Logic and Logical Philosophy
Volume 11 (2003), 277–297

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CHANGING ONE'S POSITION IN A DISCUSSION — SOME ADAPTIVE APPROACHES†

Abstract. This paper contains different approaches to solve the problem how to construct the ultimate position out of one's interventions in a discussion after possibly one or more position changes. In all approaches it is the aim to come as close as possible to human reasoning. Therefore all logics are adaptive logics. The first logic is an extension of an adaptive translation into **S5** of the Rescher-Manor mechanisms. The second one is a dynamic proof theory based on a technique using indices. In the end a satisfactory solution is given by a dynamic proof theory expressing the idea of prioritized compatibility, i.e. compatibility step by step.

1. Introduction

Apparently the position of someone in a discussion equals the consequence set of all statements he made during that discussion. Is this consequence set really the best representation for all there is to conclude from the sequence of his interventions? Next to the contents of the interventions, the sequence also contains an ordering in time. This could be more important than it seems at first sight, for there could have been an evolution in the participant's position.

The interesting point of a discussion is the confrontation with other positions and the possibility to become convinced by arguments in favour of the latter. So the temporal aspect of a discussion should not be ignored for it is

* The research for this paper was supported by the Research Fund of Ghent University and indirectly by the Flemish Minister responsible for Science and Technology (contract BIL98/37).

† I am greatly indebted to Diderik Batens for several ideas in this paper.

crucial information necessary to interpret the dynamics of a participant's position and therefore to interpret his ultimate position. This means we have to consider a person's interventions as forming a prioritized inconsistent base.[1] A technique introduced and elaborated by N. Rescher and R. Manor appropriate to this context is reasoning from consistent parts of the inconsistent whole. I shall call these mechanisms Rescher-Manor mechanisms.

Different approaches relevant to this situation are presented in [3]. The aims of this paper are first to make the Rescher-Manor mechanisms adaptive so that they link up better with human reasoning, second to extend these mechanisms in the hope that it brings a significant enrichment and third to look for other adaptive strategies that could be more efficient for the case of rational discussions. I shall restrict my attention to three types of RM consequences (to be defined in section 3). I shall translate them to the semantics of the modal logic **S5** in section 4, so they are made adaptive. The first idea is to incorporate the RM consequences in the context of the discussive logic $\mathbf{D2^r}$ Joke Meheus introduced in [4]. It comes down to making a selection on the models, this technique is already used in [1], here it will be in section 5. Section 6 is an introduction to dynamic proof theories[2]. Two such proof theories are presented here for the case of rational discussions. The first one is based on a technique using indices that is developed in [5]. It is presented in section 7, applied in section 8 and commented in section 9. The second one is an elaboration of prioritized compatibility. After the presentation in section 10 and the application in section 11, it is evaluated and found to be an acceptable solution to the problem in section 12. First I shall give some examples to get a picture of the problem.

2. Some examples

We use an ordered set $\Sigma = \langle \Gamma_1, \Gamma_2, \ldots, \Gamma_n \rangle$ of sets Γ_i to list all the statements made in the i^{th} intervention.

2.1. $\Sigma = \langle \{p\},\ \{{\sim}q\},\ \{q\},\ \{{\sim}p \vee {\sim}q \vee r\} \rangle$

We can see the speaker changed his opinion about q. Taking this into account, r is a consequence of the last intervention and should be a definite consequence.

[1] This situation is mentioned as an open problem in [1].

[2] Dynamic proof theories are the proof theories of adaptive logics, see [2].

2.2. $\Sigma = \langle \{p\},\ \{p \supset q\},\ \{\sim p\} \rangle$

Here it is obvious we should assume the person changed his opinion about p, but what about q ? From the first two interventions one can conclude q. The third intervention does not give any information about q. It can be discussed wether we should keep q or not. However in case of a rational discussion, one of the arguments for q is contradicted in the end and there is no other reason proposed to confirm q.

2.3. $\Sigma = \langle \{r \wedge s\},\ \{s \supset t\},\ \{\sim r\} \rangle$

We should lose r from $r \wedge s$ because later $\sim r$ is stated, but we should not lose s (and t) of course. So we should hold the consequences of $r \wedge s$ that are not in contradiction with later interventions. In some cases this principle needs some refinement, as explained in example 4.

2.4. $\Sigma = \langle \{\sim p\},\ \{p\} \rangle$

Although $\sim p \vee q$ is a consequence of $\sim p$ that is not contradicted by p, together with p one can derive q. Obviously it is ridiculous to believe q is really implied by the participant's statements. What is the matter here is that disjunctive syllogism should not be applied here for the reason that p is unreliable (in the context of adaptive logics).

2.5. $\Sigma = \langle \{p\},\ \{\sim p \wedge q\},\ \{\sim q\} \rangle$

It is clear the speaker changed his position twice and the definite conclusions should be $\sim p$ and $\sim q$.

2.6. $\Sigma = \langle \{p\},\ \{\sim p \wedge q\},\ \{\sim q\},\ \{r\} \rangle$

The only difference with the example above is the extra premise r in the end. We shall see that the interpretation of the other premises depends for some consequence relations on whether or not r is added.

2.7. $\Sigma = \langle \{p \wedge q\},\ \{p \supset r,\ q \supset \sim r\} \rangle$

We can not believe $p \wedge q$ is still the opinion of the speaker at the moment he claims $p \supset r$ and $q \supset \sim r$, because we then would have equally acceptable arguments for both r and $\sim r$. So though $p \wedge q$ is not literally contradicted, we should reject it because it has inconsistent consequences together with later made statements. What is a more interesting problem is whether we should keep at least one of them. In this situation both are equally acceptable, thus we should keep the disjunction $p \vee q$.

2.8. $\Sigma = \langle \{p\},\ \{q\},\ \{p \supset r,\ q \supset \sim r\} \rangle$

This is a different situation, because p and q are not claimed at the same time. Here it could be reasonable to reject p because it was the earliest statement of those that cause an inconsistency.

2.9. $\Sigma = \langle \{p,\ p \supset q\},\ \{s\},\ \{\sim(q \wedge s)\}\rangle$

Here a connected inconsistency occurs, at least one of q and s behaves inconsistently. The priority makes us conclude that q should be false. The arguments for q, p and $p \supset q$, are then no longer together acceptable either. p is not really contradicted, thus maybe we should keep p, but the same argument goes for $p \supset q$. As both have the same priority, it would be acceptable to keep the disjunction $p \vee (p \supset q)$, which does not give any information.

2.10. $\Sigma = \langle \{p\},\ \{p \supset q\},\ \{\sim q\}\rangle$

One thing is sure, namely that $\sim q$ is the case. As q is contradicted, the arguments p and $p \supset q$ are no longer together acceptable either, as in the previous example. Here it is less plausible p was meant to be kept than $p \supset q$ was, because $p \supset q$ was stated after p was. Keeping $p \supset q$ implies also $\sim p$.

