Logic and Logical Philosophy
Volume 11 (2003), 213–230

Guido Vanackere*

CHANGE IN INDIVIDUALS WITHOUT A NAME. CONTEXTUAL INDICATORS & THE FREE CHANGE-ADAPTIVE LOGIC

Abstract. Proof theory and semantics of an adaptive logic that deals adequately with change in individuals with or without a name are presented. New logical constants are introduced, *viz. indicators*. Within a given context they function as names, predicates and quantifiers at the same time. The thus extended language (of classical logic) has a big expressive power and solves—partly—the (classical) non-logical presuppositions with respect to 'the existence of individuals'. Nevertheless, from a purely logical point of view, the here presented logic requires nothing but a very intuitive selection of *classical* models of the premises, *viz.* the minimally abnormal ones.

1. Introduction

In [8] I introduced the change-adaptive logic **CAL2**. This logic deals with change in individuals, in a very natural and fruitful way. Individuals occurring in a theory are replaced by 'individuals on a given moment'. Instead of "a", we write "a^i", which stands for "a on the moment i". Obviously, we have $Pa^i \nvdash_{\mathbf{CL}} Pa^j$ (**CL** is Classical Logic) even if j comes immediately after i, in this 'chaotic' reading of the premises. The adaptive logic **CAL2** has a minimally abnormal reading of the premises. Starting from the chaotic reading, continuity is reintroduced conditionally: for every predicate P, for every individual a, for all moments i, j, we assume $Pa^i \equiv Pa^j$ unless and

*Research for this paper was supported by subventions from Ghent University and from the Fund for Scientific Research – Flanders, and indirectly by the Flemish Minister responsible for Science and Technology (contract BIL98/73). I thank Liza Verhoeven for comments on an early draft of the paper.

until the negation of this assumption is derived from the premises. I briefly present this logic in section 2. This logic has one main drawback: it cannot be applied in a theory formulated in a language without individual constants. This drawback is solved in this paper. The solution is presented in section 4: new logical constants are introduced, *viz.* contextual *indicators*, and a new and very adequate 'free' change-adaptive logic is created. Of course, this solution will be preceded by some philosophical evidence (section 3). It is easily shown that every individual constant can be replaced by an indicator.[1] This simple notational change solves the metaphysical presuppositions of classical logic, *viz.* that owners of a name *exist*. Indicators function as predicates and hence there is no reason why their assignment should be a singleton; it may contain 0, 1 or 2 or ... elements. The main philosophical idea I defend is that the individuals we describe should not appear as 'objectively existing individuals', but as individuals that are indicated by some people, within a given context.

At the mean time, the paper establishes once again that adaptive logics offer a much more powerful machinery than the non-adaptive alternatives. The power derives especially from the fact that abnormalities are no longer considered as 'bad guys' that should be avoided at all costs. The acceptance of the possible derivations of abnormalities is not only very 'human' and intuitively correct, it also allows for an interpretation of theories which is as rich and adequate as possible within the given context. This paper is an illustration of the fact that the search for the specific abnormalities in a specific context, results in a contextually adequate theory.

2. The change-adaptive logic CAL2

The language of **CAL2**, as presented in [8], contains the logical constant $\dagger$. A formula of the form $\dagger a^i$ (in which a is an individual constant), is read as "a does not exist on i". Within the context of the present paper, we are dealing with languages without individual constants. Therefore, I will present the change-adaptive logic without $\dagger$. This makes the matter even more transparent.

We can characterize the logic **CAL2** as 'the optimum' between a chaotic logic (called **CHAOS**) and a Platonic logic (called Platonic Heaven or **PH**). **CHAOS** and **PH** can be easily defined from Classical Logic **CL**. We just need to change the domain. Let $\mathcal{C}$ be the set of individual constants 'at

[1] This notational change is so simple that from my point of view it is amazing that Russell and Whitehead did not use it in their [9].

a moment', $\mathcal{V}$ the set of individual variables (referring to 'individuals at a moment'), and $\mathcal{P}^n$ the set of predicates of rank n. $\mathcal{C}^{\mathcal{P}}$ is the usual set of constants (*i.e.* without time-indices), and $\mathcal{D}^{\mathcal{P}}$ is the usual domain, *i.e* a set of (platonic forms of) individuals. Θ is the set of (shortest) periods—I call them "moments". The relations $<$ and $=$ in Θ behave exactly as the relations $<$ and $=$ in the set of integers. The domain $\mathcal{D}$ is defined as follows:

Definition 1. $\mathcal{D} = \mathcal{D}^{\mathcal{P}} \times \Theta$

Where $i \in \Theta$, $\mathcal{D}^{\mathcal{P}} \times \{i\}$ is a state of reality. Let v_p be the usual assignment such that for every $a \in \mathcal{C}^{\mathcal{P}}$: $\mathsf{v}_p(a) \in \mathcal{D}^{\mathcal{P}}$. The assignment function v we need here is such that $\mathsf{v}(a^i) = \langle \mathsf{v}_p(a), i\rangle$. For every predicate Π of rank n, $\mathsf{v}(\Pi) \subseteq \mathcal{D}^n$.

2.1. Proof theory and semantics of CHAOS

Given the new domain and the new assignment function, the proof theory of **CHAOS** is exactly the same as for **CL**. Obviously, we have the following non-theorem:

$$\nvdash_{\mathbf{CHAOS}} \alpha^i = \alpha^j \tag{1}$$

On the semantic level we have: for every α^i, α^j $(i \neq j)$ there is some **CHAOS**-model v_M such that $\mathsf{v}_\mathsf{M}(\alpha^i = \alpha^j) = 0$. It is easily seen why this logic is called chaotic: it does not allow to derive Pa^j from Pa^i even if j comes immediately after i.

