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THE DISCOVERY OF THE LAW OF GRAVITATION FROM THE LOGICAL POINT OF VIEW

Popper's Arguments Against the Logic of Scientific Discovery

Is there a logic of scientific discovery? Are there logical relations between the knowledge scientists posses as they start their investigations and new hypotheses and theories they formulate? Can such relations be *retrospectively* reconstructed?

Most philosophers of science in 20$^{\text{th}}$ century claimed that processes of inventing new hypotheses or theories are not governed by any rules of logic. They claimed that new hypotheses are products of "leaps of imagination" that cannot be logically analyzed.

Interestingly enough this view was never supported by any systematic arguments. Popper decisively announced it in his (1934, §2) giving no reasons in its favour. It is only in his paper "The Aim of Science" (1957) that he presents a case study to demonstrate that theoretical discoveries are results of "creative imagination" and not of any kind of valid inference. Popper claims about the discovery of Newton's dynamics:

> It is often said that Newton's dynamics can be induced from Galileo's and Kepler's laws, and it has even been asserted that it can be strictly deduced from them. But this is not so; from a logical point of view,

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> Newton's theory, strictly speaking, contradicts both Galileo's and Kepler's (although these latter theories can of course be obtained as approximations, once we have Newton's theory to work with). For this reason it is impossible to derive Newton's theory from either Galileo's or Kepler's or both, whether by deduction or induction. For neither a deductive nor an inductive inference can ever proceed from consistent premises to a conclusion that formally contradicts the premises from which we started. Popper 1957, p. 198

There are at least two inconsistencies between Kepler's theory and classical mechanics. Kepler assumes that the sun is motionless, whereas Newton's dynamics implies that the sun moves, together with all planets, around the centre of mass of the whole system. (One can add that in Newton's theory the sun is no longer placed in the centre of the universe.) According to Kepler, planets move in ellipses, whereas Newton's dynamics implies that the trajectories are not strictly elliptical due to the mutual interactions of planets. Of course, according to Newton's theory, planetary paths are "almost" elliptical and the sun's orbit is "very small" with respect to orbits of planets. Therefore, from the point of view of classical mechanics, Kepler's theory (and Galileo's kinematics) is at least "approximately true". But Popper points out to the fact that

> from Galileo's or Kepler's theories we do not obtain even the slightest hint of how these theories would have to be adjusted—which false premises would have to be adopted, or what conditions stipulated—should we try to proceed from these theories to another and more generally valid one such as Newton's. *Only after we are in possession of Newton's theory can we find out whether, and in what sense, the older theories can be said to be approximations to it.* [...] All this shows that logic, whether deductive or inductive, cannot possibly make the step from these theories to Newton's dynamics. It is only ingenuity which can make this step. Popper, p. 200

What Popper's remarks suggest can be presented as follows:

A, B, . . . , G, K, . . .

leap of imagination

CM

where G – Galileo's kinematics, K – Kepler's theory, A, B, . . . – other propositions accepted by Newton, CM – classical mechanics.

Popper is wrong for at least three reasons. (1) He ignores most of what Newton knew when he started his investigations. (2) He only compares the alleged starting point of Newton's investigations and their final product, ignoring all possible intermediary steps. (3) When Popper writes about logic, he means classical logic only, whereas it is possible that logics that are more sophisticated are necessary to give the rational reconstruction of the process of discovery.

I will not try to reconstruct Newton's actual reasoning—anyway it remains unknown. I would like to show that in the second half of 17th century the law of gravitation could be arrived at in a *chain of valid derivations* based on *premises that were at this time rationally acceptable*.

Newton's Premises

Let us first determine the supposed premises of Newton's reasoning.

Some assumptions were available at the market of ideas in 17th century.

T Time goes on continuously (so subsequent moments could be represented by rational numbers), uniformly and in the same way in all places.

S Space is Euclidean, three-dimensional, continuous, homogeneous and isotropic.

Kepler as Pythagorean mystic had no followers among scientists. Astronomers almost unanimously rejected his construction of five regular polyhedra inserted between planetary spheres, all relations between planetary motions and musical scales etc. Two generations of philosophers of nature that lived between Kepler and Newton separated those "mystical" elements from his "positive" achievements. What was accepted by the community of scientists Newton belonged to was expressed in so called Kepler's laws, namely:

K_1 Each planet moves in an ellipse, the motionless sun being one of the foci.

K_2 Each planet moves in such a way that a segment between it and the sun sweeps out equal areas in equal times.

