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METALOGICAL PROPERTIES, BEING LOGICAL AND BEING FORMAL*

§1. Introduction. The predicate 'being logical' has at least four applications. We can apply it to concepts, propositions, sets of propositions (systems, theories) and methods. The concepts of quantifier or disjunction are logical but those of horse or water are not. Some propositions, for instance, the principle of excluded middle, are logical, others, for instance the law of gravity, are not. Propositional calculus is a logical theory (belongs to logic), but the theory of evolution is not. In a sense, the problem of logical propositions reduces itself to the question of logical systems, because we can say that A is logical if and only if it belongs to a logical systems (however, see below). Finally, deduction is a logical method of justification, but observation is not. Of course, there are also controversial cases. Is the concept of set logical or not? Are theorems of arithmetic logical or not? Is second-order logic a logical system or not? Is induction a logical method or not? And there is also a general understanding of logic under which every intellectual or even practical activity should be logical, that is, rational, coherent, based on sound principles, strict, precise, free of ambiguities, etc. On this general understanding, definitions, classifications, explanations, predictions and many other things are items to which the label 'logical' is applied.

This variety of applications of 'being logical' as well as controversies around it reflect the complicated history of logic. Logic can be conceived

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more or less widely. Logic in the wide sense comprises formal logic, semiotics (syntax, semantic and pragmatics) and methodology of science. Logic in its narrow sense is restricted to formal logic only, where the adjective 'formal' means, roughly speaking, 'dependent only on form in abstraction from content' (I will return to 'being formal' at the end of this paper). The simple division of logic in the narrow sense and logic in the wide sense considerably alters the scope of 'being logical'. For example, it throws out various things belonging to semiotics (meanings, uses of language, etc.) or methodology of science (explanation, prediction, perhaps induction, 'perhaps' because some people try to develop logical theories of inductive inference, etc.). However, there still remain many problems about 'being logical' to be discussed even if we work with the narrow understanding of logic. My main focus in what follows concerns 'being logical' as attributed to systems or theories. On the other hand, one should remember that if we qualify a set of sentences as a logical system, we also decide that the concepts occurring in it in an essential way. Yet I will not refer here to the idea of the essential occurrence; this device was extensively used by Quine (see Quine 1970, chapter IV) in his attempts to define logical truth as invariant under substitutions of extralogical words are logical. The same also concerns the methods of inference generated by theories in question. Thus, various applications of 'being logical' are mutually interrelated and cross each other at several points.

Every study about the concept of logic that starts with a preliminary attempt to delineate its limits, points out that we have a plethora, variety or multitude of logics, and complains that the questions like "What is logic?" or "What is the scope of logic?" have no uniformly determined answers. Perhaps it helps when we distinguish here three issues or subcases of our main question, which are determined strongly by the historical development of logic itself. The first concerns the debate over the socalled first-order thesis which expresses the standpoint that logic should be restricted to standard first-order logic. The contrary view claims that the domain of logic should be extended to a variety of other systems, including, for instance, higherorder logic or infinitary logic (see Barwise and Feferman 1985; Feferman 1999, Shapiro 1991; Shapiro 1996; Woleński 1999 for discussion of several aspects of this problem). The second issue takes into account the problem of alternativeness between various logics. The typical way of stating this issue is this: Can we or should we replace classical logic by an alternative non-classical system, for instance, intuitionistic, manyvalued, relevant or paraconsistent logic? Classical logic is here the stable point of reference, and its alternatives are described as non-classical. This distinguished character of classical

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logic stems from two factors. One reason is that classical logic appeared as the earliest mature system of logic. This historical explanation is usually supplemented by an evaluation, which consists in pointing out that classical logic is the most elegant and the most effective in its service for science. The advocates of non-classical systems argue that there are various counterintuitive features of classical logic, for instance, the meaning of logical constants, particularly, negation or implication. The third issue is partly similar to the first one and partly to the second one. It also concerns an extension of logic, however not, for instance, by admission of higher order quantifiers, but rather by adding new constants, namely modalities of various sorts (alethic, deontic, epistemic, etc.) and constructing modal logical systems based on suitably enlarged vocabularies. These constructions are based on some propositional or predicate logic as the starting system, which can be classical or non-classical as well. On the other hand, new constants lack some properties possessed by classical logical items, in particular, they do not obey the principle of extensionality. It is a reason to regard modal systems as non-classical, even if they are extensions of classical basic logic.

