely plausible decision that Xs must be individuals it follows that the type of the object denoted by (D) will be  $\iota_{\tau\omega}$ : it will be a function that on a given world at a given time returns at most one individual, so it will be an *individual role*. In other words, what (D) denotes is only the condition which an individual has to fulfill if it plays the role. Now from the mere type we cannot know this condition (the same type can be ascribed to indefinitely many different individual roles, given, e.g. by

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<sup>9</sup>This concerns not only types of order 1, but also types of higher orders, see below.

descriptions like *the highest mountain*, *the President of Czech Republic*, or even *the present King of France*) but the type-theoretical analysis is only the first step. What has to follow is to find the *meanings* of the particular descriptions, which will be explained later.

The key point is now clear: if an empirical description denotes an individual role, i.e., an intension, then the meaning must determine the conditions of fulfilling this role, so that we can truly state that this individual role, and so the denotation of the respective description, is no more dependent on empirical, i.e., external facts: it holds that it is not a function what is dependent on its arguments but only its value.

### Second example: a general expression

Let it be

(G) (a) tree

Being ‘spoiled’ by predicate logics we probably tend to say that (G) denotes a set/class. Let us examine for a while, what set it could be. Let us suppose that it is a set T1 of individuals:

$$\{i1, i2, \dots, ik, \dots, i10^9\}$$

Now during a short time interval the tree represented by the individual  $ik$  is destroyed. This means that the individual  $ik$  is no more a tree. We get a set T2, which is *different from* T1. It means that (G) denotes another set in virtue of some external event. Again, this is incompatible with our sound assumption that due to the meaning of (G) the denotation cannot change as the world changes.

The arguments of this kind can be called arguments from *temporal variability*. Besides, we can apply an argument from *modal variability*. The individual  $ik$ , which is the member of T1, is always the member of T1, also at such times when it is no more a tree. Membership in a set is a necessary relation. But nobody will admit that  $ik$ ’s being a tree is necessary: if it were then no external event could change this fact.

So an individual’s being a tree changes in time and is contingent. Therefore, the denotation of (G) cannot be a class of individuals.

Now we can again apply the principle (Jans). This time—since (G) is a general expression—the Xs (see (Jans)) are no more individuals, they are classes of individuals. The type of (*being a tree*) is therefore  $(ot)_{\tau\omega}$ , i.e., (G) denotes a *property of individuals*.

No property can be identified with its ‘population’ in some  $W$  at some  $T$ . Thus properties—as every intension—are independent of any empirical fact.

*Remark.* Empirical facts can cause, e.g. that a property that was had by some objects (e.g. individuals) becomes ‘empty property’, i.e., no object will have this property. What happened? (For example the case of extinct natural kinds, see *Dodo Ineptus*.) What changed is not the property itself but the *value* of one of *its* property, viz. the property *Non-empty*, type in this case  $(oo_{\tau\omega})_{\tau\omega}$ .  $\square$

### Third example: a sentence<sup>10</sup>

Let it be

(S) *Some teachers are shortsighted.*

According to Frege and, commonly, to some contemporary Fregeans, the denotation (or: the reference, the extension) of a sentence is its truth-value. This is accepted by all contextualists, including Montague: an expression denotes one thing in one kind of context and another thing in another context. Thus the contextualist will say that **S**, indeed, denotes *its* truth-value in a ‘normal’ (or: ‘direct’) context but something other (*its* sense) in another (‘indirect’, ‘opaque’) context. (Remember Montague’s  $\wedge$  and  $\vee$ .)

Our solution is ‘transparent’, i.e., anti-contextualistic. As in the preceding examples, here too we can state that temporal and modal variability exclude thinking of the denotation of (S) as of a truth-value: Indeed, I am convinced that in the actual world just now (S) is true, but this can change: maybe that once all teachers will possess so good eyes that they will not remember what the term ‘glasses’ means (temporal variability); besides, even if (S) is true in some  $W$  at the time moment  $T$  the truth of (S) is surely not necessary, it is no mathematical or logical truth, so that there are other worlds where (S) is false *at the same time moment*  $T$  (modal variability). So we have to apply the principle (Jans): the denotation of (S) is again a function from possible worlds to chronologies of truth-values, i.e., the type of the denotation of (S) is  $o_{\tau\omega}$ . The empirical sentence (S)—and we can generalize—every empirical sentence denotes a *proposition*<sup>11</sup>.

<sup>10</sup>An important supplement to this category will be added in Section 4.

<sup>11</sup>True, the term ‘proposition’ has got many interpretations in the history of philosophy/logic. Here we choose the interpretation offered by the P(ossible)-W(orld)S(emantics).

Summarizing, in all the three examples our analysis results in stating that the denotation of all of them is an *intension*. In consequence thereof the denotation is unambiguously determined<sup>12</sup> by the meaning, i.e., is independent of empirical facts.

Let us be allowed to continue our generalization: assuming that we know, which expressions are *empirical*, we can state: *Every empirical expression denotes an intension.*

This conjecture can be made more precise after we accept the following

DEFINITION (Non-trivial intensions). An intension is *trivial* iff its value is the same in all possible worlds and times (i.e., iff it is a constant function.)

An intension is *non-trivial* iff it is not a constant function. □

The more precise formulation of our conjecture is:

(Emp) *Every empirical expression denotes a non-trivial intension.*

(Emp) together with

(Anal) *An analytic sentence is true (false) in every world-time.*

implies the claim

*No empirical sentence is an analytic  
(i.e., analytically true or analytically false) sentence.*

Notice that (Emp) is incompatible with contextualism. A contextualist could protest: “You can see”, he would say, “that the sentence *Mont Blanc is lower than MtEverest* concerns an empirical proposition in the context *Charles knows that . . .* but it is only its truth-value what is of interest in the context *. . . and higher than Zugspitze.*” Later we will be able to show that (Emp) is not threatened by such examples.

Now we return to our three examples and consider the notion of *reference*. The first example was a definite description

(D) *the head of the Catholic Church*

We tried to show that under the assumptions described in Section 2 the denotation of (D) is an intension called *individual role*, i.e., an intension, type  $\iota_{\tau\omega}$ , and that this denotation is—as a function—immune from empirical facts and is related with the expression (D) *a priori* due to its meaning.

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<sup>12</sup>Which does not mean that the respective empirical expression is not *vague*, or *fuzzy*, if you like. The problem of vagueness will not be touched here.

On the other hand, to use (D) means to identify conditions of identifying the bearer of the role in the actual world-time, i.e., identifying the value of the role in the actual world-time. To find this value requires *ex definitione* possessing a knowledge that cannot be acquired *a priori*: a probe into reality is necessary, since the *actual value* of the individual role is, of course, dependent on empirical facts. In general, the actual value of an intension is what we call *reference* (viz. of the expression that denotes the given intension). Thus *reference, unlike the denotation, is related with the respective expression contingently, since no linguistic convention can determine what happens or will happen.*

In particular, no linguistic convention can guess right who the head of the Catholic Church is, say, in 2007.

But notice that the linguistic convention is sufficient to verify the claim *The Pope is the head of the Catholic Church.*

Let us consider the second example,

(G) (a) tree

We have seen that no definite class/set can be the denotation of (G). The linguistic convention, which ensures the *a priori* character of the link between an expression and its denotation, cannot determine the actual ‘population’ of the property *being a tree*. But knowing that (G) denotes a property we know the *criterion* according to which we can tell trees from non-trees (see, however, the footnote 12). Again, to apply this criterion, we need to be acquainted with empirical facts, ‘to go into reality’. So the reference cannot be ‘empirically innocent’ like the denotation.

