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A PROFILE OF MASONIC SYNTHESIS*

Analysis and Synthesis

1. *Due sunt methodi, synthetica per artem combinatoriam et analytica...*[1]*; There are two methods: the synthetic, via the art of combination, and the analytic.*

Forma sive ordo... consistet in conjunctione duarum maximarum inventi artium, Analyticae i Combinatoriae...[2]*; Form, or order... consists in conjoining these two main arts of discovery ... Analysis and Combination ....*

And there we have, in the words of Leibniz himself, an piece of wisdom fundamental to Western Civilisation (although not only to it). Everything is a both a product of the decomposition (analysis) of a given object into simpler objects and of the synthesis (composition) of that which is composed of simpler components. In order to come to know a given object, it is necessary to reconstruct the process of analysis and synthesis, in the one and the other direction.

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[1] See L. Couturat, "La Logique de Leibniz", Math. I, 26c.

[2] See Foucher de Careil, VII, 173

Historical Sketch

2. Such a thought lies at the basis of Greek philosophy. It emerges with Empedocles and Anaxagoras. It is present in the philosophies of the Pythagoreans, of Democritus and of Plato[3]. From them it passed into in European thought where it has settled for good.

It has also appeared in Eastern thought. One may take, for example, Taoism, with its conception that everything is a compound of two primitive factors, *yang* and *ying*.

It was a central component of Western science and philosophy until the time not so much of Aristotle as the scholastics, for whom the stress shifted onto qualitative investigation and a qualitative account of phenomena, leading to a mostly futile search for essential properties and trailing off into rather fruitless further inquiry.

Let us note here that all those scholars of the structure of natural compounds, such as the alchemists, never gave up on its centrality.

3. It was with Method of Descartes that we see a certain renaissance of the paradigm of analysis and synthesis, the idea lying at the basis of his thought. Taking analysis as primary and synthesis as secondary and considering both as having conceptual and natural forms, Descartes made them the basis of his science. This is clearly stated by him, both in his "Discourse on the Method" and "Rules for the Direction of the Mind".

The full flowering of the idea, both in science and in philosophy, took place in the century after Descartes, a century dominated by the two giants, Newton and Leibniz.

4. In mathematics, two fruitful results, of many which could be given, were the Differential and Integral Calculi, of which the first consisted in the analysis (breaking-up) of a given field and the second in integration (consolidation), this being the appropriate form of synthesis. The calculi conjugate with one another and are dual.

In the 18th century, the method brought about successes in the newly-emerging discipline of chemistry, resulting in its becoming in the following century the model on which the science was built. As is no doubt familiar, the model falls apart roughly into the following pieces: a theory and a practice of chemical analysis, a theory of chemical synthesis, a theory of the chemical elements (an analytic concept, as we shall see) and a theory of chemical compounds united with a theory of the types of compound-forming connections.

[3] They all agreed on the method, although they proposed different analyses and syntheses and, in particular, they had varying opinions on the issue of what was simple.

Subsequently, at the beginning of the 20th century, the method found took root in physics and, half a century or so later, in molecular biology and genetics.

5. Somewhat different, however, appears the fate of the Cartesian and Leibnizian paradigm in philosophy and logic.

In the beginning the model was modern, and thus fashionable, and was adopted by many, including the whole Cartesian school and, from Locke onwards, Anglo-Saxon philosophy, where it fused with the type of empiricism stemming from Bacon and Locke.

Until the second half of the 18th century, the method of analysis and synthesis (for short: **AS**) was applied routinely in the examination of the world and its components. Man was considered (in accordance with common sense) as a component (part) of the world and the human world was treated naturalistically — as a part of the world as a whole.

This approach was perhaps favoured by the view that the world in its entirety was universally considered to have been created and organised by a Creator[4].

6. In the hundred or so years leading to the time of the Kantian revolution, that is the emergence of the so-called second "critical" philosophy of Kant[5], **AS** in philosophy had become mainly a method of conceptual analysis of cognition (often resting upon special cognitive intuitions).

In his second philosophy, Kant succeeded spectacularly in breaking away from the "fact-seeking" type of inquiry that had characterised traditional metaphysics, leaving the exclusive rights to form of fact-orientated inquiry to researchers in the various areas of natural science[6]. In exchange, he promoted metaphysics as inquiring into human cognition and its results. To use contemporary terminology — as suitable for conducting research in the disciplines of Cognitive Science (in "The Critique of Pure Reason" and related writings) and in Political and Axiological Science (in the remaining "Critiques").

In this, Kant was most certainly a revolutionary. He brilliantly broadened the field of human cognition. In the foreground were brought up neglected issues for treatment with his own, unusual and original solutions, which em-

[4] Such was Leibniz's God - The Creator of Heaven and Earth, He through which everything came to be.

[5] The first, "pre-critical" philosophy of Kant was directed at the world and deduction, closely connected with physics at least, in which Kant was an acknowledged expert.

[6] Let us recall that in Kant's time there was in general no such thing as the Humanities. They were to come into existence only in the 19th century.

phasised the active role of the (human) Mind. In his philosophy the Mind divided into three parts: Theoretical Understanding, Practical Understanding and individual cognitive and practical powers.

Epistemology disappeared from the foreground along with those "metaphysical" systems of epistemological and transcendental idealism that were derived from it.[7]

If one thing is certain, it is that right from the emergence of Kant's second philosophy and thanks to it, the close connection between philosophy and science was broken. The gap widened gradually becoming in some philosophical schools more of a gulf. Recently, the gap has narrowed and is expected eventually to close.

