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SOME ADAPTIVE LOGICS FOR DIAGNOSIS*

Abstract. A logic of diagnosis proceeds in terms of a set of data and one or more (prioritized) sets of expectancies. In this paper we generalize the logics of diagnosis from [27] and present some alternatives. The former operate on the premises and expectancies themselves, the latter on their consequences.

1. The Problem

In [27], two logics for diagnosis are outlined, $\mathbf{AL}_{EXP}$ and $\mathbf{AL}^{*}_{EXP}$. The idea is that the reasoning proceeds from data as well as from expectancies. The latter only have effects in as far as they are compatible with the former. The aim of the diagnosis is to locate the expectancies that fail, if some do. We refer to [27] for different types of diagnosis and for their logical features.

The difference between the two logics shows when some expectancies cannot be upheld. In such cases, the first logic merely identifies the combinations of expectancies that cannot be upheld, whereas the second logic relies on a numerical criterion for choosing a combination that saves the largest number of expectancies.

Both logics are prioritized adaptive logics. A brief introduction to adaptive logics is presented in Section 2. A logic is prioritized iff it defines a consequence relation $\Sigma \vdash A$ in which Σ is an n-tuple of sets of closed formulas, $\langle \Gamma_0, \ldots, \Gamma_n \rangle$ and each Γ_i has a different preference ranking. In the

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present paper, each Γ_i is a subset of $\mathcal{W}$, the set of closed formulas of the standard predicative language $\mathcal{L}$, and Γ_i is preferred over Γ_{i+1}.

The logics presented in [27] handle only couples of subsets: $\langle \Gamma_0, \Gamma_1 \rangle$ in which Γ_0 is the set of premises that are accepted as true and Γ_1 the set of premises that have the status of expectancies—in [27], the $A \in \Gamma_1$ are written as $E(A)$ to distinguish them from the members of Γ_0. The generalization to n-tuples is technically straightforward. Moreover, it is natural with respect to the application context. Consider the example from [27] in which the diagnosis concerns an electrical network consisting of gates and connections between them. One may expect that the gates function properly (in agreement with the tags attached to them), but expectancies may differ in strength depending on the age of the gates, on the factory that produced them, etc.

As will be explained in Section 3, the logics from [27] are specific prioritized Rescher–Manor consequence relations: they proceed in terms of maximal consistent subsets of $\Gamma_0 \cup \Gamma_1$ that contain all members of Γ_0. We generalize these logics in Section 3. In subsequent sections, we develop prioritized adaptive logics that proceed in a rather different way, and show them to be more efficient in specific circumstances. We first present the semantics of these logics, in Sections 4–7, and demonstrate some metatheoretic properties in Section 8. We present the dynamic proof theory in Section 9 and offer some examples in Section 10. Some open problems are discussed in Section 11.

2. Adaptive Logics

As the name suggests, an adaptive logic **AL** adapts itself to the premises—see the discussion of the semantics and the proof theory below. **AL** is defined from a *lower limit logic* **LLL** and a set of *abnormalities*. Extending **LLL** with (one or more) axioms that rule out the occurrence of abnormalities leads to the *upper limit logic* **ULL**. This warrants that **ULL** is an extension of **LLL**. Moreover, it warrants that $A_1, \ldots, A_n \vdash_{\mathbf{ULL}} B$ iff there are abnormalities $C_1, \ldots, C_m$ such that $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee C_1 \vee \ldots \vee C_m$.[1] The upper limit logic **ULL** presupposes that abnormalities are false, whereas the lower limit logic **LLL** drops this presupposition.

The adaptive logic **AL** interprets the premises 'as normally as possible'. This phrase is not unambiguous. It is disambiguated by choosing a specific

[1] Remark that $C_1 \vee \ldots \vee C_m \vdash_{\mathbf{ULL}} \bot$, where $\bot$ is defined by $\bot \vdash A$. This formulation presupposes the presence of a disjunction that behaves in a standard way. It is possible to get around this, see, for example, [5].

adaptive *strategy*. If the Simple strategy is chosen,[2] $Cn_{\mathbf{AL}}(\Gamma)$, the **AL**-consequence set of Γ, is closed under a set of rules of the form "if $A_1, \ldots, A_n \in Cn_{\mathbf{LLL}}(\Gamma)$ and $C_1, \ldots, C_m \notin Cn_{\mathbf{LLL}}(\Gamma)$, then $B \in Cn_{\mathbf{AL}}(\Gamma)$"—such a rule obtains just in case $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee C_1 \vee \ldots \vee C_m$ in which $C_1, \ldots, C_m$ are abnormalities. The adaptive logics articulated in the present paper are based on the Reliability strategy, on the Minimal Abnormality strategy, and on a specific Counting strategy—we shall explain these strategies in due course.

Where **CL** (Classical Logic) is taken as the standard of deduction, adaptive logics that have **CL** as their upper limit are called *corrective*; adaptive logics that have **CL** as their lower limit are called *ampliative*. This distinction is mainly introduced for pragmatic reasons. Inconsistency-adaptive logics (see [4] and many other places) are corrective: a possibly inconsistent set of premises is interpreted as consistently as possible. Examples of ampliative adaptive logics are the logic of compatibility from [12] or the logics of induction from [9] and [11] (if one disregards the background generalizations).

Adaptive logics are called *prioritized* when they apply an adaptive strategy to a sequence of sets that are ordered according to some priority ranking.

Thus, our logics of diagnosis attach the highest priority to the data—they deliver all **CL**-consequences of the data. They attach lower and descending priorities to the expectancies, starting with the ones that are most preferred or trusted. Their **CL**-consequences are adaptively derivable, but only in as far as they do not conflict with (retained) consequences of more preferred items—see Section 4 and following for formal details. With respect to the expectancies, these logics are corrective: not all their **CL**-consequences may be derivable. With respect to the data, they are obviously ampliative.[3] We shall disregard the prioritized case in the sequel of this section.

From a semantic point of view, the **AL**-models of some Γ are a subset (a selection) of the **LLL**-models of Γ, defined in terms of the abnormal parts of models—see [1] for the first application of this idea. If Γ is normal in that it has **ULL**-models, the **AL**-models of Γ coincide with these **ULL**-models of Γ. If Γ has no **ULL**-models, the **AL**-models of Γ are the **LLL**-models of Γ that are 'as normal as possible'.

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[2] The Simple strategy can only be applied for specific lower limit logics and abnormalities. For most combinations, it assigns a trivial consequence set to many sets of premises.

[3] Given the special structure of the premise sets to which prioritized adaptive logics apply, it seems advisable to consider them as a *sui generis* category.

The most fascinating feature of adaptive logics is their dynamic proof theory, first presented in [2]. Characterizations in terms of consequence sets or of models may offer precise definitions, but, unlike the dynamic proof theory, they are in themselves not computationally useful. Moreover, adaptive logics are specifically devised in order to explicate, in a formally precise way, forms of reasoning that are undecidable and for which there is no positive test. For this reason, their proof theory is necessarily dynamic.

Dynamic proofs may easily be characterized in terms of the lower limit logic and the set of abnormalities. The basic idea is that the rules of **LLL** apply unconditionally, whereas those of **ULL** apply conditionally, that is, provided some formulas behave normally. This idea is implemented by writing, at the end of each line in the proof, a *condition*—for most adaptive logics, a set of formulas. Intuitively, the formula derived at the line, is correctly derived provided all members of the condition of the line behave normally.

Given the absence of a positive test for **AL**-derivability, the proof theory is unavoidably dynamic: some formula is considered as derived at some stage of a proof, but as not derived at a later stage of the same proof—for some adaptive logics, the formula may again be considered as derived at a still later stage. This feature is implemented by the *marking* definition. This definition specifies, in terms of the formulas that are derived in the proof at a stage, which lines are marked and hence are considered as not belonging to the proof at that stage.

Clearly, we need a limit to the dynamics, even if the limit may not always be reached within a finite number of steps. This limit is defined in the same way for all adaptive logics. A is *finally* **AL**-derived at line i at a stage s of a proof from Γ iff line i is not marked at stage s, and any extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked. A is finally **AL**-derivable from Γ, $\Gamma \vdash_{\mathbf{AL}} A$, iff A is finally **AL**-derived at a stage of a proof from Γ. Needless to say, final derivability should be sound and complete with respect to the **AL**-semantics.

Although there is no positive test for (final) **AL**-derivability, there are *criteria* that enable one to decide, in some cases, that A has been finally derived from Γ. Where such criteria do not apply, it may be shown that a proof at a stage offers a good estimate concerning the final derivability of a formula from a set of premises, viz. the best estimate that is available in view of the information provided by the proof—see [3] for a formal explication of this information.

3. Consistent Chunks

The chunking approach was originated by Nicholas Rescher, partly in cooperation with Ruth Manor—see [21], [22], [23], [24], and other papers. This is why we use the term "Rescher–Manor consequence relations" to refer to such logical mechanisms. The underlying idea is to define consequence relations in terms of the **CL**-consequences of maximal consistent subsets of a set of premises. A survey of the flat (non-prioritized) Rescher–Manor consequence relations is offered in [15]; a survey of prioritized such consequence relations in [16]. All those consequence relations are characterized by inconsistency-adaptive logics as well as by ampliative adaptive logics—see [7], [14], [8], [25], and [26]).[4] All those consequence relations may be strongly enriched by relying on an idea from [19] (that enriches Jaśkowski's paraconsistent logic); see [10] for the flat and [25] for the prioritized ones. Let us start with some definitions, where Γ is a subset of $\mathcal{W}$.