Our last example,

(S) *Some teachers are shortsighted.*

has served to a demonstration of an analogous situation: The truth-value has been ascribed to this sentence not due a linguistic convention (which is not omniscient) but due to some constellation of empirical facts. The denotation has been derived as a function whose value at various world-times is a truth-value. The denotation of (S) is a proposition. The reference of (S) is again the actual value of this proposition. This actual value can be identified only empirically, so that again the reference of the empirical expression is not the same as its denotation.

*Remark.* As concerns mathematical / logical expressions, no difference between denotation and reference can be defined. The difference between a



function from worlds-times and its values is not applicable to such expressions because no possible worlds or instants of time are needed for their analyses.  $\square$

Summarizing we can say that *the denotation is a necessary link between an expression and the object denoted whereas the reference is a contingent, on empirical facts dependent link.*

From this viewpoint it is easy to refute Church's famous 'slingshot argument' in his (1956). This argument should prove that sentences denote truth-values: substituting gradually some expressions  $E_1, \dots, E_k$  for subexpressions  $F_1, \dots, F_k$  of the given sentence  $S$  transforms  $S$  to a sentence  $S'$ , where  $S$  and  $S'$  are completely distinct sentences, but if the expressions substituted for  $F_1, \dots, F_k$  are equivalent with them, respectively, then—according to the substitutability rule— $S'$  must be equivalent with  $S$ , so if we consider 'being equivalent with' as 'denoting the same object as',  $S$  must denote the same object as  $S'$ , and since these sentences are visibly distinct, the only object which both they denote must be the truth-value.

The problem with this argument is that equivalence of some  $E$  with an  $F$ , as presented in Church's example, is not equivalence in the logical sense: in particular, Church derives from the fact that the following sentences hold

“The number, such that Sir Walter Scott wrote that many Waverley Novels altogether, is twenty-nine” and “The number of counties in Utah is twenty-nine”

the conclusion that both have the same denotation because “The number, such that Sir Walter Scott wrote that many Waverley Novels altogether” has the same denotation as “The number of counties in Utah”. But this is not the case: Neither of these two expressions denotes a number—we have seen that no linguistic convention would be able to include variable empirical facts. So both they denote a 'magnitude', i.e., an intension (type  $\tau_{\tau\omega}$ ) that dependently on worlds-times returns a number. So both expressions are contingently coreferential. What Church's argument only proves is that if expressions  $E, E'$  are used *de re* and are coreferential so are the expressions  $F, F'$  that differ just by containing  $E$  instead of  $E'$ <sup>13</sup>.

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<sup>13</sup>One could ask: If an expression  $E$  denotes a magnitude rather than a number, how can we explain the fact that the sentence  $E = k$ , where 'k' denotes a number, can be true (as it is in both cases above)? We will answer this question later, now we can only informally suggest that the sentences above possessing seemingly the form  $E = k$  actually denote a proposition rather than a truth-value; the propositions denoted by both sentences take the value True in those possible worlds-times where the respective description takes the value k.

**(b) sense = intension?**

In (Kirkham 1992/1997, p. 4) we read:

The sense of an expression is often called the *connotation* or the *intension* of the expression, and the reference is often called the *denotation* or *extension* of the expression.

We have argued for the possible differentiation between denotation and reference in (a). Now we will demonstrate that if the Fregean *sense* has to fulfill the role of *mode of presentation* of the denotation then—at least for one interpretation of the term *intension*—sense cannot be an intension.

First of all, we have to state that the term *intension* has got more than one meaning in various areas but even within the philosophical framework. (Maybe because of this fact no entry “intension” can be up to now found in Stanford Encyclopedia of Philosophy.) It is for example not at all clear in which sense this term is used in Kirkham’s study (see the quotation). Here we will show that

- (i) there is (at least) one interpretation of the term *intension* which is partly compatible with the equation ‘sense = intension’, and
- (ii) the classical notion of intension used in P(ossible) W(orld) S(emantics), when applied in that equation, leads to unacceptable consequences.

*Remark.* Since our problem is the notion of *meaning* we will exploit the current interpretation of *meaning* as what Frege wanted to reach when using the term *sense*. Thus we can reformulate Kirkham’s hypothesis as *meaning = intension*? □

**Ad (i)** G. Bealer distinguishes one group of ‘intensional entities’ from another as follows:

[q]ualities, connections, and conditions are identical if and only if they are necessarily equivalent. However, though necessary equivalence is a necessary condition for the identity of concepts and thoughts, it is not a sufficient condition. (Bealer 1982, 10)

Summarizing, he then tries to express the distinction from another viewpoint:

Qualities, connections and conditions are the intensional entities that pertain to the world. Thoughts and concepts are those that pertain to thinking. And qualities, connections, conditions, thoughts, and concepts are all intensional entities there are. (Bealer 1982, 185)

The first quotation hits the essential distinction: the entities called by Bealer ‘qualities’, ‘connections’ and ‘conditions’ are *coarse-grained* while the entities

named ‘concepts’ and ‘thoughts’ are fine-grained. As for the ‘pertaining to the world’ we could say instead ‘being what our expressions are *about*’ (our *denotations!*) while ‘pertaining to thinking’ would be better replaced by ‘being *meanings*’<sup>14</sup>.

Now we could consider the possibility of identifying meanings with the second kind of Bealer’s intensions: we would like to accept that two distinct expressions sharing denotation could express distinct meanings. To build up a theory that will explain this unquestionable fact is an aim of any genuine semantic theory since Frege, who himself began to see this problem when he tried to save the principle of compositionality for the case of propositional attitudes (see his (1892)). Interestingly enough, Carnap also tried to solve the problem for the same case in (1947), but much more interestingly, one of the first logicians who accepted the distinction between coarse-grained and fine-grained semantic situations and tried to explain it logically in an ingenious way was Bernard Bolzano in (1837).<sup>15</sup>

It is unnecessary to retail again the whole story concerning the distinction between coarse-grained and fine-grained, a distinction, which most of the logicians (paradoxically, not Frege) consider to be relevant for theory of meaning. Setting aside for the present Tichý’s articles from 1968 and 1969 as well as TIL—both will be mentioned in **2**—we have to refer to David Lewis (who has presented a kind of structuredness in his (1972)), and to Max J. Cresswell with his promoting the term *hyperintensional* (1975) and *structured meaning* (1985).<sup>16</sup>

David Lewis has written:

Meanings may turn out to be complicated, infinite entities built up out of elements belonging to various ontological categories. (1972, 170)

To capture this complexity Lewis introduces categorial grammar and comes to the following characteristics of meanings: “finite ordered trees having at each node a category and an appropriate intension” (Lewis 1972, p. 182). The procedural factor is here represented by this idea of a tree (an idea playing a role after 10 years in Bealer’s *Quality and Concept*). Lewis works also with partial functions and is rather close to Tichý’s conception.

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<sup>14</sup>See (Materna 1998, pp. 75–77)

<sup>15</sup>Whereas Bolzano rightly stated (1837, §148) that the concept of triangle based on the property ‘having three sides’ differs from the concept of triangle based on the property ‘having the sum of its angles equal to 2R’ Bar-Hillel in his (1950!) suspected that this Bolzano’s statement could result in contradiction.

