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QUINE'S CRITICISM OF THE "FIRST DOGMA OF EMPIRICISM"*

Abstract. Quine's argumentation is shown to be invalid since its conclusion would need one premise more; such a premise is shown to be false.

Quine's conclusion concerning the possibility of a boundary between analytic and synthetic statements (see [Quine 1953, 37] is:

> [a] boundary between analytic and synthetic statements simply has not been drawn. That there is such a distinction to be drawn at all is an unempirical dogma of empiricists, a metaphysical article of faith.

Quine defended this conclusion consistently during all his scientific life (see, e.g., [Quine 1992, 55]. Yet the most concise and thorough argumentation can be found just in *Two dogmas of empiricism*. I will briefly recapitulate this argumentation and show that a necessary premise is missing. Then I will try to demonstrate that this premise is unjustified, and show a way how to discover the meaning of a given expression.

1. Arguments

Quine first derives the theory if meaning from Aristotle's handling 'essences' ("Meaning is what essence becomes when it is divorced from the object and wedded to the word"). Since Quine is strongly biased in his attitude to 'metaphysics',

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we can understand that this 'genealogy'of meaning makes him hostile to the category *meaning*; we will return to this point later. On the other hand, Quine admits that there are genuine semantic problems that stem from confusing "meanings with extensions". So we need a theory of meaning, but its "primary business" should be "simply the synonymy of linguistic forms and the analyticity of statements".

Now Quine, setting aside (and obviously tolerating, at least in the 'pre-holistic' phase of his arguments) the case of logically true statements, turns his attention to the 'second class of analytic sentences' represented by the paradigmatic example *No bachelor is married*. Such statements can be reduced to the unproblematic case of logically true statements if we replace some expressions by their synonyms: the analyticity itself is however not clarified thereby for we would need to explicate *synonymy*.

Quine examines another attempt at explicating analyticity: Carnap's definition in terms of state-descriptions (see [Carnap 1947]). Quine immediately finds the weakness of Carnap's attempt:

> [t]his version of analyticity serves its purpose only if the atomic statements of the language are, unlike 'John is a bachelor' and 'John is married', mutually independent.

Another possibility of clarifying analyticity is then *definition*. Quine rightly states that a definition could fulfil the required task only if it were taken to be 'reports upon usage' rather than what would be the result of a lexicographer's work. Then we can distinguish between two kinds of definition: the extreme one, 'the explicitly conventional introduction of novel notations for purposes of sheer abbreviation', and the Carnapian *explication*. As for the former kind, we cannot than appreciate Quine's pregnant formulation: such a definition "rests on synonymy rather than explaining it';. As for the explication, Quine states that any explication has "to preserve the usage of [...] favored contexts while sharpening the usage of other contexts". Then there may be 'alternative differentia' not synonymous with each other. The respective 'preexisting synonymies' are again not explicated.

Thus another possibility of explicating synonymy is investigated: the Leibnizian interchangeability *salva veritate*. The task is now defined as follows:

> [W]hat we need is an account of cognitive synonymy not presupposing analyticity.

Quine now proves that such an account cannot be acquired in terms of interchangeability *salva veritate*: the latter must be relativized to a language. Now there are two options: If the language is 'extensional', then there is

> [n]o assurance here that the extensional agreement of 'bachelor' and 'unmarried man' rests on meaning rather than merely on accidental matters of fact

so that with respect to an extensional language the Leibnizian criterion is of no use. The second option is that the language in question is an intensional one. Such a language contains (according to Quine) the term *necessarily*. Then the Leibnizian criterion

> does afford a sufficient condition of cognitive synonymy; but such a language is intelligible only in so far as the notion of analyticity is already understood in advance.

Remark: Here we could perhaps object that an intensional language could be defined not in this way, but in my opinion Quine's argument could be applied to any definition of an intensional language. □

Quine's 'afterthought' is then that a better approach to solving the problem would be to begin with defining analyticity first: synonymy could be then defined in terms of interchangeability *salva analyticitate*. Thus a last option (?) is investigated: explicating analyticity in terms of *semantical rules*. Here the problem begins with stating that we have to define the relation given by the phrase *a statement S is analytic for a language L*. Such a definition cannot be intelligibly formulated even for the seemingly simple case of artificial languages. Let L_0 be such a language and let the respective rules determine all and only analytic statements of the system. Then

> [W]e understand what expressions the rules attribute analyticity to, but we do not understand what the rules attribute to those expressions.

Indeed: The rules say that a statement is analytic for L_0 iff [...] but we cannot understand the rules before we understand the term 'analytic for', in other words, the phrase 'S is analytic for L', where S, L are variables.

If we avoid the term 'analytic' in naming the rules and are content with determining the class of *true* sentences according some *semantical rules*, then nothing has been solved again. Not all true sentences of the language can be determined by semantical rules (otherwise all true sentences would be analytic) so that

> [S]emantical rules are distinguishable [...] only by the fact of appearing on the page under the heading 'Semantical Rules'; and this heading is itself then meaningless.

Quine—after some comments to the term *postulate*—summarizes the results of his testing the possibility of defining analyticity in terms of semantical rules for artificial languages as follows:

> [S]emantical rules determining the analytic statements of an artificial language are of interest only in so far as we already understand the notion of analyticity; they are of no help in gaining this understanding.

In a short paragraph a possibility of a *pragmatic* clarification of analyticity ('mental or behavioral or cultural factors . . . ') is suggested but modeling analyticity as "an irreducible character" (read: as a semantical category) is claimed to be futile. The first "dogma of empiricism" is refused.

Or refuted?

2. A missing premise

Let us return to Quine's concluding formulation:

> [a] boundary between analytic and synthetic statements simply has not been drawn. That there is such a distinction to be drawn at all is an unempirical dogma of empiricists, a metaphysical article of faith.

Let us confront these two statements with Quine's arguments (as briefly recapitulated above):

The first statement is factual: the boundary has not been drawn. Most probably this is true at the time when Quine wrote his article. Besides, Quine's remarkable analysis of particular attempts to do it seems to be rather exhaustive (again, w.r.t. the time of writing the article).

The second sentence possesses a evaluative character: to be a 'metaphysical article of faith' is something like *anathema*. All the same, a factual thesis seems to presuppose this condemnation as well as be presupposed by it. I am convinced that it is the thesis

It is impossible to draw a boundary between analytic and synthetic statements.

Indeed, if it were possible to do it, could Quine still talk about 'metaphysical article of faith'? And if it were impossible to do it, then Quine would be right. Here it is sufficient to consider the thesis as the *sufficient* condition of Quine's conclusion.