Definition 1. The set of all *maximal consistent subsets* of Γ, denoted as $MCS(\Gamma)$, is the set of consistent subsets of Γ that are not themselves proper subsets of a consistent subset of Γ.

Definition 2. The *cardinality* of Γ, denoted as $\#(\Gamma)$, is the number of elements of Γ.

The logics $\mathbf{AL}_{EXP}$ and $\mathbf{AL}^*_{EXP}$ are easily seen to come to consequence relations that take $\langle\Gamma_0, \Gamma_1\rangle$ as their set of premises. Both $\mathbf{AL}_{EXP}$ and $\mathbf{AL}^*_{EXP}$ are determined by maximal consistent subsets containing Γ_0, the former considers all of them, the latter only those of maximal cardinality.

Definition 3. $D\langle\Gamma_0, \Gamma_1\rangle = \{\Delta \mid \Delta \in MCS(\Gamma_0 \cup \Gamma_1) \text{ and } \Delta \supseteq \Gamma_0\}$.

Definition 4. $\langle\Gamma_0, \Gamma_1\rangle \vdash_{\mathbf{AL}_{EXP}} A$ iff $\Delta \vdash_{\mathbf{CL}} A$ for all $\Delta \in D\langle\Gamma_0, \Gamma_1\rangle$.

Definition 5. $\langle\Gamma_0, \Gamma_1\rangle \vdash_{\mathbf{AL}^*_{EXP}} A$ iff $\Delta \vdash_{\mathbf{CL}} A$ for all $\Delta \in D\langle\Gamma_0, \Gamma_1\rangle$ of maximal cardinality (for which there is no $\Delta' \in D\langle\Gamma_0, \Gamma_1\rangle$ such that $\#(\Delta') > \#(\Delta)$).

For reasons explained in Section 1, it is useful to generalize $\mathbf{AL}_{EXP}$ and $\mathbf{AL}^*_{EXP}$ to make them apply to $\Sigma = \langle\Gamma_0, \ldots, \Gamma_n\rangle$. The natural place to look for such generalizations is [16], which contains a survey of prioritized

[4] A major advantage of this is the unification of a large set of non-monotonic consequence relations. A major advantage of a different sort is that all Rescher–Manor consequence relations are provided with a (dynamic) proof theory.

inconsistency-handling consequence relations that are based on consistent chunks. The most natural generalization of $\mathbf{AL}_{EXP}$ is the SMC-consequence relation. Out of the SMC-consequence relation a generalization of $\mathbf{AL}^{*}_{EXP}$ can be forged. In some diagnosis contexts, one might defend the application of the *Argued* consequence relation (which should not be confused with the flat Argued consequence relation) or of its sophistications (*SD*, *DS* and *SS*). In other diagnosis contexts, it seems sensible to apply the ℓ-consequence relation, which is not a generalization of $\mathbf{AL}_{EXP}$ and does not proceed in terms of maximal consistent subbases. In the subsequent paragraphs, we briefly characterize the SMC-consequence relation, its variant that takes the cardinality of maximal consistent subsets (or subbases) into account, and the ℓ-consequence relation. All these consequence relations will be slightly modified in view of the presupposition, which is used throughout this paper, that the set of data, viz. Γ_0, is consistent. Here are the (adjusted) definitions—we always suppose that $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$.

Definition 6. $\ell(\Sigma)$ is the subset of $\Gamma_0 \cup \ldots \cup \Gamma_n$ obtained by starting with all the elements of Γ_0 and step by step adding or not adding all the elements of the next Γ_i depending on whether consistency is or is not preserved.

In other words, one selects those Γ_i that are compatible with their selected predecessors, and calls their union $\ell(\Sigma)$. The $\mathbf{D}_{\ell}$-consequences of Σ are the $\mathbf{CL}$-consequences of $\ell(\Sigma)$:

Definition 7. $\Sigma \vdash_{\mathbf{D}_{\ell}} A$ iff $\ell(\Sigma) \vdash_{\mathbf{CL}} A$.

For the SMC-consequence relation, one starts again with all of Γ_0, but this time adds as much as is consistently possible from each subsequent Γ_i. This obviously does not result in a single set: several members of Γ_{i+1} that are compatible with the selection made up to Γ_i may be jointly incompatible with this selection. This is why one needs to define a set of SMC-subbases.

Definition 8. $\Delta_0 \cup \ldots \cup \Delta_n$ is a SMC-*subbase* of Σ iff, for all i, $0 \leq i \leq n$, $\Delta_0 \cup ... \cup \Delta_i \in MCS(\Gamma_0 \cup \ldots \cup \Gamma_i)$.

The $\mathbf{D}_{SMC}$-consequences of Σ are those formulas that are $\mathbf{CL}$-consequences of all SMC-subbases of Σ; the $\mathbf{D}^{\mathbf{P}}_{SMC}$-consequences of Σ are those formulas that are $\mathbf{CL}$-consequences of all SMC-subbases of Σ that are maximal with respect to their cardinality.

Definition 9. $\Sigma \vdash_{\mathbf{D}_{SMC}} A$ iff $\Delta \vdash_{\mathbf{CL}} A$ for all SMC-subbases Δ of Σ.

Definition 10. $\Sigma \vdash_{\mathbf{D}^{\mathbf{P}}_{SMC}} A$ iff $\Delta \vdash_{\mathbf{CL}} A$ for all SMC-subbases Δ of Σ for which there is no SMC-subbase Δ' such that $\#(\Delta') > \#(\Delta)$).

Which consequence relation is most suitable for a specific application depends on the preference ordering. Expectancies may belong to the same Γ_i because the objects to which they pertain share some relevant property. For example, in the case of an electric circuit (see [27]), the preferences may be based on the estimated reliability of the factory that produced the gates. If the data contradict an expectancy about at least one gate that was produced by a certain factory, it may be sensible to consider all expectancies about the gates produced by that factory as questionable. In such a case, the ℓ-consequence relation is appropriate. In other cases, expectancies may belong to the same Γ_i in view of less relevant properties, for example the age of the gates. That a gate of a certain age fails seems not a good reason to consider all gates of that age as unreliable—they may have been produced by different factories, be of different types, etc. In such cases, the *SMC*-consequence relation is more suitable.

In view of [26], it is obvious that all of these consequence relations are characterized by adaptive logics. One possible characterization proceeds in terms of a modal predicative language—compare [10] and [25]. A different characterization is obtained in terms of inconsistency-adaptive logics that have the basic paraconsistent logic **CLuN** as their lower limit logic—see [7] for the plot. In both cases the proof theory is dynamic. It is even simple enough to devise a 'direct' dynamic proof format (one that proceeds in terms of **CL**-formulas only) along the lines of [14]. All three proof theories (and especially the latter one) enable us to explicate the relevant human reasoning processes in a precise and adequate way.

4. From Chunking to Reasoning

From this section on, we present an approach that departs drastically from the Rescher–Manor consequence relations, and argue that it is appropriate for specific forms of diagnosis. Rather than selecting maximal consistent subsets of $\Gamma_0 \cup \ldots \cup \Gamma_n$ that contain Γ_0, we shall start from the latter and add certain **CL**-*consequences* of members of Γ_1, next certain **CL**-*consequences* of members of Γ_2, etc. In doing so, we shall require that the members of the Γ_i ($0 \leq i \leq n$) be consistent,[5] but not that the Γ_i themselves are consistent.

[5] If some $A \in \Gamma_i$ is inconsistent, the consequence relations will assign a trivial consequence set to Σ—however, see Section 11.

The general plot is to interpret expectancies within the modal logic **T** of Feys (which is von Wright's **M**).[6] Several predicative versions of **T** may do, provided $a = b \supset \Box a = b$ is not a theorem.

Let $\mathcal{L}^M$ be the standard modal language with $\mathcal{S}$, $\mathcal{P}^r$, $\mathcal{C}$, and $\mathcal{W}^M$ the sets of sentential letters, predicative letters of rank r, constants, and wffs (closed formulas). To simplify the semantic metalanguage, we introduce a set of pseudo-constants $\mathcal{O}$, *requiring that any element of the domain D is named by at least one member of* $\mathcal{C} \cup \mathcal{O}$.[7] Let $\mathcal{W}^{M+}$ denote the set of wffs of $\mathcal{L}^{M+}$ (defined by letting $\mathcal{C} \cup \mathcal{O}$ play the role played by $\mathcal{C}$ in $\mathcal{L}^M$). The function of $\mathcal{O}$ is to simplify the clauses for the quantifiers.

A **T**-model M is a quintuple $\langle W, w_0, R, D, v\rangle$ in which W is a set of worlds, $w_0 \in W$ the real world, R a binary relation on W, D a non-empty set and v an assignment function. The accessability relation R is reflexive. The assignment function v is defined by:

C1.1 $v : \mathcal{S} \times W \mapsto \{0, 1\}$
C1.2 $v : \mathcal{C} \cup \mathcal{O} \times W \mapsto D$
C1.3 $v : \mathcal{P}^r \times W \mapsto \wp(D^r)$ (the power set of the r-th Cartesian product of D)

The valuation function $v_M : \mathcal{W}^{M+} \times W \mapsto \{0, 1\}$, determined by the model M is defined by:

C2.1 where $A \in \mathcal{S}$, $v_M(A, w) = v(A, w)$
C2.2 $v_M(\pi^r \alpha_1 \ldots \alpha_r, w) = 1$ iff $\langle v(\alpha_1, w), \ldots, v(\alpha_r, w)\rangle \in v(\pi^r, w)$
C2.3 $v_M(\alpha = \beta, w) = 1$ iff $v(\alpha, w) = v(\beta, w)$
C2.4 $v_M(\neg A, w) = 1$ iff $v_M(A, w) = 0$
C2.5 $v_M(A \vee B, w) = 1$ iff $v_M(A, w) = 1$ or $v_M(B, w) = 1$
C2.6 $v_M((\exists\alpha)A(\alpha), w) = 1$ iff $v_M(A(\beta), w) = 1$ for at least one $\beta \in \mathcal{C} \cup \mathcal{O}$
C2.7 $v_M(\Diamond A, w) = 1$ iff $v_M(A, w') = 1$ for at least one w' such that Rww'.