<sup>16</sup>As for criticism of Cresswell’s theory from the viewpoint of TIL see (Tichý 1994, Jespersen 2003, Materna 2004). See also later in the present study.

Cresswell—similarly as Lewis and Kaplan—models the complex character of meanings in terms of tuples (in this respect see our footnote 16). Could be his meanings something like the second kind of Bealer’s intensions?

I don’t think so. Cresswell’s meanings respect the desideratum that they should somehow correspond to the complex character of expressions, but they are still set-theoretical objects, as we can see from his result:

The meaning  $[\dots]$  is simply the  $n + 1$ -tuple consisting of the meaning of the functor together with the meanings of its arguments.

(Cresswell 1975, 30)

In contrast Bealer’s theory (at least in his (1982)) can be interpreted as suggesting some (tree-like) procedures producing the more fine-grained ‘intensions’.

**Ad (ii)** As for the classical notion of intension as used in PWS, the essential point is that intensions are here rather well-defined as functions from possible worlds. (In what follows nothing essential changes if the dependence on time, i.e., temporality is included, either so that intensions are functions from the pairs  $\langle W_i, T_j \rangle$ , or—as in TIL—that they are functions from possible worlds to *chronologies*, where a chronology is a function from time moments.)

Now the set-theoretical character of intensions is explicitly given. Assuming that we use the term *function* in the contemporary sense, i.e., as *mappings*, we can see that any intension in the PWS conception is a set (functions in this sense are sets) and can be imagined as (possibly infinite and containing heterogeneous arguments) *tables*, *schemes*.

The *first objection* to identifying meanings with PWS-intensions follows from our requirement that meanings should be more fine-grained than denotations. Intensions are just as coarse-grained as any extension. We can see it on a simple example:

We would like to distinguish between meanings of two equivalent sentences: “The Moon is smaller than the Earth” , “The Earth is bigger than the Moon”.

Let us fix a time moment  $T$  and imagine a table that represents an intension (here a proposition) that would be expressed by any of these sentences according to the criticized conception:

poss. worlds	truth-values
$W_1$	<b>T/F</b>
$W_2$	<b>T/F</b>
$\vdots$	$\vdots$

The point is that such a table is the same for both sentences: at whatever world is one of them true (false) the other one is true (false) as well. Besides, no way how to fulfill the Compositionality principle is thinkable: you can never recognize such an element of the meaning which would correspond to some subexpression of the given expression. More succinctly: meanings as such functions/tables are *simple*.

These objections would be sufficient for refusing PWS intensions as meanings. Still other objections can be given.

The *second objection* is also deadly for anyone who pretends to preserve the spirit of Frege's schema: If PWS intensions served as meanings, what kind of object would be the denoted object? Well, there is no other possibility than that it will be the value of the intension-meaning at the actual world-time. But then the assumption that the meaning unambiguously determines the denoted object cannot be fulfilled. The denotation becomes again a contingent link.

The *third objection*: What about non-empirical, in particular mathematical expressions? Since no possible worlds-times are needed in mathematics the notion of the meaning of a mathematical expression becomes an enigmatic notion. Or would anybody be ready to say that mathematical expressions lack meanings?

We have shown that the popular interpretation of Frege's semantic schema, at least as offered in Kirkham, is probably not the most trustworthy one from the historical viewpoint (one should not wonder, though, for Frege's schema is not unambiguously interpretable) but—what is far more important—that the criticized interpretations do not fulfill our semantic intuitions and are from the viewpoint of *logical analysis* simply unacceptable. Now we will not continue historical comparisons: instead we will formulate those principles of TIL that will enable us to defend the thesis that the borderline between analytic and synthetic sentences is well-definable.

### 3.2. Meanings as Constructions

Assuming, as we do, that objects *denoted* are extensions or intensions<sup>17</sup> (independently of any context) we have to explain which kind of entity would link the expression with the denotation, in other words, what we would call *meaning*, following in some respects Frege's intuition of *sense* as “mode of presentation of the denotation”.

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<sup>17</sup>We will see that they can be also *meanings* of other expressions.

The clue might be our intuition (mentioned previously) that meaning should be ‘more fine-grained’ than denotation. Indeed: if meaning *leads* to the denotation (if any), then no simple entity would do it.

Consider first an empirical expression. The post-Fregean interpretation of *sense* is based on the following idea: let *sense* be an intension in the sense of a *function* (from possible worlds), so the denotation will be the *value* of this function in the actual world. Yet from the viewpoint of any solid theory of complexes functions are ‘complex-theoretically’ simple. They are—as mappings—not compound in the sense in which, e.g. algorithms with their steps / instructions are. To exploit a previous example with the sentences

“The Moon is smaller than the Earth”,

“The Earth is bigger than the Moon”,

nothing in the set that represents the mapping from possible worlds to (chronologies of) truth-values can be said to correspond to particular subexpressions of both sentences. The fact that both sentences somehow correspond to just this one set is not and cannot be so explained.

Besides, we would surely assume that the meaning of an expression E should be in some way derivable from the structure of E. Unless we accept the pragmatic resignation on semantic analysis we suppose that some *procedure* should realize the way from meanings of particular subexpressions of E to the procedure  $\mathbf{P}_E$  (=the meaning of E), and that the intension denoted by E should be the outcome of  $\mathbf{P}_E$ . No other option seems to be acceptable or even imaginable. (True, if we did not want to quit the area of set-theoretical objects we could try to say that the resulting intension could be the value of another function-mapping. But which one? Do we have some clues in the structure of E? And would we not start a new *regressus ad infinitum*?)

What about the meaning of simple linguistic units, say, of one-word expressions that we suppose not to be definitional abbreviations? Again, there seems not to be another option than a procedure; here the procedure has to be a simple procedure, but in any theory of procedures / complexes we have to stipulate a lowest level of simples.

Going over to mathematical expressions, the situation is very clear. A mathematical expression E denotes an object due to some *calculation*, i.e., a(n abstract) procedure that presents the object denoted (if any) in virtue of some (abstract) steps prescribed by particular subexpressions of E. We adduce an extremely simple example, which will make it possible to define

some simple procedures:

$$(M) \quad \neg(3: 2 > 2)$$

The object denoted by a mathematical sentence is best construed as a truth-value (Frege and Church were right in the area of mathematical sentences). Here we get the truth-value **T**. Now which extra-linguistic entity<sup>18</sup> links the expression (M) with the result **T**? In other words: what should we *do* if we want to come to this result once we understand (M)?

Understanding (M) involves understanding

- a) of particular ‘atomic’ subexpressions, i.e., of ‘ $\neg$ ’, ‘:’, ‘ $>$ ’, ‘2’, ‘3’; and
- b) of operations to be gradually realized, as determined by the syntax of (M).

As for a): One option (here the best one, the other options depend on the ‘conceptual system’ we use (see later)) is to use a simple procedure, say, <sup>0</sup>, such that if it is applied to an object O then the outcome is O without any change. This procedure is used in TIL and called *trivialization*.