3. Some Rescher-Manor(-like) consequences

The set of premises Σ is an ordered set of sets, say $\Sigma = \langle \Gamma_1, \Gamma_2, ..., \Gamma_n\rangle$, where Γ_i precedes Γ_j iff $i < j$. We assume that each Γ_i is a consistent set of well formed formulas belonging to the standard predicative language $\mathcal{L}$. We call a set of the form Σ consistent iff $\cup\Sigma$ is a consistent set.

A *maximal consistent subset* S of Σ is a consistent subset of Σ such that it is not contained in a larger consistent subset of Σ. We shall denote it as MCS.

Here we give the definitions of the three consequence relations we shall consider: the P-consequence relation, the π-consequence relation and the λ-consequence relation. For a set of the form of Σ we write $\Sigma \vdash_{\mathbf{CL}} A$ instead of $\cup\Sigma \vdash_{\mathbf{CL}} A$ and later we shall also use this abbreviation in the context of models.

DEFINITION 1. *$\Sigma \vdash_P A$ iff $\Delta \vdash_{\mathbf{CL}} A$ for all MCS Δ of Σ such that $\Gamma_n \in \Delta$.*

DEFINITION 2. *The set π is the consistent subset of Σ of the form $\{\Gamma_i, \Gamma_{i+1}, ..., \Gamma_n\}$ such that $\{\Gamma_{i-1}, \Gamma_i, ..., \Gamma_n\}$ is not consistent. $\Sigma \vdash_\pi A$ iff $\pi \vdash_{\mathbf{CL}} A$.*

DEFINITION 3. *The set λ is the MCS one obtains by starting from π and step by step adding or not adding the previous intervention depending on wether or not it preserves consistency. $\Sigma \vdash_\lambda A$ iff $\lambda \vdash_{\mathbf{CL}} A$.*[3]

[3]Remark that all P-consequences and all π-consequences are λ-consequences.

The P-consequence relation is most efficient when the participant summarizes his viewpoint in the end. All statements made before are handled equally, none of them is a priori more important than any other. Of course this situation is not the general course of a discussion.

The π-consequence relation is useful in a situation in which the participant changes his viewpoint very suddenly and radically, probably because he was not prepared for the discussion. The π-consequence relation will not take into account the statements made before the turning point. This is not a very realistic scenario of a discussion either.

In general the λ-consequence relation gives the best results. You can see it works good in example 1 whereas the other consequence relations do not. There q is not a P-consequence and p is not a π-consequence, but both are λ-consequences. Notice that q is a π-consequence and p a P-consequence, which illustrates none of these two consequence relations is better than the other.

Example 5 shows a situation in which the π-consequence relation is better than the λ-consequence relation, because the latter derives p undeserved whereas the former does not. Rather surprisingly example 6 has different P-consequences than example 5, nothing about p or q is P-derivable in example 6. That p is an undeserved λ-consequence in this example shows the λ-consequence relation is not always more adequate than the P-consequence relation. Thus we can conclude none of the three is efficient, nor is any of them more efficient than another one.

4. Translation to S5

Let $\mathcal{F}^p$ be the set of primitive formulas of $\mathcal{L}$, $\mathcal{L}$ being the standard language of Classical Logic, henceforth abbreviated as **CL**. A standard **CL**-model is represented by a domain D and an assignment function v and is symbolized as $M = \langle D, v\rangle$. To simplify the semantic meta-language, a non-denumerable set of pseudo-constants $\mathcal{O}$ is introduced, requiring that any element of the domain D is named by at least one member of $\mathcal{C} \cup \mathcal{O}$:

$$v : \mathcal{C} \cup \mathcal{O} \longrightarrow D, \quad \text{where } D = \{v(\alpha) | \alpha \in \mathcal{C} \cup \mathcal{O}\}.$$

Through this operation we obtain the pseudo-language $\mathcal{L}^+$. The standard modal language $\mathcal{L}^M$ is extended to $\mathcal{L}^{M+}$ in the same way.

A **S5**-model is a triple $M = \langle W, D, V\rangle$, where W is a set of **CL**-models or worlds w, D is the domain of all these worlds and V the set of valuation functions v_w determined by these worlds. The valuation function v_M determined by a **S5**-model M has also a second argument, namely a world $w \in W$, and is defined by the following clauses:

C1 where $A \in \mathcal{F}^p$, $v_M(A, w) = v_w(A)$
C2 $v_M(\sim A, w) = 1$ iff $v_M(A, w) = 0$
C3 $v_M(A \vee B, w) = 1$ iff $v_M(A, w) = 1$ or $v_M(B, w) = 1$
C4 $v_M((\exists \alpha)A(\alpha), w) = 1$ iff $v_M(A(\beta), w) = 1$ for at least one $\beta \in \mathcal{C} \cup \mathcal{O}$
C5 $v_M(\Diamond A, w) = 1$ iff $v_M(A, w') = 1$ for at least one $w' \in W$.

The other logical constants are defined as usual. First we define some useful selections of **S5** models of $\Sigma^\Diamond = \langle \Gamma_1^\Diamond, ..., \Gamma_n^\Diamond \rangle$ where $\{A_1, ..., A_n\}^\Diamond$ denotes the set $\{\Diamond A_1, ..., \Diamond A_n\}$.

DEFINITION 4. *A S5-model $M = \langle W, D, V \rangle$ is a MA-model of $\Sigma^\Diamond$ iff it is a* **S5***-model of which every world verifies a MCS of Σ.*

DEFINITION 5. *A MA-model $M = \langle W, D, V \rangle$ is a RM-model of $\Sigma^\Diamond$ iff it is a MA-model such that each MCS of Σ is verified by some world of W.*[4]

DEFINITION 6. *For a* **S5***-model $M = \langle W, D, V \rangle$ and $w \in W$: $Ab_\Sigma(w) = \{\Gamma_i \in \Sigma \mid v(A, w) = 0$ for some $A \in \Gamma_i\}$.*

DEFINITION 7. *A* **CL***-model M is maximally normal with respect to Σ iff for all $1 \leq i \leq n$ there is no* **CL***-model M' such that $Ab_\Sigma(M') \cap (\Gamma_i \cup \ldots \cup \Gamma_n) = \emptyset$ and $Ab_\Sigma(M) \cap (\Gamma_i \cup \ldots \cup \Gamma_n) \neq \emptyset$.*

4.1. The P-consequence relation

Where $M = \langle W, D, V \rangle$ is a **S5**-model and $w \in W$, extend the **S5**-semantics with the clause [5]:

$$v(\Box_n A, w) = 1 \text{ iff } v(A, w') = 1 \text{ for all } w' \in W \text{ such that } \Gamma_n \notin Ab_\Sigma(w').$$

Then we can define

DEFINITION 8. $\Sigma \models_{\mathbf{S5^P}} A$ *iff* $\Sigma^\Diamond \models_{MA} \Box_n A$.

