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VAGUENESS UNLIMITED: IN DEFENCE OF A PRAGMATICAL APPROACH TO SORITES PARADOXES

Abstract. As far as 'modern' logical theories of vagueness are concerned, a main distinction can be drawn between 'semantical' ones and 'pragmatical' ones. The latter are defended here, because they tend to retake into account important contextual dimensions of the problem abandoned by the former. Their inchoate condition seems not alarming, since they are of surprisingly recent date. This, however, could very well be an accidental explanation. That is, the true reason for it might sooner or later turn out to be bearing exactly on the fundamental human limitations, when it comes to theorizing, that these approaches are urging us to appreciate.

1. Introduction

Despite their antiquity and numerous ambitious attempts at their resolution, *Sorites Paradoxes* remain of the most astonishing philosophical knots. The famous logical 'impossibility' of turning a non-heap of wheat into a heap of it (the original *Sorites*), or a non-bald person into a bald one (sometimes referred to as the *Falakros*), when adding grains or removing hairs only one at a time, indeed seems to constitute one of man's hard confrontations with unintelligibility. For how on earth could a tiny grain make the difference

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between what is obviously a heap and what is not? Or how could we reasonably expect anyone ever to become bald merely by losing one single hair? Yet, if not so, we are driven into an equally unpleasant *regressus*: no hairy person can ever be turned into a bald one, provided the hairs (even all of them!) are plucked from his scull one by one. Clearly, from the moment we involve in 'slippery-slope' or 'little-by-little' arguments like these, something inevitably goes wrong.

In this paper, first, in section 2, some general, preparatory considerations are given concerning the puzzle of vagueness. Next, in section 3, follows a concise overview of the mainstream accounts of the Sorites Paradox, which all tend to stay within the semantical realm for a solution. It has not been the intention to develop or even refer to an elaborate critique of these accounts here (for the latter of the two tasks, however, see my [42]). The core of this paper is formed by section 4. In it, some less common approaches to vagueness are focused upon, approaches that contrast sharply with the ones of the previous section, and are hardly or not at all acknowledged in some recent monographs on the topic, viz. Burns [4], Williamson [48] and Keefe [23], but in the author's opinion really do deserve more attention and closer examination. They include contextual and paraconsistent accounts. It is argued for that the main characteristic they share, namely that in the treatment of Sorites-infected arguments, they make room for pragmatical factors, i.e. factors concerning the relations between language and its users rather than those between words and objects, is to be the key to a better understanding of and dealing with this ubiquitous phenomenon.[1] By way of conclusion, in section 5, the entire discussion is situated in a wider philosophical perspective.

2. General Considerations

One of the possible formal presentations of the 'Sorites' is most straightforward. It opposes two incompatible observations, e.g. that, *one*, a man with zero hairs on top is definitely bald, Bx(0), but that, *two*, a man with a hundred thousand hairs is clearly not, ∼Bx(100.000). Given these, it is not difficult to prove that, *three*, there must be some number i between 0 and 100.000, so that a man with i hairs is not bald, whereas a man with i+1 hairs is:

[1] It should be noted though that *Linda Burns* herself favours a pragmatically inspired approach, which is the only one of the kind briefly discussed in *Rosanna Keefe*'s survey (see 4.1). *Timothy Williamson* does not touch upon pragmatics at all. More on the preferred accounts of the latter two in 3.1.

	1.	Bx(0)	(first premise)
	2.	∼Bx(100.000)	(second premise)
∴	3.	(∃i)(Bx(i) & ∼Bx(i+1))	(quantified conclusion)

With extremely simple means, an obvious paradox has hereby been identified within the borders of classical logic (CL henceforth).[2] But one may wonder whether, as commonly assumed, it really is an expression of what is ordinarily called vagueness. On a superficial level, the answer definitely is 'no', for in everyday language, the philosophical notion of vagueness usually gets conflated with those of *generality* and *ambiguity*. To use *Max Black*'s clearcut distinction: "The former is constituted by the application of a symbol to a multiplicity of objects in the field of reference, the latter by the association of a finite number of alternative meanings having the same phonetic form" (Black [3], p.29).[3] Here, however, a deeper level should be aimed at, a level where there is no room for such terminological confusion, but where it is explicitly agreed upon what, semantically spoken, the phenomenon of vagueness has to do with: the *boundary* of a term's extension, not the extension as such.

Consider the following real-life situation. You are in your favourite pub together with a bunch of friends. Being in an experimental mood, as you have all had a few drinks, each of you tries to mark off for himself the set of bald men in the pub. What the result hereof will be, of course one cannot exactly tell, for surely that will depend on the concrete circumstances. In any case, it has to be considered doubtful that there will be a consensus among you all; i.e. *problem 1*. Furthermore, it seems unlikely that, when repeating the game an hour or so later, the demarcating behaviour of each of you will have developed in accordance with an established or establishable pattern. For in the meantime some people will have left, some will have entered, the number of customers present might differ, and each friend has presumably had a few more drinks. Brief: a number of contextual parameters will have fundamentally transformed; i.e. *problem 2*. This problem will be further touched upon in section 4. The issue at stake here is whether this fictitious experiment has got anything to do with the kind of paradoxes spelled out at

[2] The other, traditional presentations of the paradox involve, *first*, mathematical induction on a first and a universally quantified premise, followed by instantiation, or, *second*, repeated modus ponens on first premise and instantiations of the universal one (see, e.g., my [42]).

[3] From the moment one really starts paying attention to it, the frequency at which the word 'vague' is misused, both by laymen and scholars, viz. to characterize the non-specificity, i.e. generality, of otherman's arguments, proofs to be irritating. As a consequence, the word 'vague' seems to be a quite ambiguous one (sic).

the beginning of the present section. To catch this point, contrast the pub scene, which will be called here *form 1* of the Sorites argument, with the following one, matching the rigourously formulated paradox more closely. You are in a neutral environment, all by yourself, observing a single, isolated test-person. The latter is being mechanically robbed of his previously carefully counted hairs one at a time, after each removal of which you are expected to answer the question 'Is this person bald?' by a simple 'yes' or 'no'. Call this laboratory scene, which was borrowed from Schiffer [38], *form 2* of the argument. It is true that there is a certain amount of – indeed – vagueness between the two forms spelled out, but this does nothing but strenghten the case made, namely, that it is very hard, or even virtually impossible, to pin down the phenomenon of vagueness in exact terms. Which is of course the initial problem all over again. As will be argued for further on, this has everything to do with what, in technical terms, is commonly referred to as *higher order vagueness* (see the beginning of section 4).