2.2. Proof theory and semantics of PH

The syntax of **PH** is obtained by extending the syntax of **CL** with, for all $i, j \in \Theta$:

$$\text{A} =' : \ \alpha^i = \alpha^j \tag{2}$$

For the semantics, we can add the clause, for all $i, j \in \Theta$, for all $\alpha \in \mathcal{C}^{\mathcal{P}}$:

$$\text{S} =' : \ \mathsf{v}_\mathsf{M}(\alpha^i = \alpha^j) = 1 \tag{3}$$

It is easily seen that the addition of A$='$ and S$='$ results in the fact that time-indices become meaningless. Hence, we can say that applying **PH** to the 'indexed' premises and applying **CL** to the non-indexed premises give the same result. In other words: the logic **PH** deals with individuals as eternal and unchanging entities.

2.3. The change-adaptive logic CAL2

Every adaptive logic is characterized by an upper-limit logic, a lower-limit logic, a set of abnormalities, and a strategy.[2] The upper-limit logic at hand is **PH**, the lower-limit logic is **CHAOS**. The abnormalities of the change-adaptive logic **CAL2** are formulas either of the form $\sim(\alpha^i{=}\alpha^j)$ or, where Π is a predicate of rank $n+1$ ($n \geq 0$), of the form $\sim(\forall x_1)...(\forall x_n)(\Pi(\alpha^i) \equiv \Pi(\alpha^j))$ in which $\Pi(\alpha^i)$ stands for $\Pi\alpha^1 x_1...x_n$ or $\Pi x_1\alpha^1 x_2...x_n$ or ... or $\Pi x_1...x_n\alpha^1$. In general, an abnormality will be written as $?C$. $\mathcal{A}$ is the set of all abnormalities. The most credulous adaptive strategy is the minimal abnormality strategy.[3] I make use of this strategy in this paper.

An abnormality is considered to be false unless and until a contradiction of this assumption is derived from the premises. In other words: the rules valid in **CHAOS** are unconditionally valid in **CAL2**; the rules valid in **PH** but not in **CHAOS** can be applied conditionally. A condition (*viz.* "It is not shown that $?C$ is derivable.") can only be overruled if $?C$ is verified in some minimal abnormal model of the premises.

A disjunction of abnormalities is what it says, and, where $?C_1, ..., ?C_n$ are abnormalities, a disjunction of these abnormalities is written as $\mathsf{DA}\{?C_1, ..., ?C_n\}$ (a DA-formula). In general we write $\mathsf{DA}(\Sigma)$, in which Σ is a finite set of abnormalities.

2.4. Semantics of CAL2

DEFINITION 2. Where M is a **CHAOS**-model, $\mathsf{AC}(\mathsf{M}) = \{?C \mid ?C \in \mathcal{A}$ and $\mathsf{v}_{\mathsf{M}}(?C) = 1\}$.

DEFINITION 3. A **CHAOS**-model M is *minimally abnormal with respect to* Γ iff M is a **CHAOS**-model of Γ, and there is no **CHAOS**-model M' of Γ such that $\mathsf{AC}(\mathsf{M}') \subset \mathsf{AC}(\mathsf{M})$.

DEFINITION 4. M is a **CAL2**-*model of* Γ iff M is minimally abnormal with respect to Γ.

DEFINITION 5. $\Gamma \models_{\mathbf{CAL2}} A$ iff A is true in all **CAL2**-models of Γ.

All **CAL2**-models of Γ are **CHAOS**-models of Γ. Hence, $\Gamma \models_{\mathbf{CAL2}} A$ if $\Gamma \models_{\mathbf{CHAOS}} A$. Except for border cases, the **CHAOS**-consequence-set of

[2] For an introduction in adaptive logics, see *e.g* [2].

[3] For adaptive logics based on the minimal abnormality strategy, the Ghent Group usually adds the number 2 to the name of the logic. The number 1 is used for the reliability strategy. In [3], two more strategies are developed.

Γ, will be a real subset of the **CAL2**-consequence-set of Γ. In general, the set of **CAL2**-models of Γ is a real subset of the **CHAOS**-models of Γ. If for any **CAL2**-model of Γ, and for any DA-formula $\mathsf{DA}(\Sigma)$, $\mathsf{v_M}(\mathsf{DA}(\Sigma)) = 0$, then $\Gamma \models_{\mathbf{CAL2}} A$ iff $\Gamma \models_{\mathbf{PH}} A$. The definitions show that **CAL2** interprets a set of premises as normal as possible: no more $?C \in \mathcal{A}$ are true than is required by the premises. It may be informative for the reader to repeat the following theorem:

THEOREM 1. $\Gamma \models_{\mathbf{PH}} A$ *iff for some sets of abnormalities* Σ_i, $(i = 0, 1, ...)$, $\Gamma \models_{\mathbf{CHAOS}} A \vee \mathsf{DA}(\Sigma^i)$.

A will be **CAL2**-derivable from Γ if every minimally abnormal **CHAOS**-model of Γ falsifies some $\mathsf{DA}(\Sigma^i)$. The proof-theory of **CAL2** exists in a 'proof-theoretical translation' of Theorem 1.

2.5. Proof theory of CAL2

The logic **CAL2** has a dynamic proof procedure. The dynamics are due to its conditional rule, and its marking rule. The latter forces one to rule out lines derived on overruled conditions. **CAL2** contains also unconditional rules, *viz.* those of **CHAOS**. Hence, the syntax of **CAL2** is the one of **CHAOS** extended with a conditional rule (RC) and a marking rule (RM). Here is the conditional rule:

$$\text{RC : From } A \vee \mathsf{DA}(\Sigma) \text{ to derive } A \text{ on condition } \Sigma. \tag{4}$$

The format of **CAL2**-proofs is obtained from the format of **CHAOS**- or **CL**-proofs, by adding a fifth element to each line, which contains the conditions on which we rely in order for the formula in the second element to be derivable by the rule of inference mentioned in the fourth element, from the formulas of the lines enumerated in the third element. If we apply a **CHAOS**-rule to lines the fifth element of which is not empty, the fifth element of the new line is the union of the fifth element of the used lines.