THEOREM 1. *For A modality-free, $\Sigma \vdash_P A$ iff $\Sigma \models_{\mathbf{S5^P}} A$.*

PROOF. By Definition 8, $\Sigma \models_{\mathbf{S5^P}} A$ iff $\Sigma^\Diamond \models_{MA} \Box_n A$. M is a MA-model of $\Sigma^\Diamond$ iff each world of M verifies a MCS of Σ. In view of the semantic definition of $\Box_n$, it follows that $\Sigma^\Diamond \models_{MA} \Box_n A$ iff there is no **CL**-model that verifies a MCS containing Γ_n in which $\sim A$ is true. This means $\sim A$ is incompatible with the MCS of Σ that contain Γ_n. In terms of the classic consequence relation this is translated to: $\Delta \vdash_{\mathbf{CL}} A$ whenever Δ is a MCS of Σ and $\Gamma_n \in \Delta$. By Definition 1, the latter is equivalent to $\Sigma \vdash_P A$. ■

[4] Notice that a *RM*-model can contain different worlds that verify the same MCS.

[5] We will use the same notations for the extended systems of **S5**.

4.2. The π-consequence relation

Where $M = \langle W, D, V\rangle$ is a **S5**-model and $w \in W$, extend the **S5**-semantics with the clause:

> $v(\Box_\pi A, w) = 1$ iff $v(A, w') = 1$ for all $w' \in W$ that are maximally normal with respect to Σ.

Then we can define

Definition 9. *$\Sigma \models_{\mathbf{S5}^\pi} A$ iff $\Sigma^\Diamond \models_{\mathbf{S5}} \Box_\pi A$.*

Theorem 2. *For A modality-free, $\Sigma \vdash_\pi A$ iff $\Sigma \models_{\mathbf{S5}^\pi} A$.*

Proof. By Definition 9, $\Sigma \models_{\mathbf{S5}^\pi} A$ iff $\Sigma^\Diamond \models_{\mathbf{S5}} \Box_\pi A$. In view of the semantic definition of $\Box_\pi$, it follows that $\Sigma^\Diamond \models_{\mathbf{S5}} \Box_\pi A$ iff there is no **CL**-model that verifies π in which $\sim A$ is true. This means $\sim A$ is incompatible with π. In terms of the classic consequence relation this is translated to: $\pi \vdash_{\mathbf{CL}} A$. By Definition 2, the latter is equivalent to $\Sigma \vdash_\pi A$. ■

4.3. The λ-consequence relation

Where $M = \langle W, D, V\rangle$ is a **S5**-model and $w \in W$, extend the **S5**-semantics with the clause:

> $v(\Box_\lambda A, w) = 1$ iff $v(A, w') = 1$ for all $w' \in W$ such that the following holds:
> for all $w'' \in W$ and for all $2 \leq i \leq n$, if $Ab_\Sigma(w'') \cap (\Gamma_i \cup \ldots \cup \Gamma_n) = Ab_\Sigma(w') \cap (\Gamma_i \cup \ldots \cup \Gamma_n)$ and $Ab_\Sigma(w'') \cap \Gamma_{i-1} = \emptyset$, then $Ab_\Sigma(w') \cap \Gamma_{i-1} = \emptyset$.

Then we can define

Definition 10. *$\Sigma \models_{\mathbf{S5}^\lambda} A$ iff $\Sigma^\Diamond \models_{RM} \Box_\lambda A$.*

Theorem 3. *For A modality-free, $\Sigma \vdash_\lambda A$ iff $\Sigma \models_{\mathbf{S5}^\lambda} A$.*

Proof. By Definition 10, $\Sigma \models_{\mathbf{S5}^\lambda} A$ iff $\Sigma^\Diamond \models_{RM} \Box_\lambda A$. In view of the semantic definition of $\Box_\lambda$, it follows that $\Sigma^\Diamond \models_{RM} \Box_\lambda A$ iff there is no **CL**-model that verifies λ in which $\sim A$ is true. This means $\sim A$ is incompatible with λ. In terms of the classic consequence relation this is translated to: $\lambda \vdash_{\mathbf{CL}} A$. By Definition 3, the latter is equivalent to $\Sigma \vdash_\lambda A$. ■

5. A selection on the models

First we define the unreliable formulas of a set of premises. Let $\mathcal{F}^p$ denote the set of primitive formulas and let $Dab(A_1, ..., A_n)$ be a notation for $\exists(\Diamond A_1 \wedge \Diamond{\sim}A_1) \vee \ldots \vee \exists(\Diamond A_n \wedge \Diamond{\sim}A_n)$, for $A_1, ..., A_n$ elements of $\mathcal{F}^p$ and where $\exists A$ stands for the existential quantification over all free variables occurring in A[6]. Each A_i is called a factor of the *Dab*-consequence. Note that a permutation of the factors results in an equivalent formula, so it is appropriate to use sets as argument for $Dab()$. $Dab(\Theta)$ is a minimal *Dab*-consequence of Σ iff it is a **S5**-consequence of $\Sigma^\Diamond$ and any $Dab(\Delta)$ for which $\Delta \subset \Theta$ is not a **S5**-consequence of $\Sigma^\Diamond$.

Definition 11. $U(\Sigma) = \{A \mid A$ *is a factor of a minimal Dab-consequence of* $\Sigma\}$.

In the models we want to keep, only the unreliable formulas will be allowed to behave abnormally, that is to have both their confirmation and their negation possible in the model. Therefore we define the abnormal part of a model:

Definition 12. $Ab(M) = \{A \in \mathcal{F}^p \mid M \models \exists(\Diamond A \wedge \Diamond{\sim}A)\}$.

Definition 13. *M is a* $\mathbf{S5}^\star$*-model of* $\Sigma^\Diamond$ *iff M is a* **S5***-model of* $\Sigma^\Diamond$ *and* $Ab(M) \subseteq U(\Sigma)$.

Definition 14. *M is a* $MA^\star$*-model of* $\Sigma^\Diamond$ *iff M is a MA-model of* $\Sigma^\Diamond$ *and* $Ab(M) \subseteq U(\Sigma)$.

Definition 15. *M is a* $RM^\star$*-model of* $\Sigma^\Diamond$ *iff M is a RM-model of* $\Sigma^\Diamond$ *and* $Ab(M) \subseteq U(\Sigma)$.

Definition 16. $\Sigma \models_{\mathbf{S5P}^\star} A$ *iff* $\Sigma^\Diamond \models_{MA^\star} \Box_n A$.

Definition 17. $\Sigma \models_{\mathbf{S5}^{\pi\star}} A$ *iff* $\Sigma^\Diamond \models_{\mathbf{S5}^\star} \Box_\pi A$.

Definition 18. $\Sigma \models_{\mathbf{S5}^{\lambda\star}} A$ *iff* $\Sigma^\Diamond \models_{RM^\star} \Box_\lambda A$.

Now we can define the extended consequence relations:

Definition 19. $\Sigma \vdash_{P^\star} A$ *iff* $\Sigma \models_{\mathbf{S5P}^\star} A$.