What Quine's argumentation has proved was that the attempts (to draw the boundary) considered here broke down. Even when the cases *considered here* were exhaustive w.r.t. the year, say, 1953, nobody could assume that in the future no successful attempt could be made. Thus if Quine's criticism were interpreted as an (empirical!) *enumeration* of unsuccessful attempts the thesis above would not be proved. We can, of course interpret Quine's criticism as a proof that any such attempt is *logically* i.e., *a priori* doomed. Excellent as the particular steps of Quine's argumentation are nothing indicates that such a general proof has been found. Thus our thesis should be logically proved before the conclusion could be said to follow from Quine's premises. Or perhaps this thesis could stay here as another premise.

One of the ways how to realize this adding a premise could be to show that the cases *considered here* are the only possible cases. As a claim it would suffice for stating that Quine's conclusion follows from this claim (together with the obvious assumption that one should not try to do what is impossible to do). If somebody succeeded in *proving* this claim, Quine's conclusion could be said to have been proved.

Thus we can state that Quine has not proved the claim that the boundary between analytic and synthetic statements should not be drawn (from the viewpoint of empiricism). This fact itself does not say anything about the truth of the claim. If however we could show that there were a way of drawing such a boundary, then Quine's claim would be false. In next paragraphs we will try to offer such a way.

3. Determining the boundary

The circularity of the attempts at defining analyticity/ synonymy could be easily broken if *meaning* were defined independently of analyticity/synonymy. The idea is clear: suppose that we have got such an independent definition of meaning; then we can say that two expressions of a language are *synonymous* iff their meaning is the same. The rest is evident.

Unfortunately, Quine from the very beginning refuses any attempt at a definition of meaning: he says (c.d., p. 22):

> [m]eanings themselves, as obscure intermediary entities, may well be abandoned.

On p. 37 we read:

> [w]e can [...] pass over the question of meanings as entities and move straight to sameness of meaning, or synonymy.

A quotation from a more recent work (see [Quine 1992, 55]):

> [a] sentence is analytic, *in mentalistic semantics*, when it is true by virtue of the meanings of its words. (Emphasis mine.)

Quine's replacing semantics with behavioristic and pragmatic "stimulus-reaction" theory (in particular in [Quine 1960]) is a consequence of his *a priori* aversion to the category *meaning* (and thus of his essentially nominalistic ideology). Hence any attempt at defining meaning as a separate entity is refused *a limine*. For a philosopher who is ready to accept sets as the *only* abstract entities (and who thinks that properties etc. are conceived "in sin") the idea of an abstract entity playing the role of meaning is absolutely unacceptable.

This explains why Quine has not chosen the way we have suggested above. From his viewpoint any theory that accepts more abstract entities than he is ready to acknowledge is a metaphysical theory, i.e., a theory that no empiricist can accept.

Yet as soon as we accept the viewpoint that Quine's views in this respect are only an ideology[1] we are free to try to define meaning in such a way that the definition will be independent of the notions *analyticity* and *synonymy*.

(Indeed, we should remember that as a necessary counterpart of Occam's razor the "Menger's comb" can be formulated: *Entities should not be* ***omitted*** *without necessity*.)

As for terminology, we accept Church's view that what Frege meant by his *Sinn* can be called *concept* in the following context: *A Sinn (meaning for us) of an expression E is a concept of what E denotes* (see [Church 1956]).

Further, we accept that *language is a code* and, therefore,

> The notion of a code presupposes that prior to, and independently of, the code itself there is a range of items to be encoded in it. Hence meanings cannot be conceived of as products of the language itself. They must be seen as logical rather than linguistic structures, amenable to investigation quite apart from their verbal embodiments in any particular language. To investigate logical constructions in this way is the task of logic. The linguist's brief is to investigate how logical constructions are encoded in various vernaculars.
>
> ([Tichý 1996])

Thus if concepts are defined independently of language, the last point to be realized consists in defining in which way a concept can be found that is the meaning of the given expression. Here we can refer to [Tichý 1988] as to the theoretical base, to [Materna 1998] as to a respective theory of concepts, and to [Materna, Duží 2003] as to the proof that

> given a definite conceptual system *the* (i.e., optimal) analysis of the given expression can be found,

where *analyses* of an expression E are concepts that underlie E.

This is not to say that no other theories of meaning (*de facto* definitions thereof) are possible. Carnap's *intensional isomorphism* (see [Carnap 1947]) is a step towards such a definition, as well as Cresswell's attempt to model structuredness (as

[1] Well, one could object that Quine really proved what he wanted to supposing that any attempt at introducing such suspect entities as *meaning* were incompatible with *empiricism*. But then the thesis that introducing *meaning* is incompatible with empiricism would have to be proved first.

a necessary condition of meaning-like entities) in terms of tuples (see [Cresswell 1975, 1985]). In any case, nominalistic ideology aside, Quine's claim is at least unjustified, probably false.

Now we will be more explicit and show in some details how to get meaning back into semantics.

4. Meaning / Concept explicated

4.1. "Algorithmic complexity"

It can be proved that any explication of the Fregean notion *Sinn* that identifies *Sinn* (we will speak about *meaning* in harmony with the way the latter term is used) with a set-theoretical object (like intensions[2], or even Cresswell's tuples) cannot satisfy the fundamental intuitions connected therewith (see, e.g., [Materna 1998]). To recapitulate, we would like to speak about meanings of mathematical expressions (Frege himself introduced his *Sinn* in [Frege 1892] first in connection with mathematical expressions, viz. medians of a triangle) but if their meanings were intensions in the standard, PWS sense, then, e.g., all true mathematical sentences would express one and the same sense / meaning. As for Cresswell's attempt from 1975 and 1985, which is intended to capture the justified suspicion that meanings have to be *structured*, Tichý has shown in [1996, 78–80] that Cresswell's tuples cannot play the role of genuine complexes: commenting Cresswell's "subtracting example" $9 - 5$, i.e., $\langle -, 9, 5\rangle$, he says:

> There surely is a way of arriving at, or constructing, a number by means of $-$, 9, and 5; one can arrive at 4, or construct it, by *applying* the mapping—to the arguments 9 and 5 (in this order). But the triple $\langle -, 9, 5\rangle$ is not that construction. [...] *A convention is needed which interprets the triple as a proxy for the construction of* ***applying*** *the first component (that is, the subtraction mapping) to the other two as arguments. [...] The triple merely enumerates the objects of which the construction is composed; it does not combine those objects into the construction.* (Emphasis mine. P.M.)

