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Sonja Smets*

IN DEFENSE OF OPERATIONAL QUANTUM LOGIC

Abstract. In the literature the work of C. Piron on OQL, "the operational quantum logic of the Geneva School", has a few times been criticised. Those criticisms were often due to misunderstandings, as has already been pointed out in [19]. In this paper we follow the line of defense in favour of OQL by replying to the criticisms formulated some time ago in [4] and [17]. In order for the reader to follow our argumentation, we briefly analyze the basic conceptual machinery of OQL.

1. Introduction

Operational Quantum Logic (OQL) corresponds to the theory of "Property Lattices" which, developed by the Geneva School, originated in [1, 22, 23, 24, 28, 30]. OQL's aim is to characterize physical systems, ranging from classical to quantum, by means of their actual and potential properties. Concretely, it provides an operational alternative to the standard approaches on the logical status of quantum theory and based on the point of view that the set of mathematical representatives of the properties of an arbitrary physical system forms a complete lattice, the approach of the Geneva School is closely linked to the line of reasoning initiated by G. Birkhoff and J. von Neumann in [5]. This approach has been compared with those of for instance C.H. Randall and D.J.Foulis, G. Ludwig and of G.W. Mackey. It is not our intention to go into the details about the differences and similarities of those different approaches with respect to OQL, we instead refer to [7, 8, 9, 13, 18, 35] for some relevant material. In this paper we want to pay specific attention

*The author is a postdoctoral researcher at Flanders' Fund for Scientific Research.

to some points of the theory which easily lead to misinterpretations. We will go over remarks and critiques aimed at OQL. Indeed, in the papers [4, 17, 21, 38], the work of C. Piron has been criticised. Many of these criticisms were merely misunderstandings. D.J. Foulis and C.H. Randall in [19] replied explicitly to the criticisms formulated in [21, 38].[1] In this reply, they exhibited a lot of possible pitfalls one might run into when learning about OQL. We do not have the intention just to go over all the remarks D.J. Foulis and C.H. Randall made, but will concentrate on two other papers, namely, [4, 17]. Before doing so, we analyze the necessary basic concepts so the reader will be able to follow the argumentation in defense of OQL later on. Note that our analysis in section 2 is based on all the available literature from the Geneva School and doesn't restrict itself to the material present at the time when W. Balzer, W.K. Essler and G. Zoubek wrote their comments.

2. OQL's Basic Conceptual Machinery

We exhibit here some conceptual machinery of OQL and limit our exposition to the notions which are necessary for grasping the argumentation in succeeding sections, a more detailed and recent analysis of OQL focusing on several different aspects can be found in [10, 11, 25, 36, 40]. We start from the basic notion of *a particular physical system* which, within the tradition of the Geneva School, has been perceived as an isolated and characterizable part of the external world.[2] Given a particular physical system, we can now focus on the notions of yes-no question, actual and potential property and state respectively.

2.1. Yes-No Questions

A (yes-no) question or *a definite experimental project*[3] as it more recently has been called, is specified in [25]:

[1]To [19], N. Hadjisavvas and F. Thieffine replied with [20] though their paper does not add anything interesting to the discussion. Indeed, we agree with R. Piziak who reviewed [20] for the AMS' MathSciNet and states: "It is right and proper for any scientific work to be scrutinized and criticized according to its merits. Indeed, this is a main impetus to progress. But if mathematics is to be used as a tool for criticism, let it be used properly."

[2]Note that D. Aerts in [1] and his following papers, works with the notion of physical entity which is to be thought of equivalently.

[3]In this paper we take the liberty to use both the notion of definite experimental project and question freely, within citations we will sometimes insert clarifying notions between brackets.

> We take "a definite experimental project [question] relative to a physical system to be a real experimental procedure where we have defined in advance what would be the positive response should we perform the experiment." (p.65)

We think of an experimental procedure as a list of concrete actions which explicitly must or must not be performed in a specific way. Given that we execute such a procedure and that the conditions defining a positive result are known, we assign the response "yes" to the question if the conditions are satisfied, and the response "no" otherwise. The next notion we introduce is that of *a true question* or, *a certain definite experimental project* as it is called in [25]:

> "A given definite experimental project is certain [a question is true] for a particular system if it is sure that the positive response would obtain should we perform the experiment." (p.66)

Bear in mind that calling a question "true" does not presuppose that the associated experiment has to be performed. A question can be true even if one does not have the intention to ever perform the associated experimental procedure. In the earlier work of C. Piron, the definition of a true question encompassed the preparation of the physical system, e.g. in [30]:

> "When the physical system has been prepared in such a way that the physicist may affirm that in the event of an experiment the result will be "yes", we shall say that the question is certain, or that the question is true." (p.20)

The preparation procedure acts as an operational tool. A tool to obtain the answer "yes" with certainty for a prepared, non-tested, physical system with respect to a specific question. Since this operational tool is not explicitly modeled in the mathematical theory of the Geneva School, we prefer the first definition given. Although we intend to stick with the original Geneva School theory, we wish to note that C. Cattaneo and G. Nisticó focused on a syntactic scheme of the Jauch-Piron approach to the foundations of quantum physics, in which they allow as well-formed formulas predicates expressing that questions are true (respectively false) in a certain preparation [8].

