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INCONSISTENCIES AND THE DYNAMICS OF SCIENCE[*]

1. Aim and Survey

It is generally agreed upon today that scientific reasoning, like everyday reasoning, proceeds in a dynamic way: inferences derived at some stage in the reasoning process may at a later stage be rejected. This dynamics may be *extrinsic* or *intrinsic*. I shall call it extrinsic when previously derived conclusions are rejected on non-logical grounds, and intrinsic when their rejection is based on a purely logical analysis.

Historical case studies reveal that inconsistencies play a crucial role in both kinds of dynamics. An interesting example of an extrinsic form of dynamics is provided by Planck's derivation of the law that is named after him. As Smith shows in [39], Einstein discovered that Planck's derivation relied on premises that were contradicted by other findings in the domain. As these other findings were better established and more likely to survive after the inconsistencies were resolved, Einstein decided to reject Planck's derivation. Clausius' derivation of Carnot's theorem constitutes a nice example of a reasoning process that is intrinsically dynamical. As we shall see below, Clausius derived Carnot's theorem in two different ways from an inconsistent set of premises. These derivations are structurally very similar to

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one another (both are based on *Reductio Ad Absurdum* from a hypothesis). Still, Clausius rejected the first derivation, while accepting the second one. An important difference with the previous case is that Clausius' decision, unlike Einstein's, was entirely based on logical grounds—see Section 2.

The aim of this paper is threefold. First, I shall argue that we need to distinguish between two different sources of inconsistencies. An inconsistency may arise because one is reasoning from inconsistent information or because one is making plausible assumptions on the basis of incomplete information. In the former case, but not in the latter, the underlying conceptual structures necessarily preclude a consistent description of the domain at issue. Next, I shall show that both kinds of situations can adequately be dealt with by so-called adaptive logics, but that each of them requires a different type of adaptive logic.[1] Finally, I shall discuss some implications of all this for the ontological foundations of systems that can handle inconsistencies.

2. Reasoning from Inconsistent Information

The history of the sciences exhibits several examples of reasoning from inconsistent information. I already mentioned Clausius' derivation of Carnot's theorem, and Planck's and Einstein's derivation of Planck's law. Other examples are Maxwell's formulation of electrodynamics (see [36]), Einstein's account of Brownian motion (see [37]), and Bohr's theory of atomic spectra (see [39], [17], and [20]).

In each of these cases, the underlying conceptual structures prohibited a consistent description of the domain involved. Clausius, for instance, approached the domain of thermodynamic phenomena in terms of two incompatible accounts of heat. On the one hand, he relied on the theory of Sadi Carnot which stated that heat is a substance ("calorique") that can neither be destroyed nor created. On the other hand, he accepted the view, especially advocated by James Prescott Joule, that heat is a 'force' that can be converted into work. The combination of these two accounts makes it impossible to describe thermodynamic phenomena in a consistent way. It follows, for instance, that the production of heat in a heat engine results from the mere transfer of heat from a hot to a cold reservoir *and* that it results from the conversion of heat into work. The same holds true for the other examples. Planck approached the problem of black body radiation

[1] The first adaptive logic was designed by Diderik Batens around 1980—see [3]. Meanwhile, a whole variety of such logics is available—see [5] for a survey. As we shall see below, the importance of adaptive logics is that they provide a unified framework for the study of dynamic reasoning patterns.

in terms of mutually inconsistent classical and quantum hypotheses. Also Bohr combined hypotheses from classical electrodynamics with incompatible quantum hypotheses in his study of spectral emission phenomena.

Scientific reasoning processes that start from inconsistent information share some important characteristics. The first is that, although the ultimate goal is to replace the inconsistent theory by some consistent alternative, consistency is (almost) never restored by simple excision of one or more parts of the theory. As long as there are no good (logical or extra-logical) grounds to resolve the inconsistencies in a particular way, scientists prefer to work with the inconsistent theory. The reason for this seems to be at least threefold.

First, a set of inconsistent statements is only accepted when for each of them some kind of confirmation is available. Hence, prematurely resolving the inconsistencies by simple excision may deprive one of central 'elements of truth' that should follow from the consistent replacement. In some cases, these elements are derivable from a consistent subset of the inconsistent set. However, in more interesting cases, they can only be obtained by combining mutually inconsistent statements. Thus, Clausius' consistent replacement for the mutually inconsistent proposals of Carnot and of Joule is based on the idea that, in a heat engine, heat is *partially transferred* from a hot to a cold reservoir, and *partially converted* into work. As I showed in [30], this idea can neither be derived from Carnot's theory nor from Joule's view, but only from the combination of these two.