A general remark *re* sets as candidates of meanings can be found in [Zalta 1988]:

> Although sets may be useful for describing certain structural relationships, they are not the kind of thing that would help us to understand the nature of presentation. There is nothing about a set in virtue of which it may be said to present something to us. [Zalta 1988, 183]

[2] Intensions in the standard, "possible-world semantics" are simply functions, mappings (from possible worlds), thus they are set-theoretical entities.

Further, if—as Church proposes[3]—Fregean senses are identified with concepts, then we can point at Bolzano's ingenious distinguishing between concept and its content (*Inhalt*): the latter is simply the set (*Summe*) of (simple) subconcepts while the former is the way in which those parts mutually combine (*Die Art, wie diese Theile untereinander verbunden sind*) [Bolzano 1837, 244], which is much more cogent than Frege's *Die Art des Gegebenseins*.

To satisfy our (and Cresswell's) requirement that meaning should be *structured* it is necessary to become aware of the distinction between *being structured = possessing parts* (maybe a "mereological structuredness") and *being structured in the algorithmic sense*. (See also [Moschovakis 1994]!) In the former sense also Cresswell's tuples can be said to be structured; that they are not structured in the latter sense is clear from Tichý's criticism. We will show that Tichý's *transparent intensional logic* can handle structuredness in the algorithmic sense and therefore satisfactorily explicate the term *meaning*.

4.2. Transparent Intensional Logic (TIL)

4.2.1. Principles

Here we briefly mention some most important principles that underlie TIL. It is, of course, impossible to adduce arguments that ('pretheoretically') justify these principles; they are mostly contained in [Tichy 1988] and [Materna 1998] and then in many articles by Tichý, Materna, Duží, Jespersen. Here only the principles themselves.

The quasi-Fregean scheme of semantic relations differs from Frege's by an essential shift: *The object (if any) denoted is—in case of empirical expressions—an intension*, never the value of this intension in the actual world. Frege's Venus is not the denotation of 'morning star' or 'evening star', it is the reference, i.e., the value of the intension denoted by those expressions in the actual world at the given time. Since we never know which of the possible worlds is the actual one the reference can be identified only using also experience, it cannot be determined *a priori*. Thus reference—unlike denotation—is not given *a priori* and—therefore—is not handled by semantics.

The objects that can be denoted by expressions of a natural language are type-theoretically classified. They are considered to be *functions*. So classes and relations are handled as characteristic functions, intensions as functions from possible worlds to chronologies of some type, individuals as nullary functions etc.

[3] In his [1956, 6] he says: "Of the sense we say that it *determines* the denotation, or *is a concept* of the denotation."

Functions are *partial functions*, i.e., they associate every argument with *at most* ne value (so that *total functions* make up a subclass of partial functions).

The denotation of an expression is *a priori*[4] determined by the *meaning* / *construction* / *concept* that is encoded by the expression.

The meaning / etc. can be also *mentioned*, i.e., the meaning of an expression can be denoted by another expression. Thus we need a *ramified hierarchy of types*.

In general, the semantic scheme of TIL looks as follows: expression *denotes*

a) an intension (empirical expressions) or
b) an extension (non-empirical, in particular mathematical expressions) or
c) a meaning (concept) or a function with arguments/values containing meaning ("higher order expressions") or
d) nothing

Expression expresses/depicts its meaning, i.e., a concept/construction.[5]

4.2.2. Types of order 1

The lowest level of types is given by the following definition:

DEFINITON 1. Let o be the set of truth-values, $\{\mathbf{T}, \mathbf{F}\}$, ι the set of individuals, τ the set of time moments or real numbers, ω the set of possible worlds.

i) o, ι, τ, ω are types of order 1.
ii) Let $\alpha, \beta_1, \ldots, \beta_m$ be types of order 1. Then $(\alpha\beta_1 \ldots \beta_m)$, i.e., the set of partial functions from $\beta_1 \times \cdots \times \beta_m$ into α, is a type of order 1.
iii) Only what satisfies i), ii) is a type of order 1. $\square$

o, ι correspond to Montague's t, e, respectively. TIL needs the type τ as a type of time moments, since the expressions of natural languages often concern time (so, e.g., tenses can be classified as a kind of objects, see [Tichý 1980]) and numbers. Empirical expressions denote intensions; the analysis of such expressions has to take into account the fact that intensions are functions from possible worlds.[6]

[4] That an expression denotes just that object that it denotes is, of course, contingent. From the viewpoint of a theory of language no a priori is present. From the viewpoint of semantics (or: logical analysis) of natural language the fact of the given linguistic convention is, however, accepted as given. Then the genuinely semantic relations are ("relatively") a priori, of course.

[5] Tichý in his [1988] reduced semantic relations to *denoting* but he defined this relation as connecting an expression with a *construction*. We can understand and even defend this decision but it is a little bit controversial, so we accept (at least here) the scheme above.

[6] A failure made by many semanticists consists in the assumption that logical analysis needs possible worlds in "modal contexts" only. Our examples will show that this assumption is simply false.

DEFINITON 2. Let α be any type. Members of the type $((\alpha\tau)\omega)$, abbrev. $\alpha_{\tau\omega}$, are *(α-)intensions*. Objects that are not intensions are *extensions*. □

In contrast with some other intensional theories (including Montagovians) we will not say that an expression in some contexts denotes intensions , in other contexts extensions. An expression *either* denotes an intension and then it does so in any context, *or* an extension and then it does so in any context. The term *transparent* means just the *anti-contextualist* view: Every (disambiguated) expression denotes the same object in 'direct' and 'oblique' contexts. That this claim is justified will be shown as soon as we define constructions.

Examples of type-theoretical classification of objects:

Kind of object	**Type**	**Example**
class of numbers	$(o\tau)$	prime number
relation between numbers	$(o\tau\tau)$	$<$
relation between individuals	$(((o\iota\iota)\tau)\omega)$ abbrev. $(o\iota\iota)_{\tau\omega}$	taller than
relation between an individual and a proposition	$(\iota o_{\tau\omega})_{\tau\omega}$	believe that
individual role ('office')	$\iota_{\tau\omega}$	the highest mountain

(Imagine, e.g., believing. It can be really viewed as a function that, given a world, returns a chronology (a function from τ) of (binary) relations between individuals and propositions, such that at any time point we get the set of those pairs $\langle x, p\rangle$, where x, the individual, believes the proposition p in the given world and time. A proposition is a function that associates every world W with a chronology of truth-values, viz. such that at the given time point T we get **T**, **F**, **Undefined** if the proposition is true, false, undefined at T in W, respectively.