There are several implications of the depicted issues. Usually, if one discusses the first-order thesis, he or she either defends the view that first-order logic is the logic or claims that the property of being logical should be attributed to it as well as to its various extensions, like second-order logic, logic with formulas of infinite length or logic with generalized quantifiers. The situation is similar, although not so often recorded, when we go to modal logic, because the question whether extensions arising by adding modalities produce genuine logic or constitute extralogical theories is a legitimate one. The *a priori* answer is not clear, even when we decide that this or that basic system is *the* logic, that is, when we solve the second issue in a way. Now, it depends on many general options how the problem of alternativeness of logics is considered and decided. We can distinguish (see Rescher 1969, pp. 220–235, Haack 1978, pp. 221–232) absolutistic (monistic) and relativistic (pluralistic) approaches to logic. Monism insists that there is one and only one correct system of logic, and pluralism argues that many logics are to be included into the variety labelled 'logic', relatively to our cognitive needs (non-monotonic logic is perhaps the most recent spectacular case). There is also an attempt to find a compromise (van Benthem 1994, p. 135): classical logic always becomes a special case of alternative logic under definite idealised conditions. For instance, non-monotonic logic becomes monotonic in the case when perfect information is available. This situation can motivate conventionalism in defining logic: let us call 'logic' what serves as logic according to its practical

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applications. Of course, there is no reason to fight about words. Any claim that we should abandon terms like ‘second-order logic’ or ‘modal logic’ is obviously wrong. On the other hand, the properties of logic are not conventional. Logics are complete, decidable, compact, etc. or not. For example, the priority of classical logic is sometimes argued by pointing out that some properties of non-classical logic are provable only classically. It is particularly well illustrated by the case of intuitionistic logic and the question whether the completeness theorem for this logic is intuitionistically provable. Although the answer is not clear because of notorious doubts concerning the limits of methods which are intuitionistically or constructively admissible, we have at least a straigthforward way of discussing this problem.

The end of the previous paragraph suggests that metalogic is important for the question of defining logic. Moreover, the equivalence ‘A is logical if and only if it belongs to a logical systems’ does not solve the problem of the scope of logic. The reason is that it is possible that the adjective ‘logical’ as applied to single sentences refers to other properties than expressed by the same word when attributed to whole systems. For example, the completeness property is a feature of logical systems, but not particular formulas. It is a reason to look at metalogical properties as guide in defining logic. Our question is mainly a philosophical issue and we have several intuitions, which are commonly accepted as marks of essential properties of logic. According to common views, logic is formal, universal or topic-neutral, and provides sound (leading always from truths to other truths) rules of inference. We also expect that logical theorems are justified in a simple, mechanically checkable manner. It is possible to connect well — established metalogical properties of logic with intuitively accepted features of logical systems (see Woleński 1999). For instance, the completeness theorem assures, when it holds for a given logic, that the system in question proves all its tautologies and does not proceed from true to false sentences. Since, as I will argue later, this theorem leads to an important understanding of the universality of logic traditionally regarded as an essential property of logic, metalogic has a considerable philosophical merit. In fact, exact metalogical results should be considered nowadays as the main source of insights about logic. Thus, the traditional approach to logic based on some intuitions derived from the practice of doing and applying logic is to be supplemented (not replaced!) by looking at logic from the metalogical perspective. It corresponds to the change of the centre of logical gravity from logic itself to metalogic. My main task in further remarks is to show that metalogic suggests degrees of being logical even within first-order logic.

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§2. Definition of logic via the consequence operation. Having the consequence operation (Cn), that is, a mapping from $2^{\boldsymbol{L}}$ to $2^{\boldsymbol{L}}$ ($\boldsymbol{L}$ is a language understood as a set of formulas), or the consequence relation ($\vdash$) which is a subset of the set $2^{\boldsymbol{L}} \times \boldsymbol{L}$, we can define logic as the set of sentences provable from the empty set (Surma 1981; Cleave 1991, p. 76; Surma 1994; Woleński 1999). Formally speaking (I prefer the way *via* Cn; see Cleave 1991 and Gabbay 1994 for approaches *via* the consequence operator, and Hacking 1979 or Segerberg 1982 for related considerations)

(DL1) $\quad A \in \boldsymbol{LOG} \iff A \in Cn\,\emptyset$ or, equivalently, $\boldsymbol{LOG} = Cn\,\emptyset$.