Definition 20. $\Sigma \vdash_{\pi^\star} A$ *iff* $\Sigma \models_{\mathbf{S5}^{\pi\star}} A$.

[6]The abbreviation *Dab* stands for disjunction of abnormalities.

Definition 21. $\Sigma \vdash_{\lambda^\star} A$ *iff* $\Sigma \models_{\mathbf{S5}^{\lambda^\star}} A$.

Let us see what is gained by this extra selection in the examples of section 2. All three consequence sets are more complete in example 3 (s and t are held), whereas only the $\pi^\star$-consequence set is more complete in example 1 as well (p is retained). There is an overall improvement for some consequences of lost premises (e.g. in example 3 for the simplification of a conjunction), but not in all situations (e.g. not for example 5). The most problematic examples are probably the fifth and the sixth. Only the π-consequence relation in example 5 and the P-and π-consequence relations in example 6 did not derive p, which means that here the P-and the λ-consequence relations have serious shortcomings, but unfortunately the extension can not save anything. Also in examples 7, 8, 9 and 10, the extension gives no extra consequences, because all occurring primitive formulas are unreliable.

The greatest defect of the Rescher-Manor approach seems to be that it does not care about losing all consequences of rejected premises, although not all consequences lead to inconsistencies with the consequences of the other not rejected premises. Apparently it was too late here to make a saving adaptive move. So maybe we should try another approach.

6. Intro to dynamic proof theories

Dynamic proof theories are designed to be a more faithful representation of human reasoning and also to describe reasoning processes for undecidable problems. Their main characteristic is that a conclusion can be revised when necessary, that is when insight in the premises has increased in such a way that the conclusion can no longer be sustained. The tool used to express revision is marking.

The rules for a dynamic proof can be grouped in three sorts: the premise rule(s), the unconditional rule(s) and the conditional rule(s). All can be applied at any time, but not all applications remain valid. A line derived at a certain stage of the proof on a certain condition is marked when the condition is not or no longer fulfilled. Of course when the condition is fulfilled at a later stage, the line is again unmarked. In most dynamic proofs (here also) marked lines do not belong to the proof any more.

A line in a dynamic proof consists of five elements: (i) the line number, (ii) the formula derived on that line, (iii) the numbers of the lines used to derive the second element, (iv) the rule applied to derive the second element and (v) the fifth element referring to the condition on which the second element is derived. What is considered as finally derived in a dynamic proof is stated by the following.

DEFINITION 22. *A formula A is finally derived at line i at stage s of a proof iff line i is not marked at stage s and any extension of the proof in which line i is marked, may be further extended in such a way that line i is unmarked.*

7. A solution with indices

In [5] indices are used to handle theories that are inconsistent due to the ambiguity of some expression occurring in it. The idea of indexing and afterwards leaving out again the indices on certain conditions will be used here too. The purpose is not to interpret ambiguities, but to interpret changes of opinion. In the end, the formulas without indices in the proof should represent the ultimate position of the speaker.

Let $\Sigma = \langle \Gamma_1, \Gamma_2, ..., \Gamma_n \rangle$ be the set of interventions as before. Assume again that each Γ_i is consistent itself. Σ^I is constructed as follows. First all formulas in each Γ_i are transformed in such a way that only primitive formulas occur under the scope of a negation[7]. Then to every primitive formula occurring in a member of Γ_i, the index i is given. For example 2.9 Σ^I is $\langle \{p_1, \sim p_1 \vee q_1\}, \{s_2\}, \{\sim q_3 \vee \sim s_3\} \rangle$. It is obvious that Σ^I is consistent. Thus there is no danger in using **CL** to reason from Σ^I.

PREM If $A \in \cup\Sigma^I$, one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) a dash, (iv) PREM, and (v) $\emptyset$.

RU If $B_1, \ldots, B_m \vdash_{\mathbf{CL}} A$, and $B_1, \ldots, B_m$ occur in the proof on the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) the numbers of the lines on which the B_i are derived, (iv) RU, and (v) $\Delta_1 \cup \ldots \cup \Delta_m$.

Real inconsistencies can not be derived in this way, but disguised inconsistencies can. The latter are formulas that would be inconsistencies if the indices were left out. For example 2.9, $(p_1 \wedge \sim p_1) \vee (q_1 \wedge \sim q_3) \vee (s_2 \wedge \sim s_3)$ is **CL**-derivable from Σ^I and is a disguised disjunction of contradictions. Let $P^1, \ldots, P^m$ be primitive formulas. The general format of disguised inconsistencies is $\exists(P^1_{i_1} \wedge \sim P^1_{j_1}) \vee \ldots \vee \exists(P^m_{i_m} \wedge \sim P^m_{j_m})$, where $\exists A$ stands for the existential quantification over all free variables occurring in A. If Θ_1 is the set $\{P^k_{i_k} \mid i_k < j_k\} \cup \{\sim P^k_{j_k} \mid j_k < i_k\}$ and Θ_2 is the set $\{P^k_{i_k} \mid i_k \geq j_k\} \cup \{\sim P^k_{j_k} \mid j_k \geq i_k\}$, we shall note $Din(\Theta_1, \Theta_2)$ for $\exists(P^1_{i_1} \wedge \sim P^1_{j_1}) \vee \ldots \vee \exists(P^m_{i_m} \wedge \sim P^m_{j_m})$[8]. Let $def(\Theta_1)$ then be the set of the atoms in Θ_1 of which the primitive

[7] An implication should be written as a disjunction.

[8] The abbreviation *Din* stands for disguised inconsistency.

formulas have the lowest index in Θ_1 and let $saf(\Theta_1)$ then be the set of the atoms in Θ_1 that occur in conjunction in the *Din*-formula with atoms in Θ_2 of which the primitive formulas have the highest index in Θ_2. A *Din*-formula $Din(\Theta_1, \Theta_2)$ will be minimal at a stage of the proof iff no *Din*-formula $Din(\Delta_1, \Delta_2)$ is derived at that stage for which $(\Delta_1 \cup \Delta_2) \subset (\Theta_1 \cup \Theta_2)$. For example 2.9 not only $(p_1 \wedge {\sim} p_1) \vee (q_1 \wedge {\sim} q_3) \vee (s_2 \wedge {\sim} s_3)$ is **CL**-derivable, but also $(q_1 \wedge {\sim} q_3) \vee (s_2 \wedge {\sim} s_3)$ is. The latter- if derived in the proof- is a minimal *Din*-formula. It are those minimal *Din*-formulas that will indicate which indices may be left out and which formulas should be considered as representing the ultimate opinion of the speaker.

Deleting an index will be a conditional step at any stage of the proof, because at a later stage new *Din*-formulas can be derived that indicate that certain indices can not be deleted. Before leaving out the indices of a formula, again the special form is required in which only primitive formulas occur under the scope of a negation. For every deletion of an index, the atom of which the primitive formula loses its index is added to the condition. For every new derivation, the conditions of the used lines are carried over.