Next, simply eliminating some of the inconsistent statements may considerably reduce one's problem solving capacities in a certain domain. Bohr's account of atomic spectra, for instance, could not be obtained from classical electrodynamics alone (see [39]). Similarly, both Carnot's theory and Joule's view were needed for Clausius' account of thermodynamic phenomena, even though he eventually arrived at a consistent theory. This is related to the fact that reasoning from mutually inconsistent proposals may lead to reinterpretations of these proposals, and even to new concepts. In the case of Clausius, for instance, it led to the (very fruitful) idea of a partial transfer of heat from a hot to a cold reservoir (see [30]).

Finally, reasoning from an inconsistent theory usually plays an important heuristic role in resolving the inconsistencies. One reason for this should already be clear from the previous paragraphs. In interesting cases, the decision which 'parts' of an inconsistent theory should be retained and which should be rejected requires that one first analyses the theory, and even that one reinterprets some of its statements. Another reason is that, unlike what is sometimes accepted, deriving interesting results from an inconsistent theory may be far from trivial, and hence, that the mere availability of a particular derivation may provide some support for the statements used in it.

A second important characteristic of this type of reasoning process is that the occurrence of inconsistencies is not seen as a hindrance to sensible reasoning. From this, it can be inferred that the consequence relation (implicitly) used in this type of reasoning process is not that of classical logic (henceforth **CL**). Some readers may object to this that **CL** is applied to consistent subsets of the premises. Two remarks are in order here. The first is that, as we have seen above, inferences may be made from an inconsistent set of premises that cannot be made from any of its consistent subsets. The second remark is that, even in cases where **CL** is applied to consistent subsets, one needs *criteria* to decide when a statement follows from the inconsistent set. One possibility is to postulate that a statement follows from an inconsistent set of premises Γ iff it follows by **CL** from every maximal consistent subset of Γ.[2] The consequence relation thus defined is not that of **CL**.

Other readers may object that the sensible handling of inconsistent information is a matter of heuristics, not of changing the underlying logic. One may refer here to the fact that many valid derivations are in practice not made because they are heuristically useless. For instance, when trying to prove that q is a consequence of $\sim p \vee q$ and p, no sane person will infer $\sim\sim\sim\sim\sim\sim p$ from p. Analogously, in nineteenth century thermodynamics, no sane scientist inferred from the inconsistency concerning the nature of heat that the moon is made of blue cheese. This objection, however, rests on a serious misunderstanding of the problem. When dealing with inconsistent information, the problem is not that one may infer clearly false or nonsensical statements. What has to be prevented is that one infers conclusions that are *reasonable*, but that are nevertheless trivial consequences of the inconsistencies,[3] and hence, are not justified by the premises at issue. Precisely this cannot be guaranteed by **CL**.

But there is more. Even when dealing with inconsistent information, scientists try to design proofs for certain statements, and when successful, try to find acceptance for their proofs. For example, it was known that Einstein's proof of Planck's law had inconsistent premises. Nevertheless, the relevant community accepted this proof as an argument for Planck's law

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[2] This is the so-called strong consequence relation. For an overview of consequence relations defined in terms of consistent subsets of the premises, see [15] and [16]; for their reconstruction in terms of adaptive logics, see [6], [14] and [40].

[3] Intuitively, I say that A is a trivial consequence of an inconsistent set of premises Γ if it can only be derived from Γ by relying on both 'halves' of an inconsistency.

(see [39]). This is meaningless from the point of view of **CL**. If **CL** were considered as the correct logic for reasoning processes of this type, no proof with inconsistent premises would provide justification for some conclusion A. The reason is quite simple: whenever A is **CL**-derivable from an inconsistent set of premises, so is $\sim A$. Hence, if **CL** were considered as the correct logic in this type of situation, every proof for some conclusion A could be undermined by an equally valid proof for $\sim A$.