The notion of *an impossible question* or *impossible definite experimental project* is defined as follows in [25]:

> "A given definite experimental project is impossible [a question is impossible] for a particular system if it is sure that the positive

> response would not obtain should we perform the experiment." (p.66)

"Impossible" means here that it is certain that we would obtain the negative response "no", should we perform the experiment (see also [2]) — the positive response "yes" is not even a possible result. Another important notion, explicitly introduced by D. Aerts in [2], is that of a *not true question* or *not certain definite experimental project* — not to be confused with the impossible question,

> A given definite experimental project is not certain [a question is not true] for a particular system if it is not sure that the positive response would obtain should we perform the experiment.

When we are not sure of a positive response, it is a sufficient condition to state that the answer "no" is possible should we perform the experiment. When "no" can be a possible answer, the question cannot be true, and note that this is so, regardless of whether "yes" is possible or not — when not, then the question is impossible. The way in which these questions are constructed can lead to confusion. As C. Piron states in [29], "If the [positive] outcome for the question [called] β is not certain, the statement "β is true" is false, but we do not say "β is false"." Because here "β is false" means that "β is impossible".

There are some questions which have a specific status. First take a look at the *trivial question*, defined as the following specific definite experimental project in [25]:

> Trivial question: "Do whatever you wish with the system and assign the response "yes"." (p.67)

This also encompasses doing nothing with the physical system. We call this definite experimental project certain iff the system exists. Indeed, the trivial question is true when we are certain to obtain the positive answer "yes" were we to perform the question. The only condition of the trivial question is that we have a physical system to begin with. Next, consider the *absurd question* defined in [25] as the following specific definite experimental project:

> Absurd question: "Do whatever you wish with the system and assign the response "no"." (p.67)

The absurd question can never be a true question. Furthermore, we can state that the absurd question is an impossible question. Another point which deserves our focus is the *product question*, which is a question consisting of the product of a family of questions. More specifically the product question is defined as a specific definite experimental project in [25], for a specific family of questions which we call A and a question called α,

> Product question: "Choose an α from the family A as you wish, and effectuate it." (p.67)[4]

In the definitions given by C. Piron it is more explicitly stated that the answer obtained from effectively performing the arbitrarily chosen question is the answer attributed to the product question. We then conclude that the product question is true if it is sure that the answer "yes" would obtain should we perform the experiment. In other words, we follow [25, p.67],

> The product of a family of definite experimental projects called A is a "certain definite experimental project" [the product question is true] for a particular physical system iff each $\alpha \in A$ is certain [true] for the particular physical system.

And finally we obtain the inverse of a particular question, called the *inverse question*, by exchanging the responses "yes" and "no". Note that the absurd question is the inverse of the trivial question and vice versa. The inverse of a product question with respect to a certain family of questions, is then the product question with respect to the family of the inverse component questions.

Concentrating on the mathematical language of OQL, we denote questions as $\alpha, \beta, ... \in Q$, the trivial question as $I \in Q$, the absurd question as $O \in Q$, the product question with respect to a family of questions α_i as $\Pi_J \alpha_i$ and to denote the inverse of a question we use: $^{\sim} : Q \rightarrow Q$. When we consider all possible questions which could be performed on a particular physical system, we encounter a relation between questions by considering, as for example in [31], that,

> "A question α is said to be stronger than a question β, i.e. $\alpha \prec \beta$, if every time α is true, β is true as well." (p.399)

We call $\prec \subseteq Q \times Q$ a preorder relation, indeed, it is reflexive and transitive. Of any collection of questions equipped with such a preorder relation, I is the maximal (top) element and O the minimal (bottom) element. Further, we call two questions equivalent, written $\alpha \approx \beta$, iff $\alpha \prec \beta$ and $\beta \prec \alpha$. As made

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[4]Note that "choose an α as you wish" means "choose an α at random or arbitrarily".

explicit in [24], this relation is reflexive, symmetric and transitive. When a question α is true then so are all questions equivalent to α. So every always true question is equivalent to I, the trivial question. And every question α is equivalent with $\alpha \Pi I$ since $\alpha \Pi I$ is true iff α is true. Coming back to some issues concerning the product question we write:

$\Pi_J \alpha_i$ is true $\Leftrightarrow \alpha_i$ is true $\forall i \in J$
$(\Pi_J \alpha_i)^{\sim} = \Pi_J(\alpha_i^{\sim})$

2.2. Properties

In the tradition of the Geneva School, the properties a physical system possesses in reality have an objective existence. C. Piron states it in [31]:

> "The system is and it is what it is. It possesses different properties and whether these are known or not by the physicist does not change anything to the reality of the object itself. We have to describe these properties and not to justify or to explain the knowledge of the physicist about them." (p.397)

As we analyzed it in [36], with a particular physical system the physicist associates certain properties and analyzes their ontological value by means of possible experiments. More explicitly we follow [10],

> Properties are construed as candidate elements of reality corresponding to the definite experimental projects defined for a particular physical system.