A model M verifies $A \in \mathcal{W}^M$ iff $v_M(A, w_0) = 1$. A is valid iff it is verified by all models.

As the construction is somewhat unusual, a bit of explanation is useful here. One may define a function d that assigns to each $w \in W$ its domain $d(w) = \{v(\alpha, w) \mid \alpha \in \mathcal{C} \cup \mathcal{O}\}$. If an element of an r-tuple of $v(\pi^r, w)$

[6] The essential thing is that the accessibility relation R is not transitive. So, **K** would do just as well. However, **T** allows for a simpler formulation of the formal machinery below.

[7] $\mathcal{O}$ should have at least the cardinality of the largest model considered—if there is no such model, one selects a suitable $\mathcal{O}$ for each model.

does not belong to $d(w)$, then the r-tuple does not have any effect on the valuation.[8] Remark also that, if Rww', the question whether $v(\alpha, w)$ is or is not a member of $d(w')$ is immaterial for any $v(A, w)$. For example, the value of $v_M(\Diamond Pa, w)$ is determined by the values of $v(a, w')$ and $v(P, w')$ for those w' for which Rww'. Obviously, the semantics may be rephrased as a counterpart semantics: $\mathsf{a} \in d(w)$ is a counterpart of $\mathsf{b} \in d(w')$ just in case there is an $\alpha \in \mathcal{C} \cup \mathcal{O}$ such that $v(\alpha, w) = \mathsf{a}$ and $v(\alpha, w') = \mathsf{b}$. An $\alpha \in \mathcal{C} \cup \mathcal{O}$ may be seen as picking a specific counterpart 'path' on W.[9]

We now move on to express Σ in terms of $\mathbf{T}$. Where $\Diamond^i$ abbreviates a sequence of $i \geq 0$ diamonds, M is a $\mathbf{T}$-model of $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$ iff, for all i $(0 \leq i \leq n)$ and for all $A \in \mathcal{W}$, M verifies $\Diamond^i A$ if $A \in \Gamma_i$. We shall write $\Sigma \models_{\mathbf{T}} A$ to denote that all $\mathbf{T}$-models of Σ verify A.

As the accessibility relation of the $\mathbf{T}$-semantics is reflexive, a model that verifies $\Diamond^i A$ also verifies $\Diamond^j A$ for all $j > i$. Thus, some Σ have $\mathbf{T}$-models in which $W = \{w_0\}$ and hence $v_M(A, w_0) = 1$ for all $A \in \Gamma_0 \cup \ldots \cup \Gamma_n$. Such $\mathbf{T}$-models will be called *singleton models*. It is easily seen that Σ has singleton models iff $\Gamma_0 \cup \ldots \cup \Gamma_n$ is consistent.

$\mathbf{T}$ is axiomatized by its propositional axiom system together with the usual axioms and rules for quantification, the Barcan Formula, $\vdash \alpha = \alpha$, and Replacement of Identicals *outside* the scope of modalities. $\Box a = b, \Diamond A(a) \vdash_{\mathbf{T}} \Diamond A(b)$ and similar sequences are easily derivable. While soundness is as obvious as usual, the completeness proof falls beyond the scope of the present paper. However, it is quite simple in view of the present framework—if $\Gamma \nvdash_{\mathbf{T}} A$, a counter-model with denumerable domain is obtained by instantiating existentially quantified formulas by means of a denumerable set of pseudo-constants. We shall write $\Sigma \vdash_{\mathbf{T}} A$ to denote that $\{\Diamond^i A \mid A \in \Gamma_i\} \vdash_{\mathbf{T}} A$. Remember that each $\Gamma_i \subseteq \mathcal{W}$ (the set of closed formulas of the standard predicative language $\mathcal{L}$).

$\mathbf{T}$ will be our lower limit logic; let us now select the set of abnormalities. Let $\mathcal{F}^p$ be the set of *primitive formulas* (sentential letters, formulas of the form $\pi^r \alpha_1 \ldots \alpha_r$, and identities), and let $\mathcal{F}^a$ be the set of *atoms* (primitive formulas and their negations). The abnormalities will be formulas of the form $\Diamond^i A \wedge \neg A$ in which A is an *atom*. The upper limit logic, say $\mathbf{T}^+$, presupposes the normal situation, viz. the one in which all premises expressing expectancies are compatible with the members of Γ_0. $\mathbf{T}^+$ is obtained by

[8] In other words, C1.3 may just as well be replaced by "$v : \mathcal{P}^r \times W \mapsto \wp((d(w))^r)$".

[9] The technique to handle quantifiers in terms of $\mathcal{C} \cup \mathcal{O}$ is itself not related to modal logic—see, for example, the semantics for $\mathbf{P}$ in [13]. We shall not discuss here the possibility of expressing *de re* modalities in terms of the present framework.

adding to $\mathbf{T}$ the axiom "$\Diamond A \supset A$". A characteristic semantics for $\mathbf{T}^+$ is obtained by restricting the $\mathbf{T}$-semantics to singleton models.[10] As $\Box A \equiv A$ and $\Diamond A \equiv A$ are theorems of $\mathbf{T}^+$, the latter is identical to the well-known system **Triv**—see, for example, [17, p. 35]. It follows that $\Sigma \vdash_{\mathbf{T}^+} A$ iff $\Gamma_0 \cup \ldots \cup \Gamma_n \vdash_{\mathbf{CL}} f(A)$, in which $f(A)$ is obtained by deleting any occurrences of modalities in A.

We have four useful remarks on abnormalities. First, if $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$ is abnormal (in that its $\mathbf{T}^+$-consequence set is trivial), it is possible that no abnormality is $\mathbf{T}$-derivable from it, but that *disjunctions* of abnormalities are. Next, the lower limit logic $\mathbf{T}$ spreads abnormalities. If $\Diamond p \wedge \neg p$ is true in model M, then so are either $\Diamond(p \wedge q) \wedge \neg(p \wedge q)$ or $\Diamond(p \wedge \neg q) \wedge \neg(p \wedge \neg q)$, etc. This problem is well-known from other adaptive logics (see, for example, [6] and [19]). The solution is to proceed in terms of abnormal atoms (as defined above). The third remark concerns open formulas. Although diagnosis will usually concern finite systems, we need, in order to formulate the logic in a decent way, to allow for models that verify $\Diamond P\alpha \wedge \neg P\alpha$ for some $\alpha \in \mathcal{O}$ but not for any $\alpha \in \mathcal{C}$—and hence verify $(\exists x)(\Diamond Px \wedge \neg Px)$, which is a formula of $\mathcal{L}^M$. This feature too is well-known from other adaptive logics. The last remark is related to the prioritized character of the logic. If $i < j$, then an abnormality of the form $\Diamond^i A \wedge \neg A$ will count as worse than an abnormality of the form $\Diamond^j A \wedge \neg A$. In other words, the logic should first try to avoid the former, and only then try to avoid the latter.

Here is how we shall handle abnormalities. Let $\exists A$ abbreviate A preceded by a sequence of existential quantifiers (in some specified order) over all variables free in A. An abnormality is a formula of the form $\exists(\Diamond^i A \wedge \neg A)$, where $A \in \mathcal{F}^a$. For the semantics, we define, for each $\mathbf{T}$-model M of $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$ a set of abnormal parts (where $0 < i \leq n$):

Definition 11. $Ab^i(M) =_{df} \{A \in \mathcal{F}^a \mid v_M(\exists(\Diamond^i A \wedge \neg A), w_0) = 1\}$

In the logics defined in subsequent sections, the adaptive models of Σ will be obtained by making a selection of its $\mathbf{T}$-models, first with respect to the sets $Ab^1(M)$, next with respect to the sets $Ab^2(M)$, etc.

For the proof theory, we need disjunctions of abnormalities. It turns out that we may restrict our attention to disjunctions of formulas of the form $\exists(\Diamond^i A \wedge \neg A)$, with the same i in each disjunct and $A \in \mathcal{F}^a$. By $Dab^i(\Delta)$ we denote the disjunction $\bigvee\{\exists(\Diamond^i A \wedge \neg A) \mid A \in \Delta\}$. We shall say that $Dab^i(\Delta)$ is a Dab^i-consequence of Σ iff all $\mathbf{T}$-models of Σ verify $Dab^i(\Delta)$. A

[10] A different characterization is obtained by requiring, for all $A \in \mathcal{W}$ and for all $w_i, w_j \in W$, that $v_M(A, w_i) = v_M(A, w_j)$.

Dab^i-consequence $Dab^i(\Delta)$ of Σ will be called *minimal* iff there is no $\Delta' \subset \Delta$ such that $Dab^i(\Delta')$ is a Dab^i-consequence of Σ.