Further peculiarities: *first*, in the literature, theorists nearly always focus on predicate expressions, as the central or exemplary cases, and *second* and most telling, they do so by means of examples of an extraordinary variety. This is an unhappy circumstance, for it makes it hard to see what, if anything, these various instances have in common. In fact, this is the reason why, in this section, it has been opted for not to introduce the topic from the 'ordinary' point of view (form 1), but in the more artificial yet rigorous way instead (form 2). Strikingly, in the hundreds of pages of specialized literature on the subject, mostly, this 'formality' is neglected all together. Nevertheless, when done carefully, in most cases, the two forms can be (more or less) reduced to one another. Beginning with form 2 has moreover the important advantage that the reader is kept from taking a theoretical move too soon, e.g. by immediately judging *Law of Excluded Middle* and/or *Principle of Bivalence* as invalid. This illegitimate move is far more easily elicited when first confronted with real-life examples, as the latter rub us with our noses in the clear kind of *no man's land* or grey zone between the areas of 'definite' application and non-application respectively. Indeed, it is a plausible reaction then to define a vague concept (be it intuitively or implicitly) as one lacking sharp boundaries.

On the other hand, again, the non-formal expression of vagueness (form 1) is far more instructive, and indeed essential, because in the end, it is real-life situations that matter to (most of) us. Consider, e.g., the application of age limits in criminal law. There is however a deeper consideration to be made here. After all, one could say, is not CL, the language in which the Sorites Paradox was uncovered, a logical theory as good (or bad) as any of

its alternatives? Well, yes, and no. There are, admittedly, strong reasons to prefer good old two-valued CL. As one simply has no choice but to employ a particular logic, it might as well be kept as coherent and consistent as possible under the circumstances. So why not stick to powerful CL, as long as it works? We then start out dividing (well formalized) sentences, within particular theories, in true and false ones. Whenever we run into trouble, there is room for argument whether to change our logical instrument or not, and in the latter case e.g. to add extra-logical, c.q. pragmatical, constraints to our account. It has to be noted that the actual dominance of CL has allowed paradoxes like the 'Sorites' to pop up in the first place, dragging along long discarded fundamental questions such as *What's validity after all?*

It is the major, and seemingly endless (though significantly so) task of the discipline called *Philosophical Logic* to sort out essential matters like these. It therefore should and does investigate the foundations of logic(s), in their relations to other philosophical branches, such as ontology, epistemology, or ethics. In particular, this task is fulfilled in designing and promoting proper formal tools to handle phenomena such as the semantic paradoxes. There obviously is far from a consensus in doing so, as in fact this entire paper may testify. But this might be good news. At the very least, it secures us from an unfruitful, and in a sense even dangerous, monopoly of CL. Not any logic should ever be taken for granted (let alone venerated), but its very basics should be constantly open to scrutiny. As *Stephen Read* in a perhaps somewhat dramatic fashion has recently proclaimed: "Teaching in philosophy departments across the world exhibits this schizophrenia, in which the dogmatic approach to logic sits uncomfortably side by side with the ceaseless critical examination which is encouraged and demanded in philosophy" (Read [34], p.2). A final introductory point relates to this complaint. Coping with a paradox, it is one thing to provide with a neat *logical* or formal solution, but quite another to properly motivate it, by giving independent reasons for the technical interventions, i.e. to provide with a satisfactory *philosophical* solution (see, e.g., Haack [15], p.139-140). As will be argued for in the closing section, this distinction has by far received more attention in the approaches that have been labeled here as 'alternative' (section 4).

3. Mainstream Approaches

In this section, the major 'classical' recipes for dealing with the Sorites argument are briefly considered. Though an appeal to alternative logic is often made, these are analyses that have all remained strictly semantical in their approach to vagueness. The various projects can be further divided

in *coherentist* and *incoherentist* ones, depending on whether their designers believe the paradox to be of a remediable or insoluble ('genuine') kind.

3.1. Coherentism

During the last twenty-five to thirty years, various claims to a logically coherent solution for Sorites Paradoxes have been made. Arguably the most simple, yet in (not just) my view highly contra-intuitive, coherentist solution for Sorites Paradoxes is the semantically realist *Epistemic View* (EV). It has been most prominently defended by *Roy Sorensen* and Timothy Williamson (see their [39] and [48] respectively). This account allows CL to be entirely retained in the face of vagueness. It says that, *yes*, there is a fact of the matter whether a certain, problematic, amount of wheat is a heap or not, but that, *alas*, we humans are in no position (nor could ever be so) to find out. More specifically, for EV, it is due to fundamental inadequacies of our epistemic apparatus ('blindspots', Sorensen has called them) that we are denied access to some crucial piece of knowledge about the borderline case. And "once hidden lines are admitted", the argument runs further, "why should a line between truth and falsity not be one of them" (Williamson [48], p.201). In EV, bivalence holds, be it that we cannot find out which of both truth-values applies. The authors mentioned go on then justifying this essential ignorance in, it must be said, interesting independent ways. The core of their message is that vague facts are mere epiphenomena, 'supervening' on precise ones.

To date, the more popular coherentist remedies remain the many-valued approaches known as *Supervaluation* and *Fuzzy Logic*. The supervaluational technique (SV), which combines classical models with the possibility of a (non-truth-functional) higher-order indeterminacy, was initiated by *Bas van Fraassen*, but fitted for vagueness by *Kit Fine* (see his [9]). It tries to reconcile two apparently conflicting attitudes when confronted with a typical Sorites series: (1) that there admittedly is no one transition point which is valuable for each and every observer, but (2) that at the same time, for each and every observer there surely is a transition point. In reconciling these positions, one will preserve CL on the object-level (accomodating 2), while allowing dissensus on an extra meta-level (accomodating 1). If all the ways of precisifying a vague sentence lead to the same assessment of its truth or falsity, then the sentence is true or false respectively. Otherwise it is neither, and the supervaluational model remains underdetermined. Goble [13] offers a recent and elaborate criticism of SV, while the view has been eloquently defended against rival accounts lately in Keefe [23]. For an interesting variation on it, see subsection 4.4.