Of course, several questions can be associated with the same property and we can classify properties as *actual or potential*. It is important to note that as long as we deal with "the properties" of a physical system and "the associated questions", we see the system at an abstract level. This is to say that only from the moment we make the distinction between the actuality and potentiality of properties, and similarly between true, not true and impossible questions, is the particular physical system inherently conceived to be in a specific state — this notion will be elaborated further on. In that way:

> A property of a particular physical system is actual iff the questions which test it are true, and is potential otherwise.

More specifically, if a question is not true, the associated property is potential. Referring to the criterion of reality put forward in [16] together with the above given definition of a true question, it is claimed that a true question corresponds to an element of reality. To the extent that one assigns an ontological value to the elements of reality, they are what we called actual properties.

Within the mathematical language used by OQL, we represent properties as $a, b, ... \in \mathcal{L}$. About a property associated with a particular question we can now say that there is a one-to-one correspondence between: 1) the property a, associated with the definite experimental project (question) α, and 2) the equivalence class of questions $[\alpha]$ which is the collection of all definite experimental projects β such that $\beta \approx \alpha$. This correspondence is realized through the identification of a and $[\alpha]$. In other words, to each equivalence class of questions there "corresponds" a property. For our convenience we will introduce the notation $\zeta : Q \to \mathcal{L} : \alpha \mapsto a$ to express that a is associated with $\alpha \in [\alpha]$. Next, we call the property expressing the existence of a physical system $1 = \zeta(I)$ the trivial property and $0 = \zeta(O)$ the absurd property. To the use of ζ, we must add that in the Geneva School literature one originally defined a as $[\alpha]$. This is something we however try to avoid since it points to a form of operationalism which is not at all in accordance with the "operational attitude" of the Geneva School — see [36] for details.

Note that it is not possible to conceive of the set of all properties of a given system. One could here follow the point of view taken in [1, 29] according to which one can only hope for a set which is large enough to contain the useful ones with respect to what we want to do with the system. We however consider it to be a useful "working-hypothesis" that the collection of all properties constitutes a *set*, in a mathematical sense. We now obtain a partial order relation, $\leq \subseteq \mathcal{L} \times \mathcal{L}$, on that collection of properties. It is the preorder relation on questions which induces this partial order relation, (see for instance [29, p.517]):

$a \leq b$ iff $\alpha \prec \beta$ with $\zeta(\alpha) = a$ and $\zeta(\beta) = b$.

Here, $a \leq b$ states exactly that b is actual whenever a is actual. One easily verifies that this partial order relation is reflexive, transitive and antisymmetric. Within OQL one can now prove the theorem stating that the collection of all properties of a physical system, noted as $\mathcal{L}$, is a complete lattice. Quoting C. Piron, [33], this means:

> "qu'à chaque famille de propriétés $\{a_i \in \mathcal{L} \mid i \in J\}$, J quelconque, sont associés deux propriétés de $\mathcal{L}$, $\wedge_J\ a_i$ et $\vee_J\ a_i$ qui

sont respectivement la borne inférieure (le plus grand minorant):
$x \leq a_i \ \forall i \in J \ \Leftrightarrow \ x \leq \wedge_J \ a_i$
et la borne supérieure (le plus petit majorant):
$a_i \leq x \ \forall i \in J \ \Leftrightarrow \ \vee_J \ a_i \leq x$." (p.9)

For the detailed proof of this we refer to for instance [29, 30, 33]. What we do want to stress here is that the existence of a greatest lower bound or meet for any family of given properties a_i, is operationally provided by means of the product question, (see [25, p.68]):

$$\wedge_J a_i = \zeta(\Pi_J \alpha_i) \ with \ \zeta(\alpha_i) = a_i$$

The meet of physical properties as such obtains a physical meaning and its existence for any set of properties guarantees that the poset of properties is a complete meet semilattice. The join or least upper bound, on the other hand, cannot be attributed a direct physical meaning in the same sense and is defined by means of Birkhoff's theorem by which every complete meet semilattice is a complete lattice, (see [25, 29, 30]),

$$\vee_J a_i = \bigwedge\{b \in \mathcal{L} \mid \forall i \in J : a_i \leq b\}$$

When we review the use of meet and join, we see that the meet corresponds to the classical conjunction whereas the join will not always correspond to the classical disjunction, due to the possibility of superpositions for quantum systems. Hence:

$$\text{“}a \wedge b \textit{ is actual”} \Leftrightarrow \text{“}a \textit{ is actual” and “}b \textit{ is actual”} \quad \forall a, b \in \mathcal{L}$$
$$\text{“}a \vee b \textit{ is actual”} \Leftarrow \text{“}a \textit{ is actual” or “}b \textit{ is actual”} \quad \forall a, b \in \mathcal{L}$$

As is proved in [29], if the latter implication holds in both directions, $\mathcal{L}$ is a distributive lattice.

2.3. States

Traditionally *the state of a physical system* was *defined* as the collection of all its actual properties (see e.g. [24, 29]). This definition is a translation of the belief that the information which fully represents a particular physical system in a singular realization is encrypted in its actual properties. Since we do not adopt a kind of operationalism such as P.W. Bridgman's, we reconsider the concept of state as initiated in [25] and further elaborated on in [11],

> States are construed as abstract names encoding the possible singular realizations of the given particular physical system.