Still, one may object that scientists themselves often claim to be using **CL**, even in inconsistent contexts. This objection, however, should not be taken serious. As far as logical and methodological issues are concerned, a distinction has to be made between what scientists preach and what they practice. Those scientists that continue reasoning in an inconsistent context use (implicitly) a logic that does not validate *Ex Falso Quodlibet*. This can be inferred, for instance, from the fact that they do not undermine proofs with inconsistent premises by trivializing them. The latter can only be explained by assuming that they use another logic than **CL** to judge the validity of such proofs. Moreover, before the advent of paraconsistent logics, those scientists that had a good insight in the then available logics, usually objected to reasoning from inconsistent premises. At the 1911 Solvay conference, for instance, Poincaré explicitly indicated that he was well aware of the problem:

> What strikes me about the discussions that we have heard is to see the very same theory supported sometimes by principles of classical mechanics and sometimes by new hypotheses which contradict the former; *one must not forget that there is no proposition that cannot be easily demonstrated if one includes in the demonstration two contradictory premisses.*[4]

The third characteristic is that in reasoning processes of this type, the inconsistencies are resolved on the basis of *extra-logical* considerations. After analysing and possibly reinterpreting the inconsistent theory, those parts are retained that are preferred on external grounds (for instance, because they are better confirmed or because accepting their negation would cause the violation of some law that is considered to be fundamental). For example, after reinterpreting both Carnot's theory and Joule's view in the light of one another, Clausius eventually rejected the idea that heat is a substance in view of the empirically more adequate view that heat is a kind of motion (see [30]). When an inconsistency is thus resolved, inferences that were

[4] This passage is from [27, p. 451] and was found in [39, p. 431]; the italics are mine.

previously drawn may be rejected (because some premises are no longer accepted).

The fourth and final characteristic is that, in addition to extrinsic forms of dynamics, reasoning processes that start from inconsistent premises are also intrinsically dynamical. Again, this is best illustrated by means of an example. One of the central contributions to early thermodynamics was Carnot's derivation of, what was later called, Carnot's Theorem. According to this Theorem, no engine can be more efficient than a reversible one. Carnot's derivation was published in 1824 (see [18]), and was based on the calorique theory of heat. As Carnot's Theorem proved extremely successful, it was soon after its publication considered as one of the most fundamental results in the domain. Around 1840, however, it became clear that the calorique theory of heat was contradicted by some reliable experimental results concerning the conversion of work into heat. These results moreover supported the rival view that heat is not a substance, but some kind of 'force' that can be converted into other types of 'forces'. Some of the leading scholars in the domain, like Kelvin, concluded from this that one would have to restart from scratch, but that, in the meantime, it was better to stick to Carnot's theory. Others, like Clausius, tried to reason from the two incompatible approaches in the hope to arrive thus at a consistent alternative.

As may be expected, one of the main challenges concerned the problem to find a new derivation for Carnot's Theorem. In 1850, Clausius presented such a new derivation that, although the inconsistencies were not yet resolved, was accepted by the scientific community. What is even more interesting, however, is that, according to Clausius' own account (see [19]), he designed two proofs, the first of which he rejected.

As I already mentioned, both proofs are very similarly to one another—both start from the same set of premises and are moreover based on *Reductio Ad Absurdum*. This is why, at first sight, it seems highly surprising that Clausius rejected the first one, but accepted the second one. The reason for this becomes clear, however, if one takes a closer look at the proofs, and moreover takes into account that the set of premises at issue was inconsistent.

As I described in detail in [30] and [32], it is characteristic of both proofs that an inconsistency is derived from the hypothesis that some engines are more efficient than a reversible one (the negation of Carnot's Theorem) together with the set of premises. The crucial difference, however, is that, in the case of the first proof, the inconsistency at issue can be derived from the premises alone. Hence, it indeed does not make sense to reject the hypothesis on the basis of *this* inconsistency—as it already follows from the premises, it teaches nothing about the truth or falsity of the hypothesis. In the case of the second proof, however, the inconsistency at issue can only be derived in a sensible way (that is, while avoiding triviality) from the premises *together with the hypothesis*. What this comes to is that the inconsistency bears upon the truth or falsity of the hypothesis: accepting it will lead to additional inconsistencies that do not follow (in a non-trivial way) from the premises. In view of this, it makes perfectly good sense to accept the second proof as a valid derivation.

Evidently, I do not claim that Clausius was aware of the kind of logic he was using. I do claim, however, that he seemingly had some very good logical intuitions. As soon as he discovered that the inconsistency derived in the first proof follows from the premises alone, he indeed had good logical grounds to reject the derivation and to start anew.