We will now formulate a problem that will be solved at the end of the present article. The solution of this problem will be at the same time a demonstration of the way in which the meaning of an expression is sought.

PROBLEM: Consider the sentence *Charles believes that the highest mountain is taller than Mont Blanc*. What is the meaning of this sentence? □

Notice that such problems are no longer posed in the post-analytic "semantics". If what the "pre-post-analytic' semantics called 'meaning' is nothing more than the use of the given expression (according to some rules), then our question is hopelessly outdated; besides, the holistic dogma teaches us that asking for 'the meaning' of a particular expression is futile. If I were malicious, I would say that the sophisticated objections to such questions as ours are motivated by inability to answer, but I am, of course, not malicious.

TIL shows the way to finding the answer even in much more complicated cases. Finding the answer means to *present* an abstract object and prove that this object satisfies our intuitions concerning the expression 'meaning'. Among other things, being able to find the meaning of any expression (which is a non-trivial and mostly very difficult task) means being able to define analyticity and synonymy avoiding the traps detected by Quine.

The present paragraph makes it possible to prepare the solution and decompose the task to some subtasks. So let us begin: we make a *type-theoretical analysis*.

The particular expressions denote objects of following types:

Ch(arles) / ι, M(ont) B(lanc) / ι,

B(elieve) / $(o\iota o_{\tau\omega})_{\tau\omega}$,

(the) H(ighest) / $(\iota(o\iota))_{\tau\omega}$,

M(ountain) / $(o\iota)_{\tau\omega}$,

T(aller than) / $(o\iota\iota)_{\tau\omega}$.

The whole sentence / $o_{\tau\omega}$.

(As for the type of H, we can justify our proposal as follows: 'highest' is surely an empirical expression. So it denotes an intension, whose population (extension) is dependent on the state of the world (ω) at the given time moment (τ). At any respective pair $\langle$world, time$\rangle$ we then get a function that takes as an argument a class of individuals ($o\iota$) and returns as the value that individual (if any) (ι) that is the highest one in that class. An automated analysis would require that the type-theoretical information (the input) would be prepared in the automated dictionary. Here we use our linguistic intuition.)

Now our problem can be specified as follows:

We seek an abstract procedure (meaning is "algorithmically complex") whose particular steps are given by the meanings of the particular subexpressions of the sentence (see above)—compositionality is presupposed—and our composing of the particular meanings should result in a *construction* that would construct an $o_{\tau\omega}$-object, i.e., a proposition; the resulting truth-conditions (= the proposition constructed) should be in harmony with the way we understand (and normally use) the sentence. This specification requires that *constructions* were defined.

4.2.3. Constructions

The mistaken impression as if TIL were too complicated (more than, e.g., Montague) arises exclusively due to the fact that the notion of construction is in a way original: we are used to a "formalistic" approach to analysis, where there are symbols and their set-theoretical interpretation. Thus if we have some λ-term, say, $(\lambda x(2 + x)3)$ and the intended interpretation we reduce the analysis to presenting two factors: the term itself as an expression, and the result of its realisation, i.e., the number 5. Actually, there are three factors here, since the semantics of the expression (above) does not reduce to denoting (the number 5). The meaning thereof is not its denotation but the way of obtaining it. As Tichý says in [1988, 7]:

> [if] the numbers and functions mentioned in the term do not themselves combine into any whole, then the term is the only thing which holds them together. The numbers and functions hang from it like Christmas decorations from a branch. The term, the linguistic expression, thus becomes more than a way of *referring* to independently specifiable subject matter: it becomes *constitutive* of it.

The notion of construction is a successful attempt at filling the gap between the expression and the object denoted. (See also [Moschovakis 1994].) Constructions are *extra-linguistic* objects consisting of some 'steps' and such that the linguistic structure of an expression 'mirrors' in a way the structure of construction. (That distinct languages possess distinct grammars means only that the ways these languages encode the respective constructions differ.)

As soon as we become aware of the fact that *constructions are extra-linguistic entities* and that we have to distinguish between the way we fix them *via* definition and constructions themselves the impression of 'intricacy' of TIL vanishes.

The following definition *uses* "artificial" expressions to enable us to speak about constructions themselves. (So as we use the expression 'elephant' to speak about elephants.) Thus the following claims hold:

'$[{}^0{+}\,{}^0 2\,{}^0 3]$' contains brackets

$[{}^0{+}\,{}^0 2\,{}^0 3]$ constructs 5

while what follows are nonsenses:

'$[{}^0{+}\,{}^0 2\,{}^0 3]$' constructs 5

$[{}^0{+}\,{}^0 2\,{}^0 3]$ contains brackets

DEFINITON 3. i) For every type we have countably many *variables* at our disposals. Variables are *constructions* that construct objects dependently on a total function called valuation. Where v is the parameter of valuation, variables v-construct objects. The usual letters like $x, y, z, \ldots, p, q, r, \ldots, f, g, h, \ldots$ are names of variables.

ii) Let X be any object (even a construction) whatsoever. *Trivialisation*, denoted by ${}^0X$, is a *construction*; it constructs X without any change.

iii) Let X, X_1, …, X_m be *constructions* that (v-)construct objects of types $(\alpha\beta_1 \ldots \beta_m)$, β_1, …, β_m, respectively. Then $[XX_1 \ldots X_m]$, i.e., *composition*, is a *construction*. It (v-)constructs the value (if any) of the function (v-)constructed by X on arguments (v-)constructed by X_1, …, X_m, respectively. In case that the function is not defined on the given arguments the composition is (v-)improper: it (v-)constructs nothing.

iv) Let x_1, …, x_m, be distinct variables ranging over (= v-constructing objects of) the types β_1, …, β_m, respectively, and let X be a *construction* (v-)constructing objects of the type α. Then $[\lambda x_1 \ldots x_m X]$ is a *construction* called *closure*. It (v-)constructs a function F of type $(\alpha\beta_1 \ldots \beta_m)$ as follows: let b_1, …, b_m be objects of types β_1, …, β_m, respectively. Let v' be the same valuation as v with the only exception: the objects, b_1, …, b_m be assigned to variables x_1, …, x_m, respectively. Then the value of F is the object (if any) that is v'-constructed by X. If X is v'-improper, then F is undefined on $b_1, \ldots, b_m$. □

There are two "unusual" points in this definition. The point i) uses the *objectual notion of variable*. As for the details of justification see [Tichý 1988]; that this notion is necessary follows from the fact that constructions are defined as extra-linguistic entities. The point ii) introduces a notion not to be met in the logical literature. It may seem that it is not at all necessary to introduce such a 'triviality'. The important role of this notion will be obvious later, after the ramified hierarchy is introduced, but already now we can state that any theory of procedures has to start with some most simple kind of procedure. The intuitionist Fletcher says:

> If one had to define constructions in general, one would surely say that a type of construction is specified by some *atoms* and some *combination rules* of the form 'Given constructions x_1,…, x_k one may form the construction $C(x_1, \ldots, x_k)$, subject to certain conditions on $x_1, \ldots, x_k$'.
>
> [Fletcher 1998, 51]

The points iii), iv) are obvious—they are an objectual version' of the notions well-known from λ-calculi.[7]

Now the apparatus introduced till now characterises the 1st order TIL. Unfortunately, the ramified hierarchy has to be defined because

i) some expressions of natural language are "higher-order" (e.g., the attitude verbs),

[7] In [Tichý 1998] we find two other kinds of construction. We will not need them here. That TIL is an open theory can be seen in this context from the fact that some new kinds of construction (a well as of new types) have been introduced in connection with some modelling tools (see [Zlatuška 1986]).

ii) we could not define *concept* if concept—when playing the role of *meaning*—should be a *way* to the denotation.

Thus *ramified hierarchy* will get its definition now.

4.2.4. Ramified hierarchy

The purpose of defining types of higher order is to make it possible to *mention* constructions, not only to *use* them. Thus constructions will become objects *sui generis*. As a by-product we will use the higher orders for explicating *concept*.

A special form of the definition is caused by the necessity to avoid any form of a vicious circle. Thus the definition proceeds in three steps: first, the *1st order types* are recapitulated, second *constructions of order n* are defined, and till then, third, the *types of order n+1* are defined. Intuitively, the 1st order types embrace the 'normal abstract objects' like classes of individuals, classes of classes of individuals, ..., relations between individuals, classes of relations of individuals, ..., properties of individuals, properties of classes of individuals, properties of properties of individuals, ..., propositions, classes and properties of propositions, etc. Higher order types are sets of constructions of any type, sets of relations of constructions, etc. etc. Also concepts belong to higher order types.

The following definition is a slightly modified definition from [Tichý 1988].

DEFINITON 4. $\mathbf{T_1}$ *Types of order 1*: See Definition 1.

$\mathbf{C}_{\boldsymbol{n}}$ *Constructions of order n*: Let α be a *type of order n*.

i) For any variable ξ: If $\xi \rightarrow \alpha$, then ξ is a construction of order n.

ii) Let X be an α-object. Then ${}^0X$ is a construction of order n.

iii) Let X, X_1, ..., X_m be constructions of order n. Then $[XX_1 \ldots X_m]$ is a construction of order n.

iv) Let x_1, ..., x_m, X be constructions of order n. Then $[\lambda x_1 \ldots x_m X]$ is a construction of order n.

$\mathbf{T}_{\boldsymbol{n+1}}$ *Types of order n + 1*: Let $*_n$ be the collection of all constructions of order n.

i) $*_n$ and all types of order n are types of order $n + 1$.

ii) Let $\alpha, \beta_1, \ldots, \beta_m$ be types of order $n + 1$. Then $(\alpha\beta_1 \ldots \beta_m)$ (see Definition 1) is a type of order $n + 1$.

iii) Types of order $n + 1$ are only what is determined by i), ii). □

(It happens frequently that the types of particular subconstructions are distinct; Then the point $\mathbf{T}_{n+1}$(i) ("and all types of order n") makes it possible to assign a type to the whole construction, viz. the highest one among the participating types.)

Now we give some simple examples. As abbreviations we choose:

> If an object (including constructions) X is of the type (i.e., belongs to the type) α, then we write X/α.
>
> If a construction X (v-)constructs an object of type α, then we write $X \to \alpha$.

That this distinction is important will be clarified by our examples.

Let x be a numerical variable (ranging over τ). Then:

> $x \to \tau$, $x/*_1$ (it v-constructs real numbers, which are of 1st order type τ). The type of x is therefore of order 2.
>
> ${}^0x$ constructs (Definition 3) the variable x. Thus ${}^0x \to *_1$, ${}^0x/*_2$. The type of ${}^0x$ is therefore of order 3.

Some brackets can be omitted. So we can write

> $\lambda w\, \lambda t\, X$ instead of $[\lambda w\, [\lambda t\, X]]$.

We choose variables w, t, which will range over ω, τ, respectively.

If X is an intension, then applying X first to a possible world w and (the result) to a time t will be written X_{wt} instead of $[[Xw]t]$.

Consider now some simple expressions.

(1) $x + 4$

Type-theoretical analysis:

$$x \to \tau, \quad 4/\tau, \quad +/(\tau\tau\tau)$$

Synthesis:

$$[{}^0{+}x\,{}^04]$$

Let $\mathbf{v}$ be a valuation such that $\mathbf{v}(x) = 3$. Then the construction $\mathbf{v}$-constructs 7, which is the denotation of (1). Thus (1) denotes for $\mathbf{v}$ the number 7 *via* its meaning, which is not the expression '$[{}^0{+}x\,{}^04]$' but the procedure denoted by it.

(2) $27 = 5 : 0$

$$27, 5, 0/\tau, \quad -/(\tau\tau\tau), \quad :/(\tau\tau\tau)$$

$$[{}^0{-}\,{}^027[{}^0{:}\ {}^05\,{}^00]]$$

This construction is improper, since $[^0{:}\ ^05^00]$ does not construct any number. The expression (2) does not denote anything, but it possesses meaning, viz. the construction above: we know which steps are to be realised but the procedure ends in a blind alley.

(3) The Pope is indisposed.

$\mathrm{P(ope)}/\iota_{\tau\omega}, \quad \mathrm{I(ndisposed)}/(o\iota)_{\tau\omega}$

COMMENT: The Pope is not the same object as the individual who is the Pope: when we say that, e.g., Wojtyła is the Pope, we offer a non-trivial information, which would not be the case if the expression 'the Pope' denoted Wojtyła. Thus the Pope is an individual role ("office"). To be indisposed is an empirical property of individuals, so the type is that of functions associating worlds and times with classes of individuals. □

Now a problem arises: we want to predicate a property of individuals of an individual, viz. that one who occupies the role of the Pope. No such individual is however mentioned in the sentence. On the other hand, the roles are never indisposed. The solution offered by TIL is simple: the construction that constructs the Pope must be in the *de re supposition*, i.e., we get the required type of individual *via* applying the construction to w and (then) to t ("intensional descent"). Thus we can predicate the property of an individual without pretending as if the individual were known. The information identifiable from the construction will be that whoever is the Pope he is indisposed. So we have:

$\lambda w \lambda t[{}^0\mathrm{I}_{wt}\, {}^0\mathrm{P}_{wt}]$.