The concept of state is built up in such a way that it can be the abstract name corresponding to the one ontological realization of a particular system or, that it can be the name corresponding to a realization in which the system "can be" but "is not". It is important to grasp that for OQL the state is not an abstract statistical ensemble of physical systems, nor does it involve ontological probabilities — for details we refer to [3, 36]. According to OQL there should be no difference in the way in which we model the states of different physical systems ranging from classical to quantum.

We have seen that the actual properties of a physical system are "elements of reality" and that it only makes sense to talk about the actuality (potentiality) of properties if the system is conceived to be in a specific state. This insight allows us to make the link between actual properties and the state of a physical system explicit. Indeed, the state stands in a correspondence relation to the system's actual properties, though contra C. Piron and D. Aerts' earlier conceptions, we believe that the state should not be *defined* as equal to a set of actual properties. This implies that the maximum amount of information which fully represents a particular physical system in a singular realization is not called "the state of a system" but following [36] has been called "the state set" which is defined as follows:

> The state set is the set of all properties which are actual in case the particular physical system would be ontologically realized in a specified state.

From the moment that a physical system is conceived to be in a certain state or has been prepared as such, there is always at least one actual property which constitutes its state set with respect to that state. In the worst case, a state set contains only the property associated with the trivial question. It is important to mention that the notion of state set is so defined that one physical system can have several state sets; one for every of the states in which it can possibly be. Given the concept of a "state set", the greatest lower bound of such a set is one property of the system and we know that if the greatest lower bound of a state set is an actual property then all the elements of the state set — elements which are necessarily equal or greater — will be actual. This greatest lower bound of a state set is said to represent the state in which it can be actual, in other words, it is called a "state property" — the notion is a translation of C. Piron's *propriété-état* introduced in [33].

When modeling states, "possible" realizations of physical systems, within the mathematical language of OQL, we use the notation $\mathcal{E} \in \Sigma$ where Σ represents the state space. A state property of state $\mathcal{E}$ will be represented by $p_\mathcal{E} \in \Sigma_\mathcal{L}$, where $\Sigma_\mathcal{L}$ denotes the set of atoms of $\mathcal{L}$, and a state set related to $\mathcal{E}$ is modeled as $S(\mathcal{E}) = \{a \mid a \in \mathcal{L}, a \textit{ actual in } \mathcal{E}\} \in P(\mathcal{L})$.[5] In [24, 29] it is stated that if we have a complete lattice $\mathcal{L}$, $S(\mathcal{E})$ satisfies some specific properties:

(1) If $a \in S(\mathcal{E})$, $b \in \mathcal{L}$ and $a \leq b$ then $b \in S(\mathcal{E})$

(2) If $a_i \in S(\mathcal{E}), \forall i \in J$ then $\wedge_J a_i \in S(\mathcal{E})$

(3) $0 \notin S(\mathcal{E}), 1 \in S(\mathcal{E})$ for every state set $S(\mathcal{E})$

We call $p_\mathcal{E} = \wedge_{a \in S(\mathcal{E})} a$ the strongest property in $\mathcal{L}$ that is actual in the state $\mathcal{E}$. If we now look at the notion of an atom as defined below, we see that an atom is the strongest property which will be actual in a certain state, hence atoms are state properties.

> $p_\mathcal{E}$ is an atom of $\mathcal{L}$ if it is different from 0 and is such that: if there is a property $a \in \mathcal{L}$ and $a \leq p_\mathcal{E} \Rightarrow a = 0$ or $a = p_\mathcal{E}$.

Next it makes sense to postulate within OQL, that one state property should never imply the actuality of another state property. The argumentation for this postulate is traced back to Aristotle and involves the belief that a change of states implies a shift between properties which are actual and potential. Such a shift is excluded when two state sets are subsets of one another or when two state properties are less or equal to each other. From this postulate it follows that all the state properties of a physical system are atoms of a property lattice, in other words, the property lattice is atomic. Further, the property lattice can be called atomistic, i.e. each property $a \in \mathcal{L}$ can be written as $\bigvee\{p \leq a \mid p \textit{ is an atom}\}$ (see theorem 1 in [25]).

We are now in the position to redefine our notion of state set. Given the strongest actual property $p_\mathcal{E} = \wedge_{a \in S(\mathcal{E})} a$ in the state $\mathcal{E}$, the state set $S(\mathcal{E})$ has the following form (see [33]):

$$S(\mathcal{E}) = \{a \in \mathcal{L} \mid p_\mathcal{E} \leq a\}$$

Similar to how we handled the partial order relation on the set of properties of a physical system, we will now deal with an orthogonality relation on the collection of all states possible for a physical system. Following [25, 32, 34]:

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[5] We allow ourselves to drop the subscripts of state properties when no confusion is possible.

> "call $\mathcal{E}_1$ and $\mathcal{E}_2$ orthogonal, written $\mathcal{E}_1 \perp \mathcal{E}_2$, if there exists a definite experimental project α such that α is certain for $\mathcal{E}_1$ and impossible for $\mathcal{E}_2$." [25, p.69].