7. We can say now with a certain conviction, two hundred years later, that the Kantian Revolution *did not give birth to any new metaphysics which has come out or could present itself as a science*. That revolution has however for some time pushed aside classical metaphysics ever closer towards, one might say, the dustbin of philosophy. One may compare what happened in the so-called post-Kantian metaphysics of the first half of the 19th century to a regress to a very much earlier period in the Middle Ages' philosophy.

Where, then, were to be found the subtlety, momentum and reliability of Plato, Aristotle, Plotinus, Anselm, Thomas Aquinas, Duns Scotus or indeed that prince of philosophers in so many ways close to Kant — Leibniz? In recent times, such standards have been upheld in our time by thinkers no less giants but both isolated and lacking in influence. For example, Bolzano or Schopenhauer.

8. Classical metaphysics has come back to life. Yet it has most definitely become a logical, hence scientific discipline not aligned with the Kantians, but one set against them.

Those who regenerated first philosophy as a discipline were above all the fathers of contemporary logic. Conceived in the latter half of the nineteenth century, classical logic has enjoyed a era rich in developments. In chronological order, we begin with Bolzano, Boole, Frege, Peirce, and Peano, followed by Russell, Whitehead and Wittengenstein, after whom the names worthy of mention are legion.

A second figure in the revitalisation of classical metaphysics as a live scientific philosophy was Franz Brentano, and beyond him his students and

[7] Some consider that, with regard to this matter, Kant was faithfully following in Descartes' footsteps. If this is so, it takes a very peculiar reading and interpretation of him.

developers, amongst whom we find Kazimierz Twardowski, the father of Polish scientific philosophy.

Brentano united scientific philosophy with descriptive psychology and a resurrected descriptive metaphysics in the style of Aristotle. His pupils divided into schools following paths which, whilst differing from each other, always remained faithful to their source. In the case of phenomenology, special techniques of eidectic analysis were introduced into the Brentanian picture. A different case is that of Meinong and his co-workers, who worked Brentanian ideas into a general theory of objects and properties.

9. The peculiarity of the Polish school to a large extent consists in its having woven together conceptual and linguistic analyses with a structure of logico-philosophical theories based on the free and subtle use of logical tools. As a result, ontology and the other traditional components of philosophy have realised a model of philosophy maximally *more geometrico*, i.e. a paradigm of logical philosophy.

The present work is a contribution to ontology understood in precisely this way.

10. The part played by logic in the revitalisation of the Cartesian and Leibnizian paradigm determines its contemporary shape. Linguistico-logical analysis is dominant and the philosophical trend connected with the realisation of this model carries the official name of analytic philosophy.

There is however an absence of a corresponding emphasis on synthesis. Leaving aside the natural sciences and contrary to the pointers given by Descartes himself, it is as if it has been neglected. Yet analysis and synthesis go together — they complement each other. The situation in contemporary philosophy thus brings to mind the image of a mare, still fertile and fair, but fallen lame.

In short, in philosophy today, the predominance of analysis dwarfs the role of synthesis and the effect is a one-sidedness.

11. The remedy contains three ingredients: Firstly, a symmetric generalization of the method **AS**, by which I mean the introduction of a **G**eneral Theory of **A**nalysis and **S**ynthesis (or **GAS**, for short) and the consequent application of its measures.

Secondly, an emphasis on a theory of synthesis until it reaches a point of equal importance with philosophical analysis. We recall that that latter analysis is currently composed of four elements: linguistic analysis, conceptual analysis, eidectic analysis, and - last but not least - logico-mathematical analysis.

We need to complete our analytic philosophy with suitable synthetic complements. Indeed, one may say, with an suitable synthetic philosophy.

Thirdly, a convergence of the methods of philosophy with the methods of the natural sciences and a continuation of the analogy — *via* logic — with the methods of mathematics. This means placing greater emphasis on the mathematical modelling of the philosophical data supplied by everyday experience, by the natural sciences and the humanities, and, above all, by a method specific to philosophy for the preliminary analysis of all the data. The methods of phenomenology may prove in this regard to be extraordinarily helpful[8].

A reanimated phenomenology could well turn out to be a great ally of logical philosophy in it systematic research.

Content-Related Sketch

12. We are to understand ontology not with its universal, contemporary meaning[9] but the classical, Greek meaning. Ontology is therefore a theory of being. A being is thus something which is.

That which exists — a thing, a unit, a process, an event — also is, but not conversely. Not all beings exist. Existing is more than being. The concept of being is a wider concept than the concept of existing[10].

13. What is more essential is that the concept of existence is complex and multi-aspect. Four aspects are basic:

Particular Aspect: in which particular beings are given, that is particular entities as the objects which they are.

Generality (or **Totality) Aspect**: the general totality of that which is (this totality creates the so-called ontological space)

[8] I am thinking here mainly of the results of classical phenomenology. What took place in Germany, France and America, after World War II, with a few exceptions (e.g. certain analyses of M. Merleau-Ponty or H. Ey) has yielded, sorry to say, mainly rubbish.

[9] In particular, I don't identify ontology with logic (*Pace* Bocheński) but I treat the theory of sets as a mathematical discipline close to ontology, which may determine one of many ontologies coming from mathematics. I do not therefore treat it as a single modern ontology, as do Quine, Suszko and ever so many followers and imitators. It is one of many. It is true indeed to say that it occupies a special place as an ontological frame of thought.