As for b): With the exception of 2 and 3 the other objects that are outcomes of trivialization applied in (M) are best modeled as functions. We ascribe type  $\tau$  to 2 and 3, type  $(oo)$  to  $\neg$ , type  $(\tau\tau\tau)$  to  $:$  and type  $(o\tau\tau)$  to  $>$ . (Notation:  $2, 3/\tau, \neg/(oo), : /(\tau\tau\tau), > /(\tau\tau\tau)$ .) The syntax of (M) suggests that functions denoted by the respective atomic expressions have to be *applied to* their arguments. The procedure consisting in applying function to argument is called *composition* in TIL and is denoted  $[XX_1 \dots X_m]$ , where X is a procedure that constructs an  $m$ -adic function and  $X_1$  through  $X_m$  are procedures constructing the  $m$ -tuple that makes up the argument. As the result of the logical analysis of (M) we get the following procedure:

$$(M') \quad [^0\neg[^0> [^0: ^03 ^02] ^02]]$$

The terminology used in TIL calls procedures *constructions*. (M') is the way of fixing one such construction. Checking particular types and gradually building up their synthesis in (M') we get the type  $o$ , which is in harmony with the fact that (M) denotes a truth-value.

Importantly, there are cases (just in the case of composition) when nothing is constructed. Such a composition is called *improper* (or *v-improper*, see below) and its improperness is caused by the fact that the function constructed by X is not defined on the argument constructed by  $\langle X_1, \dots, X_m \rangle$ . Since TIL works with partial functions, such value-gaps frequently occur in its analyses. Easy examples: dividing by 0.

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<sup>18</sup>I.e. a *semantic* entity!

Up to now we have been acquainted with two constructions: trivialization and composition. We will need still two other from among those ones defined in TIL<sup>19</sup>:

*Variables:* To any type we have at our disposal a countably infinite set of variables. These are abstract incomplete constructions<sup>20</sup> named, as the case may be, by the well-known letters  $x, x_1, \dots, y_1, y_2, \dots$  and constructing the objects of the respective types dependently on a *valuation*. So any variable, as well as any construction containing a variable is said to *v-construct*, where  $v$  is the parameter of (Tarskian) valuation.

*Closures:* We know up to now that constructions construct particular objects (in virtue of trivializations and variables) and values (if any) of functions at their arguments. The last construction we will need here can *create functions*. Since the system of constructions has been strongly inspired by (typed)  $\lambda$ -calculus we will not be surprised when this construction (called *closure*) will correspond (on the objectual level) to *abstraction* in  $\lambda$ -calculus. So let  $x_1, \dots, x_m$  be various distinct variables *v-constructing* objects of (not necessarily distinct) types  $\beta_1, \dots, \beta_m$ , and let  $X$  be a construction that *v-constructs* objects of type  $\alpha$ . Then

$$[\lambda x_1 \dots x_m X]$$

(with possible omitting of the outmost brackets) is a construction called *closure*. This construction (*v-*)constructs a function  $F/(\alpha\beta_1 \dots \beta_m)$  as follows: Let  $b_1, \dots, b_m$  be objects of the types  $\beta_1, \dots, \beta_m$ , respectively.  $F$  returns on  $\langle b_1, \dots, b_m \rangle$  the value (if any) that is *v'-constructed* by  $X$ , where  $v'$  associates  $x_1, \dots, x_m$  with  $b_1, \dots, b_m$ , respectively, and is otherwise identical with  $v$ . If  $X$  is *v'-improper*, then  $F$  is undefined on  $\langle b_1, \dots, b_m \rangle$ .

All these constructions are procedures, i.e., they cannot be identified with the notation that fixes these procedures. Thus whereas  $[^0\text{-}[^0\text{>}[^0\text{:}^0\text{3}^0\text{2}]^0\text{2}]]$  constructs  $\mathbf{T}$  the inscription  $\text{'[^0\text{-}[^0\text{>}[^0\text{:}^0\text{3}^0\text{2}]^0\text{2}]\text{'}$  does not construct anything, and whereas  $\text{'[^0\text{-}[^0\text{>}[^0\text{:}^0\text{3}^0\text{2}]^0\text{2}]\text{'}$  contains brackets, the construction  $[^0\text{-}[^0\text{>}[^0\text{:}^0\text{3}^0\text{2}]^0\text{2}]]$  cannot contain any sign: no procedure can contain brackets.

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<sup>19</sup>The definitions of all of them can be found in Tichý (1988), Materna (2004) and in numerous papers concerning TIL, see <http://til.phil.muni.cz>.

<sup>20</sup>Since constructions are extra-linguistic procedures it is of key importance that also variables are defined as extra-linguistic entities and that the letters we are used to call 'variables' are only names of variables. See (Tichý 1988, p. 56–62).



The idea of structured, complex meaning is implicitly contained in (Bolzano's 1837). The first suggestion of a modern realization of this idea can be found in two early articles by Tichý, which appeared in 1968 and 1969. In both of them semantics was connected with the theory of recursive functions (in particular with Turing machines). In (Tichý 1968) we read:

Results of the theory of effective procedures have been intensively applied in the study of syntax of linguistic systems, when analyzing decidability and completeness problems, etc. On the other hand, the notion of an effective procedure plays an almost negligible role in current logical semantics. ... (But...) it is easy to see that, taken in an abstract way, the relation between sentences and procedures is of a semantic nature; for sentences are used to record the results of performing particular procedures. (Tichý 2004, 79–80)

Afterwards Tichý develops (as early as in 1968) a remarkable theory of language and specifies his conception by modeling meaning *via* Turing machines. In more details is this conception realized in (1969).

It is already in these two articles where Tichý basically solves the problem of explicating the semantic character of analytic sentences. We will show this solution in terms of constructions in the end of the present paper.

We have already mentioned some suggestions of doing a procedural semantics to be found in Bealer, Lewis and Cresswell.

Cresswell, Chierchia *et alii* were well aware of the necessity to let semantic structures correspond in some way to the syntactic structures of the given language. As Chierchia in (1989), considering semantics of distinct sentences  $\psi$  and  $\varphi$ , says:

[h]owever strong our notion of equivalence, we still might not see that  $\psi$  and  $\varphi$  are associated with the same region of the logical space and hence we might have different attitudes towards them. This seems to suggest that at some level of content sentence-structure has to be preserved. (Chierchia 1989, 134)

To preserve, however, the sentence-structure, does not yet mean to do procedural semantics. The way the structured meaning is modeled by Cresswell *et alii* consists in using *tuples*. A thorough criticism of the 'tuple method' can be found in (Tichý 2004, pp. 835–841), see also (Jespersen 2003, pp. 171–183). The main objection can be articulated as follows: We can find in the respective tuples elements that correspond to the respective subexpressions of a given expression, but tuples—as they are defined—represent only the particular parts of what should explicate the meaning. From the definition

of tuples we cannot derive any sequence of steps to be made in the respective *procedure*: the difference between the latter and the tuple in question reminds us of the difference between the ‘content’ of a concept and the concept itself, as shown in (Bolzano 1837, p. 244). True, the elements of the meaning are at least ordered in the tuple theory, but this improvement only duplicates the way the particular subexpressions are grammatically related, which is insufficient from the viewpoint of procedural theory.

What is closer to the idea of procedures as meanings are the systems built up by Y.N. Moschovakis (see, e.g. his (1994), the symptomatic title of which is “Sense and denotation as Algorithm and Value”, or his (2006)). We will, however, continue expounding the more general system of TIL.

To proceed further we will adduce a simple example of a construction that should model the meaning of an empirical expression, say, an empirical sentence:

(S) *Some teachers are shortsighted.*

First we determine the types of ‘atoms’:

‘*Some*’ could denote the standard existential quantifier but here it seems that another kind of existential quantifier is welcome: a function that being applied to a class  $K$  of individuals returns the class of those classes of individuals which share with  $K$  at least one member. So we have *Some* /  $((o(ol))(ol))$ .