Compare therewith the following sentence:

(4) The Pope is a distinguished office.

This time we predicate a property of an office. To be distinguished is obviously type-theoretically polymorph but here it is a property of individual offices (roles), so we have

$\mathrm{D(istinguished)} / (o\iota_{\tau\omega})_{\tau\omega}$[8]

Now the Pope will be constructed in the supposition *de dicto*: no intensional descent takes place.

$\lambda w\, \lambda t[{}^0\mathrm{D}_{wt}\, {}^0\mathrm{P}]$.

[8]Notic e that where α is a type, properties of the α-objects belong to the type $(o\alpha)_{\tau\omega}$.

Notice that the two last analyses support the intuitive fact that adding the premise 'The Pope is a Pole' to 3) we get the conclusion 'A Pole is indisposed', while adding this premise to 4) we do not get any nontrivial conclusion (and surely not 'A Pole is a distinguished office'). See [Duží 2002].

4.2.5. Concept

If *meaning* should explicate Frege's *sense* (as it seems to be intuitive), then we can quote Church (who had no reason to consider this category to be 'suspect'), who says in [1956, 6]:

> We shall say that a name *denotes* or *names* its denotation and *expresses* its sense. Or less explicitly we may speak of a name just as *having* a certain denotation and *having* a certain sense. Of the sense we say that it *determines* the denotation, or *is a concept* of the denotation.

(This is not the Fregean concept, of course, as Church states in a footnote. Church's concept is a maximal generalisation: for Bolzano every expression with exception of sentences was connected with a concept, for Frege only predicates.)

Our approach presupposes that logical entities (such as constructions) are independent of language. Going to explicate *meaning* we have introduced constructions, which are *ex definitione* independent of language. But the term 'meaning' is always connected with expressions ('meaning of ...'). To stress the fact of independence of language we will use term 'concept' (inspired by the quotation above) and define concepts so that

a) they were independent of language and
b) by attaching them to a linguistic form they would be its meaning. (See [Materna 1998].)

To motivate the following choice of *explicatum* I will consider some examples.

Take the following expressions:

1. *the father of*
2. *a father*
3. *the father of A. Einstein*
4. *the father of x*

I claim: If ${}^0\mathrm{F}$ is the simple concept of fatherhood, i.e., of a function of type $(\iota\iota)_{\tau\omega}$, then 1. expresses the concept ${}^0\mathrm{F}$, 2. expresses the concept $\lambda w\,\lambda t\,\lambda x\,[{}^0\exists\lambda y\,[{}^0{=}\,x[{}^0\mathrm{F}_{wt}y]]]$,[9] 3. expresses the concept $\lambda w\,\lambda t\,[{}^0\mathrm{F}_{wt}{}^0\mathrm{E}]$.

[9] The existential quantifier $\exists$ is—for any type α— a function of type $(o(o\alpha))$: it is a class of non-empty classes.

In all these cases the denotation of the given expression is a definite object (constructed by the concepts above): the fatherhood relation (1.), the property of being a ('happy') father (2.), the individual role of being the father of Einstein (3.). The expression 4. however has got no definite denotation; thus it does not express any concept. (This does not mean that it is meaningless, but its 'meaning' is in a sense incomplete, and such an incomplete meaning does not deserve the name 'concept'.)

Thus the intuition leads us to the following proposal of an explication of *concept*:

DEFINITON 5. Let C be a *closed construction*, i.e., a construction that does not contain any free occurrence of a variable. Then C is a *concept*. □

COMMENTS: 1. In TIL the variables can be bound in two ways, as we can see on the basis of our definitions: The variables can be "λ-bound", which is the usual kind of boundness, but they can be bound also due to trivialisation: if a variable ξ is a subconstruction of ${}^0X$, then ξ is ^{0}*bound* ("trivialisation-bound") in ${}^0X$; if it is λ-bound in X, then it is all the same ^{0}bound in ${}^0X$. To illustrate this maybe strange fact we first state that any "kind of boundness" has the following feature: the variable bound in some way is not at our disposal as for valuation. Indeed, take a construction with λ-bound variables:

$$\lambda x[{}^0\geq x\ {}^0 0]$$

(that constructs the class of non-negative reals): any particular valuation of x is excluded. But the same holds in the case of, say,

$${}^0[\geq x\ {}^0 0],$$

because the construction constructs in this case the construction $[\geq\ x\,{}^0 0]$: no manipulation with x is permitted. This holds also in the case of the construction

$${}^0[\lambda x\,[\geq x\ {}^0 0]].$$

2. The last examples can be used to show that there is a problem with Definition 5. Consider two constructions:

$$[\lambda x\,[\geq x\ {}^0 0]],$$

$$[\lambda y\,[\geq y\ {}^0 0]].$$

They are, of course, equivalent (as it is seen from Definition 3iv); see the α-rule in λ-calculi).[10] According to Definition 5 they are *distinct concepts*. This is in clear discord with our intuition: If a concept is a procedure leading (in the better case) to an object, then to say that the distinction in λ-bound variables is a distinction in procedure is not acceptable; a strong argument is that our (natural) languages simply cannot distinguish two constructions that differ just in λ-bound variables. In our case both above constructions (as well as infinitely many of other variants differing just in λ-bound variables) are in English encoded by one and the same phrase, viz. 'numbers greater than or equal to zero'. To handle this problem (and a similar problem with "η-equivalence") we can use various methods; one of them can be found in [Materna 1998], another one in Horák's dissertation (see http://www.fi.muni.cz/~hales/disert). The latter makes it possible to define 'normalisations' of such cases as above, which practically means that we can take any closed construction as either a concept or some kind of pointing at the concept. We will not embark on further details: we simply accept Definition 5. □

Defining concepts as abstract procedures makes it possible to formulate very cogent explications. We will show some examples.

Definiton 6. Let C be an object of a type of order 1. Then ${}^0C$ is a *simple concept*. □

Thus ^{0}*triangle*, ^{0}*moon*, ^{0}*the highest mountain* are simple concepts. To be a simple concept means that the construction that constructs ("identifies") the object does not "need" any other construction: it is the case of "immediate identification". The trivialisation concerns, of course the respective object, not the expression that encodes this object. Therefore our last example is a correct example of a simple concept, since the trivialisation "immediately" constructs the respective *individual role*, not "taking into account" the concepts connected with *the highest* and *mountain*. Hence compare:

^{0}*the highest mountain* (a simple concept)

$\lambda w\,\lambda t\,[{}^0\mathrm{H}_{wt}\ {}^0\mathrm{M}_{wt}]$ (not a simple concept).