Fuzzy logicians, who undoubtedly paved the way for substantial technological successes (reaching from washing machines to expert systems providing with medical diagnoses), offer a treatment for vagueness by generalizing two-valued truth-functionality *ad infinitum*, and hereby actually making any Sorites Paradox 'melt away'. This is accomplished by judging not entirely true the quantified premise, which secures that whenever an x(i) in the series is predicated B, then also x(i+1) is, and this for all i's. Though it is true enough for daily practice, where we very rarely (have to) run through an entire Sorites series, a fuzzy account will rob the statements under consideration of a tiny fraction of their truth-value whenever a single step from an x(i) to an x(i+1) is taken. The fact that this subtle mechanism 'uncovered' by fuzzy logics remains unnoticed by the ordinary speaker, explains why the latter gets embarrassed in the course of a typical Sorites argument. *Graham Priest* has recently done some peculiar work in this inspiration, devising a framework to 'fuzzify' identity (Priest [28]), the answer to a problem which he himself had formulated some years earlier (Priest [27]), introducing a 'non-standard' form of the Sorites argument (involving substitution of identicals instead of modus ponens for an inference rule).

To complete the picture, some less competing coherentist solutions should be mentioned. They include *intuitionist* ones, such as the one put forward by *Hilary Putnam* (see his [30]), and some *three-valued* strategies using *Kleene Tables* (be it the strong or weak version).[4]

3.2. Incoherentism

Contrasting sharply, then, with all the fairly optimistic approaches in the previous subsection, are inconstructive ones, from those who believe the Sorites argument as a whole to be of an irreducible incoherence. They think it forms a 'genuine' paradox, so that a coherent logical interpretation is *in principle* impossible. Part of this group, *logicists*, exemplified by *Gottlob Frege*,[5] have propagated the radical banishment of the vague, incoherent components out of logical language. The latter ought to remain precise by definition. It constitutes an ideal realm, which is aimed at by, yet finally unattainable for poor humans. These, as a consequence, have but one option: to live their miserable lives infected with such terrible natural language diseases as vagueness.

[4] For some more details, see my [42].

[5] There is even an explicit reference to vagueness, in his famous *Begriffsschrift*. See, e.g., the [11]-reprint (p.62).

Others have devoted themselves to the joyful acceptance of the semantical incoherence. It is figures in this group of 'incoherentists' that have been playing a pivotal role in the transition to most of the 'alternative' approaches gathered in the next section. This means that, from the point of view of this paper, which seeks to spotlight alternatives to the classical approaches, things are getting more interesting. There is some further subdivision in order though. Within this group of incoherentists, there are *first* those who have thought that, when facing vagueness, and matters being incoherent as they are, the quest for a sensible semantics should be abandoned all together. A *second* group (c.q. few) have opted for paraconsistent models, while a *third* have been aspiring to develop approaches much more rich and complicated than the logical ones in the strict sense, namely by allowing the field of examination to contain pragmatical criteria next to the well known semantical ones.

Members of the first group[6] are known as *nihilists*, as for them, typically, 'there is no use in trying to get a firm grasp on vague concepts'. Some of them are *global*, others *local* nihilists. The former embrace a form of scepticism, denying that there are such things as bald heads or heaps in the first place, and are nowadays considered of little interest.[7] The latter view has been most elegantly defended by *Michael Dummett* in his [8]. Since the use of vague language is intrinsically incoherent, as he claims to show, which means the paradoxes involved are indeed insoluble, it is clear, for him, that a workable logic for vagueness is illusory. This is a confirmation of what Frege said, but contrary to Frege, Dummett does not seek to eradicate vagueness, and goes on to appreciate it as an essential feature not of a defective language, but of one that truly functions. Vague fragments nonwithstanding, his message is, we speakers usually understand ourselves and one another, or at least we do so to a large enough extent so as to accomodate daily life. In other words, most of the times, language 'works'. If in effect we would demand, on top of that, mutual (or indeed even individual) consistency in the meanings of our vocabulary, all linguistic efficiency would desperately be lost, because awaiting the impossible, (internal) communication would inevitably come to a hold.[8]

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[6] Second and third group will in fact be treated in the next section.

[7] See, e.g., Wheeler [46] and [47].

[8] Here, an early trial to paint a formal picture of what happens when we run into troubles like these, might be worth mentioning, viz. Kamp [22]. It is an attempt to make a general appreciation of the explicit and implicit contextual background knowledge governing discourse. The technical translation of this, however, gets hopelessly complex. No wonder, for, as the author himself readily admits: "The contextual factors that can

It has been *Crispin Wright* in the work of whom the shift towards a positive and constructive attitude has truely taken place. More specifically, he has been increasingly promoting a non-propositional account of vagueness, implying a fierce criticism of the linguistic principle of rule-boundedness. Even admitted that the latter would in effect be acceptable, he further wondered, how could we ever, and why would we ever want to penetrate to the understanding of any such rule? Speakers, after all, implicitly follow them, no matter what. His initial position to vague concepts was very close to that of Dummett. Continued fascination by the huge actual success of the 'language-game' (Wittgenstein [49]!), made that his eventual diagnosis, fully elaborated in [51], substantially deviates from, or rather goes beyond, that of Dummett. For him, there is no fallacy in our inconsistent, yet functional fabric called natural language, but it is the very idea of incoherently codified practice that is in itself nonsensical:

> "If the rules for the use of 'red' really do sanction the paradox, why do we have absolutely no sense of disturbance, no sense that a real case has been made for the inferential ingredient at all? Are we so abjectly irrational that we cannot recognize our confusion even when it is completely explicit? A different account is called for" (Wright [51], p.213).

It is important to note that any such account must not be allowed to render the notion of meaning pointless. On the contrary, it will have to enable us to deal with non-cognitive, yet meaningful knowledge, such as the one constituting practical skills. These, we just seem to be able to perform, but we could not ever (want to) describe them in full detail. We learn them by doing, in the course of an ever-lasting ostensive training, so that their meaning is always under construction. In the course of the 1990's, on the basis of this new and penetrating philosophical idea, various researchers have set out to develop more elaborate, and what can largely be considered as 'pragmatical', logical strategies.[9]

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help to reduce vagueness are so varied as to render the task of an exhaustive analysis of contextual disambiguation [Kamp had initially dreamed of a solution along the lines of SV] a virtual impossibility" (op.cit., p.242). In spite of this illuminating insight, the aspiration of giving a full, coherentist account remains. For a 'pragmatical' comment on the framework, see Burns [4], §5.8.