3. Reasoning from Incomplete Information

The examples discussed in the previous section all have in common that the underlying conceptual structures preclude a consistent description. In many cases, however, inconsistencies arise not because one is reasoning from inconsistent information, but from *incomplete* information. The underlying mechanism is not difficult to understand. When dealing with incomplete information, scientists do not proceed in a blind way. Rather, they try to formulate plausible hypotheses on the basis of ampliative inference steps—steps that extend the information already contained in the premises. Examples of ampliative reasoning patterns are induction, abduction, analogical reasoning, and default reasoning. Inconsistencies arise when these hypotheses turn out to be in conflict with one another or with the available information. Also here, the history of the sciences offers a large number of examples.[5]

Analogical reasoning often occurs in the design of a novel model or theory. Well-known examples include Bohr's model of the atom that was designed on analogy with the solar system, and Carnot's theory of the steam engine that was formulated on analogy with hydraulic engines. Abductive reasoning typically occurs in the search for explanations for novel phenomena. When Herschel in 1781 noted an unexpectedly large object in the quartile near Zeta Tauri, he first formulated the hypothesis that the body at issue was a comet. This was inferred from the fact that the object appeared especially large

[5] As inconsistent sets of premises may be incomplete in some respects, these too may give rise to ampliative forms of reasoning. In cases like this, inconsistencies may arise from the same set of premises for two different reasons.

and that it moved among the stars. When this hypothesis was contradicted by observational data concerning the object's orbit, a new hypothesis was formulated (namely, that it was a planet). The latter too was obtained by an abductive step (see [35] for an analysis of this particular reasoning process). Also the reasoning process that led to the discovery of oxygen incorporated several abductive inferences, many of which were withdrawn in view of other information available (see [26, pp. 167–171]).

Reasoning processes that start from incomplete information share some important characteristics with those that start from inconsistent information.

A first important similarity may seem rather surprising: the inconsistencies that arise are often tolerated for a considerable period of time. This especially holds true for the case where different hypotheses turn out to be in conflict with one another. As long as there are no good (logical or extra-logical) reasons to reject one hypothesis in particular, scientists continue to reason from the mutually inconsistent hypotheses. This is nicely illustrated by Holmes' study of Lavoisier (see [25]). In his attempt to develop a unified theory for such diverse phenomena as respiration, fermentation and combustion, Lavoisier formulated different hypotheses that he himself recognized as mutually inconsistent. Still, he continued to make inferences from them until he had reasons to prefer one above the other. So, also in this type of situation the occurrence of an inconsistency is not seen as a hindrance to sensible reasoning.

A second important similarity is that the discovery of inconsistencies leads to interesting examples of dynamics. And also here, this dynamics may be extrinsic or intrinsic. The latter typically occurs when different hypotheses contradict each other. In the case of Lavoisier, for instance, the hypothesis that the reconversion of "fixed air" into "natural air" occurs by the removal of "phlogiston" was rejected in favour of the hypothesis that it occurs by the provision of "inflammable matter" when it turned out that the latter was empirically more successful than the former. When a hypothesis contradicts the available information, the dynamics is usually intrinsic. For instance, as soon as Herschel discovered that his hypothesis was in conflict with the data concerning the bodies orbit, he rejected the former. The motivation behind this rejection was logical, namely that a conclusion arrived at by some ampliative step (in this case an abductive one) was in conflict with one obtained by pure deductive means.

There are, however, also important differences between the two types of situations. A first one is that, in the case of reasoning from incomplete information, the inconsistencies one arrives at are *not* seen as an indication that the underlying conceptual structure is inconsistent. In the case of Lavoisier, for instance, it would have been foolish to conclude that his conceptual framework necessarily led to an inconsistent description of the domain. The framework was not 'overdetermined', but 'underdetermined': it was so open that mutually inconsistent hypotheses were compatible with it. Let me phrase the distinction in a different way. When dealing with inconsistent information, inconsistencies can be derived by pure deductive means. When dealing with incomplete (but consistent) information, inconsistencies can only be obtained because deductive steps are combined with ampliative steps. Thus, only in the former type of situation the inconsistencies follow *necessarily* from the available information.