Example 1. $\Sigma = \langle\{p\}, \{\neg p \vee q\}, \{\neg q, r\}\rangle$. The **T**-models of Σ verify p, $\Diamond(\neg p \vee q)$, $\Diamond\Diamond\neg q$, and $\Diamond\Diamond r$. Σ has no Dab^1-consequences. It has one minimal Dab^2-consequence, viz. $(\Diamond\Diamond\neg p \wedge p) \vee (\Diamond\Diamond q \wedge \neg q) \vee (\Diamond\Diamond\neg q \wedge q)$. This indicates that $\neg p$, q, and $\neg q$ are unreliable formulas 'at level 2' of Σ.[11]

All logics discussed below select, first, the **T**-models of Σ that verify p as well as $\neg p \vee q$. All these models verify q. From the thus selected models, the logics select those that verify r. They do not select models that verify $\neg q$ because $Dab^2\{\neg p, q, \neg q\}$ is a minimal Dab^2-consequence of Σ. Actually, there is no alternative: no **T**-models of Σ that verify $\neg p \vee q$ also verify $\neg q$.

5. Reliability

We shall first present prioritized adaptive logics that stepwise add to the **CL**-consequences of Γ_0 as many **CL**-consequences of each consecutive Γ_i as possible. Such addition may be governed by several strategies, the two most interesting of which are *Reliability* and *Minimal Abnormality*. Each of them leads to a different logic, and we shall consider both. Both logics add 'as much as possible' from each consecutive Γ_i. So, they add (each in its specific way) whatever may be added *safely*, and nothing else. This is why we shall call these adaptive logics $\mathbf{T}^{sr}$ and $\mathbf{T}^{sm}$—the "s" refers to Safety, the "r" to Reliability, and the "m" to Minimal Abnormality.

Of the two strategies, Reliability is the most cautious one. The basic idea is that any n-tuple Σ defines n sets of unreliable formulas (one for each level) and that the $\mathbf{T}^{sr}$-models of Σ are the **T**-models of Σ in which only unreliable formulas behave abnormally. The sets of unreliable formulas are defined with respect to the minimal Dab^i-consequences $(0 < i \leq n)$ of Σ:

$$U^i(\Sigma) =_{df} \bigcup\{\Delta \mid Dab^i(\Delta) \text{ is a minimal } Dab^i\text{-consequence of } \Sigma\}$$

Example 2. $\Sigma = \langle\{\neg p, \neg q\}, \{(p \vee q) \wedge r\}\rangle$. All **T**-models of Σ verify $(\Diamond p \wedge \neg p) \vee (\Diamond q \wedge \neg q)$, and some **T**-models of Σ verify $(\Diamond r \wedge \neg r)$. However, as $r \notin U^1(\Sigma)$, the latter are not selected as $\mathbf{T}^{\mathbf{sr}}$-models of Σ. As a result, all $\mathbf{T}^{\mathbf{sr}}$-models of Σ verify r.

[11] The formula $(\Diamond\neg p \wedge p) \vee (\Diamond q \wedge \neg q) \vee (\Diamond\Diamond\neg q \wedge q)$ is also **T**-derivable from Σ. It would be misleading to interpret this formula as saying that there either is a problem with $\neg p$ or with q 'at level 1' (that is, with respect to $\langle\{p\}, \{\neg p \vee q\}\rangle$), or that there is a problem with $\neg q$ 'at level 2'. There is no problem at all at level 1, as $\{p\} \cup \{\neg p \vee q\}$ is consistent. Hence it is better to consider only Dab^i-consequences of Σ, as in the discussion of the example.

Let $\mathcal{M}_\Sigma$ be the set of all **T**-models of Σ. As abnormalities of the form $\Diamond^i A \wedge \neg A$ are considered as worse than abnormalities of the form $\Diamond^j A \wedge \neg A$ whenever $i < j$, the $\mathbf{T^{sr}}$-models of an n-tuple Σ are obtained by defining $n+1$ selections of $\mathcal{M}_\Sigma$ as follows:

$$\sigma^0(\mathcal{M}_\Sigma) =_{df} \mathcal{M}_\Sigma$$

and, where $0 \leq i < n$,

$$\sigma^{i+1}(\mathcal{M}_\Sigma) =_{df} \{M \in \sigma^i(\mathcal{M}_\Sigma) \mid Ab^{i+1}(M) \subseteq U^{i+1}(\Sigma)\}$$

The $\mathbf{T}^{sr}$-models of Σ are the members of $\sigma^n(\mathcal{M}_\Sigma)$. It is possible to characterize them directly as follows:

$$M \in \mathcal{M}_\Sigma \text{ is a } \mathbf{T}^{sr}\text{-model of } \Sigma \text{ iff } Ab^i(M) \subseteq U^i(\Sigma) \text{ for } 0 < i \leq n\,.$$

As we are only interested in the members of $\mathcal{W}$ that are derivable from Σ, we define:[12]

Definition 12. Where $A \in \mathcal{W}$, $\Sigma \models_{\mathbf{T}^{sr}} A$ iff A is verified by all $\mathbf{T}^{sr}$-models of Σ.

6. Minimal Abnormality

Semantically, the logic $\mathbf{T}^{sm}$ is obtained by stepwise selecting, for each consecutive Γ_i, the minimally abnormal models of $\langle \Gamma_0, \ldots, \Gamma_i \rangle$. In other words, we define the $n+1$ selections from Section 5 as follows:

$$\sigma^0(\mathcal{M}_\Sigma) =_{df} \mathcal{M}_\Sigma$$

and, where $0 \leq i < n$,

$$\sigma^{i+1}(\mathcal{M}_\Sigma) =_{df} \{M \in \sigma^i(\mathcal{M}_\Sigma) \mid \text{for no } M' \in \sigma^i(\mathcal{M}_\Sigma),\ Ab^{i+1}(M') \subset Ab^{i+1}(M)\}$$

The $\mathbf{T}^{sm}$-models of Σ are the members of $\sigma^n(\mathcal{M}_\Sigma)$.

Definition 13. Where $A \in \mathcal{W}$, $\Sigma \models_{\mathbf{T}^{sm}} A$ iff A is verified by all $\mathbf{T}^{sm}$-models of Σ.

The difference between $\mathbf{T}^{sr}$ and $\mathbf{T}^{sm}$ is easily illustrated by a simple example.

[12] So, we define the semantic consequence relation between n-tuples of members of $\mathcal{W}$ and members of $\mathcal{W}$. Obviously, one might define the corresponding relation that obtains between subsets of $\mathcal{W}^M$ and members of $\mathcal{W}$, but we are not interested in that relation here.

Example 3. $\Sigma = \langle\{\neg p \vee \neg q\}, \{p, q\}, \{p \supset r, q \supset r\}\rangle$. $Dab^1\{p, q\}$ is a minimal Dab^1-consequence of Σ. First consider the selection functions as defined for Reliability (Section 5). $\sigma^1(\mathcal{M}_\Sigma)$ selects the models of $\neg p \vee \neg q$ that verify a proper or improper subset of $\{\Diamond p \wedge \neg p, \Diamond q \wedge \neg q\}$. These include models that verify $\neg p \wedge q$, models that verify $p \wedge \neg q$, and models that verify $\neg p \wedge \neg q$. From these models, $\sigma^2(\mathcal{M}_\Sigma)$ selects those that falsify $\Diamond\Diamond r \wedge \neg r$. Some of these models verify $\neg p$, $\neg q$ and $\neg r$, whence $\Sigma \nvDash r$. The formulas verified by all these models are the **CL**-consequences of $(\neg p \vee \neg q) \wedge (r \vee (\neg p \wedge \neg q))$.

If the selection functions are defined as in the present section, then $\sigma^1(\mathcal{M}_\Sigma)$ comprises only those models from $\sigma^0(\mathcal{M}_\Sigma)$ that verify $p \vee q$—the models that falsify both p and q are not selected because they are more abnormal than the models that verify only one of them. As a result, $\sigma^2(\mathcal{M}_\Sigma)$ comprises only models that verify $p \vee q$, $p \supset r$, $q \supset r$, and hence r. The formulas verified by all these models are the **CL**-consequences of $(\neg p \vee \neg q) \wedge (p \vee q) \wedge r$.

The Minimal Abnormality strategy is semantically simple and transparent, and it delivers a richer consequence set than the Reliability strategy. However, as we shall see in Section 9, its proof theory is complicated.

It is possible to characterize the minimally abnormal models of Σ in terms of the minimal Dab^i-consequences of Σ. Although the matter is a bit complicated, it is useful to spell this out in view of subsequent sections.

Let $f(A)$ be the result obtained by relettering the free variables in A in such a way that they occur in some standard order (the first occurring free variable is always x, the second always y, etc.), let $A \prec B$ denote that $\exists B$ follows by (non-zero applications of) existential generalization from $\exists A$, and let $g(\Delta) = \{f(A) \mid A \in \Delta$; for no $B \in \Delta$, $f(B) \prec f(A)\}$. Let Φ^{oi}_Σ be the set of all sets $g(\Delta)$ such that Δ contains, for each minimal Dab^i-consequence $Dab^i(\Theta)$ of Σ, at least some $A \in \Theta$, and let Φ^i_Σ be obtained by eliminating from Φ^{oi}_Σ those members that are proper supersets of other members. It can easily be shown that $M \in \sigma^{i+1}(\mathcal{M}_\Sigma)$ iff $M \in \sigma^i(\mathcal{M}_\Sigma)$ and $Ab^{i+1}(M) \in \Phi^{i+1}_\Sigma$. The proof proceeds wholly as the one presented in [4, §7] for the non-prioritized case. As a result, the $n+1$ selections for $\mathbf{T}^{sm}$ may be defined as follows:

$$\sigma^0(\mathcal{M}_\Sigma) =_{df} \mathcal{M}_\Sigma$$

and, where $0 \leq i < n$,

$$\sigma^{i+1}(\mathcal{M}_\Sigma) =_{df} \{M \in \sigma^i(\mathcal{M}_\Sigma) \mid Ab^{i+1}(M) \in \Phi^{i+1}_\Sigma\}$$

This result will prove useful to define the logic presented in the next section.