[9] Strangely enough, and contrary to his philosophical plea, Wright himself has not seemed able to let go 'simple' semantics. In the article cited above, he is still struggling to get a bivalent grip on the phenomenon by means of a definitely-operator. Very recently then, he has most earnestly been promoting an intuitionist path, contrasting it to Williamson's EV (see Wright [52]).

4. Alternative Approaches

4.1. Introduction

As early as in 1975, Crispin Wright noted:

> "The utility and point of the classifications expressed by many vague predicates would be frustrated if we supplied them with sharp boundaries. [...] It is not generally a matter simply of lacking an instruction where to draw the line; rather the instructions we already have determine that the line is not to be drawn" (Wright [50], p.330).

It has taken some fifteen more years though, before dissatisfaction with the aforementioned approaches, the coherentist as well as the incoherentist ones, has led a number of logicians to still take a step further than Dummett, and to finally heed Wright's call to consider fundamentally other, notably pragmatical strategies to get a grip on Sorites-infected arguments. Their common inspiration has been most beautifully expressed by *Mark Sainsbury* in an inaugural speech. The too strict and therefore allegedly contra-productive adherence to a form of *referential semantics* the former treatments exhibit, drives him to promote a radically different view on Sorites-related issues, including a severe critique of our classifying ability. What he suggests "is that almost all concepts lack boundaries, so that the classical picture is of very little use to us" (Sainsbury [36], p.7). "Classical" is here clearly meant to apply to much more than just the two-valued approaches, as to include at least all solutions gathered in the previous section, in fact all those not prepared to give up at any price the idea of labeling propositions with a fixed, definite meaning, be it in principle. In this section though, some proposals will be presented that do take a further step, and expand the field of logical flexibility beyond that of the traditional 'non-classical', i.e. multi-valued, logics.

Theorists belonging to the semantical camp have been confronted with one particularly tough problem over and over again: that of higher order vagueness (HOV henceforth). It pops up whenever one tries to attribute the available truth-values to statements, i.e. when trying to link the vague object-language with the precise meta-language, answering the question of when we are supposed to switch values. The availability of more of those values might very well soothe certain practical problems, and may particularly be extremely welcome in applied contexts, thus providing with a neat technical solution. The *philosophical* problem however, i.e. that of justifying a transition between truth and falsehood, is far from met, but on the contrary gets duplicated with each introduction of an additional truth-value!

Philosophically speaking, fuzzy logic, in this sense, is nothing but a radicalization of the very idea behind bivalence. Sainsbury's sharp comment on this is that "you do not improve a bad idea by iterating it" (op.cit., p.11). Hence, the argument in favour of pragmatical accounts, concerned also with language-users rather than just the world they talk about, will be the absolute pointlessness of rigid linguistic boundaries, supported by a most straightforward observation: that of the apparent unwillingness and/or inability of speakers to draw them. Similar pragmatical considerations can, e.g., take the form of contextual constraints, whether internal, as in the case of *Diana Raffman*, or external, as in the case of *Ruth Manor*.

First, however, some considerations about the one 'pragmatical' view that hás received a bit of mainstream publicity, e.g. in Keefe's recent survey ([23]): that of Linda Burns (see [4]), which explicitly draws upon *David Lewis*'s semantical theory. Briefly, with respect to vagueness, a multiplicity of precise languages is considered that are at the disposal of speakers. According to the circumstances then, they engage in one of these alternative languages. Consequently, on the individual level, vagueness is said to be non-existent: in principle, there are always relevant differences to be discovered so as to draw a boundary.[10] The reluctance to do so is a matter of incomplete knowledge, echoing the epistemic view touched upon in section 3. When urged to, this *informational* form of 'unreal' vagueness can easily be sidestepped by stipulation. It is, conversely, only on the interpersonal level that 'genuine' vagueness turns up. At this point, Keefe, the only commentator undertaking a general appreciation of any pragmatical approach, makes the move to recycle Burns within her own favoured, supervaluational account (see section 3). However, even if one could readily accept the conditional that *if* there were no vagueness but on the interpersonal level, *then* a supervaluational account would very much do (an equivocation problematical enough), surely this does not mean that its antecedent, all too easily accepted by Keefe, has been settled. Especially when this apparently seems to cut off the way for any pragmatical approach:

> "[A]ny attempt to treat vagueness as an entirely pragmatic matter must be of broadly the same form as Burns's account. For if vagueness is a pragmatic matter, then languages are not themselves vague. But then [...] the view would collapse into the epistemic view [...]. So a cluster of precise languages must be involved" (Keefe [23], p.141).

[10] Burns's arguments against the possibility of a genuine Sorites Paradox have been criticized at length in Varzi [43].

After which these apparently 'collapse' into SV. Together with Burns, Keefe is simply neglecting here this single most important preoccupation shared by pragmatist approaches: the striving for a satisfactory philosophical explanation of the Sorites Paradox, taking it at face value. She has the right to do so, lest I be misunderstood, but in the same move, Keefe illegitimately reduces pragmatical views of vagueness to the one championed by Burns. So let me now turn to a number of proposals that, in my view, do have the right 'feel' about, from the common sense point of view. One might say that they accept the inevitability of running into inconsistency, and either don't bother much about it, or seek an elegant way to cope with it, rather than theorize it away.

4.2. Internal Contextualism

What will be called here *internal contexts* are in fact *psychological states*. In trying to 'humanize' the field of investigation, it should indeed be justifiable to bring psychology in. Remember Williamson said that the problem was epistemical, so that CL could be entirely saveguarded, and that alternative logicians, who renounced this, instead claimed it was a semantical indeterminacy or variety it was all about. In contrast then, some have urged to view vagueness as a psychological phenomenon, taking the human paralysis of judgment when confronted with borderline cases most seriously. One of them, *Stephen Schiffer*, has recently developed a theory of what are called *vagueness-related partial beliefs*. In contradistiction to standard partial beliefs, these beliefs are not subject to subjective probability theory, for there is no possible way of improving or even idealizing their epistemic circumstances.[11] Clearly, this condition blocks the way for hidden precise facts, susceptible to bivalence, to become available for vagueness to supervene on, as well as it does for semantical pluralist diagnoses: "The propositional attitude we have towards a borderline case can't be *constitutive* of a thing's being a borderline case of a property; it must merely be a *reflection* of, or *response to*, the thing's status as a borderline case" (Schiffer [38], p.230).