Definiton 7. Let C be an improper closed construction (Def. 3). Then C is a *strictly empty concept*. □

Let ι_α be "α-singulariser" (for any type α), i.e., a function of type $(\alpha(o\alpha))$, which is *defined on singletons only* and returns as value that object of type α which

[10] Notice that the respective trivialisations are *not*: they construct two *distinct*, albeit equivalent constructions.

is the only member of the singleton. (Thus where A is a class of α-objects we can read the scheme $[{}^0\iota\,{}^0A]$ “the only x such that x is A”.) Observe now the construction

$$[{}^0\iota\,\lambda y[{}^0\forall\,\lambda x\,{}^0\geq y\,x]]].$$

Clearly, there is no such (real) number that would be greater than or equal to each number. Thus the construction above does not construct anything: it is a strictly empty concept (THE NUMBER THAT IS GREATER THAN OR EQUAL TO ANY NUMBER).

Two other kinds of empty concepts can be defined: ‘quasi-empty concepts’ (that construct empty classes/relations) and ‘empirically empty concepts’ (that construct intensions whose value in the actual world + time is either missing or is an empty class/relation). (See [Materna 1998].)

The ramified hierarchy enables us to realise very fine analyses. Consider, e.g., the sentence

The concept of the number that is greater than or equal to each number is an empty concept.

Let $\mathbf{E_1}$ be the class of all empty concepts of order 1. So we have $\mathbf{E_1}/(o*_1)$. Our sentence gets the following analysis:

$$[\mathbf{E_1}\,{}^0[{}^0\iota\,\lambda y[{}^0\forall\,\lambda x\,{}^0\geq y\,x]]].$$

4.2.6. Conceptual systems

Definition 6 makes it possible to define *conceptual systems*. The idea is that we can easily define systems that result in generating the class

$$\{C_1,\ldots,C_k\}\cup\{C_{k+1},\ldots\}$$

such that the set $\{C_1,\ldots,C_k\}$ (it may be called P_S, **p**rimitive concepts of S) contains only *simple concepts* and $\{C_{k+1},\ldots\}$ (let it be D_S, **d**erived concepts of S) just those (complex) concepts whose simple subconstructions are exclusively variables and members of the class $\{C_1,\ldots,C_k\}$. (The respective type-theoretical basis can be said to contain *preconcepts*, like *individuals*, *real numbers*, …).

I mention here the notion of conceptual systems because it can be shown that detecting the *meaning* of an expression is non trivial also in the following respect: *It is in general not the case that simple expressions express simple concepts.*

Many expressions used by a language at a time T are abbreviations, which are composed from simple expressions used by a language at some time T', $T' < T$.

Whoever really understands such a simple expression E, does so because (s)he understands the less complex subexpressions of E. The fragment of the language used at T' may be construed as being based on a conceptual system S', in which some members of $D_{S'}$ have not be connected with an expression, which happened at T. Example: Let $P_{S'}$ contain following concepts:

$$\{\ldots, {}^0\mathrm{I(nteger)}, {}^0\mathrm{D(ivide)}, \ldots\}$$

and some logico-mathematical concepts like connectives and numerical quantifiers. The set $D_{S'}$ surely contains the concept

$$\lambda x[{}^0\&\ [{}^0\mathrm{I}x]\ [{}^0\exists!2y\ [{}^0\&\ [{}^0\mathrm{I}y]\ [{}^0\mathrm{I}\ [{}^0\mathrm{D}xy]]]]]$$

('${}^0\exists!2y$' is 'there are exactly two numbers such that ...'). (THOSE INTEGERS THAT POSSESS EXACTLY TWO DIVISORS). Since the class of those numbers turned out to be very interesting and began to be frequently mentioned an abbreviation has been introduced: *prime numbers*, *primes*. Thus we can say that—*ceteris paribus*—the system S can be identical with S' and the only change concerns the language: in the later stage of the development of the respective language the complex concept from $D_{S'}$ (see above) will get a name, viz. *prime* in English, *Primzahl* in German, etc. In this case it is practically excluded that a user of a natural language understood this expression due to possessing the simple concept ^{0}prime: only in so far as (s)he understands the definition (s)he can be said to understand the simple word *prime*. Thus when we attempt at an "ideal", "the best" etc. analysis of an expression we cannot proceed simply *via* associating simple concepts to simple expressions (and, of course, obeying some rules connecting grammar with constructions): or, when doing it, we have to attach a warning/condition: 'Our analysis of the expression presupposes that the conceptual system on which the language is based contains following primitives: ...'; the list has to contain trivialisations of the objects that are denoted by those simple expressions which we decided to analyse as expressing simple concepts. (See [Materna, Duží 2003, in particular 174–175].

5. Finding *the* meaning

In 3. we have claimed that

> *given a definite conceptual system* **the** *(i.e., optimal) analysis of the given expression can be found*;

this would mean that Quine would have to prove that our finding were incompatible with empiricism. Such a proof would be extremely dubious when "empiricism" is not connected with a clear concept, but Quine obviously did not suspect that "meaning" could be defined in an exact manner. We now will show that if "meaning" is what we defined as "concept" then a general method of obtaining such a meaning for a given expression is at our disposal. Analyticity, as well as synonymy, then can be defined in terms of meaning while concept has been defined independently of both of them.

5.1. Bachelors are men

We begin with an easy example, which can be a good illustration of dependence of an analysis on conceptual systems.

The classical sentence is

Bachelors are men.

This sentence can be read either as *Every bachelor is a man* or as stating a relation between two properties (saying that the property *bachelor* is a subproperty of *man*). The first reading hides somewhat the fact of analyticity of the sentence while this fact is clearly suggested by the second reading.

Let us begin with the first reading. Types:

$$\mathrm{B}/(o\iota)_{\tau\omega}, \quad \mathrm{M}/(o\iota)_{\tau\omega}, \quad \forall/(o(o\iota)), \quad \supset/(ooo)$$
$$\lambda w\, \lambda t\, [{}^0\forall\, \lambda x\, [{}^0\supset [{}^0\mathrm{B}_{wt}x]\, [{}^0\mathrm{M}_{wt}x]]]$$

(The *class* $\forall/(o(o\iota))$ *of classes of individuals* contains just one member: the universe. Another type can be associated with $\forall$, here we choose the "more standard one".)