RC If B occurs in the proof on the condition Θ and there occurs an atom A in B of which the primitive formula is indexed, then one may add a line consisting of (i) the appropriate line number, (ii) the formula obtained from B by replacing all occurrences of A outside the scope of a negation by A with the index left out, (iii) the number of the line on which B is derived, (iv) RC, and (v) $\Theta \cup \{A\}$.

How the *Din*-formulas should interfere with the conditional derivations, can be approached in several useful ways. The reliability strategy is the one in which every possible abnormality (disjunct of a minimal *Din*-formula) is a reason not to omit certain indices. The minimal abnormality strategy is the one in which a minimal set of abnormalities obstructs the unindexing. The defeasibility strategy is the one in which only the most defeasible formulas can not lose their indices. The safety strategy is a little different because it depends on the defeating formulas instead of the defeated ones. It is the one in which only the most strongly defeated formulas keep their indices.

First we introduce some sets. Let $Min_s(\Sigma)$ be the set of Θ_1 for which $Din(\Theta_1, \Theta_2)$ for some Θ_2 is a minimal *Din*-formula derived unconditionally at stage s of the proof. Now we can define the set $\Phi_s(\Sigma)$. Let the sets ϕ_i contain at least one element from each member of $Min_s(\Sigma)$. $\Phi_s(\Sigma)$ is the set of those ϕ_i that are not supersets of any other ϕ_i.

The **marking definitions** are the following:

- for reliability: a line on condition Θ is marked at a stage s of the proof iff there is a $\Theta_1 \in Min_s(\Sigma)$ for which $\Theta_1 \cap \Theta \neq \emptyset$.
- for minimal abnormality: a line on which A is derived on condition Θ is marked at a stage s of the proof iff there is no $\phi \in \Phi_s(\Sigma)$ such that $\Theta \cap \phi = \emptyset$ or there is a $\phi \in \Phi_s(\Sigma)$ such that there is no line on which A is derived on a condition Θ' for which $\Theta' \cap \phi = \emptyset$.
- for defeasibility: a line on condition Θ is marked at a stage s of the proof iff there is a $\Theta_1 \in Min_s(\Sigma)$ for which $def(\Theta_1) \cap \Theta \neq \emptyset$.
- for safety: a line on condition Θ is marked at a stage s of the proof iff there is a $\Theta_1 \in Min_s(\Sigma)$ for which $saf(\Theta_1) \cap \Theta \neq \emptyset$.

Let us call these proof theories $\mathbf{POCH^I1}$, $\mathbf{POCH^I2}$, $\mathbf{POCH^I3}$ and $\mathbf{POCH^I4}$. I give only the definition for the reliability approach, the other ones are completely analogous.

DEFINITION 23. $\Sigma \vdash_{\mathbf{POCH^I1}} A$ *iff* A *is finally derived in a* $\mathbf{POCH^I1}$*-proof from* Σ^I.

The consequence relations to interpret the ultimate position of the speaker will be called $\vdash_{\mathbf{POCH1}}$, $\vdash_{\mathbf{POCH2}}$, $\vdash_{\mathbf{POCH3}}$ and $\vdash_{\mathbf{POCH4}}$.

DEFINITION 24. $\Sigma \vdash_{\mathbf{POCH1}} A$ *iff* $\Sigma \vdash_{\mathbf{POCH^I1}} A$ *and* A *is free of indices.*

It is clear that the consequence sets of Σ for **POCH1**, **POCH2**, **POCH3** and **POCH4** are consistent and closed under **CL**.

8. Examples

8.1. $\Sigma^I = \langle\{p_1\},\ \{{\sim}q_2\},\ \{q_3\},\ \{{\sim}p_4 \vee {\sim}q_4 \vee r_4\}\rangle$

The only minimal *Din*-formula is $({\sim}q_2 \wedge q_3)$. In all approaches all premises except ${\sim}q_2$ may be used without indices and the **CL**-consequences of $\{p,\ q,\ r\}$ are the consequences.

8.2. $\Sigma^I = \langle\{p_1\},\ \{{\sim}p_2 \vee q_2\},\ \{{\sim}p_3\}\rangle$

The only minimal *Din*-formula is $(p_1 \wedge {\sim}p_3)$. In all approaches the **CL**-consequences of ${\sim}p$ are the consequences.

8.3. $\Sigma^I = \langle\{r_1 \wedge s_1\},\ \{{\sim}s_2 \vee t_2\},\ \{{\sim}r_3\}\rangle$

The only minimal *Din*-formula is $(r_1 \wedge {\sim}r_3)$. In all approaches the **CL**-consequences of $\{s,\ t,\ {\sim}r\}$ are the consequences.

8.4. $\Sigma^I = \langle\{\sim p_1\},\ \{p_2\}\rangle$

The only minimal *Din*-formula is $(\sim p_1 \wedge p_2)$. In all approaches the **CL**-consequences of p are the consequences.

8.5. $\Sigma^I = \langle\{p_1\},\ \{\sim p_2 \wedge q_2\},\ \{\sim q_3\}\rangle$

The minimal *Din*-formulas are $(p_1 \wedge \sim p_2)$ and $(q_2 \wedge \sim q_3)$. In all approaches the **CL**-consequences of $\{\sim p,\ \sim q\}$ are the consequences.

8.6. $\Sigma^I = \langle\{p_1\},\ \{\sim p_2 \wedge q_2\},\ \{\sim q_3\},\ \{r_4\}\rangle$

The consequences only differ from the ones of the previous example in that r is added to $\{\sim p,\ \sim q\}$.

8.7. $\Sigma^I = \langle\{p_1 \wedge q_1\},\ \{\sim p_2 \vee r_2,\ \sim q_2 \vee \sim r_2\}\rangle$

The *Din*-formula $(p_1 \wedge \sim p_2) \vee (q_1 \wedge \sim q_2) \vee (r_2 \wedge \sim r_2)$ is derivable, but it is not a minimal *Din*-formula. The disjunct $(r_2 \wedge \sim r_2)$ can be omitted, because it is a real contradiction and in **CL** everything is derivable in that case. The only minimal *Din*-formula is $(p_1 \wedge \sim p_2) \vee (q_1 \wedge \sim q_2)$. For the reliability, the defeasibility and the safety strategy, the lines in the proof that have p_1 or q_1 in their condition, are marked. For the minimal abnormality strategy the situation is different. Formulas that are derived on a condition that does not contain p_1 and on possibly another condition that does not contain q_1 are saved. An illustration of a $\mathbf{POCH^I2}$-proof from Σ^I follows.