The orthogonality relation is symmetric and antireflexive [25]:

> "If $\mathcal{E}_1 \perp \mathcal{E}_2$ then $\mathcal{E}_2 \perp \mathcal{E}_1$ and $\mathcal{E}_1 \neq \mathcal{E}_2$" (p.69)

We call a question [definite experimental project] α true [certain] for a state $\mathcal{E}$ in case the property $a = \zeta(\alpha)$, is an element of $S(\mathcal{E})$. A question α is impossible for a state $\mathcal{E}$ in the case $\alpha^{\sim}$ is true [certain] for $\mathcal{E}$ which implies that the property $a = \zeta(\alpha)$ is not an element of $S(\mathcal{E})$. Note that when $\mathcal{E}_1$ and $\mathcal{E}_2$ are represented by the state properties p and q respectively, we represent $\mathcal{E}_1 \perp \mathcal{E}_2$ also by $p \perp q$. Within OQL one then introduces the axiom stating that for every given state $\mathcal{E}$ there exists at least one question which is true if and only if the state is orthogonal to $\mathcal{E}$. Exactly this axiom allows us to postulate the existence of "opposite" properties, i.e. for every state property p in $\Sigma_{\mathcal{L}}$, there exists a property $p^\sharp \in \mathcal{L}$ which is actual if and only if the state is orthogonal to the one represented by p. The property $p^\sharp$ is the opposite of the state property p while the opposite of any other property of $\mathcal{L}$ is defined by means of the meet of such opposite properties $p^\sharp$, i.e. we introduce the map $' : \mathcal{L} \to \mathcal{L} : a \mapsto \bigwedge\{p^\sharp \mid p \leq a\}$. Another axiom insures that each property is the opposite of another one, in other words, the map $' : \mathcal{L} \to \mathcal{L}$ is surjective. Without going into further details we are now allowed to call $' : \mathcal{L} \to \mathcal{L}$ an orthocomplementation, see for instance [33].

Returning to the concept of state, if T is a subset of Σ, we use the notation $T^\perp = \{\mathcal{E} \in \Sigma \mid \mathcal{E} \perp \eta, \textit{for all } \eta \in T\}$ for the set of all states orthogonal to those in T. Note that such orthogonality relations were, within the context of the Geneva School, explored first by D. Aerts in [1]. In a way this forms the cornerstone of a "state space description" of a physical system, parallel to its property lattice description. This parallelism is called the state-property duality and can be characterized once we can associate with each property a the set of states $\mu(a) = \{\mathcal{E} \in \Sigma \mid a \in S(\mathcal{E})\}$ in which it is actual and to each state $\mathcal{E}$ the set $S(\mathcal{E})$ of its actual properties. Formally, $\mu : \mathcal{L} \to P(\Sigma) : a \mapsto \mu(a)$ is called the Cartan map and is injective, order and meet-preserving. This Cartan map is surjective if we restrict the co-domain to the biorthogonally closed subsets of Σ, i.e. when we take subsets $T \subseteq \Sigma$ which satisfy $T^{\perp\perp} = T$. Further we note that if the system satisfies the already mentioned axioms, the application of the Cartan map determines an isomorphism between $\mathcal{L}$ and $(\Sigma, \perp)$ the lattice of the biorthogonally closed subsets of Σ, ordered by inclusion, with the singletons $\{\mathcal{E}\}$ as its atoms (see [33]).

The above mentioned dual state-property description for a particular physical system is very broad, both classical and quantum systems can be described by it. Of course, OQL would not have the known impact were it not the case that there is a representation theorem which proves the generality of the OQL-formalism with respect to the Hilbert space formalism. Indeed, C. Piron proved in [28] that a property lattice which is complete, atomic, orthomodular, which satisfies the covering law and some additional properties[6], can be represented in the form of the lattice of closed linear subspaces of a generalized Hilbert space. The crucial axioms at stake in Piron's representation theorem — see [39] for a recent concise analysis — and which can not be stated to hold in general, are weak modularity and the covering law. Indeed, those two quantum axioms as we may call them, are of a more mathematical nature than the others, and according to D. Aerts [1] they form the core-reason why it is impossible to describe separated systems by means of quantum theory. It is not at all our intention to elaborate on any more details here, hoping to have pointed out enough basic conceptual machinery to refer the interested reader to the OQL-literature.

3. A Reply to Criticism

In [17] we witness some confusion concerning C. Piron adhering to the stance of realism, we quote:

> "His [Piron's] epistemological or metaphysical position is unclear. He adapts a realistic position without telling whether this realism — nowadays called "physicalism" in order to distinguish it from ontological or "Platonic" realism — is an absolute (or naive) or hypothetical (or critical) one." (p.411)

First of all, we would like to object to the point of view that the realistic position of the Geneva School would be some kind of physicalism or materialism — the view in which reality is reducible to, or emergent from, the realm of physical things and processes [26].[7] OQL's realistic position also is not naive in the sense that certified truth is believed to be easily accessible,

[6] The property lattice has to be irreducible and must be of a sufficient dimension.