While Schiffer seems not at all preoccupied with filling up the psychological black box that our head is considered to be, most certainly Diana Raffman has been.[12] In fact, the term *internal context* is hers, and her ac-

[11] The laboratory scene in section 2, borrowed from Schiffer [38], will do as a nice example. The paper mentioned is followed by some vivid discussion, which I cannot go into here.

[12] Not surprisingly, both Schiffer and Raffman point back to Crispin Wright (see the end of section 3). Raffman, e.g., confides to have been inspired by his urge for "a more

count indeed begins where the previous one came to a halt, assuming "an adequate treatment of vague predicates and their Sorites puzzles must appeal to the character of our judgments about the items in the series" (Raffman [31], p.44). Let us depict this series, for a change, as a red to orange 'continuum' of colour patches.[13] It can and will be easily agreed upon that any subject doing the 'forced-march' of this spectrum, simply must report a shift in colour sometime sooner or later, in order to avoid the paradoxical conclusion. Does this inevitably mean then, that two particular adjacent patches will be reported to have a different colour? Raffman says not, and claims that neighbouring patches can and will never be in different categories when judged pairwise, only when judged singly.

To be able to explain this, she trades in the traditional (cartesian) picture of the integrated mind for a so-called *modularity thesis*, as at least two different mental personae are said to be at the basis of any observational decision: one categorizing, i.e. 'vertically' comparing with a (possibly imaginary) model, another discriminating, i.e. 'horizontally' comparing two objects. The latter of the two is said only to come into play from the moment the patches are (to be) considered pairwise. "Finding a pair to be marginally different, he will constrain his colleague to categorize them identically: 'Categorize them as you like,' he'll say, 'but categorize them together'" (op.cit., p.47). A suitable diagnosis for the 'Sorites' is straightforward: if judgments alongside the series are made pairwise, a shift in colour cannot happen but pairwise. Hence, it becomes possible that two adjacent patches, say x_5 and x_6, are said to be red, while just after (or before) that, the patches x_6 and x_7 jointly receive(d) the label 'orange'. And there is more to it. A phenomenon called judgmental inertia prevents categorical shifts from being local. As long as pairs of patches are judged as red, the 'polar' attraction of the original red reference will remain fairly strong.

The latter phenomenon is also the reason why, once a shift to orange has occured, it will often be observed that the brand new, orange reference spreads its force way back in the series, so that 'red' pairs now have become 'orange' ones. This does not however, at any moment, imply that couples of patches are seen as red and orange at the same time. Perception is episodic, and it is precisely because of this kind of shifts that no discontinuities whatsoever are perceived. In this respect, it is also to be noted

behaviouristic semantics", while "for the most part, assaults on the paradox have been strictly 'logico-semantic' in their methods and presuppositions" (Raffman [31], p.43).

[13] It was to this manifestation of the troublesome argument that the quote of Crispin Wright, at the end of section 3, referred.

that the presentation of the colour-spectrum can turn out to be extremely important. Just think about the force of the initial red reference when all of the patches (also the clear orange ones at the extreme of the continuum we are moving towards) are exposed during the entire experiment, as compared to its force when only a couple of patches are uncovered at a time. Also, the importance of the predicates dealt with becomes increasingly clear. Not all types of opposites will exercise the same force on an observer. This suggests that arguably, we should not just inquire subject's anticipations of situations, but also the objective side of the contextual coin. The account that will be presented next, exactly does that.

4.3. External Contextualism

Ruth Manor, who in her turn explicitly challenges the sharp distinction between semantics and pragmatics, says the relevant contexts are not primarily internal or psychological, as Raffman supposes, but rather external or physical. More precisely, she considers the domain of objects to which properties are (not) predicated to be playing the crucial part in the varying meanings vague sentences exhibit. As Manor, in her [26], has proposed a formal framework that should do justice to this particular contextual constraint, I shall briefly set it out, meanwhile allowing the underlying idea gradually to become clear.[14] One of the basic ideas is that vague predicates may denote so-called 'foggy' objects. These are objects whose parts cannot unequivocally be determined. In formal terms, objects d are represented by sets of (alternative) sets of atoms belonging to the domain $A : d \in \mathcal{PP}(A)$.

As a set of subsets of A, any object d is characterized as a set of possible delineations. It will then be called a *distinct* object d_d, if it has only one such a delineation ($d_d = \{s\}$, with $s \subseteq A$), and a *foggy* object d_f, if it has at least two ($d_f = \{s_1, s_2, \dots\}$, with $s_1, s_2, \dots \subseteq A$). The *opposite* $\sharp d$ of an object d relative to A is defined as $\sharp d = \{A \backslash s \text{ for all } s \in d\}$, with $A \backslash s$ reading: the complement of s relative to A. An object d_1 is then considered a *boundary case* of an object d_2, *iff* every member of the former is a subset of some member(s) of the latter as well as a subset of some member(s) of its complement $\sharp d_2$. So the boundary case d_1 for an object d_2 is partly enclosed in that object d_2 (more exactly, to remain faithful to the terminology: some of the delineations of d_1 are enclosed in delineations of d_2), and partly enclosed in its complement $\sharp d_2$.

[14] A philosophical defence of her contextual view has been published as [25].

I shall be skipping some additional notions here, such as parthood, n-ary objects, universal vs. restricted domains, as well as the outline for a fully fledged formal model Manor builds on top of all this. Nevertheless, we are already in a position to get an idea of how a similar model can and will be used in dissolving Sorites Paradoxes. Consider the domain of people all over the world. In it, for every n, there is at least one person with n hairs on his scull. Let us therefore call H_n the non-empty extension of 'has n hairs'. Now consider an object $p = \{H_0, H_0 \cup H_1, \ldots, H_0 \cup H_1 \cup H_2 \ldots \cup H_n, \ldots\}$, the elements of which are the sets of people with a number of hairs less than or equal to n. Obviously, one can define its complement $\sharp p$. In this particular domain, p and $\sharp p$ then denote the bald and non-bald people respectively. One can verify that they are both vague (or foggy) in nature, for counting different subsets of the domain as their members (as the delineation of the n-haired clearly cannot be unequivocal). But more importantly, every set of people with a particular number of hairs, e.g. $\{H_n\}$, neither belongs to p nor to $\sharp p$. This means that the corresponding object $\{\{H_n\}\}$ can never belong to the extension of 'is bald' or 'is non-bald', or, put differently, that it is always a boundary case of both. This contextual account thus proves to be fruitful, for it appears to depend on how the domain gets divided, i.e. by determining n (from occasion to occasion), whether the group of n-haired will count as bald (together with the rest of the bald) or non-bald (together with the rest of the non-bald), while taken on its own this group cannot be but borderline. Any which case, things remain nicely under control.