DEFINITION 6. A *occurs unconditionally* at some line of a proof iff the fifth element of that line is empty.

Suppose that A is derived on one or more lines the fifth element of which is not empty. A is considered as derived at a stage of a proof and the lines become a full part of the proof if A comes out true under a maximally normal 'interpretation' of the DA-formulas (at that stage). "Interpretation" should refer to formal properties of the formulas that occur in the proof.

The role of DA-formulas is crucial in **CAL2**-proofs. Clearly, when a DA-formula occurs unconditionally at some line of a proof, at least one of its disjuncts is true. Some DA-formulas occurring unconditionally in a proof may be disregarded. As a first step, we only have to consider *minimal* DA-*formulas*. Where Σ and Δ are sets of abnormalities, let us stipulate that

DEFINITION 7. $\mathsf{DA}(\Sigma)$ is a *minimal* DA-*formula at a stage of a proof* iff (i) it occurs unconditionally in the proof at that stage, and (ii) there is no $\Delta \subset \Sigma$ for which $\mathsf{DA}(\Delta)$ occurs unconditionally in the proof at that stage, and (iii) there is no $\mathsf{DA}(\Delta)$ that occurs unconditionally in the proof at that stage such that $\mathsf{DA}(\Delta) \vdash_{\mathbf{CHAOS}} \mathsf{DA}(\Sigma)$, whereas $\mathsf{DA}(\Sigma) \nvdash_{\mathbf{CHAOS}} \mathsf{DA}(\Delta)$.[4]

Next, let Φ_s^* be the set of all sets that contain one factor out of each minimal DA-formula at stage s of the proof. Φ_s^* may contain redundant elements: the same factor may occur in different minimal DA-formulas. If $\mathsf{DA}\{?C_1, ?C_2\}$ and $\mathsf{DA}\{?C_1, ?C_3\}$ are minimal DA-formulas, then $\Phi_s^* = \{\{?C_1\}, \{?C_1, ?C_2\}, \{?C_1, ?C_3\}, \{?C_2, ?C_3\}\}$. Of these $\{?C_1, ?C_2\}$ and $\{?C_1, ?C_3\}$ are redundant. Both $\mathsf{DA}\{?C_1, ?C_2\}$ and $\mathsf{DA}\{?C_1, ?C_3\}$ are true if $?C_1$ is true; there is no need that also $?C_2$ and $?C_3$ be true. So, let Φ_s be obtained from Φ_s^* by eliminating elements from it that are proper supersets of other elements. Hence, the members of Φ_s are sets of formulas, such that, if all members of such a set are true, then all DA-formulas that occur unconditionally in the proof at stage s are true.

DEFINITION 8. Where Φ_s is as defined above and A is the second element of line j, line j *fulfils the integrity criterion at stage s* iff (i) the intersection of some member of Φ_s and of the fifth element of line j is empty, and (ii) for each $\varphi \in \Phi_s$ there is a line k such that the intersection of φ and of the fifth element of line k is empty and A is the second element of line k.

Now we can introduce the marking rule:

RM : A line is marked OUT at a stage iff it does not fulfil the integrity criterion. (5)

If the fifth element of a line is empty, *i.e.*, when the formula in its second element is a **CHAOS**-consequence, the integrity criterion is obviously fulfilled. All formulas in lines with an empty fifth element are **CAL2**-consequences.

[4] Condition (iii) excludes that, *e.g.*, $\sim(a^i = a^j)$ can be a minimal DA-formula if $\sim(Pa^i \equiv Pa^j)$ is one, for $\sim(Pa^i \equiv Pa^j) \vdash_{\mathbf{CHAOS}} \sim(a^i = a^j)$, whereas $\sim(a^i = a^j) \nvdash_{\mathbf{CHAOS}} \sim(Pa^i \equiv Pa^j)$.

Definition 9. A is *finally derived* at some line in a **CAL2**-proof iff it is the second element of that line and any (possibly infinite) extension of the proof can be further extended in such way that the line is unmarked.

Definition 10. $\Gamma \vdash_{\mathbf{CAL2}} A$ (A is **CAL2**-*finally derivable* from Γ) iff A is finally derived at some line of a **CAL2**-proof from Γ.

It may be informative for the reader to repeat the following theorems:

Theorem 2. *$\Gamma \vdash_{\mathbf{CAL2}} A$ iff there are one or more (possibly empty) finite sets $\Sigma_1, \Sigma_2, \ldots$ of abnormalities, such that $\Gamma \vdash_{\mathbf{CAL2}} A \vee \mathsf{DA}(\Sigma_1)$, $\Gamma \vdash_{\mathbf{CAL2}} A \vee \mathsf{DA}(\Sigma_2)$, ..., and for any $\varphi \in \Phi_\Gamma$, one of the Σ_i is such that $\Sigma_i \cap \varphi = \emptyset$.*

Theorem 3. *If $\Gamma \vdash_{\mathbf{CAL2}} A$, then it is possible to extend any proof from Γ into a proof in which A is finally derived.*

2.6. Drawback of CAL2

In order to construct a theory about individuals that may change (in that some of them meet a predicate P at time i but do not meet P at time j), in a language without individual constants, the logic **CAL2** as it is, is useless. If no constant α occurs, we cannot express that $\sim\alpha^i = \alpha^j$, or that $\sim(P\alpha^i \equiv P\alpha^j)$.

One could think of introducing time-indices over variables, and allow for formulas like:

$$(\exists x)\sim(Px^i \equiv Px^j) \tag{6}$$

This approach would force us to use quantifiers over time indices, if we want to express rules, laws or definitions. An example:

$$(\forall y \in \Theta)(\forall x)(Px^y \supset Qx^y) \tag{7}$$

Suppose we observe an element that meets P on i but does not meet Q on j;

$$(\exists x)(Px^i \& \sim Qx^j) \tag{8}$$

i.e, we observe an element that changes with respect to Q. (7) and (8) would allow us to derive:

$$\sim(\forall x)(Px^i \equiv Px^j) \& \sim(\forall x)(Qx^i \equiv Qx^j) \tag{9}$$

The derivation of these abnormalities has an unwanted result: we can no longer assume that other elements that meet P on i also meet P on j, and we can no longer assume that other elements that do not meet P on j do not meet P on i. Analogously for Q. This approach would force us to conclude that *all* elements change with respect to P and Q between i and j, as soon as one element is shown to do so.