[7] Although one will nowadays perhaps not anymore put the thesis of physicalism forward in the same strong terms, we still adopt its "old" meaning, i.e. "Physicalism asserts that every term of the language of science — including beside the physical language those sub-

for details concerning this kind of debate in favor for a more critical realism we refer to for instance [26, 27]. And indeed, as we explained in [36, 37], the metaphysical stance of the Geneva School lies in the nature of a critical scientific realism. W.K. Essler and G. Zoubek are not the only ones who thought Piron's realistic position to be somewhat unclear. In [4] we see some confusion concerning realism and the EPR-principle of reality:

> "By a realistic interpretation he [Piron] means an interpretation under which the objects of the theory can be described *completely* by their *actual* properties." (p.403-404)

It is obvious that W. Balzer who wrote this comment in 1981, was mainly focusing on C. Piron's earlier work. We think that in the meantime things have been put in a larger perspective. But it is indeed so that the state of a physical system has been conceived by C. Piron as the collection of its actual properties which describe the system completely. It is important however to see that the actual properties of a physical system only count as a complete description of it in Piron's sense when we see it relative to a specific "singular realization" of that system. We mean this exactly in the sense that we can conceive a state set to describe a physical system. A more general and complete description of a physical system is however given by its property lattice consisting of *all* its properties. If we want to focus on states instead of properties, the closest we get to obtain a complete description of the system *in general* is by taking into account all possible states of the system, in other words, its state space. So we have to disagree with W. Balzer if he means that the realistic approach of the Geneva School only allows us to focus on actual properties to describe a physical system. We do retain the criterion of reality as put forward in the EPR-paper, though this does not exclude potential properties. There is a difference between the lattice of all properties which characterizes the system in general and the set of actual properties which describes the system in a certain state! We think this may be a misconception in [4]. In his analysis of C. Piron's work, he also tries to see properties and questions as sets of physical systems. This is obviously a picture which will not work, there is no need to give another interpretation of properties than the one we analyzed above. A property corresponds to an equivalence class of questions and a question is operationally characterized. Contrary to what is often thought, a question is not something vague but rather very-well specified. There is another misconception concerning properties and questions in [17], we quote:

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languages which are used in biology, in psychology, and in social science — is reducible to terms of the physical language." [6, p.467].

> "Since he [Piron] presupposes an *absurd question* 0 he has to assume an *absurd property* 0 also which has then to be, according to his distinction, either an actual or a potential property. But obviously 0 is neither an actual nor a potential property of an object. He therefore has to distinguish the *impossible* properties both from the *actual* and from the *potential* properties." (p.412)

In the sense that potentiality of a property is defined within OQL, 0 is and will always be a potential property. Especially because "0 is potential" only means that it is "not actual". Of course there is more to calling a property "potential", within OQL a deterministic or indeterministic change of a particular physical system entails a shift between certain of its actual and potential properties. It is therefore that we analyzed the notion of a potential property in [36] as a capability with respect to actualization, however we wish to stress that such an analysis can only be applied to potential properties different from 0. If one thinks it would improve the theory it is of course possible to introduce the notion of an *impossible* property. In that case an impossible property could correspond to an equivalence class of impossible questions and an impossible property then implies that the property is potential, but not vice versa. Concerning the concept of a question, defined in [31] as an experiment leading to a well defined alternative of which the terms are "yes" or "no", the "yes" corresponding to the expected result and the "no" to all the others; there also exists some confusion. It has maybe not been very clear in [31] but as we analyzed it above, using [25], it concerns an experimental procedure which is performable and is such that we can interpret the outcome of it in terms of yes and no — exactly which outcome corresponds to "yes" is decided upon in advance. There is even no need to specify which outcome of the experiment we interpret as "no" since it will be everything not leading to the response "yes". If we concentrate on the possible confusion which arises here, we quote [17]:

> "Other concepts of his approach are ambiguous and vague, especially his central concept of an "experiment leading to an alternative". This concept of an experiment may be understood in at least three different ways:
> (1) An experiment is a device that tells the scientist how to handle things by observing them and especially how to use an apparatus for this purpose.
> (2) An experiment is a single performance of such an experimental device together with its result.

(3) An experiment is a basic (or minimal covering) law that generalizes inductively the connections between performance and result of that device." (p. 412)

Firstly note that there is more to a question than just performing an experiment. In any case, we believe that for C. Piron an experiment is similar to a procedure which we *can* perform. OQL clearly distinguishes an experiment and its performance, this means indeed that *something along the lines* of (1) is the case and not (2). However, W.K. Essler and G. Zoubek think that a reconstruction of C. Piron's theory along the lines of (1) would make no sense of Piron's use of the terms "actual" and "potential" and therefore they exclude interpretation (1) as a possibility. This seems to be a mistake, regarding questions we first of all do not use the terminology of actuality and potentiality which is reserved for properties. But if W.K. Essler and G. Zoubek have the terminology of "true" and "not true" questions in mind, we still do not think there is a problem with accepting interpretation (1). A question is true if in case we would perform it, "yes" is a sure answer. Since this is a counterfactual definition, it is not in disaccordance with disconnecting experiments from their performance. However Essler and Zoubek are convinced that (3) is the case, questions are strict laws and therefore properties too are identified with a law or a theory [17, p.413-414]. One of their arguments concerns the counterfactual definition of a true question, [17]:

> "The statement "If we performed the question the answer "yes" would be certain" cannot be justified by referring to some few experiments and its results but only either to some law inductively based on those results or gained by other laws or to some inductive method expressing the belief in uniformity of the performances of this experiment." (p.414)

We agree with W.K. Essler and G. Zoubek if their statement points to the connection between the lawlike character of utterances and the decidability of those counterfactual stated utterances. Indeed, when some kind of question is always true on some kind of physical systems, the question is connected to some law-like behavior of the system. The same is the case for impossible questions, but not true questions are then linked to utterances which are definitely not law-like. Now, we still have to be careful with such generalizations here, for between utterances with an explicit law-like character and those without there is still a whole gray-scale left. We do not intend to enter the debate on the status of laws in connection with counterfactuals within philosophy of science, let us just mention instead that "experiments" are certainly not laws. We also do not believe one can so easily generalize and say that any question or property is a law. Maybe W.K. Essler and G. Zoubek have a small point here but according to us it will only involve true or impossible questions. Note that this also does not exclude that experiments can be interpreted as disconnected from their performances.

Another critical point concerns the product question. We quote [17]:

> "In his book [30, p.20] he [Piron] tells us: "... one measures an arbitrary one of the α_i and attributes to $\Pi_i\alpha_i$ the answer thus obtained ... By starting from the definitions it is easy to verify the following rule: $(\Pi_i\alpha_i)^\sim = \Pi_i(\alpha_i^\sim)$". The equation "$(\Pi_i\alpha_i)^\sim = \Pi_i(\alpha_i^\sim)$" holds only in very few degenerate lattices, e.g. if the lattice contains only the elements I and 0; in every other case we obtain: "$(\Pi_i\alpha_i)^\sim = \Sigma_i(\alpha_i^\sim)$". So Piron obviously takes "$\Pi_i(\alpha_i^\sim) = \Sigma_i(\alpha_i^\sim)$" to be true." (p.415)

It is not clear what W.K. Essler and G. Zoubek are getting at here; perhaps they have something like the De Morgan Laws in mind which of course are not applicable here. But since we are talking about operational definitions, what is their interpretation of the Σ of a family of questions? Especially taking into account that it has to be operationally defined ! The product questions are operationally precisely defined and therefore "$(\Pi_J\alpha_i)^\sim = \Pi_J(\alpha_i^\sim)$" does make sense. W.K. Essler and G. Zoubek try to analyze their confusion and assume two possible cases. One in which the members of the family on which a product is defined have to be questions of a similar kind and one in which they can be different. In the first case they use their previous law-argumentation which does not take them very far and in the second case they say that "Piron's rule of handling a product is of course wrong since a product may be false even if the arbitrary chosen member is true, namely, if one of the remaining members is false" [17, p.415]. Here we believe they pretty well mix up some fundamental ideas. There is a difference between obtaining the answer "yes" for a question and a "true" question. A product question gives the result "yes" if we perform it, this means exactly that the arbitrarily chosen question gives "yes" if we perform it. So, even when all questions of a specific family are impossible except for one, the product can give the result "yes". It is something different to say that a product question is true or not true. It is true when every question which we could pick out of the specific family — relative to which the product is defined — is true. And a product question is not true when not every question which we could pick out of the specific family is true; it is impossible when every question which we could pick is impossible. This leads us to conclude that W.K. Essler and G. Zoubek's analysis is simply wrong, a product question can never be "false" (impossible) when the arbitrary chosen member is true. A product question is "not true" in case the one chosen member is true *and* if at least one of the remaining members is not true. The main point here is that we cannot claim in the latter case that any "arbitrarily chosen" member is true. Truth and falseness — where false means impossible — of a product question does not have anything to do with the truth or falseness of one chosen member, but has to do with the status of the arbitrarily chosen member, in other words, with every member in the family. We agree that a product of questions is a confusing concept, being constructed such that we can still talk about the product of two questions even if their performances wouldn't be possible together. Let us now concentrate on $(\Pi_J \alpha_i)^\sim = \Pi_J(\alpha_i^\sim)$. A few possible situations indicate this equality:

1) Suppose we have a family of questions $\{\alpha_i \in Q \mid i \in J\}$ with J given. If now every α_i we could select, gives "yes" if we would perform it, then $\Pi_J \alpha_i$ is true and $(\Pi_J \alpha_i)^\sim$ impossible (or false). If every arbitrary question we could pick out of $\{\alpha_i^\sim \mid \alpha_i \in Q, i \in J\}$ gives "no" if we would perform it, then we say that $\Pi_J(\alpha_i^\sim)$ is impossible (or false).
2) If on the other hand not every α_i we could select gives "yes", $\Pi_J \alpha_i$ is not true and so is $(\Pi_J \alpha_i)^\sim$ not true. If every α_i which we could choose gives "no", $\Pi_J \alpha_i$ is impossible and so $(\Pi_J \alpha_i)^\sim$ is true. If any arbitrary $\alpha_i^\sim$ which we could have selected gives "yes", then we say that $\Pi_J(\alpha_i^\sim)$ is true. If not every arbitrary $\alpha_i^\sim$ which we could have selected gives "yes", then we say that $\Pi_J(\alpha_i^\sim)$ is not true.