An obvious virtue of (DL1) is that it defines logic as something independent of particular assumptions, which agrees with a wellestablished intuition that logic is universal. However, this definition depends on the accepted properties of Cn. The matter is important also because we have infinitely (in fact, uncountably) many mappings that transform $2^{\boldsymbol{L}}$ into $2^{\boldsymbol{L}}$, assuming that $\boldsymbol{L}$ is infinitely denumerable. Tarski characterised Cn axiomatically (the axiom set given below is adapted from Borkowski 1991). The axioms are as follows (the symbol '**fin**' abbreviates 'finite' as applied to sets):

(C1) $\quad 0 \leqslant |\boldsymbol{L}| \leqslant \aleph_0$

(C2) $\quad X \subseteq Cn\,X$

(C3) $\quad X \subseteq Y \;\Rightarrow\; Cn\,X \subseteq Cn\,Y$

(C4) $\quad Cn\,Cn\,X = Cn\,X$

(C5) $\quad A \in Cn\,X \;\Rightarrow (\exists Y \subseteq X \wedge Y \in \mathbf{fin})(A \in Cn\,Y)$

(C6) $\quad B \in Cn(X \cup \{A\}) \;\Rightarrow\; (A \to B) \in Cn\,X$

(C7) $\quad (A \to B) \in Cn\,X \;\Rightarrow\; B \in Cn(X \cup \{A\})$

(C8) $\quad Cn\{A, \neg A\} = \boldsymbol{L}$

(C9) $\quad Cn\{A\} \cap Cn\{\neg A\} = \emptyset$

(C10) $\quad A(v/t) \in Cn\{\forall v A(v)\}$, if the term t is suitable for v.

(C11) $\quad A \in Cn\,X \Rightarrow \forall v A(v) \in Cn\,X$, if v is not free in X, for every $B \in X$.

The axioms (C1)–(C5) establish general properties of Cn: the cardinality of $\boldsymbol{L}$ (C1), belonging of any set to the set of its consequence, (C2), monotonicity (C3), idempotence (C4), and finiteness (C5). The general axioms do not generate any logic; these conditions (or reduced, for example, by skipping monotonicity or finiteness) are often regarded as providing abstract logic(s), prelogic(s) or protologic(s). The logical, more precisely, inferential

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machinery is given by the remaining axioms. Of course, (C6)–(C11) are related to classical first-order logic (without identity, but adding this predicate does not change anything in my further considerations) based on negation, implication and the universal quantifier as primitive terms. Moreover, the formalization is Hilbertian, that is, by schemata. It is easy to generate other logics, alternatives as well as extensions, in a similar way (see e.g. Pogorzelski 1994, pp. 190, 296 for intuitionistic and minimal logic).

Although (DL1) can be used in defining various logics, it does not solve the problem which logic is proper. Moreover, this definition looks as arbitrary to some extent, because it seems that the most essential features of logic are related to selected conditions attributed to Cn. Clearly, the logical machinery represented by the consequence operation, in particular, the deduction theorem, is responsible that we can derive something from the empty set; hence, the empty set serves in (DL1) as a convenient metaphor. Therefore, we cannot avoid the problem whether that or other stipulation or constraint concerning the consequence operation proper for logic(s). I will not enter into the discussion of the first-order thesis, which I defended elsewhere (Woleński 1994; Woleński 1999) or considerations whether modal extensions of classical logic (or its alternatives) are logics or not. Let me also leave general axioms without any further comments, although it should be always remembered that they are motivated by obvious analogies derived from topology.

An additional motivation for (DL1) is provided by the fact that it is equivalent to other statements, in the particular following (Surma 1981):

(DL2) $A \in \boldsymbol{LOG}$ if and only if $\neg A$ is inconsistent.

(DL3) $\boldsymbol{LOG}$ is the only non-empty intersection of all deductive systems.

Both (DL2) and (DL3) express properties which are surely desirable for any logic, because we expect that negations of logical principles are inconsistencies and that logic belongs to any system, that is, deductively closed set of sentences. Moreover, (DL3) entails that logical laws are derivable from arbitrary premises. This last circumstance shows more deeply the sense in which logic is universal.

The characterisation of logic by (DL1) is purely syntactical because the concept of consequence operation belongs to the vocabulary of syntax. Another definition of logic is provided by semantics and is expressed by the statement

(DL4) $A \in \boldsymbol{LOG}$ if and only if for every model $\mathfrak{M}$, A is true in $\mathfrak{M}$.

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The account of logic given by (DL4) sees logic as universal for truth (validity) of its theorem in every model (domain) which displays another aspect of the universality of logic, namely its topical neutrality. Now we can expect a link between (DL1)–(DL3) on one hand, and (DL4) on other hand. The connection in question is established by the completeness theorem which, in the terminology of this paper, is given by

(CT) $A \in Cn\,\emptyset$ if and only if for every model $\mathfrak{M}$, A is true in $\mathfrak{M}$.