The problem is that, whereas α^i stands for exactly one element on one moment if α is an individual constant, α^i stands for possibly all elements on that moment if α is an individual variable. It is my proposal to leave the problem for a moment, and to take a closer look at the relation between elements of the domain and the way people indicate them.

3. Indicated individuals

A name is not a property of an individual *an sich*. A name functions as an indicator of an individual within a specific context. This context guarantees that everyone who uses some name —within this context— indicates the same individual.

Within a given context, people have more at their disposal than language, if they want to indicate an individual, or want to keep track of some individuals. People may hold individuals in their hands, look at individuals, isolate individuals, point at individuals, recognize individuals, *etcetera*. In general, we can say that a specific action or whole of actions, may function as an indicator of individuals.

Let X be a context. Let Σ_{X} be a set of people who act and communicate within X. Let A_{X} be the set of all possible actions of members of Σ_{X}. Let $\textsc{Indic}_{\mathsf{X}} \subseteq \mathsf{A}_{\mathsf{X}}$ be the set of all actions that function as indicators of individuals. Now, we can introduce for every $\mathbf{a} \in \textsc{Indic}_{\mathsf{X}}$ a new logical constant $I_{\mathbf{a}}$, that I call *an indicator*. The behaviour of indicators keeps the middle between the behaviour of quantifiers and the behaviour of predicates of rank 1. Before I give an exact definition of *indicators*, let me first remind you that within **CL** names for individuals can be written as predicates. Indeed, where β is an individual constant, and B is the same name written as a predicate of rank 1, every expression of the form $P\beta$ can be written as

$$(\exists x)(\forall y)((Bx \& Px) \& (By \supset x = y)) \tag{10}$$

If we leave names out of the language of **CL** and introduce formulas of the form of (10), we can extend the set of predicative names to the set of indicators. However, this move would not solve our problems. Suppose we know that β changes with respect to P between i and j. Using constants and **CAL2**, we can write:

$$P\beta^i \& {\sim} P\beta^j \tag{11}$$

Using predicative names, we would get either:

$$(\exists x)(\forall y)(((B^i x \ \&\ Px) \ \&\ (B^i y \supset x = y)) \ \&\ ((B^j x \ \&\ {\sim}Px) \ \&\ (B^j y \supset x = y))) \tag{12}$$

or:

$$(\exists x)(\forall y)((B^i x \ \&\ Px) \ \&\ (B^i y \supset x = y)) \ \&\ (\exists x)(\forall y)((B^j x \ \&\ {\sim}Px) \ \&\ (B^j y \supset x = y))) \tag{13}$$

Formula (12) however is plainly false, and formula (13) forces us to derive:

$${\sim}(\forall x)(B^i x \equiv B^j x) \tag{14}$$

instead of a formula equivalent to ${\sim}(A(\beta^i) \equiv A(\beta^j))$. This means that we can no longer talk about change of individuals with respect to predicative expressions.[5]

As an introduction to the solution I propose, let me mention that the formula (10) reminds us of the weird presuppositions of the use of names within **CL**.

1. If one uses a constant β (resp. a predicative name B), the domain should contain at least one individual a such that for any assignment v: $\mathsf{v}(\beta) = a$ (resp. $\mathsf{v}(B) \neq \emptyset$).
2. The domain should not contain more than one individual a such that $\mathsf{v}(\beta) = a$ (resp. if $a \in \mathsf{v}(B)$ then $\mathsf{v}(B) - \{a\} = \emptyset$).

We can get rid of the latter presupposition if we replace (10) by

$$(\exists x)(Bx \& Px) \tag{15}$$

and we can get rid of *both* presuppositions if we replace (10) by

$$(\forall x)(Bx \supset Px) \tag{16}$$

[5] If one hangs on to the latter approach, one has to assume that every change 'kills' the old individual and gives birth to a new one. Maybe this approach would get a warm welcome from René Descartes and other rationalists, who said that God creates the world as new on every moment.

Both (15) and (16) however have the drawback that the predicate B cannot be recognized as a name. Moreover, the replacement of $P\beta$ by either (15) or (16) would leave us with the same problems as indicated by means of formulas (12) and (13). Still, from a logical point of view, it would be great to have a formula that states: "β meets P" without presupposing the existence of the owner of the name β, and without presupposing the unique identity of 'all' owners of the name. My proposal is to formalize "β meets P" as follows: $(Bx)Px$. This way, (Bx) functions as a quantifier. The formula $(Bx)Px$ can be true even if $\mathsf{v}(B)$ is not a singleton, *i.e.*, it may be the case that the cardinality of $\mathsf{v}(B)$ is $0, 1, 2, 3, \ldots$. Moreover, if we want to talk about change, we can write:

$$(B^i x)Px \& (B^j x){\sim}Px \tag{17}$$

Suppose we have the rule, law or definition that 'all P are Q', and we know that β^i meets P, whereas β^j does not meet Q. We can formalize this as:

$$(\forall x)(Px \supset Qx),\ (B^i x)Px,\ (B^j x){\sim}Qx \tag{18}$$

The semantical and proof-theoretical definitions of *indicators* will allow us to derive:

$${\sim}((B^i x)Px \equiv (B^j x)Px)\ \&\ {\sim}((B^i x)Qx \equiv (B^j x)Qx) \tag{19}$$

which is exactly what we want. The reader has noticed that I refer to "$(B^i x)$" and "$(B^j x)$" by means of the term "indicator". Indeed, 'using a name β' can be considered as one of the members of $\textsc{Indic}_{\mathsf{X}}$, let us say $\mathbf{b}$, such that B is nothing but $I_{\mathbf{b}}$. Hence, I have illustrated the meaning of *indicators* by means of a specific kind of indicators we all know, *viz.* 'using names'. We are now ready to deal with individuals that change, even if we have no names for the individuals. By doing this we gain a lot: we have contextualized classical logic, and we got rid of weird non-logical presuppositions.