1	$p_1 \wedge q_1$	-	PREM	$\emptyset$
2	$\sim p_2 \vee r_2$	-	PREM	$\emptyset$
3	$\sim q_2 \vee \sim r_2$	-	PREM	$\emptyset$
4	$(p_1 \wedge \sim p_2) \vee (q_1 \wedge \sim q_2)$	1, 2, 3	RU	$\emptyset$
5	p_1	1	RU	$\emptyset$
6	$p_1 \vee q$	5	RU	$\emptyset$
7	$p \vee q$	6	RC	$\{p_1\}$ ($\surd$)
8	q_1	1	RU	$\emptyset$
9	$p \vee q_1$	8	RU	$\emptyset$
10	$p \vee q$	9	RC	$\{q_1\}$
11	$\sim p \vee r$	2	RC	$\{\sim p_2,\ r_2\}$
12	$\sim q \vee \sim r$	3	RC	$\{\sim q_2,\ \sim r_2\}$
13	$\sim p \vee \sim q$	11, 12	RU	$\{\sim p_2,\ r_2,\ \sim q_2,\ \sim r_2\}$
14	$(p \wedge \sim q) \vee (\sim p \wedge q)$	10, 13	RU	$\{q_1,\ \sim p_2,\ r_2,\ \sim q_2,\ \sim r_2\}$ ($\surd$)
15	$(p \wedge \sim q) \vee (\sim p \wedge q)$	7, 13	RU	$\{p_1,\ \sim p_2,\ r_2,\ \sim q_2,\ \sim r_2\}$

Line 7 is derived as a marked line. As soon as line 10 is derived, line 7 is unmarked. Line 14 is also derived as a marked line, but it is unmarked when line 15 is derived. $p \vee q$ and $(p \wedge \sim q) \vee (\sim p \wedge q)$ are both **POCH2**-consequences of Σ that are not consequences for any of the other strategies.

8.8. $\Sigma^I = \langle \{p_1\},\ \{q_2\},\ \{\sim p_3 \vee r_3,\ \sim q_3 \vee \sim r_3\} \rangle$

The only minimal *Din*-formula here is $(p_1 \wedge \sim p_3) \vee (q_2 \wedge \sim q_3)$. For the reliability, the minimal abnormality and the safety strategy, the results are the same as in the previous example. For the defeasibility approach, the situation is different. Only lines that have p_1 in their condition are marked. Let us look at a **POCH^I3**-proof from Σ^I.

1	p_1	-	PREM	$\emptyset$
2	q_2	-	PREM	$\emptyset$
3	$\sim p_3 \vee r_3$	-	PREM	$\emptyset$
4	$\sim q_3 \vee \sim r_3$	-	PREM	$\emptyset$
5	$(p_1 \wedge \sim p_3) \vee (q_2 \wedge \sim q_3)$	1-4	RU	$\emptyset$
6	p	1	RC	$\{p_1\}$ ✓
7	q	2	RC	$\{q_2\}$
8	$\sim p \vee r$	3	RC	$\{\sim p_3,\ r_3\}$
9	$\sim q \vee \sim r$	4	RC	$\{\sim q_3,\ \sim r_3\}$
10	$\sim r$	7, 9	RU	$\{q_2,\ \sim q_3,\ \sim r_3\}$
11	$\sim p$	8, 10	RU	$\{\sim p_3,\ r_3,\ q_2,\ \sim q_3,\ \sim r_3\}$

Line 6 is a marked line and is never unmarked. q, $\sim r$ and $\sim p$ are all **POCH3**-consequences of Σ that are not consequences for any of the other strategies.

8.9. $\Sigma^I = \langle \{p_1,\ \sim p_1 \vee q_1\},\ \{s_2\},\ \{\sim q_3 \vee \sim s_3)\} \rangle$

The only minimal *Din*-formula here is $(q_1 \wedge \sim q_3) \vee (s_2 \wedge \sim s_3)$. For every strategy p will be a consequence. For the reliability and the safety strategy, q_1 in a condition and s_2 in a condition leads to marking. For the minimal abnormality strategy, formulas that are derived on a condition that does not contain q_1 and on another condition that does not contain s_2, are saved. For example $q \vee s$ and $(q \wedge \sim s) \vee (\sim q \wedge s)$ are **POCH2**-consequences of Σ. For the defeasibility strategy, only lines that have q_1 in their condition are marked. For example s and $\sim q$ are **POCH3**-consequences of Σ that are not consequences for any of the other strategies.

8.10. $\Sigma^I = \langle \{p_1\},\ \{\sim p_2 \vee q_2\},\ \{\sim q_3\} \rangle$

The only minimal *Din*-formula is $(p_1 \wedge \sim p_2) \vee (q_2 \wedge \sim q_3)$. The results for the reliability and the minimal abnormality strategy are the same. Only $\sim q$

and its **CL**-consequences are **POCH1**, resp. **POCH2**-consequences of Σ. For the defeasibility strategy $\sim q$ and $\sim p$ are finally derivable, whereas for the safety strategy $\sim q$ and p are finally derivable.

9. Comparing the strategies

In example 7 the minimal abnormality strategy gives the most acceptable result. For examples 8, 9 and 10, the best results are obtained by the defeasibility strategy. Whether the safety strategy can be the most efficient, we can not see from these examples, but we can imagine another one.

$$\Sigma = \langle \{q \supset \sim s,\ q \supset p\},\ \{q\},\ \{s\} \rangle$$

$$\Sigma^I = \langle \{\sim q_1 \vee \sim s_1,\ \sim q_1 \vee p_1\},\ \{q_2\},\ \{s_3\} \rangle$$

The only minimal *Din*-formula is $(\sim q_1 \wedge q_2) \vee (\sim s_1 \wedge s_3)$. Here the safety strategy gives the most consequences: $\mathbf{Cn_{CL}}(\{s,\ q,\ p\})$. The reliability strategy is clearly the least efficient and is in no situation preferable to the other ones. Choosing one of the logics **POCH2**, **POCH3** or **POCH4** as the best in all cases is impossible. In examples 7, 8 and 9, the consequence set of the most efficient logic coincides with the richest consequence set. The idea arises here that we have to choose for every situation the logic giving the richest consequence set. This is also supported by the fact that in a rational discussion one only reconsiders a conclusion when necessary, we should keep as much as possible. We can suggest the following

Choice of the logic:

If there is an $i \in \{2,3,4\}$ s.t. $\mathrm{Cn}_{\mathbf{POCHj}}(\Sigma) \subseteq \mathrm{Cn}_{\mathbf{POCHi}}(\Sigma)$ for $j \in \{2,3,4\}$, then **POCHi** is the appropriate logic.

Unfortunately, there could be cases in which there is no such logic. The latter is confirmed by example 10. Let us have a closer look at this situation. Stating p, later $p \supset q$ and eventually $\sim q$ is not that transparent. In real discussions it is even an obscure evolution. The speaker in question, let's call him Ché, does not tell us what was wrong with the arguments p and $p \supset q$. This could of course be said by another participant. By not reacting Ché implies his agreement. Maybe we should have incorporated the interventions of the other participants on which Ché agrees. In the supposition we have done so, the evolution is rather incomprehensible, but one is inclined to think that rejecting p is the appropriate interpretation in view of the least priority.