[10] According to the Scholastic way of thinking, a failure to distinguish existence from being leads to that great artificial problem of the ways of being or the paralogisms of the sort "how can that which not exist exist?". We should rather ask "how can that which does not exist be?". We observe that, on the basis of Parmenides' statement that there are no non-beings, one may not ask "how can that which is not be?" precisely because there is nothing such. (see [3])

Ontic Aspect: the entirety of that which is; that is, the totality of particular beings and logical space taken as a whole, as **One**.

Onto\logical Aspect: the principle of integration of that entirety, the uniting principle, **Logos**.

It may still be necessary to distinguish further aspects but those I have listed above will provide us with sufficient difficulties for the present. Let us therefore confine our attention to them for now.

14. The Polish language is poor in names for types of being. Greek, English, German and other languages have at least two basic forms and two articles — a definite and a indefinite, which allows us to create at least four names, which is as many as we will need.

I shall therefore establish the following convention[11]: "being" signifies any particular individual, that is each entity taken under the particular aspect and "Being" in turn I will use to signify any being taken under the generality aspect. Being[12] is simply the ontological space. "the Being" signifies being under the ontic aspect. The word "BEING" or the Greek "Logos" signifies being taken under the onto\logical aspect.

15. Ontology is thus the most general theory of being, that is, taken under each of the four aspects.

If we take into consideration the distinction between planes of being (there are at least five of such) which I sketched in §12 of my "On Philosophy" (see [4]), with which human beings are involved with, then we will get as a result at least twenty types of being and in turn at least as many varieties of ontology.

He who would say that ontology is a straightforward discipline would thus be lying.

16. Let us concentrate on the plane of being itself. Our considerations here will bear on general ontology with particular ontologies being thus bracketed for the moment.

The path to such considerations is opened by the introduction of a sufficiently general and subtle conceptual network. This is usually determined by those basic categories which are given by FUNDAMENTAL OPPOSITIONS.

17. The ontological universe is ordered by certain fundamental relations.

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[11] See [1]

[12] There is of course the danger here of ambiguity when the word "being" occurs at the start of a sentence. Furthermore, the word "being" used without any additional definitions or used in the absence of an explicitly introduced convention is that typical everyday word with its many variants and meanings.

With regard to the relation *being simpler* (a basic relation of any analysis) and *being complex* (a typical relation for synthesis), we obtain *via* their contrast the opposition below:

simpler — (more) complex

following this thought through to its limits we obtain the opposition:

simple — maximally complex (possible world)

We observe that in the case of analysis that which is call simple is most often *an atom* or *element*; in the case of synthesis, however - *an element.*

In view of what is logically primary, we have the opposition:

primitive — secondary (derivative)
in terms of being

In view of the number of primary elements we obtain the fundamental opposition of Parmenides:

One — Many

It would be possible to delineate the whole set of oppositions. These four shall however suffice for our present inquiries.

18. The opposition *primitive — secondary* is the generator (and indeed the effect) of a basic Greek intuition that that which is given in our everyday life is - in view of its transience and accidentiality — the result of some more primary relationship and foundation of being. That primary order is the Logos.

Both by definition and by the Axiom of Extensionality, there is one BEING. And ONE is one.

19. Let us now ask what is first in terms of being. ONE or something else? And how many such objects primary in terms of being are there: one or at least two? These questions lead us directly to two opposing ontological positions:

monism *versus* pluralism

Monists state that there is only one object primary in terms of being and it is ONE. Pluralists state that there are at least two objects primary in terms of being and that ONE may be either of them (cf. Plato, Plotinus)

20. We observe that monism leads to a problem: how do we obtain many from one? This is the problem of PLURALISATION. Conversely, Pluralism asks how do we obtain one from many? How do we integrate the many? That is, how do we get ONE from many? This leads us to the problem of UNIFICATION.

Both operations — pluralisation and unification — are conjugate and presumably dual with respect to each other.

$$\text{ONE} \underset{\text{pluralisation}}{\longmapsto} \text{MANY} \underset{\text{unification}}{\longmapsto} \text{ONE}$$

21. A natural pluralitic ontology is therefore an ontology of integration of objects, connecting them into compounds, combinations. From this we take the name **combination ontology**[13] — in short: **CO**. Among the ontological pluralists we find Plato, Aristotle and all those counting themselves Aristotelians, Leibniz, Russell, Wittgenstein, Husserl, Hartmann, Ingarden, and others. In a word, rather a lot of working ontologists.

In the other direction, a natural monistic ontology is a transformational ontology, in short: **TO**. It is very easy to imagine pluralisation at the level of the primary beings as a result of suitable transformations (modi) of a ONE. Amongst the monists, we find Parmenides, Spinoza (the first conscious transformational monist), Hegel, Bradley and, last but not least, Einstein.

22. It is my fundamental intuition that both types of ontology in a certain sense complement each other. A general theory of Analysis and Synthesis, **GAS**, may serve as a framework for the combined generalisation of these two fundamental types of ontologies of the planes of being, that is of being itself. This is a topic for future research.

I shall here be developing **GAS** in a rather one-sided fashion — keeping an eye on a general combination ontology.

Three Approaches to a General Theory of Analysis and Synthesis

23. The space of analysis and the space of synthesis, taken individually or together, may be described from the inside or the outside (the latter being the easier of the two).