4. The change-adaptive logic FreeCAL2

"The expression 'free logic' is an abbreviation for the phrase 'free of existence assumptions with respect to its terms, general and singular'."[6] Hence, if I leave constants out of **CL** and import indicators, we obtain a partially free logic. We can use names and other indicators and we do not have to assume that the owner of the name (or the indicated individual) does exist. And if

[6] See [4] (p. 104 for the quote) and [5].

we assume that some indicated individual exists, we do not have to assume that it is really an *in-dividual*. There may be plenty of it, *i.e* it may be divided into several entities.[7]

4.1. The language

Let $\mathcal{I}_{\mathsf{X}}$ be set of indicators, used by the members of Σ_{X} within a context X, such that for all $I_{\mathbf{a}} \in \mathcal{I}_{\mathsf{X}}$, $\mathbf{a}$ is a specific set of actions, and "$(I_{\mathbf{a}}x)$" can be read as "the individual(s) (if any) indicated by action(s) $\mathbf{a}$ performed by some member(s) of Σ_{X}, within context X". Let $\mathcal{I}^{\Theta}_{\mathsf{X}}$ be defined from $\mathcal{I}_{\mathsf{X}}$ by replacing every $I_{\mathbf{a}} \in \mathcal{I}_{\mathsf{X}}$ by all $I^i_{\mathbf{a}}$ $(i \in \Theta)$.

$\mathcal{V}$ is a set of variables. $\mathcal{P}^n$ is a set of predicates of rank n, which are all assumed to be contextually well-defined.[8] Let $\mathcal{L}_{\mathsf{X}}$ be the language containing nothing but the members of $\mathcal{V}$, $\mathcal{P}^n$ and $\mathcal{I}^{\Theta}_{\mathsf{X}}$, the logical constants $\supset$, &, $\vee$, $\equiv$, $\sim$ and $\forall$, and brackets.[9]

The set of well-formed formulas (wffs) is defined as usual for the usual constants. There is one extension: If $I^i_{\mathbf{a}} \in \mathcal{I}^{\Theta}_{\mathsf{X}}$, and $A(\alpha)$ is a formula in which the variable α (and no other variable) occurs free, then $(I_{\mathbf{a}}\alpha)A(\alpha)$ is a wff.

4.2. Semantics of the underlying logic FreeCHAOS

With respect to $\mathcal{V}$ and $\mathcal{P}^n$, the assignment v is defined as usual. The domain $\mathcal{D}$ can be as simple as one wants it to be: it exists of individuals (if any).[10]

$$\mathsf{v} : I^i_{\mathbf{a}} \in \mathcal{I}^{\Theta}_{\mathsf{X}} \longrightarrow \mathbf{P}(\mathcal{D}) \text{ (the powerset of } \mathcal{D}) \tag{20}$$

With respect to formulas of the form $A \supset B$, $A \& B$, $A \vee B$, $A \equiv B$, $\sim A$, and $(\forall\alpha)A$, the semantical clauses defining the valuation function $\mathsf{v_M}$ are classical. The clause for formulas of the form $(I^i_{\mathbf{a}}\alpha)A(\alpha)$ is as follows:

$$\mathsf{v_M}((I^i_{\mathbf{a}}\alpha)A(\alpha)) = 1 \text{ iff, if } \mathsf{v}(\alpha) \in \mathsf{v}(I^i_{\mathbf{a}}), \text{ then } \mathsf{v_M}(A(\alpha)) = 1 \tag{21}$$

---

[7] This may open perspectives to quantum-physics, where it seems to be the case that *'one'* individual is at several places simultaneously.

[8] If we allow for vague or ambiguous predicates, or for any kind of predicates that is subject to interpretation, we are better of using an ambiguity-adaptive logic. See [6] and [7].

[9] Notice that we do not need "=" !

[10] We do not need the domain required for **CAL2** and **CHAOS**, in which the members of the domain are couples of the form $\langle \mathsf{v}_p(a), i \rangle$ (see section 2).

It is easily seen that the following sentences are equivalent:

$$\mathsf{v}_{\mathsf{M}}(\sim(I^i_{\mathbf{a}}\alpha)A(\alpha)) = 1 \tag{22}$$

$$\mathsf{v}_{\mathsf{M}}((I^i_{\mathbf{a}}\alpha)A(\alpha)) = 0 \tag{23}$$

$$\mathsf{v}(\alpha) \in \mathsf{v}(I^i_{\mathbf{a}}) \text{ and } \mathsf{v}_{\mathsf{M}}(A(\alpha)) = 0 \tag{24}$$

$$\mathsf{v}(\alpha) \in \mathsf{v}(I^i_{\mathbf{a}}) \text{ and } \mathsf{v}_{\mathsf{M}}(\sim A(\alpha)) = 1 \tag{25}$$

$$\mathsf{v}_{\mathsf{M}}((I^i_{\mathbf{a}}\alpha)\sim A(\alpha)) = 1 \tag{26}$$

At the mean time, this shows that **FreeCHAOS** is not completely ‘free’: if a sentence of the form of (22) or (26) is true, then the domain cannot be empty.[11] If not, the sentences (24) and (25) would be false.

It is also easily seen that we can derive a specific instantiation rule. If $\mathsf{v}_{\mathsf{M}}((\forall\alpha)A(\alpha)) = 1$, then $\mathsf{v}_{\mathsf{M}}((I^i_{\mathbf{a}}\alpha)A(\alpha)) = 1$ for all $I^i_{\mathbf{a}}$.