It is also worth mentioning that the **POCH**-logics handle this situation differently when the arguments p and $p \supset q$ are stated in the same intervention. Then p_1 and $\sim p_1 \vee q_1$ are equivalent to $(p_1 \wedge \sim p_1) \vee (p_1 \wedge q_1)$. Because of the consistency of **CL**, the latter is equivalent to $p_1 \wedge q_1$. We can conclude that the intervention $\{p,\ p \supset q\}$ gives the same results as the intervention $\{p,\ q\}$. This occurs in example 9. Therefore p is a consequence for all strategies, though it should not belong to the ultimate position of the speaker.

In view of the above comments it is clear that these logics are an improvement, but certainly not a satisfactory solution. It could possibly be more efficient to proceed step by step by looking at each stage what is compatible with the results of the previous stages.

10. Compatibility step by step

This idea could be handled semantically, but as it is my purpose to look after representations of human reasoning, I shall only explain the proof theory.

The strategy here will be to consider step by step which consequences of a lower level can be added to the previous considered levels. The aim is to keep as much **CL**-consequences as possible, that is to keep all **CL**-consequences that are compatible with the previous level. This is justified by the fact that in a rational discussion one only reconsiders a conclusion when necessary, as was mentioned before in section 9. As underlying logic we take **CL**. Let $\Sigma = \langle \Gamma_1, \Gamma_2, ..., \Gamma_n \rangle$ be the set of interventions as before. Now which should be the first level to be considered? If Γ_n is inconsistent, we do not want to add all its **CL**-consequences of course. Therefore we shall introduce a $\Gamma_{n+1} = \emptyset$. From Γ_{n+1} all **CL**-theorems are **CL**-derivable and no inconsistencies are compatible with them.

It could be that premises are not compatible with the previous level. Because here we do not want to change the form of the premises, we shall have to introduce them conditionally. The condition will be an ordered set containing the premise itself and the stage to which it belongs (the index of the Γ_i to which it belongs).

PREM If $A \in \Gamma_i$, one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) a dash, (iv) PREM, and (v) $\{\langle A, i\rangle\}$.

To have at each moment the possibility to derive **CL**-consequences, we need an unconditional rule.

RU If $B_1, \ldots, B_m \vdash_{\mathbf{CL}} A$, and $B_1, \ldots, B_m$ occur in the proof on the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) the numbers of the lines on which the B_i are derived, (iv) RU, and (v) $\Delta_1 \cup \ldots \cup \Delta_m$.

Now when should a line be marked? A line should certainly be marked when it is not compatible with the previous level. Also the lines of which the derivation is based on marked lines should be marked. In view of the rules PREM and RU, a line should be marked when the condition contains an element $\langle A, i\rangle$ for which $\neg A$ is derived on a previous level. The level on which a formula is derived can be formally defined as follows.

DEFINITION 25. *If A is derived on condition $\{\langle B_1, i_1\rangle, \ldots, \langle B_m, i_m\rangle\}$, then the level of this derivation is $min\{i_1, \ldots, i_m\}$.*

It could also be that some formulas are compatible with the previous level, but that they are not jointly compatible with it! In that case the disjunction of the negations of these formulas is derivable on the previous level. To recognize such a situation it is necessary to check whether the disjunction of negations is minimal (at that or a higher level), that is whether no disjunction of a subset of these negations is derivable at that or a higher level.

DEFINITION 26. *$\neg A_1 \vee \ldots \vee \neg A_m$ (we shall write $Don(A_1, \ldots, A_m)$) is a* ***minimal disjunction of negations*** *of level i at stage s iff at stage s, $\neg A_1 \vee \ldots \vee \neg A_m$ is derived at level i on an unmarked line and for no $\Theta \subset \{A_1, \ldots A_m\}$, $Don(\Theta)$ is derived at stage s at a level higher than or equal to i.*

DEFINITION 27. *If at stage s, $\langle B_1, i\rangle, \ldots, \langle B_m, i\rangle$ all occur in conditions of lines in the proof and $Don(\{B_1, \ldots, B_m\})$ is a minimal disjunction of negations of a level higher than i, then all lines containing such a $\langle B_j, i\rangle$ for some $1 \leq j \leq m$ in their condition are marked.*

This is already something, but our aim is not yet completely achieved. There may be consequences of marked premises that are compatible (also in the broad sense) with the previous level. These must be saved in some way. The most straightforward way to do this is to introduce these formulas on their own conditions, as we did for the premises. Now not only consequences of a single marked line should be considered, but also consequences of several marked and unmarked premises of the same level.

RC If $B_1, \ldots, B_m \vdash_{\mathbf{CL}} A$, and $B_1, \ldots, B_m$ occur in the proof on marked or unmarked lines with the conditions $\{\langle B_1, i\rangle\}, \ldots, \{\langle B_m, i\rangle\}$ respectively, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) the numbers of the lines on which the B_i are derived, (iv) RC, and (v) $\{\langle A, i\rangle\}$.

We shall call this logic **PCOM**, referring to prioritized compatibility.

DEFINITION 28. $\Sigma \vdash_{\mathbf{PCOM}} A$ *iff* A *is finally derived in a* **PCOM**-*proof from* Σ.

It is again clear that the consequence set of Σ for **PCOM** is consistent and closed under **CL**.

11. Examples

11.1. $\Sigma = \langle \{p\},\ \{\sim q\},\ \{q\},\ \{\sim p \vee \sim q \vee r\} \rangle$

Let us have a look at a proof and see the interaction between RC and the marking, i.e. the marking of applications of RC.

1	p	-	PREM	$\{\langle p, 1\rangle\}$	$\checkmark$
2	$\sim q$	-	PREM	$\{\langle \sim q, 2\rangle\}$	$\checkmark$
3	q	-	PREM	$\{\langle q, 3\rangle\}$	
4	$\sim p \vee \sim q \vee r$	-	PREM	$\{\langle \sim p \vee \sim q \vee r, 4\rangle\}$	
5	$\sim q \vee \sim p$	2	RC	$\{\langle \sim q \vee \sim p, 2\rangle\}$	
6	$\sim p$	3, 5	RU	$\{\langle q, 3\rangle, \langle \sim q \vee \sim p, 2\rangle\}$	

Line 2 was already marked from the introduction of line 3. At this stage line 1 gets marked, though it looks rather obscure to do so. The point is that by the application of RC some arbitrariness sneaks in that should not be handled as real information. This arbitrariness can be expressed by also applying RC for the derivation of its counterpart. In this way the balance is restored.