4.4. Subvaluationalism

Reviewing Williamson's monograph [48], *Dominic Hyde* judged: "The broad scope of the historical survey, extending to a comprehensive bibliography, is marred only by the omission of dialectical or paraconsistent analyses" (Hyde [20], p.925).[15] This is strange, for our daily use of vague vocabulary more often has an overdetermined than an underdetermined feel about. Indeed, people tend to answer borderline questions such as 'Is it raining?' more easily with a 'yes *and* no' rather than with a 'neither'. The neglect is less disturbing however, once one realizes that there has hardly been done any profound research in this sense.[16] Hyde himself has then tried to fill the 'gap' (sic),

[15] As this very paper may testify, by now, a lot more would have to be regretted.

[16] At least to his [and my] knowledge. In the famous 'black book' (or 'bible', if you like) of paraconsistency ([29]), only one reference can be found, viz. to a tiny note of Arruda and Alves ([1]). This, in its turn, simply suggests some logical possibilities, but is not at all concerned with any philosophical justification of a paraconsistent treatment.

relying on a logical framework developed by *Achille Varzi* "from analyses of vagueness already in the philosophical literature - supervaluational analyses. A simple dualisation of supervaluational semantics results in a paraconsistent logic based on what has been termed *subvaluational* semantics" (Hyde [21], p.646), which Hyde has applied to the case of vagueness.[17]

In a subvaluation (SbV henceforth), a sentence is true (false) if it is true (false) for *at least one* precisification (compare SV in section 3: for *all*). Knowing that indeterminate sentences receive contradictory precisifications, it is clear that, in SbV, "indeterminate sentences take on both truth-values" ([21], p.647). The exclusivity of truth-values, i.e. the principle that for all sentences A, either A is true or A is false, no longer holds. But as in SV, CL's validity conditions remain intact. An inference is SbV-valid whenever *if* the premises are true in *a* precisification, *then* the conclusion is valid in *a* precisification. Just as the Law of Excluded Middle (LEM), for all $A : A \vee \sim A$, holds in SV, the Law of Non Contradiction (LNC), for all $A : \sim (A \,\&\, \sim A)$, does in SbV, as boundary cases, where $A \,\&\, \sim A$, clearly do not render invalid $\vDash_{SbV} \sim (A \,\&\, \sim A)$.

There is however an asymmetry compared with SV. It concerns one of the clauses characterizing the inference-relation, giving rise to a non-standard conjunction, and thus yielding a possibility for an alternative treatment of the 'Sorites', rendering its argument invalid. This is the clause: $A_1, \ldots, A_n \vDash_{CL} B$ iff $A_1 \& \ldots \& A_n \vDash_{SbV} B$, expressing the SbV-validity of LNC without exclusivity of truth-values.[18] The alternative way of disarming the paradox exploits the fact that Modus Ponens is not unrestrictedly SbV-valid. A boundary case Bx_n (e.g. 'a man with n hairs is bald') is true ánd false. Because it is false, the material conditional $Bx_n \supset Bx_{n+1}$ is always true. Because it is true, we are together with this true conditional able to infer Bx_{n+1}. However, it is in no way to be excluded that Bx_{n+1} can be clearly ('definitely') false. As a consequence: $Bx_n, Bx_n \supset Bx_{n+1} \nvDash_{SbV} Bx_{n+1}$ (for an n). The one exception to this is when both conditional and antecedent are the case within one and the same precisification of the premises. At this

[17] Varzi has recently presented his most instructive original nicely in [45], while in [44], he has provided with a more general, viz. "universal" framework, "within which a large variety of semantic theories naturally fall and from which the semantics of a wide class of logics (standard and non-standard) may be obtained as special cases" (p.4). Thus trying to regiment the large number of mainly *ad hoc* semantics that have been developed in the course of the last thirty years, he has been wanting to promote a more liberal attitude towards all kinds of problems concerning natural language (such as vagueness).

[18] In SV, we have a mirroring limitation though: a non-standard disjunction, as LEM is valid without bivalence. For proofs, see [21], p.648n.

point, an obvious connection becomes clear with the notions of 'local' vs. 'global' validity.[19] Also, it recalls Manor's strategy of properly marking out the domain under consideration in order to dissolve any apparent paradoxes. Pragmatically spoken, the major merit of paraconsistent logic in general has been to take incoherence or overdetermination in our daily language for what it is, trying to get it formally under control instead of theorizing it away.[20] In the case of vagueness, the latter attitude is precisely what has been so characteristic for the bulk of semantically coherentist solutions, which in spite have all proven unable to escape the drawback of HOV conclusively.[21]

4.5. Transvaluationalism

Terry Horgan has tied up the following two assertions to characterize the nature of vagueness: (T1) *The Incoherence Thesis*, drawing upon the Fregean school: vagueness is incoherent, *but* (important qualification) only weakly so, not strongly; and (T2) *The Legitimacy Thesis*, drawing upon the Wittgensteinean school: vagueness is a viable, acceptable and even essential aspect of human thought and speech.[22] Horgan explicitly declines nihilism. There is no point in despair in the face of vagueness. Yet, combining the above theses, while sticking to a *referential* semantics, one can hardly avoid running into what he calls the 'Fregenstein' conceptual monster. So instead of that, he favors the idea of a *contextual* or *psychologistic* semantics, in which operational standards may vary according to the internal and external circumstances at hand. Epistemological and ontological commitments are to be kept as strict as possible, but never add up to ultimate, reductive, absolute or universal ones. It is only when engaging in the latter that the incoherence harbored by vagueness has the disastrous nihilistic consequences Dummett has pointed us to. Rather, we should heed the calls of philosophers as Wright and Sainsbury to embrace the *boundarylessness* of vague concepts: there are no definite semantical transitions between true and false statements in a Sorites series, which constitutes the *robustness* of

[19] Surprisingly, 'dialetheist' Priest has been looking for a translation of these notions in fuzzy terms rather than in paraconsistent ones (see his [27] and [28]).