In analogy with **CHAOS**, $\mathsf{v}_{\mathsf{M}}((I^j_{\mathbf{a}}\alpha)A(\alpha)) = 1$ does not follow from $\mathsf{v}_{\mathsf{M}}((I^i_{\mathbf{a}}\alpha)A(\alpha)) = 1$, even if j comes immediately after or before i.

4.3. Proof-theory of FreeCHAOS

The Axiom Schemas for $\supset$, &, $\vee$, $\equiv$ and $\sim$ are classical.

$$\mathrm{A}\forall : (\forall\alpha)A \supset (I^i_{\mathbf{a}}\alpha)A \tag{27}$$

$$\mathrm{R}\forall : \text{from } \vdash A \supset (I^i_{\mathbf{a}}\alpha)B \text{ to derive } \vdash A \supset (\forall\alpha)B \text{ provided } I^i_{\mathbf{a}} \text{ does not occur in } A \tag{28}$$

$$\mathrm{R}I^i_{\mathbf{a}} : \text{If } A_1, ..., A_n \vdash_{\mathbf{CL}} A, \text{ then } (I^i_{\mathbf{a}})A_1, ..., (I^i_{\mathbf{a}})A_n \vdash_{\mathbf{FreeCHAOS}} (I^i_{\mathbf{a}})A \tag{29}$$

$$\mathrm{A}I^i_{\mathbf{a}}\sim : (I^i_{\mathbf{a}})\sim A \supset \sim(I^i_{\mathbf{a}})A \tag{30}$$

If one feels like it, one can define $(\exists x)A =_{\mathsf{def}} \sim(\forall x)\sim A$. But if one does so, one has to keep in mind that there is no wff of the form $A(\beta)$ from which one may derive $(\exists\alpha)A(\alpha)$. Formulas of the form $(\exists\alpha)A(\alpha)$ can be useful if one wants to express that there is some entity that meets A, whereas one can not indicate this entity.

---

[11] Which sheds a remarkable light on metaphysics: *Denying truth implies asserting existence!*

4.4. Semantics of the upper-limit logic FreePH

The semantics of **FreePH** is obtained from the semantics of **FreeCHAOS** by adding the clause, for all $i, j \in \Theta$, for all $I \in \textsc{Indic}_{\mathsf{X}}$:

$$\mathrm{S}I_{\mathbf{a}} : \ \mathsf{v}_{\mathsf{M}}((I^i_{\mathbf{a}})A \equiv (I^j_{\mathbf{a}})A) = 1 \tag{31}$$

4.5. Proof theory of FreePH

The syntax of **PH** is obtained by extending the syntax of **FreeCHaos** with, for all $i, j \in \Theta$, for all $I \in \textsc{Indic}_{\mathsf{X}}$:

$$\mathrm{A}I'_{\mathbf{a}} \ : \ (I^i_{\mathbf{a}})A \equiv (I^j_{\mathbf{a}})A \tag{32}$$

In the same way as for **PH**, the time-indices become meaningless in **FreePH**. Hence **FreePH** may be considered as a logic of too fast induction. From the fact that some indicated individual meets A at one moment, **FreePH** derives that the individual always meets A.

4.6. Semantics of FreeCAL2

Given the definitions of **FreeCHAOs** and **FreePH**, the definition of **FreeCAL2** is completely analogous to the definition of **CAL2**. The upper-limit logic of **FreeCAL2** is **FreePH**, the lower-limit logic is **FreeCHAOS**. The abnormalities of the 'free' change-adaptive logic **FreeCAL2** are formulas of the form $\sim\forall((I^i_{\mathbf{a}}\alpha)\Pi(\alpha) \equiv (I^j_{\mathbf{a}}\alpha)\Pi(\alpha))$ in which α is a variable and Π is a predicate of rank n and $\forall$ is an abbreviation of $(\forall x_1)...(\forall x_m)$ (in which $0 \leq m \leq n-1$), a universal quantification of all free variables in $\Pi(\alpha) \equiv \Pi(\alpha)$, in some preferred order. In general, an abnormality is (again) written as $?C$. $\mathcal{A}$ is the set of all abnormalities. Again, I make use of the minimal abnormality strategy.

The rules valid in **FreeCHAOS** are unconditionally valid in **FreeCAL2**; the rules valid in **FreePH** but not in **FreeCHAOS** can be applied conditionally. A condition (*viz.* "It is not shown that $?C$ is derivable.") can only be overruled if $?C$ is verified in some minimal abnormal model of the premises.

A disjunction of abnormalities $\mathsf{DA}\{?C_1, ..., ?C_n\}$ is still what it says.

Definition 11. Where M is a **FreeCHAOS**-model, $\mathsf{AC}(\mathsf{M}) = \{?C \mid ?C \in \mathcal{A}$ and $\mathsf{v}_{\mathsf{M}}(?C) = 1\}$.

Definition 12. A **FreeCHAOS**-model M is *minimally abnormal with respect to* Γ iff M is a **FreeCHAOS**-model of Γ, and there is no **FreeCHAOS**-model M' of Γ such that $\mathsf{AC}(\mathsf{M}') \subset \mathsf{AC}(\mathsf{M})$.

Definition 13. M is a **FreeCAL2**-*model of* Γ iff M is minimally abnormal with respect to Γ.

Definition 14. $\Gamma \models_{\mathbf{FreeCAL2}} A$ iff A is true in all **FreeCAL2**-models of Γ.

4.7. Proof theory of FreeCAL2

As for the proof theory of all adaptive logics, we rely on concrete proofs in order to define the consequence-relation. The proof format is the same as for **CL** be it that every line has an extra (fifth) element, in which the conditions (on which the formula in the second element of the line is derived) are written down.