K_3 For each planet the ratio between the square of its sidereal period and the cube of its average distance from the sun is approximately of the same value.

Newton—in spite of what Popper's remarks may suggest—did not try to unite or to generalize Galileo's and Kepler's theories. He based his investigations on physical principles that were very different from basic concepts that were applied by his two great predecessors.

Kepler was using almost-Aristotelian concept of force: force is what moves a body and keeps it in motion. If a force is constant, then the velocity is constant and proportional to the force. Kepler suitably constructed the causal model of solar system: something (*anima motrix*) is emanated from the sun in the plane of ecliptic, rotates together with it and pushes planets around. Kepler also tried to explain, by attributing magnetic properties to the sun and planets, why planetary orbits are not strictly circular.

At the same time Galileo developed the principle of circular inertia. In the light of this principle planetary motions around the sun and also moon's motions around planets are inertial and does not need any further explanations. (Galileo simply ignored the fact that planetary orbits are not strictly circular.) Moreover, Galileo's physics was kinematical rather then dynamical.

In the 17th century various concepts of force were available at the market of ideas. *There are no criteria of rational choice of basic principles* (as the failure of Lakatos' or Laudan's efforts shows). One never knows in advance whether a research program will find successful applications in a given range of phenomena. But one can try and see what will happen.

Newton was using the concept of force that was developed during a long and complicated process. In the 14th century Buridan and his nominalistic colleagues modified Aristotelian physics by adding impetus to the system to explain some previously anomalous data. Impetus was to be *internal force* that a body acquires when it is set into motion and that maintains its motion in the absence of external forces. It followed from Buridan's theory that if there were no resistance of the medium, then the body would *continue its motion with constant velocity.* Impetus was to be equal to the product of the amount of primary matter the body is composed of (we can call it "mass") and its speed. But it was not "momentum" of modern physics. Momentum in Newtonian physics can informally be called "the quantity of motion in a body", whereas impetus of Buridan's physics was to be "the cause of motion". Yet, as impetus was introduced into the framework of Aristotelian physics, the function of *external force* was deeply transformed: it became an agent changing impetus, so *changing the product of mass and velocity.*

In this way formulas quite similar to what we now call the first and the second principle of Newton's mechanics were arrived at. They were expressed in the language of Aristotelian physics. New interpretation of them was necessary to pave the way toward modern science. It took another two centuries to transform impetus from internal force into momentum. Consequently, external force became an agent changing momentum.

If no external force acts on a body, then, as Buridan claimed, it moves with constant velocity. There are two possible kinds of paths for inertial motion: straight line or circle. (Only in those two cases if there are no external forces, then all phases of motion are strictly similar to each other.) Galileo developed the principle of circular inertia. Descartes, Huygens and others claimed that

N_1 If no external force acts on a system of bodies, then its centre of mass moves with constant velocity in a straight line.

The principle can be interpreted either as *factual statement* or as *partial definition* of inertial frame of reference. The choice of definition is not a matter of arguments (based e.g. on the results of experiments) but a matter of decision. You never know in advance which definition will be fertile. The only way to find out is to try. Newton chose N_1 and he did not have to justify it.

The second pillar of the framework of classical mechanics was the principle

N_2 The body of a mass m moves with an acceleration a if and only if an external force $\boldsymbol{F}$ acts on a body. The relation between force, acceleration and mass is given by the equation $\boldsymbol{a} = \boldsymbol{F}/m$.

This also can be understood as *factual statement* or as *partial definition* of concepts of force and mass.

The third law of dynamics followed from N_1 and N_2. If there were two bodies in the universe acting on each other with unbalanced forces, then in the absence of external forces the centre of mass of the system would move with some acceleration—and this would contradict N_1. So

N_3 If a body A acts on a body B with a force $\boldsymbol{F}_{AB}$, then the body B acts on A with the force $\boldsymbol{F}_{BA} = -\boldsymbol{F}_{AB}$.

Two rival hypotheses about the nature of interplanetary and interstellar space were available at the market of ideas of 17$^{\text{th}}$ century. According to Descartes the whole space is filled by an invisible medium (ether). According to Tycho Brahe, who based his claim on observations of a 1577 comet's motion:

E interplanetary space is empty.

Newton's Problem

Newton invented neither N_1 nor N_2, although he gave them better formulations that his predecessors. Accepting those principles Newton faced a new problem not existing for either Galileo or Kepler. From N_1 it followed that planetary motions, as described in Kepler's model, are not uniform. So, according to N_2, *there are forces acting on planets.* The problem was to determine what those forces are. *The solution was not to be found by creative imagination but by valid derivations.*

First Step

From S and T it followed mathematically that if a body moves with velocity v in a circle of a radius r, then its acceleration defined, in the light of N_1, as $d^2\boldsymbol{r}/dt^2$ is

$$a_r = v^2/r\,,$$

vector of acceleration being directed towards the centre of the circle.