A second important difference is that, in the case of reasoning from incomplete information, the inconsistencies that arise are often *resolved* on logical grounds. This typically happens in the case of conflicts between ampliative conclusions and deductive conclusions. As soon as an ampliative conclusion is contradicted by a deductive one, the former is withdrawn in favour of the latter and the inconsistency is resolved. The reason is that, whereas the latter is necessary with respect to the premises, the former is only plausible in view of them. This is completely different in the case of reasoning from inconsistent information. As the inconsistencies follow necessarily from the premises, they cannot be resolved on logical grounds, but only on the basis of extra-logical criteria.

A final difference is related to this. In the case of inconsistent information, the inconsistencies that arise can only be resolved by eliminating or modifying some of the premises (for instance, reinterpreting some of the predicates that occur in them). Such modifications are not necessary in the case of reasoning from incomplete information. Here, the inconsistencies are immediately resolved in view of the available information (when an ampliative conclusion turns out to be incompatible with it) or can be resolved by further *extending* one's information (when two or more ampliative conclusions are in conflict with one another). In neither of these cases, it is necessary to reject or modify some of the available premises.

4. Corrective and Ampliative Adaptive Logics

From the point of view of logic, one of the main challenges posed by the reasoning processes discussed above concerns the fact they are intrinsically dynamical. As may already be clear from the examples, this type of dynamics is not related to the rejection or modification of some of the premises, but to the fact that some predetermined set of logical presuppositions is followed 'as much as possible'—this is, unless and until they are explicitly violated. When one of these presuppositions is violated, conclusions that were previously inferred may be withdrawn.

In the case of reasoning from inconsistent information, for instance, it may be presupposed that sentences behave consistently with respect to the premises, unless and until proven otherwise. This enables one to apply inference rules that rely on the consistent behaviour of some of their premises. In most contexts, for instance, it is accepted that Disjunctive Syllogism—to derive B from $\sim A$ and $A \vee B$—only makes sense if A behaves consistently (if both A and $\sim A$ are true, $A \vee B$ is true even if B is false).[6] If in such a context B is derived from $\sim A$ and $A \vee B$ on the presupposition that A behaves consistently, the former will be withdrawn as soon as A turns out to be inconsistent. Also the example of Clausius may be remembered here. In both his proofs, *Reductio Ad Absurdum* is applied on the presupposition that the sentence for which a contradiction is derived (from the premises together with the hypothesis) behaves consistently with respect to the premises. When this presupposition proves false in the first proof, the conclusion thus derived is withdrawn.

A similar pattern can be found in ampliative forms of reasoning. In the case of analogical reasoning, for instance, one of the presuppositions may be that all information concerning the source domain can be transferred to the target domain. When this presupposition is violated—for instance, when it turns out that some of the information concerning the source domain is incompatible with available information on the target domain—previously derived conclusions may be withdrawn.

From a formal point of view, reasoning processes that are intrinsically dynamical have two important characteristics. The first is that the dynamics they exhibit may be *external* or *internal*. It is said to be external when conclusions are withdrawn because new premises are added, and internal when the mere analysis of the available information leads to such withdrawal.[7] The second characteristic is that (in general) they are not only undecidable, but even lack a positive test (see also below).

[6] Which inference rules rely on the consistent behaviour of their premises depends on the context. In [31], for instance, it is argued that, in contexts governed by the principle of maximal informativeness, it is justified to derive B from $\sim A$ and $A \vee B$, even if A behaves inconsistently. The condition, however, is that Addition—to derive $A \vee B$ from A—is made dependent on the consistent behaviour of A.

[7] Reasoning patterns of the former type are also called *non-monotonic*. Note that not all dynamical reasoning patterns are non-monotonic—an interesting example of a reasoning pattern that is intrinsically dynamical yet monotonic is studied in [7].

The consequence relation that captures a specific reasoning pattern of this type may be defined in various ways—for instance, with respect to some monotonic logic, some semantics or some set of criteria. As I shall argue below, such definitions are important. However, if we want to understand dynamical reasoning processes, we need something more. The reason for this is twofold. First, the definitions only indirectly explicate the external dynamics, and moreover completely fail to capture the internal dynamics. Next, in view of the undecidability of the reasoning patterns at issue and the fact that they even lack a positive test, the definitions are of limited use in actual reasoning. Let me try to illustrate this with some simple examples.