W.K. Essler and G. Zoubek clearly did not understand that the product question has an operational definition and gives rise to a property called the meet. Contrary to what is claimed in [17], in order to test the affirmation "a is actual and b is actual" we do not have to "perform" two experiments on the same system! We only have to consider the product question $\alpha \Pi \beta$ with $\zeta(\alpha) = a$ and $\zeta(\beta) = b$. On the other hand it is clear that the join, defined by means of Birkhoff's theorem, admits of no *direct* physical meaning [10]. More explicitly,

$$\vee_J a_i = \zeta(\Pi\{\beta \in Q \mid \forall i \in J : \alpha_i \prec \beta\}) \text{ with } \zeta(\alpha_i) = a_i.$$

When we look at the disjunction, the classical "or", we see that a OR b is a property of a physical system *iff* it is possible to construct a question that tests this property [1] — something which will in general not be the case.

Finally we want to say something with respect to ideal, first kind measurements. A recent and detailed analysis of the part of OQL dealing with such measurements can be found in [36]. To be brief we mention that OQL paid special attention to the kind of questions which allow one to deal with properties — being actual or potential — after the associated experimental procedures have really been performed. It is obvious that not just any question is fitted for this purpose, indeed performing an arbitrary given experimental procedure may destroy the particular physical system. So, what is called an ideal measurement of the first kind of a property a is a definite experimental project [a question] α which satisfies the following conditions [25, 33]:

(i) $a = \zeta(\alpha)$ and $a' = \zeta(\alpha^{\sim})$;
(ii) if the positive response is obtained then a is actual immediately after the measurement;
(iii) if the positive response is obtained then the perturbation suffered by the system is minimal.

In [4, p.405] W. Balzer thinks that C. Piron in his definition of such kind of measurements uses basic concepts which are new relative to the underlying theory he starts with. Especially the notion of a question which is "true afterwards" seems to be confusing. Note that C. Piron in [31] intended to talk about properties which were actual or not afterwards, but we could indeed consider questions as well. There is however nothing special going on here; what seems to be forgotten by W. Balzer is the fact that when we consider a question to be true, not true or impossible, the physical system is always in a specific *singular realization.* And it is of course always with respect to a specific state of the system that we can say that the property associated with a question is (potential) actual. As such, given a particular physical system we can ask whether a specific question is true or not and it makes sense to ask the question again after the state of the system has changed. What also may cause the confusion is that an ideal first kind measurement α is true and a is actual when it is sure that the mentioned conditions would be fulfilled should we perform the experiment and exactly those conditions refer again to a being actual, so there is some recursion involved. When focusing on condition (ii) which is responsible for the label "first kind", α is true and a is actual if a remains actual and α remains true should we repeatedly perform the associated experimental procedure. Further note that in case we deal with a quantum system, an ideal measurement of the first kind allows us to calculate the perturbation suffered by the system in course of the associated action if it yields a positive response. This gave rise to the formulation of theorem 4.3 in [30, p.68] stating that if the response of β, ideal and first kind, is "yes" then the state of the system immediately after the experiment is $(p \vee b') \wedge b$, where p is the state before and $b = \zeta(\beta)$. So actually we are puzzled by W. Balzer's remark where he says:

> "...I think the basic vocabulary must be enlarged in a nontrivial way and new axioms must be adduced. If this is not done I find it difficult to *prove* assertions like the following (compare also [30]):
> If a system is in state q, if question α is compatible with p and if α is performed and yields the answer "yes" then the state of the system after the experiment is $(p \vee a') \wedge a$ where a is the proposition defined by α." (p.406)

Since this "assertion" is quite badly formulated we can only ponder what W. Balzer means. How are p and q related? What kind of question is α and what does it mean for questions to be compatible? C. Piron only talked about compatible properties where compatibility is used to explain the above mentioned condition (iii) formally. We can admit however that ideal and first kind questions belong to a part of OQL which deals with aspects of dynamics and that is a part which allows itself to further explorations (see for instance [12, 14, 15]).

Finally we may affirm that W.K. Essler and G. Zoubek did not understand C. Piron's work very well, so we hope to have answered also some of their questions in this reply to criticism, especially when one reads the following in [17]:

> "As long as Piron does not answer these and similar questions in a way satisfactory to logicians and especially as long as he does not determine the logical structure of the elements of his class Q of all questions his attempt remains as obscure as Heidegger's approach to a new ontology: he is able then to immunize his theory against any critique by pretending that he has been misinterpreted, and no one will then be able to prove the interpretation's correctness." (p.417)

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SONJA SMETS
Center for Logic and Philosophy of Science
Free University of Brussels
Pleinlaan 2, 1050 Brussels
Belgium