‘*teacher*’ denotes a property of individuals. So we have *Teach* /  $(ol)_{\tau\omega}$ .

‘*shortsighted*’ denotes a property of individuals as well. So we have *Shrts* /  $(ol)_{\tau\omega}$ .

(S), as an empirical sentence, denotes a proposition, so the meaning of (S) (a construction) must construct a function, type  $o_{\tau\omega}(= ((o\tau)\omega))$ .

A typical construction of a proposition begins with  $\lambda w \lambda t, w \rightarrow \omega, t \rightarrow \tau$ ; the rest must *v*-construct (see above) a truth-value. We now present such a construction for (S); since no added information is available we are bound to analyze simple expressions *via* trivialization.

We get the figure 1 (see p. 29).

The suggested procedure constructs a proposition; a function that—in our case—takes  $\mathbf{T}$  at such worlds-times WT where the class of individuals that are shortsighted at WT has at least one member in common with the class of individuals that are teachers at WT.

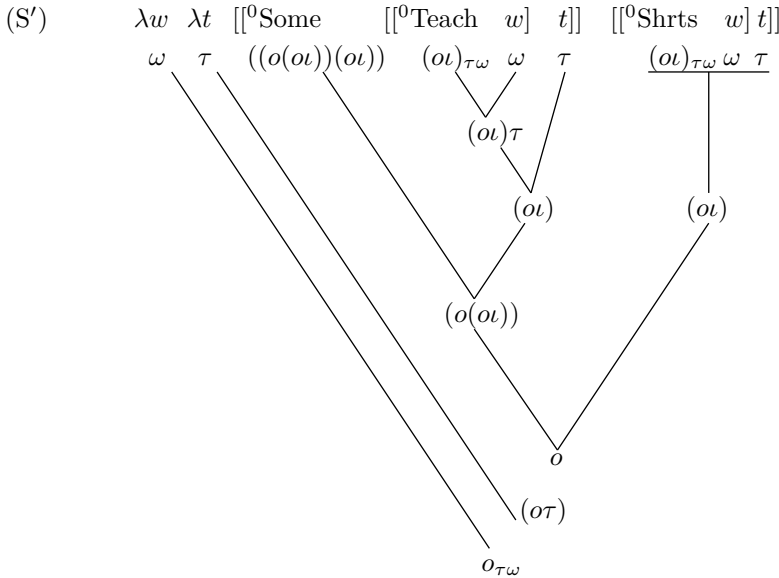


Figure 1.

We can see that some evident rules (not to be formulated here) would help us to find and with respective expressions associate such constructions—at least if the expressions in question are not in an up to now unpredictable way complex.

Now we will show that our definitions would not suffice to analyze in this way some kind of expressions. Expressions of that kind traditionally serve as indicators of our ability to build up sufficiently fine-grained means of logical analysis. It is this kind of expressions that has signaled a problem for Frege (leading to his unfortunate contextualism), Carnap (‘intensional isomorphism’), Church (‘synonymous isomorphism’)<sup>21</sup> and numerous other contemporary logicians. I mean *attitudes*, in particular propositional attitudes.

So our example will be of this kind. Consider the sentences

(E2) *Charles does not believe that 2 is a prime.*

Ch(arles) /  $\iota, not = \neg/(oo), 2/\tau, Pr(ime)/(o\tau)$ . What type would we associate with Bel(ieve)? It is an empirical relation of an individual to X. What can be X?

Options: a) X / o, b) X is an expression (sentence), c) X is a construction.

<sup>21</sup>Cf. Anderson’s (1998) for this history.

We can immediately edge out the option a): believing etc. surely is no attitude to a truth-value. Besides, which truth-value would come into question?

As for b), Charles' attitude does not concern the English sentence. Charles may not understand English and all the same believe / not believe that 2 is a prime.

What remains is the option c); obviously this is the right option: Charles does not believe that the procedure connected with verifying the claim 2 *is* a *prime* results in **T**.

But whereas we know that the type of the object constructed by

$$[{}^0\text{Pr}^0 2]$$

is, of course, *o*, we do not know the type of the construction itself.

*Abbreviation.* We choose a symbolic fixation of the difference between *X is of the type  $\alpha$*  and *(the construction) X constructs an object of the type  $\alpha$* : : In the former case we write

$$X/\alpha,$$

in the latter case

$$X \rightarrow \alpha.$$

□

Thus we know that  $[{}^0\text{Pr}^0 2] \rightarrow o$  but we do not know the type  $\alpha$  such as  $[{}^0\text{Pr}^0 2]/\alpha$ .

To fill up this gap we have to extend the set of types. We have to define *types of higher order*, which will make it possible to ascribe types also to constructions and enable us not only to *use* constructions but also to *mention* them.

The exact inductive definition of the resulting *ramified hierarchy of types* can be found in Tichý (1988) as well as in other TIL works. Here we suggest only briefly the core of the definition:

*Types of order 1* have been already defined. Now first *constructions of order  $n$*  are defined (they always construct objects of order  $m, m < n$ ) and then the set  $*_n$  is defined as the set of all constructions of order  $n$ .  $*_n$  and all types of order  $n$  are *types of order  $n + 1$* . Further, if among the types  $\alpha, \beta_1, \dots, \beta_m$  there is some type of order  $n + 1$ , then the type  $(\alpha\beta_1 \dots \beta_m)$  is a *type of order  $n + 1$*  as well.

From the exact formulation of the definition it follows that if  $X / \alpha$  and  $\alpha$  is a type of order  $n$ , then  ${}^0X$  is of a type of order  $n + 1$ .

Now we are able to write down the construction that is expressed by the sentence (E2)<sup>22</sup>:

$$(E2') \quad \lambda w \lambda t [{}^0\neg [{}^0\text{Bel}_{wt} {}^0\text{Ch} {}^0[{}^0\text{Pr} {}^02]]]$$

Notice that the construction  $[{}^0\text{Pr} {}^02]$  (i.e. the meaning of the sentence 2 *is a prime*) is here *mentioned*, i.e., this procedure is not to be realized if the procedure (E2') is realized. If it were not trivialized, we would get a non-construction because the believing relation would connect Charles with the result of  $[{}^0\text{Pr} {}^02]$ , i.e., with a truth-value, which would be absurd, of course. The relation Bel gets thus the type  $(oi*_1)_{\tau\omega}$ .

Due to the suggested extension of the set of types TIL becomes very expressive, and it can be proved, among other things, that it is immune from various slingshot arguments (in particular Gödel's and Church's)<sup>23</sup> trying to reduce the amount of entities relevant for logical analysis of language.

In particular, attitudes of various kind are analyzable by TIL even when hyperintensional level is necessary (this point is in favor of TIL as compared with, say, Montague's IL).

In Section 1, point b, we have promised to show that

*a purely semantic notion of meaning is definable so that more pretentious commitments than those ones connected with mathematics and/or logic are not needed.*

In our opinion this point has been fulfilled: We have defined meaning as an *abstract* procedure so that we do not transcend the area of semantics; the respective commitment consists in admitting that there are objective abstract procedures. True, this could be a dubious commitment for Quine, for whom even objective intensions like properties were 'conceived in sin', but is this famous Quine's aversion something more than a kind of ideology? Are sets—as the only abstract entities approved by Quine—better defined than functions? And are objective algorithms—as senses of programs (as Moschovakis would have it)—less admissible than sets?