One of the basic forms of ampliative reasoning concerns the inference of sentences that are (merely) compatible with a set of premises.[8] A consequence relation for this type of reasoning can easily be defined with respect to a monotonic logic. For instance, if one is interested in a consequence relation "$\vdash_{\mathbf{COMPAT}}$" for classical compatibility, the latter may be defined as "$\Gamma \vdash_{\mathbf{COMPAT}} A$ iff $\Gamma \nvdash_{\mathbf{CL}} \neg A$".[9] This definition clearly captures the meaning of the phrase "A is compatible with Γ". However, as the definition is static, it does not enable one to model (in a direct way) what happens when Γ is extended to Γ'. Moreover, as it refers to **CL**-derivability, it can neither explain what happens when a person draws a conclusion on a *partial understanding* of Γ (and withdraws this conclusion when Γ is further analysed) nor provide guidance in the case of undecidable fragments. Indeed, according to the definition, a sentence A *is* or *is not* compatible with some set of premises Γ independently of the question whether this can be established.

Two other important forms of ampliative reasoning are abduction and induction. The former is usually defined by summing up a set of criteria that should be satisfied for A to be 'abducible' from a theory T and an explanandum B (for instance, that A is compatible with T, that B does not follow from T, that B follows from $T \cup \{A\}$, that A is as parsimonious as possible, ...). The latter is sometimes defined with respect to some kind of preferen-

[8] A fundamental requirement for ampliative consequence relations is that they lead to conclusions that are compatible with the premises. Different ampliative consequence relations can be distinguished from one another with respect to the additional requirements they impose on their conclusions. In the case of inductive reasoning, for instance, one of the requirements may be that the conclusions should be jointly compatible with the premises—see, for example, [10]—and in the case of abductive reasoning that the conclusions explain one or more explananda—see, for example, [35].

[9] It is easily seen from these definition why compatibility lacks a positive test: as **CL** is undecidable, it may be the case that A follows from Γ by **COMPAT**, whereas no finite construction exists that establishes this.

tial semantics. For instance, one may stipulate that A follows by induction from Γ iff A is true in all **CL**-models of Γ that have the smallest possible domain. Also reasoning from inconsistent premises has been defined with respect to various types of preferential semantics. For instance, to capture the idea that an inconsistent set of premises Γ is interpreted 'as consistently as possible', one may select those models of Γ (of some monotonic paraconsistent logic), that are minimally inconsistent (in some specific sense),[10] and define a consequence relation with respect to these. In each of these cases, however, one obtains a definition that does not capture the dynamics of the reasoning patterns at issue, and that is of limited use in actual reasoning.

An interesting way out of all this is offered by adaptive logics, and more specifically by their dynamic proof theory. As is argued in [9], adaptive logics provide a unified framework for the formal study of reasoning processes that are intrinsically dynamical. What makes adaptive logics suitable for such reasoning processes is that they 'adapt' themselves to specific violations of presuppositions. Where a presupposition is violated, the rules of inference are restricted in order to avoid triviality. However, where this is not the case, the rules can be applied in their full strength.

All currently available adaptive logics are defined in terms of three elements: an upper limit logic, a lower limit logic, and an 'adaptive strategy'. The upper limit logic is an extension of the lower limit logic. The former thus introduces a set of presuppositions on top of those of the latter. These additional presuppositions are the ones that are defeasible: they are followed 'as much as possible', but are abandoned when necessary to avoid triviality. The third element, the adaptive strategy, determines the interpretation of the ambiguous phrase "as much as possible". When a set of premises violates one of the presuppositions of the upper limit logic, it is said to behave abnormally with respect to the upper limit logic.[11]

The semantics of an adaptive logic is a preferential one: it is obtained by selecting a subset of the models of the lower limit logic. The selection is determined by the adaptive strategy. The Minimal Abnormality Strategy, for instance, selects those models that are minimally abnormal (in a set-theoretical sense) with respect to the upper limit logic. If some theory Γ

[10] Such a selection may either be linguistic or ontological. In the former case, it refers to the inconsistencies that are verified by the model, in the latter to the 'inconsistent objects' in the domain. For an interesting comparison of the two types of approaches, see [4].

[11] It is important to note that "abnormality" does not refer to the purported standard of reasoning, say **CL**. It refers to properties of the application context—to presuppositions that are considered desirable, but that may be overruled.

behaves normally with respect to the upper limit logic, the adaptive models of Γ coincide with the models of the upper limit logic that validate Γ.