From this analytical formula and K_3

$$r_{pS}^3/T_{pS}^2 = \mathrm{const}$$

it follows, by a series of trivial substitutions ($T = 2\Pi r/v$ etc.), that accelerations of all planets are inversely proportional to the square of their distance from the sun

$$a_{pS} \sim 1/r_{pS}^2$$

and are directed towards the sun. At this stage we ignore the fact that planetary paths are not strictly circular.

There is no mass of a planet in the formula for a_{pS}. It seemed almost obvious to presuppose that

M masses of planets are different.

So, according to N_2, the force acting on planets is proportional to their masses:

G_S $\quad F_{pS} = k_S m_p/r_{pS}^2$

and is directed towards the sun.

The whole argument was purely deductive. Unfortunately, it was based on premises known to be only "approximately true".

Second Step

Formula G_S *contained more than one variable*—and such formulae are *candidates for laws of nature.* To be a law of nature a formula containing variables must find diverse successful applications.

Newton probably arrived at G_S in 1665 or 1666 and then tried to apply G_S to the system consisting of the Earth, projectiles, pendulums and the Moon. He failed, partly because of mathematical difficulties and partly because the empirical data at his disposal were wrong (see below). Eventually Newton stopped his investigations in the field of mechanics for the next thirteen years.

He returned to it after receiving in 1679 the letter from Hooke who, on the basis of K_3, obtained the inverse-square proportionality of the gravitational force to the distance. This is quite typical in the history of science that different theoreticians, working independently, arrive at the same formula: theoretical discoveries are products of valid derivations rather then of irrational "leap of imagination". If two scientists work within the same research programme and have the same empirical data at their disposal, they should obtain formulas that are identical or at least similar.

About 1680 Newton overcame mathematical difficulties and demonstrated that the formula G_S follows from T, S, N_1, N_2, K_1, K_2, K_3 and M. Newton's derivation (it can be found in N. R. Hanson 1958, §V, C) was again purely deductive (not retroductive, as Hanson seems to suggest); this time all premises could be treated as "strictly true".

N_3 was not taken into account yet. At this stage, there was no rational way to apply it: it was not known whether the mass of the sun is finite or not, whether the sun is a ball of pure light situated in the centre of a vortex of ether etc. Maybe it is the ether that pushes planets toward the sun.

Third Step

This time the theoretical framework constituted by S. T, N_1, N_2, and G_S quickly found new successful applications.

Improved measurements of Earth's radius and the distance between the moon and the Earth confirmed that for the system consisting of the Earth, free falling bodies, projectiles and the Moon the following formula holds (within the limits of experimental errors):

$$a_{xE} \sim 1/r_{xE}^2$$

where a_{xE} – the acceleration of a body x towards the Earth, r_{xE} – the distance between a body x and the centre of the Earth. Hence,

$$\mathrm{G_E} \qquad F_{xE} = k_{\mathrm{E}} m_x / r^2_{xE}$$

The movements of the moons of Jupiter and Saturn satisfied (within the limits of observational errors) formulae analogous to K_3, so for the system consisting of Jupiter (Saturn) and its moons it is at least approximately true that

$$\mathrm{G_J} \qquad F_{mJ} = k_{\mathrm{J}} m_m / r^2_{mJ}$$

$$\mathrm{G_{St}} \qquad F_{mSt} = k_{\mathrm{St}} m_m / r^2_{mSt}$$

The constants k_{S}, k_{E}, k_{J} and k_{St} were of different values.

However, the mathematical structure of $\mathrm{G_S}$, $\mathrm{G_E}$, $\mathrm{G_J}$ and $\mathrm{G_{St}}$ was identical and this suggested that they were particular cases of a general law of nature.

Fourth Step

Newton demonstrated in his *Principia* (1687) that Descartes' ether theory is inconsistent. We do not know when he found the proof. Anyway, faced with two rival hypothesis—interplanetary space is either filled with ether or is empty—he could choose E even at random.

It was only after introducing E that N_3 could be taken into account. If the solar system as a whole is to obey N_1, then forces act on the sun of magnitudes equal to forces of the form $\mathrm{G_S}$ but in opposite directions.