There are two types of **external** account: **ordinal** and **operational**. A pure ordinal account in effect assumes that the ontological space of all beings is given and explicitly ordered in view of the first two oppositions listed above, which express appropriate ordering relations, respectively analytic and synthetic.

An operational account is an indirect account, in part external and in part internal. We describe the ontological space as a space equipped with two appropriate operators — **an analyser**, which decomposes complex objects into simpler ones (into parts) and a **synthesiser**, which integrates groups of objects into their compounds, synthesising them.

[13] We observe that the so-called combinatorial logic is a simplified version of combination onto\logic.

This account assumes, just the ordinal account does, that the ontological space is given and also that its (hidden) order is given. In both accounts, it is a matter of grasping and examining that hidden order: in the first case, by the methods of the theory of relations and in the second, by the methods based on the theory of sets and a suitable calculus of operators.

Both of the aforementioned accounts thus rest on the assumption that the ontological space along with its natural orders is given and our main task is to unearth its hidden order.

On the side remain fundamental onto\logical questions: Where does this ontological space come from? What is its source? What constitutes it? What is the source of and principle behind this hidden order?

24. A partial attempt to answer these questions is given by an **internal** (**modal**) account of the space of analysis and synthesis. We shall treat it as a complex of primary and secondary objects, which are **marked** (modalised) in such a way: that simpler objects naturally join together in compounds of increasing magnitude, constituting in this fashion the ontological space and generating both fundamental orders of analysis and synthesis of this space.

I have given a preliminary sketch of the ordinal and modal approach in [2] and I intend to publish a final account of the whole combinatory ontology in [7].

Now, I shall concern myself with a semi-formal outline of an account of three approaches to the General Theory of Analysis and Synthesis.

The Ordinal Approach

25. Let OB be a family of all objects in general. Let the universe of considerations (of a given area of study) be a subset of it: $U \subseteq OB$

We shall examine in turn three ordered spaces.

Space of Analysis, $AS := \langle U, < \rangle$
where $<$ is the relation is *simpler than*

Space of Synthesis, $SS := \langle U, \prec \rangle$
where $\prec$ is the relation *to be a component of*

Space of Analysis and Synthesis, $SS := \langle U, <, \prec \rangle$

26. These approaches immediately generate a series of questions. What are the natural axioms for the relation is *simpler than* $<$? Aren't the usual mereological axioms too strong?

Analogously, what are the natural axioms for the fundamental relation of synthesis, *to be a component of* $\prec$? In turn, what axioms bind both relations, that is both analysis and synthesis taken as a relational whole?

We observe first of all, that the simplest solution, relatively speaking, that both relations, although orientated contrary to one another — the order of analysis aims “downwards”, to the simpler, the order of synthesis being the other way around, aiming “upwards” to the more complex — are co-extensive, that is that synthesis is a simple reversal of analysis, goes too far in the direction of simplification. It is clear that it may happen, but in natural cases it is found very rarely. If, however, our minds are set on an account and analysis of the plane of being, being itself, then we must take into account that both analysis and above all synthesis splendidly broaden our universe.

And besides, analysis is simpler than synthesis — it's easier to take a watch apart than to put it back together! The conviction that synthesis widens the human universe demarcated by introductory analysis is therefore natural. Analysis drives synthesis!

Synthesis of objects separated by analysis via their recombination creates not only straightforward outputs from inputs but also produces new objects — **possibilities**.

27. One may therefore picture the space of analysis and synthesis ASS (on the assumption of an ontological foundation) exactly as having the shape of an “ontological dustbin”, presented in figure 1.

This analysis may lead, though need not, to **simple objects**, which form the **substance S**. Coming from them (or from a different family of simpler objects), synthesis produces ever more complex compounds until we have maximal objects given in a limit crossing — **possible worlds**. The support of x is denoted by $S(x)$. It is the totality of simple objects below x or, to put it more simply — the totality of simple objects from which x is built.

The unshaded circles signify indirect situations (compounds) neither too big nor too small. The shaded circles signify maximal situations, or possible worlds.

That which is given — a DATUM — is a part of the real world (in Fig. 1 it is pictured by the big shadow circle with DATUM inside), to which is via suitable relations (causal and otherwise) expandable. The real world itself is not given to us — human beings — directly. Only fragments of it reach us directly. Such is the case with other worlds, possible worlds. The world is our construct, indeed.

In the next paragraph I shall discuss the concepts introduced here more precisely.

28. Objects simple and maximal. Supports and substance.

For each of the two considered relations, $<$ and $\prec$, we separately distinguish simple objects (in one of the five listed below meanings of that word) and appropriately distinguish co-simple objects, or those that are maximal. We thereby obtain:

simple analytic objects: $s_a(x)$ *means that* x *is simple with respect to* $<$*, in one of five ways*

simple synthetic objects: $s_s(x)$ *means that* x *is simple with respect to* $\prec$*, also in one of five ways.*

By generalising the above, we may simplify the notation to bring about univocality Namely:

simple: $s(x)$ *means that* x *is simple in an appropriate sense*

(1) *At least five kinds of simple object are distinguishable — both for analysis and synthesis.*

Throughout all the considerations to follow, the general ontological relation E, read "is", is introduced here. Depending on the context, this means either of our two established relations, that is $<$ "is simpler" and $\prec$ "is component of ".