[20] It has even been argued for some years ago, jointly by *Newton da Costa*, *Otávio Bueno*, and *Steven French*, that the proper logic of pragmatic truth is in fact a paraconsistent one (see their [6]).

[21] Recently, *JC Beall* and *Mark Colyvan* have confirmed this point, but also raised an interesting one against Hyde's approach, on the very existence of a ponable, Sorites-supporting conditional in natural language, which seems a prerequisite for his proposed disarmament of the paradox (see [2]).

[22] The sources for this entire paragraph are Horgan [16] and [17].

this peculiar linguistic phenomenon. As a consequence of this, a qualification was built in in (T1): a set of mutually unsatisfiable standards (weak incoherence) never leads to the confirmation of two contradictory statements (strong incoherence), because somehow, e.g. when forced through a Sorites series, the discourse ultimately gets regimented around a 'collectivistic' pole (to be compared with Raffman's pairwise constraint). As a matter of fact, daily speech is packed with examples hereof, and we never seem to bother very much (see again the citation of Wright at the end of the previous section). In contrast with Manor and Raffman, Horgan doesn't see the point of elaborating ingenious philosophical, logical or psychological theories. For him, vagueness is *dephtless*, in that no theory could ever bring to the surface a coherent set of principles underlying the phenomenon. The proper attitude when confronted with the paradox is simply refusing to play the game, and that is it. On the technical front, generally CL does a very fine job, and when it comes to grasping vagueness, intuitionist *weak negation* is about as far as Horgan would want to go.[23]

This reluctance is not so exceptional. At the beginning of 4.2, I have already touched upon how Stephen Schiffer philosophically approaches the vagueness matter, and why, in his eyes, alternative semantics are prevented to get a grip on it. He has, however, also added a positive technical note to this, suggesting that no harm would be done considering the meta-question of whether borderline propositions have a truth value as an indeterminate one itself. "For the most part, we simply act as though excluded middle always held and that all propositions were bivalent. Our use of CL may be strained when we confront arguments, such as the 'Sorites', which explicitly turn on the status of borderline propositions, but I'm suggesting that vagueness will on occasion strain us no matter what logic we use" ([37], p.211). This, again, recalls the specific propagation of a four-valued variant of the Kleene Strong Tables with respect to vagueness, by *Michael Tye*. According to Tye, the universal conditional premise should not be judged false, but indefinite, as for some of its instantiations, "there are assignments under which both its antecedent and its consequence are indefinite, since there are borderline bald men who would not cease to be borderline by gaining a hair" (Tye [41], p.548). Of course, with this, Tye runs directly into HOV, but he persists.

[23] In a recent paper, Horgan has brought this account under the wings of an encompassive contextual semantics. Here is a version of the main thesis, most relevant for the present discussion: "Which semantic standards are the operative ones, in any given context of discourse, depends largely on the contextually attuned, socially coordinated, truth-judging and falsity-judging dispositions of competent speakers" (Horgan [18], p.76). I come back to this in the closing section.

Indeed, if there are first true, then indefinite, and finally false statements in the Sorites series, this doesn't mean that speakers will agree where to draw the boundaries. It is even highly unprobable that one person will behave consistently on various occasions. For that reason, no statement of the form 'Bx_n is true and Bx_{n+1} is not true' can be true itself. Neither can all of them be false, provided we want to saveguard the transitions between truth, indefiniteness and falsehood. That is why at least some of them have to be indefinite. This, again, would only be possible in case at least some of the statements 'Bx_n is true' are indefinite. But Tye denies the latter, and shows the following ingenious way out:

> "[T]here is no determinate fact of the matter about whether there are any statements of the form ['Bx_n'] that are indefinite" (op.cit., p.550). This is the same as saying that "the truth-value predicates [...] are vaguely vague: there simply is no determinate fact of the matter about whether the properties they express have or could have any borderline instances. So, it is indefinite whether there are any sentences that are neither true nor false nor indefinite" (op.cit., p.551).

Ingenious as it may be, this introduction of a fourth truth value, *indefinitely indefinite*, thereby circumventing HOV, rather obscures then clarifies the matter at hand. The unwillingness to 'play the game', reduced to a superficial phenomenon as it is, gets semantically covered up instead of receiving the proper examination it deserves as a deep and serious phenomenon. Consequently, the genuinely pragmatical character of the account can be called into doubt. Part of this criticism can be directed towards transvaluationism as well, for although it does not proceed with a similar technical cover-up, it also urges to take our hands off the matter philosophically.

5. Conclusion: Continental Fever

It is not the intention in this closing section to weigh pros and cons of the traditional and alternative accounts briefly set out in sections 3 and 4. The meticulous dissection needed for that would truly require a booklength presently unavailable. But even apart from this limitation, it should not be of primary interest here, on pain of missing the main point anyway. That is, it would particularize, viz. 'logicize', the discussion further (a notable trend of still rising specialization in all scientific branches, despite various multi- and interdisciplinary dreams), while an important part, if not all, of its future presumably lies exactly in the opposite, generalizing and thus

'truely' philosophical, direction. As it has been noted earlier on, programmatically, the main point uniting the various accounts offered in section 4, is the enhancement, beyond a technical solution, of the philosophically satisfying *explanation* or *understanding* of the phenomena under investigation, thereby not eschewing an interdisciplinary approach. Certain psychological considerations have already been brought in. These might be enlarged, so as to include, next to 'hardware' or perceptional ones, also 'software', e.g. motivational, ones. Sociological, educational, or anthropological ones might as well be good candidates. After all, we learn our mother tongue largely by ostension, alternatively immersed in the various social contexts we are members of. And as this pragmatical talk is not so unfamiliar to linguists or cognitive psychologists, why should it be so to philosophical logicians?