7. Counting

The logic $\mathbf{AL}^*_{EXP}$ *minimizes*, at each level, the *number* of expectancies. Incorporating this idea in the logics $\mathbf{T}^{sr}$ and $\mathbf{T}^{sm}$ results in minimizing the *number* of abnormalities (that are in general subformulas of expectancies). This actually leads to a single logic, which we shall call $\mathbf{T}^c$.

In view of the preceding Section, the matter is straightforward. Each member of Φ^i_Σ provides us with a hypothesis on what went wrong at level i, provided we take it for granted that we have reasons to (set theoretically) minimize the number of things that went wrong. All we have to do is to select the members of Φ^i_Σ that are *numerically* smaller than other members. The following example illustrates the matter.

Example 4. Consider $\Sigma = \langle\{p,\ q,\ r\}, \{p \supset \neg r,\ q \supset \neg r,\ p \supset s\}\rangle$. The minimal Dab^1-consequences of Σ are $(\Diamond\neg p \wedge p) \vee (\Diamond\neg r \wedge r)$ and $(\Diamond\neg q \wedge q) \vee (\Diamond\neg r \wedge r)$. Hence, $\Phi^1_\Sigma = \{\{\neg r\}, \{\neg p, \neg q\}\}$. As $\{\neg r\}$ is numerically smallest, $p \supset s$, and hence s, are verified in all $\mathbf{T^c}$-models of Σ.

For other examples, several members of Φ^i_Σ may be numerically minimal, and hence all of them have to be treated on a par. Let $\Phi^{i\#}_\Sigma$ denote the members of Φ^i_Σ that do not have a larger cardinality than any other members of Φ^i_Σ. We then select the $\mathbf{T}^c$-models of Σ as follows:

$$\sigma^0(\mathcal{M}_\Sigma) =_{df} \mathcal{M}_\Sigma$$

and, where $0 \leq i < n$,

$$\sigma^{i+1}(\mathcal{M}_\Sigma) =_{df} \{M \in \sigma^i(\mathcal{M}_\Sigma) \mid Ab^{i+1}(M) \in \Phi^{i+1\#}_\Sigma\}$$

The $\mathbf{T}^c$-models of Σ are the members of $\sigma^n(\mathcal{M}_\Sigma)$ and, where $A \in \mathcal{W}$, $\Sigma \models_{\mathbf{T}^c} A$ iff A is verified by all $\mathbf{T}^c$-models of Σ.

Example 4 illustrates the gain obtained by moving from $\mathbf{T}^{sm}$ to $\mathbf{T}^c$: r is not a $\mathbf{T}^{sm}$-consequence of $\langle\{p,\ q,\ r\}, \{p \supset \neg r,\ q \supset \neg r,\ p \supset s\}\rangle$. This gain is obtained by introducing a numerical criterion and in this respect differs from the gain obtained by moving from $\mathbf{T}^{sr}$ to $\mathbf{T}^{sm}$, which is obtained by introducing a logical criterion.

8. Some metatheory

In this section, we show some basic properties of the semantics, and at once prepare the proof theory. We shall rely freely on the Soundness and Completeness of both $\mathbf{T}$ and $\mathbf{T}^+$ with respect to their semantics.

The models that belong to the selections have some striking properties. We cannot fully report on these properties here, and merely pick out two striking ones. The first property is that, if $A \in \mathcal{F}^p$ and $A, \neg A \notin U^i(\Sigma)$ ($i \geq 1$), then any $M \in \sigma^i(\mathcal{M}_\Sigma)$ verifies either $\Box^i A$ or $\Box^i \neg A$ (in which $\Box^i$ abbreviates i occurrences of $\Box$).[13] A further striking property is the following:

Theorem 1. *If $Dab^i(\Delta)$ is a minimal Dab^i-consequence of Σ, then there is a (non-empty) $\Delta' \subseteq \Delta$ such that $Dab^{i+1}(\Delta')$ is a minimal Dab^{i+1}-consequence of Σ.*

Proof. If the antecedent is true, $\Sigma \models_{\mathbf{T}} Dab^i(\Delta)$ and hence $\Sigma \models_{\mathbf{T}} Dab^{i+1}(\Delta)$. It follows that there is a (non-empty) $\Delta' \subseteq \Delta$ such that $Dab^{i+1}(\Delta')$ is a minimal Dab^{i+1}-consequence of Σ. ■

This means that it does not make any difference, for the Reliability strategy, whether we make the selections with respect to $U^i(\Sigma)$ or with respect to $U^{\widehat{i}}(\Sigma) =_{df} U^i(\Sigma) \cap U^{i+1}(\Sigma) \cap \ldots \cap U^n(\Sigma)$. Remark that $U^{\widehat{1}}(\Sigma) \subseteq U^{\widehat{2}}(\Sigma) \subseteq \ldots \subseteq U^{\widehat{n}}(\Sigma)$. As the matter is somewhat complicated, an example seems useful.

Example 5. $\Sigma = \langle \{p, q, r\}, \{(\neg p \vee \neg q) \wedge (\neg p \supset s)\}, \{\neg p, \neg r\}\rangle$. As $U^1(\Sigma) = \{\neg p, \neg q\}$, $\sigma^1(\mathcal{M}_\Sigma)$ comprises the **T**-models of Σ in which the only abnormalities of level 1 are $\Diamond\neg p \wedge p$, $\Diamond\neg q \wedge q$ or both. Selected models that verify $\Diamond\neg p \wedge p$ verify $\Diamond s$. As $s \notin Ab^1(M)$, *these* models verify s as well as $\Box s$. However, some models that falsify $\Diamond\neg p \wedge p$ verify $\neg s$ as well as $\Box\neg s$.

As $U^2(\Sigma) = \{\neg p, \neg r\}$, $\sigma^2(\mathcal{M}_\Sigma)$ comprises no models that verify $\Diamond\Diamond\neg q \wedge q$. It follows that $\sigma^2(\mathcal{M}_\Sigma)$ comprises no models that verify $\Diamond\neg q$ (because $\Diamond\neg q \vdash_{\mathbf{T}} \Diamond\Diamond\neg q$ and all models of Σ verify q). Hence all models in $\sigma^2(\mathcal{M}_\Sigma)$ verify s as well as $\Box s$. If the models had been selected in terms of $U^{\widehat{i}}(\Sigma)$, no models verifying $\Diamond\neg q$ would have been selected in the first run: $U^{\widehat{1}}(\Sigma) = \{\neg p\}$ and $U^{\widehat{2}}(\Sigma) = \{\neg p, \neg r\}$.

The theorem has similar consequences for the Minimal Abnormality strategy and for the Counting strategy of $\mathbf{T}^c$.

As is usual for adaptive logics, the dynamic proof theories for $\mathbf{T}^{sr}$, $\mathbf{T}^{sm}$, and $\mathbf{T}^c$ are based on a specific relation between derivability by the upper limit logic $\mathbf{T}^+$, and derivability by the lower limit logic $\mathbf{T}$. We now prove the lemmas and theorems that warrant this.

[13] This holds for all three logics. Stronger properties hold for $\mathbf{T}^{sm}$ and $\mathbf{T}^c$.

Lemma 1. *For any* $\mathbf{T}$*-model* M*, any primitive formula* A*, and any* i*,* M *verifies* $\Diamond^i A \wedge \Diamond^i \neg A$ *iff* M *verifies* $(\Diamond^i A \wedge \neg A) \vee (\Diamond^i \neg A \wedge A)$*.*

Proof. Obvious in view of the $\mathbf{T}$-semantics. ■

Let $prim(A)$ be the set of all primitive formulas that occur in A, and let $at(A) = prim(A) \cup \{\neg B \mid B \in prim(A)\}$.

Lemma 2. *For any* $\mathbf{T}$*-model* M*, and any* $A \in \mathcal{W}^M$*, if there is no* $B \in prim(A)$ *such that* M *verifies* $\exists(\Diamond^i B \wedge \Diamond^i \neg B)$ *for some* i*, then there is a* $\mathbf{T}^+$*-model* M' *such that* M *verifies* A *iff* M' *verifies* A*.*

Proof. Suppose that the antecedent holds true.

Case 1: M verifies A. Suppose that no singleton model M' verifies A. It follows that, for some $B \in prim(A)$, B is true at some world of M, and $\neg B$ at another. But then, M verifies $\exists(\Diamond^i B \wedge \Diamond^i \neg B)$, for some i, which contradicts the main supposition. Hence, some $\mathbf{T}^+$-model verifies A.