The proposition denoted by our bachelor sentence and constructed by the above construction (= concept) holds, of course, in every WT-pair, but one cannot see it when looking at the concept. Naturally, as soon as we really understand the words *bachelor* and *man*, we can see that the concept is an analytical one: nobody will ask particular bachelors whether they are men.

The other reading is more explicit. It can be captured by introducing an auxiliary notion *requisite*. In the case of properties we define:

Let P, Q be properties (of, say, individuals). P is a *requisite* of Q iff the construction ${}^0\forall\, w\, {}^0\forall t\, [{}^0\supset\ {}^0Q_{wt}\, {}^0P_{wt}]$. We write $[{}^0\mathrm{Req}\, {}^0P\, {}^0Q]$.

In our case we get

$$[{}^0\mathrm{Req}\, {}^0\mathrm{M}\, {}^0\mathrm{B}].$$

Now the independence of WT-pairs is explicit.

Now we exploit the obvious fact that *bachelor* is an abbreviation. The meaning of *bachelor* is given by a definiens, say, *a man who has never married.* The analysis of this definiens gives (M(an)/$(o\iota)_{\tau\omega}$, Ma(rry)/$(o\iota)_{\tau\omega}$)

$$\lambda w\, \lambda t\, \lambda x\, [{}^0\forall\, \lambda t' [{}^0\&\, [{}^0\mathrm{M}_{wt} x]\, [{}^0\neg\, [{}^0\mathrm{Ma}_{wt'}\, x]]]]$$

Now the analyticity of our sentence is explicit: it can be seen simply as a logical truth (rule of & elimination). Thus the fact of analyticity of the sentence can be reduced to the fact of logical validity, and this reduction is made possible due to the transition to another conceptual system, where the concept ^{0}bachelor has been "decomposed" to "simpler" concepts.

5.2. Solution of our PROBLEM (4.2.2). Parmenides Principle

We now show the way to obtaining the meaning of the sentence

Charles believes that the highest mountain is taller than Mont Blanc.

using the conceptual system where the primitives are

$${}^0\mathrm{Ch},\, {}^0\mathrm{B},\, {}^0\mathrm{MB},\, {}^0\mathrm{H},\, {}^0\mathrm{M},\, {}^0\mathrm{T}$$

(see 4.2.2). The types of the respective objects are adduced in 4.2.2 too. The following process is based on an intuitive exploitation of the type-theoretical analysis; a system of rules should replace the intuition, of course, but although it is in principle clear that such a system of rules can be built up we do not solve this problem here.

The type of *believe* has been determined as $(o\iota o_{\tau\omega})_{\tau\omega}$: it is an empirical relation between an individual and a proposition. The whole sentence is empirical, so it denotes a proposition. Thus we have to compose the above primitives so as to get a construction of a proposition. We have

$$\lambda w\, \lambda t\, [{}^0\mathrm{B}_{wt}\, {}^0\mathrm{Ch}\ \mathrm{A}],$$

where A is a construction of a proposition. Now we can analyse the proposition constructed by A. i.e., the proposition denoted by the sentence *the highest mountain is taller than Mont Blanc*. We get

$$\lambda w\, \lambda t\, [{}^0\mathrm{T}_{wt}\, {}^0\mathrm{HM}_{wt}\, {}^0\mathrm{MB}],$$

where HM is a construction that constructs the highest mountain, i.e., an individual role, type $\iota_{\tau\omega}$. But the highest mountain can be constructed as applying H to w and (then) to t and (then) to the class of mountains in w at t, so that HM is constructed as follows:

$$\lambda w \lambda t\, [{}^0\mathrm{H}_{wt}\ {}^0\mathrm{M}_{wt}]$$

Thus we can analyse our sentence as follows:

(5) $\lambda w \lambda t\, [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{Ch}\ \lambda w \lambda t\, [{}^0\mathrm{T}_{wt}\ {}^0\mathrm{HM}_{wt}\ {}^0\mathrm{MB}]]$,

(6) $\lambda w \lambda t\, [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{Ch}\ \lambda w \lambda t\, [{}^0\mathrm{T}_{wt}\ \lambda w \lambda t\, [{}^0\mathrm{H}_{wt}\ {}^0\mathrm{M}_{wt}]_{wt}\ {}^0\mathrm{MB}]]$,

(7) $\lambda w \lambda t\, [{}^0\mathrm{B}_{wt}\ {}^0\mathrm{Ch}\ \lambda w \lambda t\, [{}^0\mathrm{T}_{wt}\ [{}^0\mathrm{H}_{wt}\ {}^0\mathrm{M}_{wt}]\ {}^0\mathrm{MB}]]$,

To give an example of an extremely poor analysis we state that according to our definitions also the following construction is an analysis of our sentence:

(8) ^{0}Charles believes that the highest mountain is taller than Mont Blanc.

Indeed, this trivialisation constructs just the proposition constructed by (5)–(7). Only it is an “amorph” concept, which is so “flat” that it can serve only as “the extreme case”.

Now we want to compare all these analyses: we would like to call *the meaning* of our sentence just one of them (we will see however that the problem is still more complicated).

What we need for the above purpose is some *ordering relation*, i.e., such a relation which would be reflexive, transitive and anti-symmetric, and correspond to our linguistic intuition. In [Materna, Duží 2003] we have defined such a relation as an anti-symmetric closure of the relation *worse than* between analyses:

> *An analysis A of an expression E is worse than an analysis B iff some semantically self-contained subexpression of E has not been analysed in A and—ceteris paribus—has been analysed in B.*

Applying this ordering principle to our analyses (5)–(8) we get (let $\leq$ be the ordering relation):

$$(8) \leq (5),\ (8) \leq (6),\ (8) \leq (7),\ (7) \leq (6),\ (5) \leq (6).$$

So the result is a lattice, and it can be proved (see [Materna, Duží 2003]) that every such set of analyses of a given expression can be ordered in this way and contain the best analysis as a vortex of the lattice.

The problem is that—as we already stated—*it is in general not the case that simple expressions express simple concepts*. Hence the concept that is expressed, e.g., by the expression *believe* or *mountain* need not be a simple concept and that therefore our lattice would have to change once we accepted such a conceptual system where ^{0}believe or ^{0}mountain etc. would be replaced by complex concepts from the D-part (and the primitive concepts would be other, "less complex" ones). So yes, the best analysis, **the** meaning of an expression can be always found but only relative to a conceptual system.

This result is surely not a great revolution in semantics, but in connection with Quinean sceptical views it justifies, as I hope, the following claim: *The assumption that there are abstract objects that can play the role of meanings can be defended without losing rigorous character of argumentation*.

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