We must therefore distinguish the following:

super-elements, $se(x)$: $\neg\exists y\, \neg xEy$, that is $\forall y\, xEy$

x is a super-element if x is simpler than every object

true simple object, $ss(x)$: $\neg\exists y\, yEx$

x is simple if there is no object simpler than it, or preceding it.

atoms $a(x)$: $\neg\exists y \neq x\ yEx$, that is $\forall y(yEx \rightarrow\ y = x)$

x is an atom if there is no object simpler than it, with the exception of at most the object itself

elements $e(x)$: $\neg\exists y(\neg se(y) \wedge y \neq x \wedge yEx)$, that is $\forall y(yEx \rightarrow\ y = x \vee se(y))$

e is an element if the only objects that can be simpler than it, beside that object itself, are super-elements.

hyper-elements, $he(x)$: $\forall y(yEx \rightarrow se(y))$

x is a hyper-element if the only objects simpler that can be simpler than it are super- elements

Relations between the five concepts of simple objects above are rather complicated and an account of them constitutes an essential part of the ordinal version of **GAS**.

We observe that appropriate simple objects for the converse of the relation E, namely co-simple objects that are precisely those maximal objects in view of one of the considered relations, being in the case of the relation of analysis $<$ or being in the relation of synthesis $\prec$, are appropriate **possible worlds**, respectively for the spaces of analysis and synthesis.

We may form appropriate abbreviations for the preceding with by prefixing then with the letter "c", to express "co". We thus have $cse(x)$, $cs(x)$, $ca(x)$, $ce(x)$, and $che(x)$.

We add that objects which are not simple are to be called **complexes**. We shall not differentiate them into kinds, signifying them just by $C(x)$.

29. We distinguish generally two concepts of substance — analytic and synthetic. These are to be distinguished for each of the five kinds of simple objects.

Analytic substance — this is the family of all simple objects of a given analysis — S_a. **Synthetic substance** — this is the family of all simple objects of a given synthesis — S_s. **General Substance** — this is the family of all simple objects in general — S.

We observe that in view of the great number of simple objects of various kinds of their generality, substance may be (and usually is) heterogeneous. Generally, one must distinguish several dozen sorts of substance.

The substance (or **support**) of an object x is in turn a family of appropriate simple objects enmeshed in x. Let $(x]$ signify appropriately (either analytically or synthetically) the ordered ideal generated by x. We distinguish:

analytic support: $S_a(x) := S_a \cap (x]$
synthetic support: $S_s(x) := S_s \cap (x]$
support in general: $S(x) := S \cap (x]$.

30. It is easy to see that we have a series of issues relating to the use of a substance and its role both in analysis and synthesis. Substance plays a particularly great role in various **founded** ontologies, namely those in which each object is demarcated and (completely?) determined by its support.

This issue of substance is regulated by numerous axioms and their consequences from which presently — for example — I shall give only the most famous, this being the axiom of **involvement** (or **non-inertia**) of Franz Brentano

$$\text{(AI)} \qquad \forall e(x) \exists C(y)\ xEy$$

Each element is involved in a certain complex, that is, there are no "flying" non-inert elements.

31. We observe finally that the ordinal approach allows us to describe the universe of analysis and synthesis from the outside. It allows only for the possibility of a superficial discussion of the ASS space from "the outside", and not for the recognition of its machinery.

The Operator Approach

32. This approach is based on the introduction of two operators able to work on the universe U: **analyser** α and **synthesiser** σ. The first of these works in the following way: if "the input" is a given object x, "the output" is the collection of all its **parts**, **fragments**, or **bits**. In turn and conversely, if the synthesiser σ has a certain collection of objects as "inputs", it yields their compounds or the collection of compounds obtained from it or — for a single object x — the family of all objects **synthesisable** from x (more precisely — from the substance of x, $S(x)$)

These operators α and σ are rather like "black boxes". We don't know that much about how they work (as regards ontology, that is, and not, for example, chemistry) and we can only sometimes guess at their operation observing the results they produce.

33. In each instance, they are assigned the following classes:

$\alpha(X) := \{Y : Y \textit{ is obtained from } X \textit{ by } \alpha\}$
$\sigma(X) := \{Y : Y \textit{ is obtained from } X \textit{ by } \sigma\}$.

Straightaway arise a series of obvious questions. Is $\alpha(x)$ a family of all the parts of x? Is $\sigma(x)$ a family of all that lets itself be obtained from x, either in **practice** or in **a given period of time** obtained from x? Or is rather a family of all that is **in general possible** to obtain from x?

What are the general and particular properties of both operators? In particular, what relationships link them? What would it mean, if one operator were the reverse of the second? (In this case, the answer is easy: in our field of research there would not be genuine possibilities, only facts.)

I will consider some of these questions in the next few paragraphs, reserving one of them as the title and substance of the third part of this article.

34. In each case, the operator account of the space of analysis and synthesis $\langle U, \alpha, \sigma \rangle$ is also an external account and, in view of the appropriate and sufficiently simple resources of the theory of sets, an extensional account. It opens the way, however, to an internal, modal account.

35. Global conditions. The reader will find below the output axioms which will be in force in the course of the considerations to follow.

(A0)
$\sigma(x) := \sigma(S(x))$ - *to synthesise from x is to synthesise from a substance x*
$C(y) \to S(y) \neq \emptyset$ - *complexes have non-empty substance*

For $X \subseteq OB$ we put $\sigma(X) := \sigma(\bigcup\{S(x) : x \in X\})$.