Similarly for planets and their moons. If the system consisting of the Earth (Jupiter, Saturn) and its moon(s) is to obey N_1, then force(s) act(s) on the Earth (Jupiter) of magnitude equal to forces expressed by $\mathrm{G_E}$ ($\mathrm{G_J}$), but in opposite direction(s).

Fifth Step

So, are $\mathrm{G_S}$, $\mathrm{G_J}$, $\mathrm{G_E}$ particular cases of a general formula?

To answer this question one should make an *inductive generalization* by replacing variables referring to finite sets of objects (planets, moons and also projectiles, pendulums etc. already examined during earthly experiments) by variables applicable to any systems of bodies. (There is no abstract justification for inductive steps. However, this it the way our knowledge grows.)

Let us now consider an isolated system consisting of two bodies of masses m_A and m_B placed in a distance r_{AB} from each other. According to G_i, a force directed towards B acts on A:

$$F_{AB} = k_B m_A / r_{AB}^2$$

and a force directed towards A acts on B:

$$F_{BA} = k_A m_B / r_{AB}^2$$

According to N_3,

$$k_B m_A / r_{AB}^2 = k_A m_B / r_{AB}^2$$

so

$$k_B m_A = k_A m_B\,.$$

This is in general true if and only if

$$k_B = G m_B$$
$$k_A = G m_A\,.$$

In this way we arrive at

G $\quad F_{AB} = G m_A m_B / r_{AB}^2,$

F_{AB} being always attractive and acting along the line joining A and B.

Sixth Step

At this stage it became possible to determine "how big" the contradiction was between Kepler's laws and Newton's model of the solar system constructed on the basis of T, S, N_1, N_2, N_3, G and E. It followed form N_2, G etc. that the acceleration of a body A towards a body B is

$$a_{AB} = G m_A / r_{AB}^2$$

As $a = v^2/r$ we have

$$G m_A = v_A^2 r_{AB} = 4\Pi^2 r_{AB}^3 / T_{AB}^2\,.$$

It means that the ratio of the mass of the sun to planetary masses is given by

$$m_S/m_P = (r_{PS}^3/T_{PS}^2)/(r_{MP}^3/T_{MP}^2),$$

where r_{PS} – the mean distance between a planet P and its moon M.

This equation and empirical data give the mass of the sun so big in comparison with planetary masses that Kepler's model could be accepted as "approximately true" in the light of Newton's dynamics. In this step the possible value of systematic error in deriving G was estimated.

Seventh Step

The essence of science lies at the systematicity of experimental research and theoretical investigations (Hoyningen-Huene 2001). One of the aspects of systematicity finds its expression in the challenge to expand the range of applications of a theory. So after arriving at G, one should ask whether G applies also to systems planet- planet. Thus far such systems were not mentioned at all: G was applied to systems consisting of the sun and planets, or of a planet and its moon(s). It was still not known whether forces of the form G act between planets.

Newton asked Flamsteed to provide him the data about positions of Jupiter and Saturn. Then he could—on the basis of N_1, N_2, N_3 and G—calculate suitable corrections and introduce them into Kepler's model. The calculated deviations from ellipticity were confirmed by observations. At this moment the process of discovering the law of gravitation came to an end: the formula G proved to be the candidate for the law of nature.

Final Comments

In the reconstruction of the chain of inferences leading to the law of gravitation presented above, which remains in unknown relation to Newton's actual reasoning, fifth step was inductive, all others were deductive. However, what about inconsistencies Popper was pointing to?

The inconsistency connected with the motion of the Sun was passed by in the way that is quite typical for human thinking. We never make inferences based on everything we know. The problem is that before we arrive at some conclusions and introduce them into the body of our knowledge, we do not know how to apply some claims known to us at the very beginning of an investigation. N_3 could not be applied before E had been introduced. The truth of E was uncertain. But the derivation of G_S, G_E, G_J and G_{St} was independent of E. After G_S etc. had been generalized, E could be used and the final step leading to G could be made.

The inconsistency connected with mutual interactions of planets does not appear in my reconstruction at all. All derivations were made on the assumption (generally held in 1666 or 1680) that planetary paths are strictly elliptical. Was a hidden inconsistency involved in the whole process? No. G_S, G_J etc. were not intended to be applied to pairs of planets at all: they were referring only to systems consisting of a central body and its satellites or projectiles in its neighbourhood. It was also not *a priori* clear whether systems consisting of pairs of planets belong to the range of applications of G. Our seventh step was not a (Popperian) severe test of a new law: its result was the discovery that G applies to systems like planet-planet, planet-comet etc.

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