Case 2: M verifies $\neg A$. As $prim(A) = prim(\neg A)$, this case is entirely analogous to the previous one. ■

Theorem 2. $\vdash_{\mathbf{T}^+} A$*, iff there are* $B_1, \ldots, B_m \in \mathcal{F}^a$ *and there is an* i *such that* $\vdash_{\mathbf{T}} A \vee Dab^i\{B_1, \ldots, B_m\}$*. (Theorem Adjustment Theorem.)*

Proof. We first consider the left–right direction. If $\vdash_{\mathbf{T}} A$, the theorem obviously holds. So, suppose that $\vdash_{\mathbf{T}^+} A$, and $\nvdash_{\mathbf{T}} A$. As $at(A)$ is finite, $A \vee Dab^j(at(A))$ is a wff (for any j). Consider any $\mathbf{T}$-model M.

Case 1: M verifies $\exists(\Diamond^i B \wedge \neg B)$ for some $B \in at(A)$, and some i. It follows that M verifies $Dab^i(at(A))$.

Case 2: M falsifies $\exists(\Diamond^i B \wedge \neg B)$ for any $B \in at(A)$ and for any i. Suppose that M falsifies A. By Lemma 1, M then falsifies $\exists(\Diamond^i B \wedge \Diamond^i \neg B)$, for any $B \in prim(A)$. Hence, by Lemma 2, some $\mathbf{T}^+$-model falsifies A. But this contradicts the supposition that A is $\mathbf{T}^+$-valid.

For the right–left direction, suppose that there are $B_1, \ldots, B_m \in \mathcal{F}^a$ and that there is an i such that $\vdash_{\mathbf{T}} A \vee Dab^i\{B_1, \ldots, B_m\}$. As $\mathbf{T}^+$ extends $\mathbf{T}$, $\vdash_{\mathbf{T}^+} A \vee Dab^i\{B_1, \ldots, B_m\}$. Any $\mathbf{T}^+$-model falsifies $Dab^i\{B_1, \ldots, B_m\}$, and hence verifies A, whence $\models_{\mathbf{T}^+} A$ and $\vdash_{\mathbf{T}^+} A$. ■

Theorem 3. $A_1, \ldots, A_n \vdash_{\mathbf{T}^+} B$*, iff there are* $C_1, \ldots, C_m \in \mathcal{F}^a$ *and there is an* i *such that* $A_1, \ldots, A_n \vdash_{\mathbf{T}} B \vee Dab^i\{C_1, \ldots, C_m\}$*. (Derivability Adjustment Theorem.)*

Proof. As both $\mathbf{T}^+$ and $\mathbf{T}$ are derivability-compact, this follows from Theorem 2. ■

Theorem 3 will provide the basis for the dynamic proof theories. It warrants that whenever B is $\mathbf{T}^{+}$-derivable from $A_1, \ldots, A_n$, there are $C_1, \ldots, C_n$ such that B is $\mathbf{T}$-derivable from $A_1, \ldots, A_n$ or one of the C_i behaves abnormally with respect to $A_1, \ldots, A_n$. This suggests that we derive B from $A_1, \ldots, A_n$, on the condition that none of the C_i behaves abnormally. The following examples illustrate Theorem 3:

(1) $\diamond^i\neg p \vdash_{\mathbf{T}} \neg p \vee (\diamond^i\neg p \wedge p)$
(2) $p \vee q, \diamond^i\neg p \vdash_{\mathbf{T}} q \vee (\diamond^i\neg p \wedge p)$
(3) $\diamond^i(p \vee q) \vdash_{\mathbf{T}} (p \vee q) \vee (\diamond^i p \wedge \neg p) \vee (\diamond^i q \wedge \neg q)$
(4) $\diamond^{i-1}(p \vee q), \diamond^i\neg p \vdash_{\mathbf{T}} q \vee (\diamond^i p \wedge \neg p) \vee (\diamond^i\neg p \wedge p) \vee (\diamond^i q \wedge \neg q)$

The proof of the following theorem is nearly identical[14] to the proof presented in [19] (which relies on the simplified version of the method first presented in [6]).

Theorem 4. *If M is a $\mathbf{T}$-model of Σ, then there is a $\mathbf{T}^{sm}$-model M' of Σ such that $Ab^i(M') \subseteq Ab^i(M)$. (Strong Reassurance)*

Corollary 1. *If Σ has $\mathbf{T}$-models, then Σ has $\mathbf{T}^{sm}$-models. (Reassurance)*

The proof of the Reassurance Theorem and Strong Reassurance Theorem for $\mathbf{T}^{c}$ follows immediately from the above. The proof of these theorems for $\mathbf{T}^{sr}$ is obtained by the reasoning from [6, §4].

Given these theorems and corollaries, one easily proves a set of interesting properties of the considered adaptive logics. We prove some of them for $\mathbf{T}^{sr}$. The proofs for $\mathbf{T}^{sm}$ are analogous. The proofs for $\mathbf{T}^{c}$ are nearly obvious in view of the latter (the $\mathbf{T}$-models of Σ that verify the numerically smallest set of abnormalities are all Minimally Abnormal models of Σ).

Theorem 5. *If Γ_0 is consistent and no $A \in \Gamma_0 \cup \ldots \cup \Gamma_n$ is inconsistent, then $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$ has $\mathbf{T}^{sr}$-models.*

Proof. If the antecedent is true, Σ obviously has $\mathbf{T}$-models. By the Reassurance Theorem for Reliability, Σ has $\mathbf{T}^{sr}$-models. ■

Theorem 6. *If $\Gamma_0 \cup \ldots \cup \Gamma_n$ is consistent, then $\Sigma \models_{\mathbf{T}^{sr}} A$ iff $\Gamma_0 \cup \ldots \cup \Gamma_n \models_{\mathbf{CL}} A$.*

[14] The only change required with respect to the proof in [19] is that, in the definition of Δ_{i+1}, one replaces $\neg\exists(\diamond A \wedge \diamond\neg A)$ by $\neg\exists(\diamond^i A \wedge \neg A)$, where i is the largest j ($1 \leq j \leq n$) for which M' verifies $\neg\exists(\diamond^j A \wedge \neg A)$. Theorem 1 warrants that, if there is such a j, then M' verifies $\neg\exists(\diamond^k A \wedge \neg A)$ for all $k < j$, as required.

Proof. If $\Gamma_0 \cup \ldots \cup \Gamma_n$ is consistent, $U^i(\Sigma) = \emptyset$ for all i $(0 < i \leq n)$. As the $\mathbf{T}^{sr}$-models of Σ only verify abnormalities that are members of some $U^i(\Sigma)$, no abnormalities are verified by any $\mathbf{T}^{sr}$-model of Σ. It follows that $v_M(A, w_i) = v_M(A, w_0)$ for all $A \in \mathcal{W}$ and for all $w_i \in W$. But then the set of $\mathbf{T}^{sr}$-models of Σ coincides with the set of $\mathbf{T}^{+}$-models of Σ, and hence with the set of $\mathbf{CL}$-models of Σ. ■

$\mathbf{T}^{sr}$ is non-monotonic. So, it is important that one can prove the following theorem:

Theorem 7. *If* $\langle \Gamma_0, \ldots, \Gamma_n \rangle \models_{\mathbf{T}^{sr}} A$, *then* $\langle \Gamma_0, \ldots, \Gamma_n, \Gamma_{n+1} \rangle \models_{\mathbf{T}^{sr}} A$.

Proof. Obvious in view of the fact that $\sigma^{n+1}(\mathcal{M}_\Sigma) \subseteq \sigma^n(\mathcal{M}_\Sigma)$. ■

Not all $\mathbf{CL}$-consequences of Γ_{i+1} that are compatible with the $\mathbf{T}^{sr}$-consequences of $\langle \Gamma_0, \ldots, \Gamma_i \rangle$ can be $\mathbf{T}^{sr}$-consequences of $\langle \Gamma_0, \ldots, \Gamma_{i+1} \rangle$. This is easily seen from Example 1. The $\mathbf{T}^{sr}$-consequences of $\langle \{p\}, \{\neg p \vee q\} \rangle$ are the $\mathbf{CL}$-consequences of $\{p, q\}$; both $\neg q \vee s$ and $\neg q \vee \neg s$ are compatible with these but their conjunction is not. For this reason, it is important to spell out which $\mathbf{CL}$-consequences of Γ_i do enter into the $\mathbf{T}^{sr}$-consequences of $\langle \Gamma_0, \ldots, \Gamma_i \rangle$.

The proof of the following lemma is immediate in view of the proof of the left–right direction of Theorem 2.

Lemma 3. $\vdash_{\mathbf{T}} (\Diamond^i A \supset A) \vee Dab^i(at(A))$.

Theorem 8. *Where* $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$, *if* $A \in \Gamma_i$ $(1 \leq i \leq n)$, $A \vdash_{\mathbf{CL}} B$, *and* $at(B) \cap U^i(\Sigma) = \emptyset$, *then* $\Sigma \models_{\mathbf{T}^{sr}} B$.

Proof. If $A \in \Gamma_i$ and $A \vdash_{\mathbf{CL}} B$, then all $\mathbf{T}$-models of Σ verify $\Diamond^i B$. Hence, by Lemma 3, they all verify $B \vee Dab^i(at(B))$. So, if $at(B) \cap U^i(\Sigma) = \emptyset$, then all $\mathbf{T}^{sr}$-models of Σ falsify $Dab^i(at(B))$ and hence verify B. ■

The full effect of this theorem appears from:

Theorem 9. $Cn_{\mathbf{T}^{sr}}(\Sigma)$ *is* $\mathbf{CL}$*-closed.*

Proof. A $\mathbf{T}$-model verifies A iff $v_M(A, w_0) = 1$ and it is obvious in view of the $\mathbf{T}$-semantics that $\{A \mid v_M(A, w_0) = 1\}$ is $\mathbf{CL}$-closed. ■

9. Dynamic Proof Theories

As is usual for adaptive logics, lines of a proof have five elements: (i) a line number, (ii) the formula A that is derived, (iii) the line numbers of the formulas from which A is derived, (iv) the rule by which A is derived, and (v) a (possibly empty) 'condition'. The condition specifies which formulas have to behave normally in order for A to be so derivable.