Now we can try to fulfill our promise from Section 1. Point c to show that

*the difference between analytic and synthetic (empirical) sentences is definable on the basis of the points a) and b) so that the positive core of Quine's theory is inapplicable as a criticism of the resulting theory.*

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<sup>22</sup>We use the abbreviation introduced in TIL, viz.  $X_{wt}$  instead of  $[[Xw]t]$ .

<sup>23</sup>See (Neale 1995) for more details.

Before then, however, i) one point concerning contextualism should be elucidated and ii) the category of *concepts* introduced.

Ad i): Returning to a remark in III.1) we will consider two sentences:

- A. *Charles knows that Mont Blanc is lower than MtEverest*
- B. *Mont Blanc is lower than MtEverest and higher than Zugspitze.*

We have suggested that a contextualist could claim that in A. the sentence *Mont Blanc is lower than MtEverest* denotes a proposition (knowing is a relation that links individuals with propositions) whereas in B. this sentence denotes a truth-value (the truth function denoted by *and* concerns truth-values rather than propositions). TIL refutes this claim and defends the following principle holding for all non-indexicals:

*What is dependent on context is a supposition, never the denotation or meaning.*

A modification of the old category *de dicto* vs. *de re* makes it possible to argue that an expression always denotes an extension or always denotes an intension and that the meaning of an expression is the same in all contexts. Meaning—a construction in TIL—either constructs an intension without applying it to a world-time (then we mention the intension, not being interested in its value in a given world-time; the case *de dicto*) or the intension is constructed together with its application to world-times *via* composing it with the variables  $w$  or  $w, t$  (then we are after the value of the intension in the given world-time, the case *de re*). Exploiting this distinction we analyze A. and B. as follows:

(A')  $\lambda w \lambda t [{}^0 \text{Kn}_{wt} {}^0 \text{Ch} \lambda w \lambda t [{}^0 \text{Lower}_{wt} {}^0 \text{Mb}_{wt} {}^0 \text{Mte}_{wt}]]$

(B')  $\lambda w \lambda t [{}^0 \wedge [\lambda w \lambda t [{}^0 \text{Lower}_{wt} {}^0 \text{Mb}_{wt} {}^0 \text{Mte}_{wt}]_{wt}] [\lambda w \lambda t [{}^0 \text{Higher}_{wt} {}^0 \text{Mb}_{wt} {}^0 \text{Z}_{wt}]_{wt}]]$

( $\wedge / (ooo)$ , Ch/ $\iota$ , Mb, Mte, Z/ $\iota_{\tau\omega}$ , Lower, Higher/ $(ou)_{\tau\omega}$ , Kn/ $(oio_{\tau\omega})_{\tau\omega}$ .)

The meaning of the sentence *Mont Blanc is lower than MtEverest* is the same in A. and B.: it is the construction

(C')  $\lambda w \lambda t [{}^0 \text{Lower}_{wt} {}^0 \text{Mb}_{wt} {}^0 \text{Mte}_{wt}]$

The difference (given by the context) does not concern meaning, it concerns exclusively the *supposition*: In (A') this construction is in the supposition *de dicto*: no 'intensional descent' (i.e., application to  $w, t$ ) is present. In (B') the context requires that truth-values come into play, but it does not mean

that the sentence *Mont Blanc is lower than MtEverest* will begin to denote a truth-value: the requirement is fulfilled even so, due to the *de re* supposition of (C') in (B').

Ad ii): Since our main problem concerns the possibility of 'hard and fast line' between analytic and synthetic sentences, where the problem of indexicals is not relevant, a special class of meanings is important for us: the class of *concepts*. We dedicate to this class a separate section:

### 3.3. Concepts

The meaning of any such expression that does not contain any indexical is a closed construction, i.e., a construction that does not contain any free variable.

Informally, a variable is *bound* in (a formal expression or a) construction if it cannot be logically handled: No valuation can be considered, nothing can be substituted for it. In predicate logics variables are bound by quantifiers and—as the case may be—by the descriptive operator. In TIL there are two kinds of boundness:

DEFINITION ( $\lambda$ -bound and  $^0$ bound variables). If X is any construction then any occurrences of variables (even those ones where the variable is bound by  $\lambda$ ) in  $^0X$  are  *$^0$ bound* ("bound by trivialization"). If X is  $\lambda x_1 \dots x_m Y$  then any variable  $x_i$ ,  $1 \leq i \leq m$ , in Y is  $\lambda$ -bound in X unless it is  $^0$ bound in Y.  $\square$

For illustration, consider the following constructions:

- |      |                                 |
|------|---------------------------------|
| (C1) | $\lambda x_1[^0 > x_1^0 0]$     |
| (C2) | $\lambda x_2[^0 > x_2^0 0]$     |
| (C3) | $^0[\lambda x_1[^0 > x_1^0 0]]$ |
| (C4) | $^0[\lambda x_2[^0 > x_2^0 0]]$ |

Notice that while (C1) constructs the same object as (C2) (the class of positive numbers) we cannot say the same about (C3) and (C4): the former constructs the construction (C1), the latter the construction (C2), and even when (C1) is equivalent to (C2), they are distinct constructions. The difference between them is, of course, trifling, and therefore the  $\lambda$ -terms of the forms (C1), (C2) are considered to be identical in  $\lambda$ -calculus (the ' $\alpha$ -rule').

Since trivialization means always that what is under trivialization is mentioned we can say that the <sup>0</sup>bound variables are mentioned; therefore they cannot be substituted for by anything.<sup>24</sup>

The simplest definition of concepts would be: *Concepts* are closed constructions. (So they can be meanings of expressions that do not contain indexicals.) A proviso is, however, necessary, otherwise we would have to say that distinct but ‘ $\alpha$ -equivalent’ (closed) constructions are distinct concepts. Thus (C1), (C2) and infinitely many similar ( $\alpha$ -equivalent, i.e., only by bound variables differing) constructions would be distinct concepts, which contradicts our intuition—here all such ‘(C1)-like’ constructions represent one and the same concept expressed by the expression *numbers greater than zero*. Simplifying a little we can therefore define:

DEFINITION (Concept). A *concept* is a procedure represented by any two  $\alpha$ -equivalent (closed) constructions.  $\square$

This definition is a simplified version of the definition in (Materna 2004, HDefinition 12, p. 56).<sup>25</sup>

This explication makes it possible to say that there are more (theoretically infinitely many) concepts of one and the same object, which cannot be said if concept is explicated as a set-theoretical entity (classical case: Frege’s *Begriff*, see Frege (1891, 1892a)). It is in harmony with Bolzano’s *Begriff* from his (1837) and satisfies some intuitions in Bealer (1982). It would meet some sympathy on the part of intuitionists and constructivists if their ‘constructions’ were not defined in an anti-realistic way. A good non-circular explication of *synonymy* as distinct from a mere *equivalence* is now feasible in terms of concept and denotation.

DEFINITION (Synonymous expressions). An expression E is *synonymous with* an expression E’ iff E and E’ express the same concept.  $\square$

DEFINITION (Weakly equivalent expressions). An expression E is *equivalent with* an expression E’ iff E and E’ denote the same object. An expression E is *weakly equivalent with* an expression E’ iff they are equivalent but not synonymous.  $\square$

DEFINITION (Coreferential expressions). An empirical expression E is *coreferential with* an expression E’ iff E denotes an intension I, E’ denotes an

<sup>24</sup>Some combination of procedures can neutralize this limitation.