RU : RU: If $A_1, ..., A_n$ occur in a proof on resp. conditions $\Sigma_1, ..., \Sigma_n$, and $A_1, ..., A_n \vdash_{\mathbf{FreeCHAOS}} A$, then write a new line with A as second element and $\Sigma_1 \cup ... \cup \Sigma_n$ as fifth element. (33)

RC : From $A \vee \mathsf{DA}(\Sigma)$ to derive A on condition Σ. (34)

Definition 15. *A occurs unconditionally* at some line of a proof iff the fifth element of that line is empty.

Definition 16. $\mathsf{DA}(\Sigma)$ is a *minimal* DA*-formula at a stage of a proof* iff (i) it occurs unconditionally in the proof at that stage, and (ii) there is no $\Delta \subset \Sigma$ for which $\mathsf{DA}(\Delta)$ occurs unconditionally in the proof at that stage, and (iii) there is no $\mathsf{DA}(\Delta)$ that occurs unconditionally in the proof at that stage such that $\mathsf{DA}(\Delta) \vdash_{\mathbf{CHAOS}} \mathsf{DA}(\Sigma)$, whereas $\mathsf{DA}(\Sigma) \nvdash_{\mathbf{CHAOS}} \mathsf{DA}(\Delta)$.

Let Φ^*_s be the set of all sets that contain one factor out of each minimal DA-formula at stage s of the proof. Let Φ_s be obtained from Φ^*_s by eliminating elements from it that are proper supersets of other elements. Again,[12] the members of Φ_s are sets of formulas, such that, if all members of such a set are true, then all DA-formulas that occur unconditionally in the proof at stage s are true.

Definition 17. Line j with A as second element *fulfils the integrity criterion at stage s* iff (i) the intersection of some member of Φ_s and of the fifth element

[12] See section 2.5.

of line j is empty, and (ii) for each $\varphi \in \Phi_s$ there is a line k such that the intersection of φ and of the fifth element of line k is empty and A is the second element of line k.

RM : A line is marked OUT at a stage iff it does not fulfil the integrity criterion. (35)

DEFINITION 18. A **FreeCAL2** proof is a proof with the required format, in which—apart from the PREMISE-rule—only the rules RU, RC *may* be applied, and in which the rule RM *must* be applied at every stage.

DEFINITION 19. A is *finally derived* at some line in a **FreeCAL2**-proof iff it is the second element of that line and any (possibly infinite) extension of the proof can be further extended in such way that the line is unmarked.

DEFINITION 20. $\Gamma \vdash_{\mathbf{FreeCAL2}} A$ (A is **FreeCAL2**-*finally derivable* from Γ) iff A is finally derived at some line of a **FreeCAL2**-proof from Γ.

4.8. Meta-theory

It is easily seen that every formula of the form $(I^i_{\mathbf{a}}\alpha)A(\alpha)$ is equal to a formula of the form $(\forall\alpha)(I^i_{\mathbf{a}}\alpha \supset A(\alpha))$, in which $I^i_{\mathbf{a}}$ functions as an ordinary predicate of rank n. The new notation simply avoids that formulas of the form $(\forall\alpha)(I^i_{\mathbf{a}}\alpha)$ can be considered as wffs. The latter sequence is indeed meaningless; it just says "All thus indicated individuals". It is easily seen that the semantics and the proof theory of both **FreeChaos** and **FreePH** capture this notational change in a straightforward and equivalent way, the soundness and completeness theorems for these logics follow immediately from the soundness and completeness theorem of **CL**.

Given this new notation of some classical predicates, the adaptive logic **FreeCAL2** is nothing a but a special application of the ambiguity-adaptive logic **AAL2**. The first ambiguity-adaptive logic was presented in [6]. I refer to [6] and [7] for the meta theory. The ambiguity-adaptive logic **AAL2** starts from a maximally ambiguous interpretation of the premises: all non-logical constants get a different index. The interpretation of the upper-limit logic is such that for every non-logical constant C, all C^i and C^j are identified. The ambiguity-adaptive logic assumes that C^i and C^j can be identified unless and until this assumption leads to the derivation of an abnormality.

4.9. An example of a FreeCAL2-proof

For aesthetic reasons I will write $(\mathbf{F}^i x)$ instead of $(I^i_{\mathbf{f}}x)$, and $(\mathbf{Q}^i x)$ instead of $(I^i_{\mathbf{Q}}x)$. "†16" stands for: "this line is marked out on stage 16 of the proof".

$i, j \in \Theta$, $i < j$.

1.	$(\forall x)(Bx \supset (Mx\&Px))$	—	PREM	$\emptyset$	
2.	$(\mathbf{F}^i x)Bx\&(\mathbf{Q}^i x)Bx$	—	PREM	$\emptyset$	
3.	$(\mathbf{Q}^j x){\sim}Mx$	—	PREM	$\emptyset$	
4.	$(\mathbf{F}^i x)(Mx\&Px)$	1,2	RU	$\emptyset$	
5.	$(\mathbf{Q}^i x)(Mx\&Px)$	1,2	RU	$\emptyset$	
6.	$(\mathbf{F}^j x)(Bx \supset (Mx\&Px))$	1	RU	$\emptyset$	
7.	$(\mathbf{Q}^j x)(Bx \supset (Mx\&Px))$	1	RU	$\emptyset$	
8.	${\sim}((\mathbf{F}^i x)Bx \equiv (\mathbf{F}^j x)Bx) \vee (\mathbf{F}^j x)(Mx\&Px)$				
		2,6	RU	$\emptyset$	
9.	$(\mathbf{F}^j x)(Mx\&Px)$	8	RC	$\{(\mathbf{F}^i x)Bx \equiv (\mathbf{F}^j x)Bx)\}$	
11.	$(\mathbf{Q}^j x)Mx$	2,7	RC	$\{(\mathbf{Q}^i x)Bx \equiv (\mathbf{Q}^j x)Bx)\}$	†14
12.	$(\mathbf{Q}^j x)Px$	2,7	RC	$\{(\mathbf{Q}^i x)Bx \equiv (\mathbf{Q}^j x)Bx)\}$	†14
13.	$(\mathbf{Q}^j x){\sim}Bx$	3,7	RU	$\emptyset$	
14.	${\sim}((\mathbf{Q}^i x)Bx \equiv (\mathbf{Q}^j x)Bx)$	2,13	RU	$\emptyset$	
15.	$(\mathbf{Q}^j x)Mx$	5	RC	$\{(\mathbf{Q}^i x)Mx \equiv (\mathbf{Q}^j x)Mx)\}$	†17
16.	$(\mathbf{Q}^j x)Px$	5	RC	$\{(\mathbf{Q}^i x)Px \equiv (\mathbf{Q}^j x)Px)\}$	
17.	${\sim}((\mathbf{Q}^i x)Mx \equiv (\mathbf{Q}^j x)Mx)$	3,5	RU	$\emptyset$	