Syntactically, an adaptive logic is obtained by taking the rules of the lower limit logic as unconditional (as unconditionally valid), and the rules of the upper limit logic as conditional. The adaptive strategy determines a marking rule (see below). As mentioned above, the proof theory of adaptive logics is *dynamic*. Sentences derived conditionally at some stage in a proof may at a later stage be rejected, namely, when the condition is no longer satisfied. The mechanism by which this is realized is quite simple. If a formula is added by the application of a conditional rule, a 'condition' (set of formulas) that is specified by the rule, is written to the right of the line. If a formula is added by the application of an unconditional rule, no condition is introduced, but the conditions (if any) that affect the premises of the application are conjoined for its conclusion. The members of the condition set have to behave normally for the formula to be derivable. At each stage of the proof—with each formula added—the marking rule is invoked: for each line that has a condition attached to it, it is checked whether the condition is fulfilled or not. If it is not, the line is considered as not (any more) belonging to the proof. The formulas *derived at a stage* are those that, at that stage, occur on non-marked lines.

Thanks to this dynamic proof theory, adaptive logics enable one to model in a direct way both the external and the internal dynamics that is typical of the reasoning processes discussed in the previous paragraphs. It can moreover be shown that the proof theory leads to the best possible conclusions in view of the understanding of the premises at a given stage (see [9]). This makes the proof theory of adaptive logics more realistic than the above mentioned definitions. Indeed, it not only enables one to come to justified conclusions for undecidable fragments, it can also account for the fact that, even for decidable fragments, inferences are drawn on the basis of a partial understanding of the premises (because of limited resources, for instance). Still, one obviously wants that the dynamics is *sensible*—that, for instance, different dynamic proofs 'eventually' lead to the same results. In an adaptive logic, this is guaranteed by the notion of *final derivability* which is sound and complete with respect to the semantics. Intuitively, a sentence is said to be finally derived in a dynamic proof iff it is derived on a line i that is not marked and is such that any extension of the proof in which i is marked, can be further extended in a way that i is unmarked.[12] For all currently

[12] This notion of final derivability corresponds to the static definitions discussed above.

available adaptive logics, it can be shown that the order in which inferences are made does not affect what is finally derivable from the premises.[13]

Adaptive logics can be divided into two categories: corrective and ampliative. In a corrective adaptive logic, the standard of reasoning is determined by the *upper limit logic*; specific *deviations* from this standard are *minimized*. All currently studied corrective adaptive logics have **CL** as their upper limit logic, and hence, adapt themselves to specific violations of **CL**-presuppositions. Inconsistency-adaptive logics, for instance, interpret inconsistent theories 'as consistently as possible'. Examples in this category are **ACLuN1** and **ACLuN2** (see especially [3]), and **ANA** (see [31]). In an ampliative adaptive logic, the standard of reasoning is determined by the *lower limit logic*; specific *extensions* of this standard (that are considered desirable within the application context at issue) are *maximized*. Examples of ampliative adaptive logics are logics of compatibility (see [11]), of metaphorical reasoning (see [23]), of diagnosis (see [12]), of induction [10], of abduction (see [35]), and of inference to the best explanation (see [34]). In each of these logics, **CL** is the *lower limit logic*.

Although from a formal point of view corrective and ampliative adaptive logics are very similar to one another,[14] the former are best suited for reasoning from inconsistent information and the latter for reasoning from incomplete information. One reason for this is that according to an ampliative adaptive logic, but not according to a corrective one, inconsistencies may arise even if the set of premises is consistent. Another reason is that corrective logics merely localize the inconsistencies (and leave their resolution to extra-logical considerations). In an ampliative adaptive logic some inconsistencies are resolved by the logic itself (for instance, when there is a conflict between a deductive inference and an ampliative one). Both differences coincide with central differences between the two types of situation (see the previous section).

5. Inconsistencies and Ontological Presuppositions

As is argued convincingly in [8], the question whether the world (or some domain) is consistent is a confused one. Consistency refers to negation, and

[13] There is only one exception: the adaptive logic for default reasoning that is presented in [1]. This, however, is related to the aim of this logic, namely to reconstruct standard default logics (see, for instance, [28]). According to these logics the order in which default rules are applied indeed makes a difference for the set of sentences that is finally derivable.