<sup>25</sup>In particular, also  $\eta$ -equivalent constructions can be identified in this sense, e.g. <sup>0</sup>sister and  $\lambda w \lambda t \lambda x y [^0 \textit{sister}_{wt} xy]$ . HDefinition reproduces a proposal in Horák (2001).



intension  $I'$ ,  $I \neq I'$  and the value of  $I$  in the actual world-time is the same as the value of  $I'$ .  $\square$

Once concept has been exactly defined all the three definitions above are exact. And in principle—that is, assuming that the given language is understood (see Section 2)—we can check for any pair of expressions whether they are synonymous, equivalent or only coreferential.

#### 4. Analytic vs. Synthetic: Definability of the borderline vindicated (ad c)

It is obvious that Quine's arguments in (1953), where circularity—and thus impossibility—of defining analyticity is argued for, is as for its cogency dependent on Quine's refusal to accept the notion of meaning as a notion of a legitimate and independently of notions of analyticity and synonymy definable entity<sup>26</sup>. Quine says:

Once the theory of meaning is sharply separated from the theory of reference, it is a short step to recognizing as the primary business of the theory of meaning simply the synonymy of linguistic forms and the analyticity of statements; *meanings themselves, as obscure intermediary entities, may well be abandoned.* (Emphasis mine, P.M.)  
(Quine 1953, 22)

But indeed, if you wave aside meanings as *intermediary entities* (because they are *obscure*) you should not wonder that neither analyticity nor synonymy can be satisfactorily (or at all) defined. Our intuition is that analyticity as well as synonymy can be defined just in terms of *meaning*. An easy definition of synonymy has been formulated in Section 3.3, as for analyticity we can offer the following definition:

**DEFINITION** (Analyticity). A sentence is *analytically true* iff its meaning constructs **T** (mathematical sentence) or the proposition **TRUE**.<sup>27</sup>

A sentence is *analytically false* iff its meaning constructs **F** (mathematical sentence) or the proposition **FALSE**.

A sentence is *analytic* iff it is analytically true or analytically false.  $\square$

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<sup>26</sup>See also Materna (2004a)

<sup>27</sup>Proposition **TRUE** is a constant function whose value is **T** in every possible world-time.

*A simplifying convention:* I will use the term *analytic sentence* instead of *analytically true sentence*. This convention is often made implicitly, and Quine, e.g. says:

[a] sentence is analytic, in mentalistic sense, when it is true by virtue of the meanings of its words.<sup>28</sup> (Quine 1992, 55)

Thus it seems now that as soon as we refuse to believe that meanings are obscure entities and offer a definition of meaning independent of the other two semantic notions we have won our struggle for definability of the distinction between *analytic* and *synthetic*. Yet the problem is not that simple. To show this we again quote Quine:

There are those who find it soothing to say that the analytic statements of the second class<sup>29</sup> reduce to those of the first class, the logical truths, by *definition*; ‘bachelor’, for example, is *defined* as ‘unmarried man’. But how do we find that ‘bachelor’ is defined as ‘unmarried man’? Who defined it thus, and when? (Quine 1953, 24)

Notice that Quine’s rhetoric question is justified as soon as problems we want to solve are problems concerning a general (empirical) theory of language. As soon, however, as we want to do logical semantics or LANL we have to accept the LANL PRINCIPLE (see the outset of Section 2). Quine’s question is incompatible with this principle.

All the same, the general problem of the relation between simple and compound concepts (meanings)<sup>30</sup> is an interesting problem even from the viewpoint of LANL. So let us first define:

DEFINITION (Simple concepts). Let  $X$  be any object of a type of order 1. Then  ${}^0X$  is a *simple concept*.  $\square$

According to our definitions a simple concept constructs the object  $X$  without any change. The problem mentioned by Quine consists in determining the relation between the simple concept  ${}^0$ bachelor and the compound

<sup>28</sup>A characteristic feature of Quine’s philosophy is Quine’s rather implicit belief that what we call ‘semantic’ notions can be either mentalistic (and thus unacceptable) or behavioristic notions. The latter are accepted by Quine (remember *stimulus meaning*); the possibility of *logical analysis* is inaccessible to him.

<sup>29</sup>I.e., analytic sentences as compared with logically true sentences (the latter belonging to the ‘first class’).

<sup>30</sup>As far as our considerations concern non-indexical expressions we can use *meaning* and *concept* as synonyms, at least if the object denoted by the respective expression is not an infinite function. See below.

concept that underlies the expression *unmarried man*. Let us try to find this latter concept.

We can do it *via* combining some other simple concepts. So the following simple concepts will be the particular components:  ${}^0\neg(\rightarrow (oo))$ ,  ${}^0\wedge (\rightarrow (ooo))$ ,  ${}^0\text{Married}(\rightarrow (o\iota)_{\tau\omega})$ ,  ${}^0\text{Man}(\rightarrow (o\iota)_{\tau\omega})$ ,  $x \rightarrow \iota$ . The result will be

$$(MM) \quad \lambda w \lambda t [\lambda x [{}^0\wedge [{}^0\neg [{}^0\text{Married}_{wt}x]] [{}^0\text{Man}_{wt}x]]].$$

To be able to rationally solve the problem of the relation between the simple concept  ${}^0\text{bachelor}$  and the compound concept (MM) we have to say something more about concepts and language.

Not only mathematics but also any natural language is full of *abbreviations*. Applying the LANL PRINCIPLE we have to take this fact into account. Indeed, otherwise we could not understand the intuitive principle according to which

**(Underst)** *understanding an expression E means knowing the underlying construction, i.e., knowing the procedure* (Tichý: “intellectual journey”) *whose realization identifies what E denotes.*<sup>31</sup>

For if you neglect the fact of existence of abbreviations (and, therefore, accept the wrong hypothesis that the meaning of any simple expression is a simple concept) you cannot explain that you understand, say, the expression *prime number*: this expression is a simple expression<sup>32</sup> and our wrong hypothesis would mean that the simple concept

$${}^0\text{Prime\_number}$$

would be the meaning of the expression *prime number*, which would mean (according to **(Underst)**) that we understand the expression because we know the respective procedure. But this procedure consists in taking the (infinite) class of prime numbers without any change and without any help of other concepts. This is what we mortals are unable to do. So why do we all the same understand the expression *prime number*?

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<sup>31</sup>This principle is well compatible with the fact that understanding an expression E does not automatically mean knowing the denotation of E. (Do you understand Goldbach’s conjecture? And do you know whether it is true?) An important exception: *knowing the procedure given by a simple concept we automatically know the respective denotation, for to know  ${}^0X$  means to know X.*

<sup>32</sup>‘Compound’ idioms like *old maid* are only seemingly compound: they do not obey the compositionality principle (thus inspiring many jokes). Therefore *prime number* is an idiom and so a simple expression.

The answer is: *prime number* is an abbreviation; the reason why we do understand it is that we are acquainted with the procedure that underlies the *Definiens* of some of definitions of prime numbers. Take the following one:

(Def) *a natural number possessing just two factors*

The respective construction is

(Divisible/  $(o\tau\tau)$ , Nat(ural number)/ $(o\tau)$ , Card(inality)/ $(\tau(o\tau))$ ,  $\wedge/(ooo)$ , =  $/(o\tau\tau)$ ,  $2/\tau$ ,  $x, y \rightarrow \tau$ ):

(Def')  $\lambda x[{}^0\wedge [{}^0\text{Nat}x][{}^0 = [{}^0\text{Card } \lambda y[{}^0\text{Divisible } x y]]{}^0 2]]$

Now the harmony with (**Underst**) is attainable: Let  $a$  be any number. The procedure (Def') consists in following steps:

i) Is  $a$  a natural number? If not,  $a$  is not a prime number. If so, ii) calculate the number of factors of  $a$ . ( ${}^0\text{Divisible}$ ). iii) Is this number distinct from 2? If so,  $a$  is not a prime number. If not,  $a$  is a prime number.