1	p	-	PREM	$\{\langle p, 1\rangle\}$	$(\checkmark)$
2	$\sim q$	-	PREM	$\{\langle \sim q, 2\rangle\}$	$\checkmark$
3	q	-	PREM	$\{\langle q, 3\rangle\}$	
4	$\sim p \vee \sim q \vee r$	-	PREM	$\{\langle \sim p \vee \sim q \vee r, 4\rangle\}$	
5	$\sim q \vee \sim p$	2	RC	$\{\langle \sim q \vee \sim p, 2\rangle\}$	$\checkmark$
6	$\sim p$	3, 5	RU	$\{\langle q, 3\rangle, \langle \sim q \vee \sim p, 2\rangle\}$	$\checkmark$
7	$\sim q \vee p$	2	RC	$\{\langle \sim q \vee p, 2\rangle\}$	$\checkmark$
8	$(q \wedge p) \vee (q \wedge \sim p)$	3	RU	$\{\langle q, 3\rangle\}$	
9	$\sim(\sim q \vee \sim p) \vee \sim(\sim q \vee p)$	8	RU	$\{\langle q, 3\rangle\}$	

10	$\sim p \vee r$	3, 4	RU	$\{\langle q, 3\rangle, \langle \sim p \vee \sim q \vee r, 4\rangle\}$
11	r	1, 10	RU	$\{\langle p, 1\rangle, \langle q, 3\rangle, \langle \sim p \vee \sim q \vee r, 4\rangle\}$

As soon as line 9 is derived, the marking pattern changes. Lines 5, 6 and 7 are marked, line 1 is again unmarked and line 2 stays marked. The **PCOM**-consequences of Σ are the **CL**-consequences of $\{p, q, r\}$.

11.2. $\Sigma = \langle\{p\},\ \{p \supset q\},\ \{\sim p\}\rangle$

The **PCOM**-consequences of Σ are the **CL**-consequences of $\{\sim p\}$.

11.3. $\Sigma = \langle\{r \wedge s\},\ \{s \supset t\},\ \{\sim r\}\rangle$

The following proof illustrates the use of RC.

1	$r \wedge s$	-	PREM	$\{\langle r \wedge s, 1\rangle\}$	$\surd$
2	$s \supset t$	-	PREM	$\{\langle s \supset t, 2\rangle\}$	
3	$\sim r$	-	PREM	$\{\langle \sim r, 3\rangle\}$	
4	$\sim(r \wedge s)$	3	RU	$\{\langle \sim r, 3\rangle\}$	
5	r	1	RC	$\{\langle r, 1\rangle\}$	$\surd$
6	s	1	RC	$\{\langle s, 1\rangle\}$	
7	t	2, 6	RU	$\{\langle s \supset t, 2\rangle, \langle s, 1\rangle\}$	

Line 1 gets marked by the derivation of line 4. Line 5 is derived as a marked line in view of line 3. Thanks to RC s is derivable and can be used to derive t. The **PCOM**-consequences of Σ are the **CL**-consequences of $\{\sim r, s, t\}$.

11.4. $\Sigma = \langle\{\sim p\},\ \{p\}\rangle$

The **PCOM**-consequences of Σ are the **CL**-consequences of $\{p\}$.

11.5. $\Sigma = \langle\{p\},\ \{\sim p \wedge q\},\ \{\sim q\}\rangle$

The **PCOM**-consequences of Σ are the **CL**-consequences of $\{\sim q, \sim p\}$.

11.6. $\Sigma = \langle\{p\},\ \{\sim p \wedge q\},\ \{\sim q\},\ \{r\}\rangle$

The **PCOM**-consequences of Σ are the **CL**-consequences of $\{r, \sim q, \sim p\}$.

11.7. $\Sigma = \langle\{p \wedge q\},\ \{p \supset r,\ q \supset \sim r\}\rangle$

The following proof again illustrates the use of RC.

1	$p \wedge q$	-	PREM	$\{\langle p \wedge q, 1\rangle\}$	$\surd$
2	$p \supset r$	-	PREM	$\{\langle p \supset r, 2\rangle\}$	
3	$p \supset \sim r$	-	PREM	$\{\langle p \supset \sim r, 2\rangle\}$	

4	$\sim(p \wedge q)$	2, 3	RU	$\{\langle p \supset r, 2\rangle, \langle p \supset \sim r, 2\rangle\}$
5	p	1	RC	$\{\langle p, 1\rangle\}$ $\surd$
6	q	1	RC	$\{\langle q, 1\rangle\}$ $\surd$
7	$\sim p \vee \sim q$	4	RU	$\{\langle p \supset r, 2\rangle, \langle p \supset \sim r, 2\rangle\}$

Line 1 is marked by the derivation of line 4. Line 5 and 6 are marked by the derivation of line 7. Note that the formulas derived on line 4 and line 7 are equivalent, but the marking can only occur when the formula $\sim(p \wedge q)$ is recognized as being equivalent to $\sim p \vee \sim q$. Thanks to RC the disjunction of the two jointly incompatible formulas is derivable. The **PCOM**-consequences of Σ are the **CL**-consequences of $\{(p \wedge \sim q) \vee (\sim p \wedge q)\}$.

11.8. $\Sigma = \langle \{p\},\ \{q\},\ \{p \supset r,\ q \supset \sim r\}\rangle$

The **PCOM**-consequences of Σ are the **CL**-consequences of $\{q, \sim r, \sim p\}$.

11.9. $\Sigma = \langle \{p,\ p \supset q\},\ \{s\},\ \{\sim(q \wedge s)\}\rangle$

The following proof illustrates the marking of premises of the same level that are not jointly compatible.

1	p	-	PREM	$\{\langle p, 1\rangle\}$ $\surd$
2	$p \supset q$	-	PREM	$\{\langle p \supset q, 1\rangle\}$ $\surd$
3	s	-	PREM	$\{\langle s, 2\rangle\}$
4	$\sim(q \wedge s)$	-	PREM	$\{\langle \sim(q \wedge s), 3\rangle\}$
5	$\sim q$	3, 4	RU	$\{\langle s, 2\rangle, \langle \sim(q \wedge s), 3\rangle\}$
6	$\sim(p \wedge (p \supset q))$	5	RU	$\{\langle s, 2\rangle, \langle \sim(q \wedge s), 3\rangle\}$
7	$\sim p \vee \sim(p \supset q)$	6	RU	$\{\langle s, 2\rangle, \langle \sim(q \wedge s), 3\rangle\}$

Lines 1 and 2 are marked by the derivation of line 7. An equivalent formula to the one derived on line 7 was already derived on line 6. The **PCOM**-consequences of Σ are the **CL**-consequences of $\{s, \sim q\}$.

11.10. $\Sigma = \langle \{p\},\ \{p \supset q\},\ \{\sim q\}\rangle$

The **PCOM**-consequences of Σ are the **CL**-consequences of $\{\sim q, \sim p\}$.

12. Evaluation and open problems

For all the examples the system **PCOM** gives precisely the desired results. The only limitations it has are due to our intuitions and demands. We have been looking for a consequence relation of which the consequences represent the construction of the ultimate position of a participant in a discussion after possibly one ore more position changes. The construction here is based on our intuitions, so the system can only be considered satisfactory as far as it expresses our expectations. One of our presuppositions was that statements of other participants on which Ché agrees were incorporated in Σ. When this is not fulfilled, we need the interventions of the other participants to figure out ourselves on which parts Ché agrees or to question him about it to obtain really certain information. The latter situation contains two open problems: one similar to the one solved here, but more complex, and one of erotetic logics.

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