Complexes (compounds) are built of their own substance.

In the axioms below, R signifies, so long as there is no specific necessity present, one of the relations already considered: $E, <, \prec$. We add that the definitions below are entirely general and can be considered appropriate for all relations in general.

(T) R is transitive: $xRy \wedge yRz \to xRz$ **Transitifity**

(AS) R is anti-symmetric: $xRy \wedge yRx \to x = y$ **Antisymmetry**

35.1 The meanings of the axioms below are clear. I am endeavouring, in various ways and *via* the variations and different shades of meaning, convey the "economy" of synthesis.

Regularity: *Synthesis founded on elements from X creates complexes built from the simple elements taken from X.*
(REG) $\emptyset \neq X \subseteq S \to \sigma(X) = \{C(y) : S(y) \subseteq X\}$

Monotonicity: *The greater the quantity of material, the greater the number of objects synthesised from it.*
(MON) $X \subseteq Y \subseteq S \to \sigma(X) \subseteq \sigma(Y)$

Compactness: *Compact syntheses only use the simple objects of their components.*

(COMP) $C(x) \to S(x) = \bigcup\{S(y) : yRx\}$

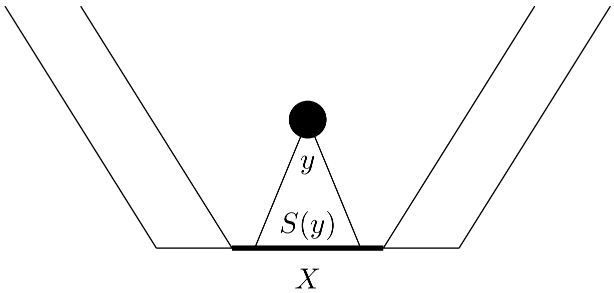
Economy: *The greater is not synthesisable from the simpler.*

(SE) $C(x) \wedge y \in \sigma(x) \wedge xRy \to x = y$,
resp. $C(x) \wedge xRy \wedge x \neq y \to y \notin \sigma(x)$

Extensionality: *Extensional combinations are determined in full by their reference.*

(ES) $S(x) = S(y) \to x = y$.

The following Figure 2 shows a monotonic regular space of synthesis



36. Local conditions. With these conditions, I am trying in effect to "localise" synthesis, meaning that I am trying to make it exclusively dependent on a synthesised object and the materials that compose it.

(SL) $x \in \sigma(x)$, *i.e.* $x \in \sigma(S(x))$
x is synthesisable from its own substance.

In a weaker form

(wSL) $C(x) \to x \in \sigma(x)$
The same, but for complexes.

37. Axioms for fusions (sums). It is well known that the problem of sums (unions, fusions) is one of the most subtle that the theory of synthesis has to deal with.

What may we fuse? Everything? So say the majority – and they are wrong!

Or only those objects which "go with themselves", attractive one to another, by which I mean either those appropriately marked in a way determining connections or those that fulfil special extra conditions?

Furthermore, as is shown by the subtle discussion of the matter by Andrzej Pietruszczak in his [8], the concept of fusion itself is ambiguous and its theory demands a certain awareness of different nuances of meaning.

For the further part of my considerations I shall be styling myself as a "Boolean" somewhat less subtly, as my goal is to characterise a Masonic Boolean Synthesis!

The first three axioms below state that a fusion $\circ$ is a lattice meet in virtue of the order R.

(F0) $\circ : \langle x, y\rangle \to x \circ y$
the fusion operator is a function
(F1) $xR(x \circ y), yR(x \circ y)$
(F2) $xRy \leftrightarrow (x \circ y) = y$

The remaining two axioms, in a stronger and weaker form, express the natural conviction that the support of a sum of two objects is itself a plural sum of their supports.

(F3) $S(x \circ y) = S(x) \cup S(y)$
Or in a weaker form:
(F3^w) $S(x \circ y) \subseteq S(x) \cup S(y)$

The Internal or Modal Approach

38. In order to at least describe the necessary conditions for a favourable synthesis, it is useful to employ two basic ontological modalities (for a wider account, see [2] and [7]): $MP(\ ,\ )$, which abbreviates **making possible**, and $MI(\ ,\ )$, which abbreviates **making impossible**.

Their proper theory is sufficiently complex and ontologically rich for a proper discussion to have to be set aside for another occasion. The interested reader may consult [2] now and shortly [7].

For our present purpose, that of clarifying what the relations are between particular approaches to analysis and synthesis, I shall give here only those **axioms connecting the second and third approaches**.

(RS1) $y \in \sigma(x) \to MP(x, y)$
That which is synthesisable from x is made possible by x.

In an indirect form:

(RS2) $y \in \sigma(x) \to \neg MI(x, y)$.
That which is synthesisable from x is not made impossible by x.

Masonic Boolean Synthesis

39. Boolean syntheses are associated with the ontology of mind, hence with the being plane of thought and culture. They are certainly also associated with the plane of being itself, as both we and our brains are a part of the world, as the theory of Boolean algebra is in essence the most general theory of networks and the world may certainly be presented as the network of networks.

40. Masonic Boolean Syntheses are, by the nature of their being, deeply combinatory. Their character is captured by the famous Stone Representation Theorem, whose essence is given in the form of the following equivalence.

(BR) $\quad x < y \quad$ *iff* $\quad [x) \subseteq [y)$.