However, their reluctance to 'naturalistically' engage in these matters is huge. Indeed, the more complex and esoteric you allow the field of investigation to become, the more difficult it becomes to 'define your terms'. As we know, the analytical tradition, where modern logic largely has risen and resided, has been obsessed with language throughout its entire development during the former century. Or more specifically, for, in itself, there can be nothing wrong with that: it has been obsessed with (analyzing) the *formal* aspects of language, thereby counting out most of the 'intensional' aspects, having to do with its contents, and all of the 'intentional' ones, having to do with its use. With a recent book, containing a wealth of material supporting my cry for a pragmatical turn in the logical theory of language, away from abstraction and mathematical analysis, *Keith Devlin* has stressed once more that "in human terms logical behavior is not an absolute notion; it is relative. It depends on what the individual wants to achieve" (Devlin [7], p.12). Nevertheless, in logic, we are apparently still not ready, in both – related – senses of not able and not willing, to properly deal with the factors mentioned. The *unwillingness* has undoubtedly had much to do with a fear of getting adrift, of loosing oneself in 'non-scientific' speculation, and with it the seriousness that was precisely gained by restricting the philosophical task to formal linguistic analysis (the very heart of the analytical program). And the *unability* seems not so strange, in its turn. Predicating is the outcome of such complex a process, that it is in most, if not all, cases no less than unique. Moreover, normally, in daily life situations, a cluster of predicates is attributed fairly simultanuously, in order to nuance the message carried over, again, according to contextual factors, internal (ranging from conscious, e.g. intentional, ones, to sub- or unconscious, e.g. emotional, ones) as well as external (both objects *and* subjects). In other words: the whole of linguistic practice is imbedded in a vast communicative setting.

Which theorist will dare say to have listed all its relevant factors, let alone taken them into formal account?

Brief, where we are right now, and to use an understatement, a satisfactory solution seems not directly at hand. *Jamie Tappenden*, in his turn, has argued some time ago that, as yet, "ways of setting up issues represent language too statically" (Tappenden [40], p.193). He pointed to a 'bad' habit in logical circles to freeze the pragmatical dimension, e.g. concerning relevance, and continued: "We should not forget that these jurisdictional divisions are highly theoretical ones: if the consequences are too wild, it should prompt us to rethink the boundaries between pragmatics and semantics/logics" (op.cit., p.194). I think the consequences of what has been gathered in section 3 are generally indeed pretty wild. But, to be fair, where we are currently, pragmatically spoken, is not far from nowhere. We humans operate in psychological and social environments populated by various and no more than partially controlable *personae*, which, in principle, all have to be taken into full account before linguistic utterances can be said with certainty to have (or lack) this or that genuine meaning. To get a grip on the proper role of these various contextual factors in reasoning processes, an enormous amount of work is still left to be done, such that at present "we really do not understand enough about vagueness to be more than exceedingly tentative in any claims as to what 'the logic of vagueness' could be" (op.cit., p.193). From here, one should be inclined to encourage all kinds of investigation into the matter, and to let a thousand flowers bloom, before ever thinking about synthesizing them, or making resolute theoretical choices. There is an additional pragmatical reason in favor of such a methodological richness, be it in a quite different sense of the word. Every day, we humans are confronted with a myriad of problems. Not all of them, thank heavens, are philosophical ones. On the contrary: most are 'just' practical ones, 'merely' in need of practical solutions. Hence, e.g., hooray for fuzzy logic, which seems to be able to operate the dishwasher pretty well while these very lines get written. To put it more generally: whereas it actually goes without saying that truely philosophical solutions should have proper explanatory power, and thus add up to a better understanding of phenomena, a criterion unfulfilled by fuzzy logic when it comes to vagueness (not addressing the philosophically pressing question of HOV), one should guard himself from being taken for a philosophical reductionist.

As far as that philosophical knot is concerned, then, I think we should take very seriously the suggestion by *Nicholas Humphrey* that "it may well be that in some areas of discourse our linguistic culture is still at the infant stage" (Humphrey [19], p.32). Exploiting the homological nature of theories

of language, viz. that, as linguistic contructs, they should be applicable to themselves, this line of thought leads to the conclusion that an exhaustive logical account of the phenomenon will be possible *iff* the problem of vagueness will have disappeared in a natural way out of language. In other words: a satisfactory or complete theory is only possible when its arrival is rendered obsolete (when the problem has first dissolved).[24] We have no window to the future, probably fortunately so, but, then, it surely looks as if it will always be impossible to pinpoint the phenomenon.[25] The reason for this appears to be that this is exactly the point about vagueness after all. *Nicolas Goodman*, in another context (then again, perhaps not quite so), viz. that of mathematical epistemology, has noted: "A completely precise description of any macroscopic physical situation would require more bits of information than we ever have at our disposal.[26] *Ordinary language copes with this problem by being vague*. For many purposes this solution is adequate, but for scientific purposes it has the disadvantage that vague concepts do not support long chains of reasoning" (Goodman [14], p.65, my emphasis). The logical fallacy would then precisely amount to persisting in an idealized, scientific account of ordinary language.[27] But in order to do so, the integral problem of vagueness has to be chopped up first and narrowed down too much to retain any of its initial, multidimensional significance.

This could very well be where philosophical logic in the analytic tradition, past the linguistic turn, has run afoul. Overestimating the rigidly logical side of the coin, while largely losing out of sight the truely philosophical one, it has allowed the instrument of analysis to turn into an object in itself. A rare sensitivity for this apparent perversion has come to surface nevertheless, giving rise to the evolution that has been sketched from sections 3 to 4. It is an evolution in the wake of that much more encompassing transition throughout the 20th century, in the philosophy of language, namely from a preoccupation with ideal-language (mirroring reality) to ordinary-language (in its own right). In both movements, strikingly, one and the same person, *Ludwig Wittgenstein*, has been a frontrunner. In spite of the decline of the

[24] Again, it is nothing but HOV showing one of its many faces here.

[25] Making it a philosophical issue *par excellence*, as, since their first recorded appearance, in early Greece (*including vagueness*; see my [42]), the major problems on the philosophical agenda have generally proven to be of this extraordinary obstinacy.

[26] 'Tychism', pragmatist *C.S. Peirce* labeled this indeterminist phenomenon, expressing the approximate or probabilistic nature of any theory.

[27] While I shall not be persisting here in the fact that scientific concepts too are as much susceptible to the very phenomenon under investigation.

latter school,[28] the continued reading of the later him, the one of [49], so often jeered off, and fellow linguistic pragmatists, such as *Richard Rorty*,[29] should probably inspire the exploration, only started off a decade ago and still hardly on the increase, by more philosophical logicians, of the numerous areas yet uncharted of this tough, multi-facetted problem at the very heart of the theory of language.

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[28] See, e.g., Forguson [10].

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