If A is conditionally derived in the proof (that is, on a line the fifth element of which is not $\emptyset$), then the condition will be a couple: a set of formulas and a 'level' i ($0 < i \leq n$), indicated by a subscript (as in Δ_i). Sometimes a condition will be compounded from several other conditions by taking the union of their first members and the maximum of their second members, which will be denoted by max(...).

We now list the generic rules that govern dynamic proofs from $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$. They are the same for all three logics.

PREM If $A \in \Gamma^i$, then one may add a line consisting of
- (i) the appropriate line number,
- (ii) $\Diamond^i A$,
- (iii) "–",
- (iv) "Prem", and
- (v) $\emptyset$.

RU If $B_1, \ldots, B_m \vdash_{\mathbf{T}} A$ and $B_1, \ldots, B_m$ occur in the proof with the conditions $\Delta^1_{j_1}, \ldots, \Delta^m_{j_m}$ respectively,[15] then one may add a line consisting of
- (i) the appropriate line number,
- (ii) A,
- (iii) the line numbers of the B_i,
- (iv) "RU", and
- (v) $(\Delta^1 \cup \ldots \cup \Delta^m)_{\max(j_1, \ldots, j_m)}$.

RC If $B_1, \ldots, B_m \vdash_{\mathbf{T}} A \vee Dab^k(\Theta)$ and $B_1, \ldots, B_m$ occur in the proof with the conditions $\Delta^1_{j_1}, \ldots, \Delta^m_{j_m}$ respectively, then one may add a line consisting of
- (i) the appropriate line number,
- (ii) A,
- (iii) the line numbers of the B_i,
- (iv) "RC", and
- (v) $(\Theta \cup \Delta^1 \cup \ldots \cup \Delta^m)_{\max(k, j_1, \ldots, j_m)}$.

[15] If $\Delta^i = \emptyset$, we shall consider it to have the (invisibly written) subscript 0.

It is obvious in view of the rules that A is derivable on the condition Δ_i in a proof from Σ iff $A \vee Dab^i(\Delta)$ is $\mathbf{T}$-derivable from Σ.

The three logics are distinguished from each other by the marking definitions. While the selection of models proceeds in terms of the minimal Dab^i-consequences of Σ, the marking definitions proceed in terms of the minimal Dab^i-formulas that have been derived in the proof (at the stage). Obviously, $Dab^i(\Delta)$ is a minimal Dab^i-formula at a stage iff, at that stage, it has been derived on the condition $\emptyset$ and $Dab^i(\Theta)$ has not been derived on the condition $\emptyset$ for any $\Theta \subset \Delta$.

From the set of minimal Dab^i-formulas at stage s, one defines $U^i_s(\Sigma)$, $\Phi^i_s(\Sigma)$, and $\Phi^{i\#}_s(\Sigma)$ in the same way as $U^i(\Sigma)$, Φ^i_Σ, and $\Phi^{i\#}_\Sigma$ were defined from the minimal Dab^i-consequences of Σ. Next, we define the marked lines (for each stage) for the three logics.

Definition 14. Marking for $\mathbf{T}^{sr}$: Line i is marked at stage s iff, where Δ_j is its fifth element, $\Delta_j \cap U^j_s(\Sigma) \neq \emptyset$.

Definition 15. Marking for $\mathbf{T}^{\mathbf{sm}}$: Line i, with A as its second element and Δ_j as its fifth element, is marked at stage s iff (i) there is no $\varphi \in \Phi^j_s(\Sigma)$ such that $\varphi \cap \Delta_j = \emptyset$, or (ii) for some $\varphi \in \Phi^j_s(\Sigma)$, there is no line k that has A as its second element and has as its fifth element some Θ_j such that $\varphi \cap \Theta_j = \emptyset$.

Definition 16. Marking for $\mathbf{T}^c$: Line i, with A as its second element and Δ_j as its fifth element, is marked at stage s iff (i) there is no $\varphi \in \Phi^{j\#}_s(\Sigma)$ such that $\varphi \cap \Delta_j = \emptyset$, or (ii) for some $\varphi \in \Phi^{j\#}_s(\Sigma)$, there is no line k that has A as its second element and has as its fifth element some Θ_j such that $\varphi \cap \Theta_j = \emptyset$.

Definition 15 is most easily understood with respect to the semantics. For all the proof tells one,[16] each $\varphi \in \Phi^j_s(\Sigma)$ comprises the abnormalities verified by a minimally abnormal model of Σ. So, line i is marked iff (i) it does not witness that A is verified by some minimally abnormal model of Σ (if there is no minimally abnormal model of Σ in which all elements of Δ_j are normal), or (ii) the proof does not witness that A is verified by all minimally abnormal models of Σ (there is a minimally abnormal model of Σ for which it has not been shown in the proof that it verifies A). Similarly for Definition 16.

[16] Actually, this may be made precise in terms of block-formulas—see [3]. Each $\varphi \in \Phi^j_s(\Sigma)$ comprises the abnormalities verified by a minimally abnormal model of the block formulation of Σ determined by the stage of the proof.

If, at stage s of a proof from Σ, A is the second element of an unmarked line, then we say that A is *derived at stage s*. Suppose that A is derived on an unmarked line i of a proof. Obviously, it is possible that line i is marked in an extension of the proof. If any extension of the proof can be further extended in such a way that line i is unmarked, we say that A is *finally derived* at line i. And we say that A is *finally derivable* from Σ iff there is a proof from Σ in which A is finally derived. For all three logics we finally define:

Definition 17. Where $A \in \mathcal{W}$, $\Sigma \vdash A$ iff A is finally derivable from Σ.

10. Some Applications

We first present a proof from $\Sigma = \langle \{p, q\}, \{(\neg p \wedge r) \wedge (q \supset s)\}\rangle$. A boxed number s at the far right of the line indicates that the line is marked at stage s of the proof.

1	p	–	Prem	$\emptyset$	
2	q	–	Prem	$\emptyset$	
3	$\Diamond((\neg p \wedge r) \wedge (q \supset s))$	–	Prem	$\emptyset$	
4	$\Diamond\neg p$	3	RU	$\emptyset$	
5	$\Diamond r$	3	RU	$\emptyset$	
6	$\Diamond(q \supset s)$	3	RU	$\emptyset$	
7	$\neg p$	4	RC	$\{\neg p\}_1$	$\boxed{10}$
8	r	5	RC	$\{r\}_1$	
9	$q \supset s$	6	RC	$\{\neg q, s\}_1$	
10	$\Diamond\neg p \wedge p$	1, 4	RU	$\emptyset$	
11	s	2, 9	RU	$\{\neg q, s\}_1$	

As $\Diamond\neg p \wedge p$ is the only minimal Dab^1-consequence of Σ, and this formulas has been derived at line 10, no other Dab^1-formula can possibly be derived in the proof. It follows that, except for line 7, none of these lines will be marked in any extension of the proof. Remark that it does not make any difference which of our logics is applied: $U^1(\Sigma) = \{\neg p\}$ and $\Phi^1_\Sigma = \Phi^{1\#}_\Sigma = \{\{\neg p\}\}$. As the premises have obviously been 'exhausted', $Cn_{\mathbf{T}^{sr}}(\Sigma) = Cn_{\mathbf{T}^{sm}}(\Sigma) = Cn_{\mathbf{T}^{c}}(\Sigma) = Cn_{\mathbf{CL}}(\{p, q, r, s\})$. The reader may easily check that $Cn_{\mathbf{AL}_{EXP}}(\Sigma) = Cn_{\mathbf{AL}^*_{EXP}}(\Sigma) = Cn_{\mathbf{CL}}(\{p, q\})$.

It is useful to construct a proof from $\Sigma = \langle \{p, q\}, \{(\neg p \wedge r) \wedge (p \supset s)\}\rangle$ or from $\Sigma = \langle \{p, q\}, \{(\neg p \wedge r), (p \supset s)\}\rangle$. In these cases $Cn_{\mathbf{T}^{sr}}(\Sigma) =$

$Cn_{\mathbf{T}^{sm}}(\Sigma) = Cn_{\mathbf{T}^{c}}(\Sigma) = Cn_{\mathbf{CL}}(\{p, q, r\})$; s is not finally derivable from Σ because $\Diamond\neg p \wedge p$ is a minimal Dab^1-consequence of Σ.

Let $\Sigma = \langle\{Pa, Qa, Pb\}, \{(\forall x)(Px \supset \neg Rx)\}, \{(\forall x)(Qx \supset Rx)\}\rangle$. This simple predicative example illustrates expectations with exceptions. We shall see that the proof displays a very interesting feature. To improve readability, we abbreviate $\exists\Diamond^i A \wedge \neg A$ by $!^i A$. Also, we do not present an elegant proof, but rather one that illustrates what is going on.