Boolean ordering is isomorphically representable by the set-theoretical inclusion between appropriate principle filters from a universe of a Boolean algebra under consideration. In this way, each Boolean algebra is isomorphic with a certain Boolean algebra of sets.

The above more clearly follows in the case of Boolean power algebras, being those Boolean algebras of all subsets of a given collection. These algebras, as is known, correspond perfectly with complete and atomic Boolean algebras (see [9]).

In this case, the dependence takes the form:

(PBR) $\quad x < y \quad$ *iff* $\quad a(x) \subseteq a(y)$.

In other words, the order of a given algebra is fully determined by the set-theoretical inclusion between families of atoms involved into appropriate objects: the greater the number of atoms, the greater the object.

41. This also corresponds perfectly to the fundamental principle of Masonic Synthesis (or Mosaical Synthesis): *the more bricks you use, the more you can build.*

Each traditional brick-built structure, or mosaic one is its model, as is each structure built from bricks of Lego.

Hence we consider the following **principles of a Boolean Masonic Representation**.

(BMR1) $\quad xRy \to S(x) \subseteq S(y)$

to be called, for simplicity, a Semi-Masonic Boolean Representation

(BMR2) $\quad S(x) \subseteq S(y) \to xRy$

(BMR) $\quad xRy \leftrightarrow S(x) \subseteq S(y)$

PROBLEM OF THE PRESENT WORK: Indicate the "rational" profile of Masonic Boolean Syntheses.

Departing Observations

42. Recall first that, in general, we assume two conditions of (Ax0) saying respectively that to synthesise from an object is to synthesise from its substance and that any complex has non-empty substance.

Now, we observe that *regularity implies weak locality*:

(2) $\quad \mathrm{REG} \vdash \mathrm{wSL}$

Assume REG and that y is complex: $C(y)$. By Ax0, $S(y) \neq \varnothing$. Now REG gives that $y \in \sigma(S(y))$ iff $C(y)$ and $S(y) \subseteq S(y)$. But the right side of the equivalence is true. Hence $y \in \sigma(S(y))$. Applaying again Ax0 we obtain $y \in \sigma(y)$. QED

Observe also that regularity implies that *in regular spaces of synthesis simples are objects without substance (support)*, i.e., *that simples are not sythasisable form anything.*

(3) $\quad \mathrm{REG} \vdash \mathrm{S(y)} = \varnothing$ iff y is simple

43. On the other hand, *transitivity implies the Semi-Masonic Boolean Representation* BMR1:

(4) $\quad \mathrm{T} \vdash \mathrm{BMR1}$

Let R be transitive, $xRy \wedge yRz \to xRz$ and xRz. Assume additionally that $u \in S(x)$, that is, that u is a simple object and uRx. We have therefore $uRxRyRz$, hence by (T): uRz. Hence $u \in S(z)$ and therefore $S(x) \subseteq S(z)$, which was what we needed to prove.

44. Let us move now to a consideration of the axioms relating to fusion.

The condition of absorption for fusion F1 and the principle of Semi-Masonic Boolean Synthesis (BMR1) imply, that both forms of the axiom describing the substance of fusion - the stronger F3 and the weaker $\mathrm{F3^w}$ - are equivalent to one another.

(5) $\quad \mathrm{F1}, \mathrm{BMR1} \vdash \mathrm{F3} \leftrightarrow \mathrm{F3^w}$

We need to prove the left-hand implication only. Observe first that *via* F1 we have $xR(x \circ y)$ and $yR(x \circ y)$, hence via BMR1 we obtain $S(x), S(y) \subseteq S(x \circ y)$, that is $S(x) \cup S(y) \subseteq S(x \circ y)$.

$F3^w$ gives us the reverse inclusion, from which we obtain the intended equality F3.

It follows from the above proof that in syntheses respecting the absorption condition F1 and the condition BMR (*a fortiori*, the condition of transitivity) elements of compounds do not "perish" during synthesis, as expressed by the first of the inclusions in the proof. In the condition characterising the substance of a fusion of objects as an plural sum of their substances, it is thus essential that no new element appears in the substance of the object composed through fusion, as expressed by $F3^w$.

On the other hand, we observe by accepting that fusion fulfils the lattice conditions F2 and F3, the Semi-Masonic Boolean Synthesis BMR1 follows immediately.

(6) $\quad \mathrm{F2}, \mathrm{F3} \vdash \mathrm{BMR1}$

Assume that xRy. Via F2 we obtain $(x \circ y) = y$. It follows from the axioms of the logic that $S(x \circ y) = S(y)$. By F3 we obtain $S(x \circ y) = S(x) \cup S(y)$. It follows from both proceeding identities, that $S(y) = S(x) \cup S(y)$, hence $S(x) \subseteq S(y)$, which was what we wanted to show.

We have therefore given two sufficiently natural conditions (4) and (6) for Semi-Masonic Boolean Representation.

45. Let us now return momentarily to a consideration of the type (5) and ask what other consequences follow from an acceptance of BMR1 in conjunction with the appropriate axioms.

(7) $\quad \mathrm{ES}, \mathrm{BMR1}, \mathrm{REG} \vdash \mathrm{SE}$

Extensional, semi-masonic and regular syntheses are economic.

Assume the antecedent of SE, namely $y \in \sigma(x)$ and xRy. By BMR1 and the second of our assumptions we obtain $S(x) \subseteq S(y)$. Also $C(y)$, that is, through (Ax0) $S(y) \neq \emptyset$.