$\Phi_{14} = \{\{{\sim}((\mathbf{Q}^i x)Bx \equiv (\mathbf{Q}^j x)Bx)\}\}$, and hence lines 11 and 12 do not fulfil the integrity criterion at stage 14, for condition (i) of Definition 17 is overruled. $\Phi_{17} = \{\{{\sim}((\mathbf{Q}^i x)Bx \equiv (\mathbf{Q}^j x)Bx), {\sim}((\mathbf{Q}^i x)Mx \equiv (\mathbf{Q}^j x)Mx)\}\}$, and hence lines 15 and 16 meet the condition (i) of Definition 17, but line 15 does not fulfill condition (ii) whereas line 16 does. Indeed, the intersection of Φ_{17} and the fifth element of line 16 is empty. Hence at stage 17, $(\mathbf{Q}^j x)Px$ is derived, whereas $(\mathbf{Q}^j x)Mx$ is not. Even this simple proof illustrates the dynamics of **FreeCAL2**-proofs: $(\mathbf{Q}^j x)Mx$ is derived on stage 11, no longer derived on stage 14, again derived on stage 15, and no longer derived at stage 17. As no other minimal DA-formula is derivable from the premises, we may conclude that $(\mathbf{Q}^j x)Mx$ is not **FreeCAL2**-derivable from the premises, and thus that the individual(s) indicated by $\mathbf{Q}$ has/have changed with respect to M between i and j. We also see that the formula $(\mathbf{Q}^j x){\sim}Bx$ is finally derived on stage 13. $(\mathbf{Q}^j x)Px$, that was derived on stage 12 and that was no longer derived on stage 14, is finally derived at stage 16. This is an interesting result. Although change is proved with respect to some predicates (B and M), continuity of the same individual(s) is proved with respect to some other predicate! We also see that the individual(s) indicated by $\mathbf{F}$ are not subject to change. Concerning this/these individual(s) **FreeCAL2** concludes continuity with respect to all predicates.

It may be interesting for some readers that $(\mathbf{Q}x)$ can be read as "the individuals indicated by means of some quantum-operation", while $(\mathbf{F}x)$

can be read as "the individuals indicated by means of some (macro-)physics-operation". "B" stands for "is a body", "M" stands for "has an observable momentum", "P stands for "has an observable place".

5. Concluding remarks

We have created a logic that allows for change in individuals, even if they have no name. The logic allows for change, but it assumes continuity unless and until change is proved. The derivation of 'change' of some individual with respect to some predicate has no implications on the continuity of the same individual with respect to other predicates, nor on the continuity of other individuals with respect to this specific predicate. The proof written in section 4.9 is a good illustration of this feature.

The philosophical implications of the use of indicators instead of the use of names are not to be underestimated. Individuals do no longer appear as existing *an sich*, but as individuals that are distinguished and indicated by people acting and communicating within a given context. We must realize that all our knowledge is contextual, and that whenever we create theories about individuals, we start by indicating them. There is no point in trying to get rid of this 'subjective' aspect of the indications of individuals. To the contrary, this 'subjective', or 'collective' aspect guarantees us that our theories are of importance for us, people who indicate individuals of all kinds through our action and communication.

References

[1] Batens, Diderik, 'Inconsistency-adaptive Logics', in Ewa Orlowska, (ed.), *Logic at Work. Essays Dedicated to the Memory of Helena Rasiowa.* Heidelberg, New-York, Physica Verlag, Springer, 1998, pp. 445–472.

[2] Batens, Diderik, 'Extending the realm of logic. The adaptive-logic programme', in P. Weingartner, (ed.), *Alternative logics. Do sciences need them?*, Springer Verlag, in print.

[3] De Clercq, Kristof, 'Two new strategies for inconsistency-adaptive logics', *Logic and Logical Philosophy* 8 (2001), 65–80.

[4] Lambert, K., *Meinong and the Principle of Independence*, Cambridge University Press, Cambridge, 1984.

[5] Lambert, K., 'The Nature of Free Logic', in K. Lambert, (ed.), *Philosophical Applications of Free Logic*, Oxford University Press, New York-Oxford, 1991, pp. 3–13.

[6] VANACKERE, GUIDO, 'Ambiguity-adaptive Logic', *Logique* & *Analyse* 159, sept. (1997), (appeared in 1999), 261–280.

[7] VANACKERE, GUIDO, 'Minimizing ambiguity and paraconsistency', *Logique & Analyse* 165–166, (1999), (appeared 2002), 139–160.

[8] VANACKERE, GUIDO, 'Ontological causes of inconsistencies, and a change-adaptive logical solution', in: W. Carnielli, M. Coniglio, and I. D'Ottavio (eds.), *P*araconsistency: The Logical Way to the Inconsistent, Proceedings of the II World Congress on Paraconsistency (WCP'2000), Marcel Dekker inc. New York, april 2002, pp. 151-165.

[9] WHITEHEAD, ALFRED NORTH, and RUSSELL, BERTRAND, *Principia Mathematica Volume I*, second edition, Cambridge University Press, 1950.

GUIDO VANACKERE
Center for Logic and Philosophy of Science
University of Ghent
Belgium
Guido.Vanackere@rug.ac.be