1	Pa	–	Prem	$\emptyset$	
2	Qa	–	Prem	$\emptyset$	
3	Pb	–	Prem	$\emptyset$	
4	$\Diamond(\forall x)(Px \supset \neg Rx)$	–	Prem	$\emptyset$	
5	$\Diamond\Diamond(\forall x)(Qx \supset Rx)$	–	Prem	$\emptyset$	
6	$(\forall x)(Px \supset \neg Rx)$	4	RC	$\{\neg Px, \neg Rx\}_1$	
7	$\neg Ra$	1, 6	RU	$\{\neg Px, \neg Rx\}_1$	
8	$\neg Rb$	3, 6	RU	$\{\neg Px, \neg Rx\}_1$	
9	$(\forall x)(Qx \supset Rx)$	5	RC	$\{\neg Qx, Rx\}_2$	$\boxed{10}$
10	$!^2\neg Px \vee !^2\neg Qx \vee !^2 Rx \vee !^2\neg Rx$	1, 2, 4, 5	RU	$\emptyset$	
11	$Qa \supset Ra$	5	RC	$\{\neg Qa, Ra\}_2$	$\boxed{12}$
12	$!^2\neg Pa \vee !^2\neg Qa \vee !^2 Ra \vee !^2\neg Ra$	1, 2, 4, 5	RU	$\emptyset$	
13	$Qb \supset Rb$	5	RC	$\{\neg Qb, Rb\}_2$	
14	$\neg Qb$	8, 13	RU	$\{\neg Qb, Rb, \neg Px, \neg Rx\}_2$	

No unmarked line will be marked in any extension of the proof. Although the union of the premises is inconsistent, $\neg Ra$ is finally derivable because Γ_1 has a higher priority than Γ_2. A more important feature concerns the abnormality derived on line 10. This shows that, given the priorities, the premise $(\forall x)(Qx \supset Rx)$ does not hold in general. There are objects for which it does not hold, and, as appears from 12, a is such an object. This, however, does not prevent one from deriving $\neg Qb$. The reason is that $Qb \supset Rb$ cannot only be derived on the condition $\{\neg Qx, Rx\}_2$, which leads to marking in view of 10, but also on the more specific condition $\{\neg Qb, Rb\}_2$ which does not lead to marking.

It is instructive to look at Example 5 from a proof theoretic point of view.

1	p	–	Prem	$\emptyset$
2	q	–	Prem	$\emptyset$
3	r	–	Prem	$\emptyset$
4	$\Diamond((\neg p \vee \neg q) \wedge (\neg p \supset s))$	–	Prem	$\emptyset$
5	$\Diamond\Diamond\neg p$	–	Prem	$\emptyset$

6	$\Diamond\Diamond\neg r$	–	Prem	$\emptyset$	
7	$\Diamond(\neg p \vee \neg q)$	4	RU	$\emptyset$	
8	$\Diamond(s \vee \neg q)$	4	RU	$\emptyset$	
9	$s \vee \neg q$	8	RC	$\{s, \neg q\}_1$	$\boxed{11}$
10	s	2, 9	RU	$\{s, \neg q\}_1$	$\boxed{11}$
11	$!^1\neg p \vee !^1\neg q$	1, 2, 7	RU	$\emptyset$	
12	$\Diamond\Diamond(s \vee \neg q)$	4	RU	$\emptyset$	
13	$s \vee \neg q$	12	RC	$\{s, \neg q\}_2$	
14	s	2, 13	RU	$\{s, \neg q\}_2$	
15	$!^2\neg p$	5, 1	RU	$\emptyset$	
16	$!^2\neg r$	6, 3	RU	$\emptyset$	

The interesting point here is that s is finally derivable because it is derivable on a condition of level 2 and, at level 2, $\neg q$ behaves normally. Indeed, although $!^2\neg p \vee !^2\neg q$ is obviously derivable, it is not a minimal Dab^2-formula in view of 15. Please compare this with the comment in Example 5.

Finally, let us consider the Σ from Example 4 to illustrate the difference between the Minimal Abnormality strategy and the Counting strategy from Section 7. We shall not introduce any marks but comment on them after the proof.

1	p	–	Prem	$\emptyset$
2	q	–	Prem	$\emptyset$
3	r	–	Prem	$\emptyset$
4	$\Diamond(p \supset \neg r)$	–	Prem	$\emptyset$
5	$\Diamond(q \supset \neg r)$	–	Prem	$\emptyset$
6	$\Diamond(p \supset s)$	–	Prem	$\emptyset$
7	$p \supset \neg r$	4	RC	$\{\neg p, \neg r\}_1$
8	$q \supset \neg r$	5	RC	$\{\neg q, \neg r\}_1$
9	$p \supset s$	6	RC	$\{\neg p, s\}_1$
10	s	1, 9	RU	$\{\neg p, s\}_1$
11	$!^1\neg p \vee !^1\neg r$	1, 3, 4	RU	$\emptyset$
12	$!^1\neg q \vee !^1\neg r$	2, 3, 4	RU	$\emptyset$

Remark that $\Phi^1_{12}(\Sigma) = \{\{\neg p, \neg q\}, \{\neg r\}\}$ and that $\Phi^{1\#}_{12}(\Sigma) = \{\{\neg r\}\}$, and that the sets remain the same in all possible extensions of the proof. $p \supset s$ and s have not been derived, and actually cannot be derived, on some condition that does not contain any member of $\{\neg p, \neg q\}$. So, lines 9 and 10 are marked on the Minimal Abnormality strategy. As the conditions of line 9 and 10 do not overlap with the only member of $\Phi^{1\#}_{12}(\Sigma)$, these lines are not marked on the Counting strategy from Section 7. It follows that

s (and hence also $p \supset s$) is finally $\mathbf{T}^c$-derivable from Σ but is not finally $\mathbf{T}^{sm}$-derivable from Σ.

11. Some Open Problems

We hope to have shown that the adaptive logics $\mathbf{T}^{sr}$, $\mathbf{T}^{sm}$, and $\mathbf{T}^c$ are attractive for explicating specific forms of diagnostic reasoning. We also made clear the difference with chunking logics (which are also characterized by adaptive logics and) which are attractive for different forms of diagnosis.

The confines of this paper did not enable us to spell out all the proofs. Most central are the soundness and completeness of the dynamic proof theory with respect to the semantics. Their proofs proceed by techniques from [4], [10] and [19], but it seems interesting to spell them out. In Section 8, we mentioned some interesting properties of the logics. It seems worthwhile to perform a systematic study of the logics in this respect.

The import of the present results does not only depend on the application of the logics to diagnosis. The adaptive logics $\mathbf{T}^{sr}$, $\mathbf{T}^{sm}$, and $\mathbf{T}^c$ have different applications as well[17] and deserve to be studied from an abstract point of view, as prioritized consequence relations. Special attention should be paid to computational aspects, for example to criteria that enable one to decide, in specific cases, that some formula is finally derived in a proof at a stage. The consequences of Theorem 1 may be explored to obtain useful derivable rules for the dynamic proof theories.

Our last comment concerns some interesting further prioritized consequence relations. The central difference between the Rescher–Manor consequence relations on the one hand and the adaptive logics $\mathbf{T}^{sr}$, $\mathbf{T}^{sm}$, and $\mathbf{T}^c$ on the other, is that the former proceed by selecting certain *members* of the different Γ_i whereas the latter proceed by selecting certain **CL**-*consequences* of those members. Remark, however, that the **CL**-consequences of the members of the Γ_i do not comprise all **CL**-consequences of the Γ_i themselves. Forthcoming work by Liza Verhoeven concerns an adaptive logic that selects certain **CL**-consequences of the Γ_i.[18] The price to be paid is that this logic delivers the trivial consequence set as soon as one of the Γ_i is inconsistent.

The situation suggests that a very different approach is attractive. The matter is most easily explained in semantic terms. Let a **PT**-model be like a **T**-model, except that any world that is different from w_0 is governed by

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[17] See [18] on the generation of questions in an inconsistent environment.

[18] Intuitively, according to this approach a **T**-model M verifies $\Sigma = \langle \Gamma_0, \ldots, \Gamma_n \rangle$ iff M verifies $\{\Diamond^i A \mid \Gamma_i \vdash_{\mathbf{CL}} A\}$ $(0 \leq i \leq n)$.

some paraconsistent logic **P**. A **PT**-model M may then be said to verify $\Sigma = \langle \Gamma_1, \ldots, \Gamma_n \rangle$—remark that we start with Γ_1—iff M verifies $\{\Diamond^i A \mid A \in \Gamma_i\}$ $(1 \leq i \leq n)$. From **PT**, an adaptive logic is build by selecting models. This selection should fulfil two requirements. First, a model of Σ should only be selected if it verifies $\Diamond^i A$ whenever A is an inconsistency-adaptive consequence of Γ_i. Next, the selections should agree with one of the selections from Sections 5–7. This construction has the following advantage. If some Γ_i is consistent, then its inconsistency-adaptive consequences are identical to its **CL**-consequences. If some Γ_i is inconsistent, many **CL**-consequences of subsets of Γ_i are still derivable. The presuppositions of this construction clarify the situations in which its application is suitable. Most importantly, the members of some Γ_i are considered as forming a coherent whole, not just a set of suppositions that turn out to have the same preference.

On the Rescher–Manor consequence relations, either some Γ_i is rejected or accepted as a whole, or each member of the Γ_i is accepted or rejected as a whole. The consequence relations from Sections 5–7 save as much as possible from each member of some Γ_i. The approach from the previous paragraphs saves as much as possible from each Γ_i. Given the variety of the prioritized consequence relations that are available, it is important to spell out such features. Whether a consequence relation is suitable for a certain application depends on them.[19]

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