[14] Formally, both corrective and ampliative logics are realized by choosing some upper limit logic that incorporates the desired presuppositions, and by minimizing the deviations from these presuppositions.

hence to a description of the world, not to the world itself. As every description of the world presupposes a language L and a correspondence relation R that ties the language to the world, the real question is whether there exists a language L and a correspondence relation R such that the true description of the world as determined by L and R is consistent. In [8], it is argued that all attempts to answer this question in the positive are flawed. Here, I shall focus on a different aspect, namely that if some researcher (temporarily) accepts inconsistencies with respect to some domain, this does not necessarily indicate that he or she is using a language L and correspondence relation R that together preclude a consistent description of that domain. I shall argue that if the latter is not taken into account, one easily misjudges the way in which the world is categorized by the researcher at issue.

I mentioned in the previous section that, from a formal point of view, corrective and ampliative adaptive logics are very close to one another. In view of this, it is possible to design corrective and ampliative systems that are equivalent (in the sense that they lead to the same consequence set when applied to the same set of premises). What is important, however, is that the ontological presuppositions behind the two types of systems are different. This is why, in some contexts, an ampliative system leads to a better reconstruction than a corrective one (and vice versa), even if the two are equivalent. Let me illustrate this with an example.

In [2], Batens presented a reconstruction of circumscription[15] in terms of an inconsistency-adaptive logic. This was followed by a reconstruction of Reiter's default logic in terms of an inconsistency-adaptive logic (see [21] and [22]) and one in terms of an ampliative adaptive logic (see [1]). As far as the consequence relation is concerned, both these reconstructions of default logic are equivalent to one another. There are, however, some important ontological and epistemological differences.

In the first reconstruction, it is presupposed that a person who applies default rules (general rules that may have exceptions, such as "Birds fly") categorizes the world in such a way that he or she necessarily comes to an inconsistent description. This is illustrated in Figure 1, where "F" stands for "flying" and "B" for "bird". The assumption here is that the extensions of "flying" and "not-flying" overlap. In this way, it can be ensured that the sentences "birds fly" and "some birds do not fly" can both be satisfied. The price to be paid, however, is that some entities (like penguins) can only be

[15] Circumscription is a non-monotonic formalism that is very popular in Artificial Intelligence. The intuitive idea behind it is that the extension of predicates is minimized (or that predicates are 'circumscribed'): objects that can be shown to satisfy a certain predicate are presupposed to be the only object that satisfy it.

described in an inconsistent way. This is also why the first reconstruction necessarily proceeds in two steps: first localizing the inconsistencies that follow from the theory, and next resolving them by means of extra-logical criteria.

Figure 1.

The second reconstruction proceeds in a totally different way. The assumption behind it is that a person who applies default reasoning categorizes the world by means of (what I call) graded concepts. This is illustrated in Figure 2: there is no overlap of extensions, but a distinction is made between 'typical' instances of entities that have property B, and 'non-typical' ones; typical instances have property F, non-typical ones do not. In line with this assumption, the dynamical character of default reasoning is linked to a lack of information. Suppose, for instance, that one uses a conceptual framework as presented in Figure 2, and that one hears the sentence "Tweety is a bird". As typical birds fly, it seems justified to conclude that Tweety flies. This, however, is not a deductive consequence, but a plausible hypothesis on the basis of incomplete information (one does not know whether or not Tweety belongs to the exceptions). Hence, as soon as one hears that Tweety is a penguin, the hypothesis will be withdrawn. This is precisely how the second reconstruction works: conclusions can be inferred on the basis of default rules, but as soon as one of these conclusions contradicts a deductive one, the former is withdrawn.

In the case of default reasoning, a reconstruction in terms of an ampliative adaptive logic seems more justified. Default reasoning is a typical case of reasoning from incomplete information: one makes plausible assumptions on the basis of what typically or normally holds true. Assuming that in this type of process the underlying conceptual structures are inconsistent is a form of 'paraconsistencitis'. The same holds true for other forms of

Figure 2.

ampliative reasoning. Also these should not be modelled by means of a corrective logic. In other cases, however, there is every reason to accept that the underlying conceptual structures are inconsistent (see, for instance, the examples in Section 2). If one reconstructs processes like this on the basis of an ampliative system,[16] the inconsistencies are simply explained away. This inevitably leads to a serious underestimation of the difficulties one is dealing with in such situations.

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