Every step is a finite calculation, and there are always finitely many such steps. This explains why we are able to understand the expression *prime number*.

(Well,  ${}^0\text{Divisible}$ , as well as  ${}^0\text{Nat}$  and  ${}^0\text{Card}$ , also identify infinite classes, but using them in (Def') requires only applying them to a particular argument, and this is feasible due to the recursive character of both concepts. See (Duží, Materna 2004).)

We can summarize:

*If an expression E denotes the object X and if we consider some definition D of X then  ${}^0X$  is a simple concept equivalent to the meaning of E, whereas the meaning of E is the construction that underlies the Definiens of D.*

Accepting LANL PRINCIPLE we can now solve our problem and answer, in a sense, Quine's rhetorical question. LANL PRINCIPLE presupposes that we understand the given language (which is given as a set of lexical units together with grammatical and semantic rules) and so the simple expressions either primarily or (the case of abbreviations) on the base of definitions. To model this situation we can define what we have called *conceptual systems* in (Materna 1998, 2004).

**DEFINITION** (Conceptual system). Given an infinite set of types (in TIL see Definition of types of order 1 and ramified hierarchy of types) and an infinite

set of variables ranging over this set a *conceptual system* is determined by a finite set  $Pr$  of simple concepts (called ‘*primitive concepts*’). If a non-empty subset of  $Pr$  contains logical/mathematical concepts then an infinite set  $Der$  of *derived concepts* is unambiguously determined by  $Pr$ : its members are compound concepts whose subconstructions that are simple concepts are members of  $Pr$ .  $\square$

Now suppose (a non-realistic example) that a part of a conceptual system underlying the contemporary English contains in  $Pr$  concepts

$${}^0\neg, {}^0\wedge, {}^0\text{Married}, {}^0\text{Man};$$

among the members of  $Der$  we certainly find

$$\text{(Bach)} \quad \lambda w \lambda t [\lambda x [{}^0\wedge [{}^0\text{Man}_{wt} x] [{}^0\neg [{}^0\text{Married}_{wt} x]]]].$$

As every normal speaker of English knows, the word *bachelor* has been chosen (by the anonymous linguistic convention) as an abbreviation for the expression *unmarried man*, whose meaning is (Bach). Thus the simple concept  ${}^0\text{bachelor}$  is equivalent to (Bach), and the speakers of English, i.e., the participants of the linguistic convention, understand the expression *bachelor*: they know the procedure (Bach). And so the sentence

$$\textit{Bachelors are men}$$

needs no empirical tests to be verified, unlike such sentences as *The prime minister of Thailand is a bachelor*.

Other examples concerning analyticity based on definitions can be easily found.

Other kinds of analyticity can be also explained once we accept the LANL PRINCIPLE. Let us adduce a paradigmatic example.

Again: As every normal speaker of English knows, any sentence of the form

$$\textit{A is higher than B}$$

implies and is implied by the sentence of the form

$$\textit{B is lower than A.}$$

That is, verifying the sentence

$$\text{(If)} \quad \textit{If A is higher than B then B is lower than A}$$

we do not need any empirical test. Face to face of our linguistic evidence it is hard to believe that the boundary between sentences of this last kind and those ones like

$$\textit{A is high}$$

could be impugned. And yet we read:

In repudiating such a boundary I espouse a more thorough pragmatism.  
(Quine 1963, 46)

I will return to this Quine's explanation below; now let us explain why LANL can claim that sentences like (If) are analytic.

Our semantics is a procedural semantics. We have at our disposal some basic criteria/tests, which make up what Tichý later (in his pre-theoretical explication) called *intensional base* (see, e.g. his (1988, p. 199)). The members of this base are 'primary intensions', which become building stones of compound intensions. Some intensions are however pairwise connected in the following sense: applying a procedure to an intension  $I_1$  at a possible world  $W$  we get, e.g. exactly opposite results than if this procedure is applied to an intension  $I_2$  in  $W$ , this dependence being given only by the respective language, i.e., not by the state of the world. Thus if we use a sentence like (If) we exploit the linguistic convention that has associated *higher* with an intension  $I_1$  while associating *lower* with an intension  $I_2$ , where these two intensions are connected by the just mentioned world-independent relation.

A historical irony consists in the following fact: Arguments adduced in the present paper (written in 2007) are not a new discovery. Forty years ago we can read:

[u]ntil now there has been no theory of concepts which would adequately explain, in particular, the phenomenon of analytical truth. The analytical truth of sentences like, for instance, "If X is higher than Y then X is not lower than Y" consists obviously in a certain relation between concepts that are senses of the expressions "higher" and "lower". If we want to describe this relation, we cannot regard these concepts as being atomic, unanalyzable entities, but as entities that have a structure. Employing an analysis and mutual comparison of these concept structures, we can explain why the considered sentence is analytically true. The main idea of this article consists in a proposal for using the notion of procedure... so as to perform such an analysis.

(Tichý 1968, 86)

(Tichý's analysis in that article is based on the notion of Turing machine<sup>33</sup>; later Tichý builds up TIL, where the notion of constructions is used instead.)

Let us now return to Quine. In the last quotation the core of the problem is named. The whole Quine's indubitable charisma has been since 1953

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<sup>33</sup>As well as in his article (1969).

used to *change the subject*. What Carnap intended in his *Meaning and Necessity* was to give semantics (or better what we call LANL) good logical foundations. Quine's famous criticism of Carnap raised a false impression as if it were not particular shortcomings in Carnap's attempt what deserves critique but his decision to do *semantics*. No proposals how to do semantics better have been given; instead the intention to do semantics has been rejected. The only 'scientific' study of language can be a pragmatic one, best in a behaviorist spirit (see (Quine 1960)). What Quine would never accept is just our LANL PRINCIPLE.

The reasons of this change of the subject are ideological. Quine—as also other members or friends of the Vienna Circle—feared first of all any scent of platonic realism, but you cannot work out a genuine logical semantics without being a kind of a realist. So if you do not like it, you should take recourse to an essentially empirical study of language behavior. Meaning as an abstract entity is an “obscure entity”: the notion of “stimulus meaning” does not require a realistic abstraction, you can use only empirical generalizations.

If you accept this Quinean turning point then you are no longer doing semantics (neither the later Wittgenstein does do it). To do semantics in the sense of LANL means to be after the logical structures which are encoded by the natural language, which is well possible, understandable (see LANL PRINCIPLE) and in full harmony with our intuition, which will always see a principal, not only gradual difference between claims verifiable just due to their meaning and those ones that can be verified only if empirical factors are taken into account.

I hope that it is not “politically incorrect” to sum up as follows: Quine is an original and inspiring thinker. He is, however, responsible for a colossal misunderstanding that dominated (and still to a large extent dominates) the analytic and especially post-analytic philosophy. In particular we can read that Quine *refuted* rather than *refused* the idea of doing logical analyses of natural language and that he *teaches us* that the distinction between analytic and synthetic is *passé*. But it is not.

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