We now have two situations: $S(x) = \emptyset$ but then x is simple, hence the antecedent SE is false, which means that the synthesis is extensional via classical logic. In the second situation, we shall first take the case when $S(x) \neq \emptyset$. Applying (REG) for $X = S(x)$ we immediately obtain $S(y) \subseteq S(x)$. By (ES) it follows immediately that $x = y$, which was what we wanted to show.

This result, however, is not overly satisfying. It is true that economy is a desirable property for a synthesis, but the assumption of extensionality is a strong assumption. Perhaps too strong!

46. Let us therefore keep looking for conditions which determine that a synthesis is economical. We observe first of all that *(full) Masonic Boolean Representation and anti-symmetry entail the extensionality of synthesis.*

(8) $\quad \mathrm{BRM}, \mathrm{AS} \vdash \mathrm{ES}$

Indeed, when $S(x) = S(y)$ then by BMR we have that $xRyRx$, from which it follows via anti-symmetry that $x = y$.

Joining (7) and (8) we obtain the following result: *regular anti-symmetric synthesis with a Masonic Boolean Representation is economic.*

(9) $\quad \mathrm{AS}, \mathrm{BMR}, \mathrm{REG} \vdash \mathrm{SE}$

Essentially, by (8): AS, BMR, REG $\vdash$ ES, BMR1, REG hence, via (7), we obtain the expected conclusion.

A Characterisation of Masonic Boolean Synthesis

47. We will demonstrate first the following theorem: *the axioms for fusion in conjunction with weak locality and economy imply* BMR

(10) $\quad \mathrm{F0–F3}, \mathrm{wSL}, \mathrm{SE} \vdash \mathrm{BMR}$

In view of (6), it suffices to check BMR2. We assume, therefore, $S(x) \subseteq S(y)$. Thus $S(x) \cup S(y) = S(y)$. But via F3, $S(x) \cup S(y) = S(x \circ y)$. Hence $S(x \circ y) = S(y)$. Applying wSL, we obtain $x \circ y \in \sigma(S(x \circ y))$. Therefore $x \circ y \in \sigma(S(y))$. For sure, $x \circ y$ is complex. Then, by Ax0, $S(x \circ y) \neq \varnothing$ Therefore $S(y) \neq \varnothing$, hence y is complex as well. Clearly $yR(x \circ y)$. Now, by economy axiom SE, we have, $y = x \circ y$. Thereby, using F3, we have xRy. QED.

Similarly, *the axioms of fusion and extensionality imply* (BMR):

(11) $\quad \mathrm{F0–F3}, \mathrm{ES} \vdash \mathrm{BMR}$

In view of (6), it suffices to demonstrate BMR2. We assume therefore, that $S(x) \subseteq S(y)$. Acting as before, we obtain $S(x \circ y) = S(y)$. Now, via extensionality ES we obtain $x \circ y = y$, hence by F3 we have that xRy.

48. We now move into the final part of the present section, asking this time after the natural equivalents of BMR?

(12) $\quad \mathrm{F0–F3}, \mathrm{REG}, \mathrm{wSL}, \mathrm{AS} \vdash \mathrm{BMR} \leftrightarrow \mathrm{SE}.$

In regular, weakly-local and antisymmetric fields with fusion, Masonic Boolean Syntheses agree with economic syntheses.

We obtain the proof of this by applying (9) and (10) directly.

(13) $\quad$ $\text{F0–F3}, \text{AS} \vdash \text{BMR} \leftrightarrow \text{ES}.$

In antisymmetric fields with fusion, Masonic Boolean Syntheses agree with extensional syntheses.

Proof follows directly from (10) and the fact that $\text{BMR}, \text{AS} \vdash \text{ES}$.

Conclusion

49. The work presented in this article shows that a Masonic Boolean Synthesis is closely associated with both economy and extensionality. It is, however, those axioms characterising fusion that play the key role, such axioms being both of a standard form and seemingly rather natural. Many would certainly be prepared to accept them without raising the slightest objection.

The axioms are meanwhile very strong and somehow they extract an acceptance of a combinatory ontology (set-theoretical ontology). The axiom of transitivity also shows its ontological strength in implying BMR1.

If we want to venture outside of the narrow confines of an combinatory ontology as a overly-simple theory of being, we must modify either the conception of fusion (the more preferable move) or give up on economy (which we would not willingly do) or also give up on the transitivity of the relation R. Or indeed take more that one of the steps at the same time!

50. And now to the most important conclusion. If, in a Masonic Boolean Synthesis, just as in every combinatory synthesis, the chosen elements and how of them there are play the key role, then

(i) there will always be finitely many objects in an ontological universe and moreover in the World, being a fragment of the former, so long as substance is finite. More precisely, there will be $2^n - 1$ possible combinations if the substance S has n elements. The infinitude of an ontological space will then only be secured if the substance S is infinite.

(ii) the remaining null object is the empty subset of the set S. We certainly do not know *a priori* whether from $\emptyset$ it is possible to generate another object (*creatio ex nihilio*)

(iii) the ontological space will always be isomorphic to an atomic (even more: power) Boolean algebra, ultimately without a zero.

51. Finally, let me emphasise the high degree of generality with which our inquiries have been conducted. Axioms are expressed for any binary relation R and whether we understand it as one of the basic relations of **GAS**, analytic or synthetic, the matter is, from a formal point of view, of secondary importance.

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