Although (CT) does not provide in itself any definition of logic, it is easy to see how and why it is important for the philosophy of logic. It is so because the completeness theorem formally expresses the equivalence of two senses of the universality of logic. Parenthetically speaking, (CT) in its fully general form does not hold for higher-order logic and modal logic. These systems require some additional, essentially extralogical constraints on models. The case is represented, for example by the distinction between general and normal models for the Henkin proof of the completeness of second-order logic or definite conditions for the accessiblity relation in modal frames, except the system **K**, which for this reason serves as the basic normal modal logic. It provides a strong motivation for any logic for which (CT) holds in its unrestricted form. However, the fact that metatheory, necessary for proving metalogical properties (or their lack) of alternative logics, is based on classical first-order logic strongly motivates this system as *the* logic. Anyway, my further remarks are restricted to classical logic.

§3. Are there degrees of being logical? The former considerations suggest that the property expressed by the predicate 'being logical' is formally articulated by (DL1)–(DL3) and their equivalence with (DL4). It is correct but we can go further in characterising being logical. In order to do it let us consider two other metalogical properties, namely Post-completeness and decidability, assuming (C1)–(C5) as abstract properties of any logical consequence operation. A system $\boldsymbol{S}$ is Post-complete if and only if $Cn(\boldsymbol{S} \cup \{A\}) = \boldsymbol{L}$, for any formula A which is not a theorem of $\boldsymbol{S}$ (I neglect here some subtleties connected with various ways of formalizing logic depending on using concrete formulas or schemata and on the stock of rules; strictly speaking, the above definition applies to propositional logic with the rule of substitution, but there is also a corresponding property for systems presented by schemata). Further, $\boldsymbol{S}$ is decidable if and only if the set of its theorems is recursive.

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A natural stratification of first-order (elementary) logic is to divide it into propositional calculus and (first-order) predicate calculus. Both are consistent, semantically complete (that is, obey (CT)) and syntactically incomplete (a system $\boldsymbol{S}$ is syntactically incomplete if and only if there are the formulas A and $\neg A$ belonging to the language of $\boldsymbol{S}$, such that both are not provable in this system. We also have that for any formula of the language of propositional (predicate) logic, it is a tautology (theorem) or non-tautology (non-theorem). However, different results arise when non-tautologies are added to propositional calculus and when we add non-tautologies to predicate logic. The resulting system is inconsistent in the first case, but does not need to be such in the second. For instance, we can add a sentence, which asserts that there are n objects to predicate logic without producing its inconsistency. The reason is that propositional calculus is Post-complete, but predicate logic is not. Let us agree that that Post-completeness expresses an aspect of being logical. Hence, we can conclude that propositional calculus (defined by (C1)–(C9)) is logical in a stronger sense than predicate logic. Of course, this view is related to the considered metalogical property, but we have reasons to look at the systems becoming inconsistent by adding non-theorems to them as somehow more logical than the systems behaving otherwise. The same (see above) concerns decidability, because propositional calculus is decidable, but predicate logic is not. Still another difference consists in methods of proving (CT) for propositional calculus and predicate logic. First of all, (CT) for propositional calculus is equivalent to the theorem that this system is Post-complete. Moreover, both theorems are constructively provable even under the most severe understanding of constructivity (perhaps except the ultraintutionistic sense on which only "small" finite methods are admissible). A completely different situation holds for predicate logic. It is true that we can prove by constructive methods, that is, formalizable in the finitary syntax that (CT) is equivalent to the Gödel–Malcev theorem ($\boldsymbol{S}$ is consistent if and only if it has a model), which is another version of the completeness theorem. On the other hand, proofs of both forms of the completeness theorem require non-constructive methods. In fact, opposite views expressed for example in Grandy 1977, pp. 21–36 or Stekeler-Weithofer 1986, p. 520, that (CT) in predicate calculus has an effective proof are due to a too wide understanding of constructivity; this fact confirms the earlier mentioned vagueness concerning the scope of methods regarded as constructive). The indicated differences between both parts of elementary logic might be interpreted as degrees of being logical, because non-constructive proofs involve semantic ingredients exceeding purely syntactic devices. The latter are traditionally

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considered as the paradigm of logical methods. According to this interpretation, the property of being logical has, so to speak, a different intensity in both parts of elementary logic, greater in the case of propositional calculus and lesser in the case of predicate logic.

It seems that these degrees of being logical depend upon the relation between syntax and semantics in logical theories. The completeness theorem established the parity or equivalence between syntax and semantics in the case of first-order logic. However, this harmony is not equal in its concrete features in the whole domain of first-order logic. Decidability and Post-completeness, as metalogical properties, decide that semantics of propositional calculus is fully constructive and can be replaced without any remainder by the syntax of this system. In fact, truth-tables as devices of checking whether formulas are tautologies or not, admit a semantic as well as syntactic interpretation. On the other hand, the parity of syntax and semantics in predicate logic is not full in the sense that the latter is not replaceable by the former. Thus, although we know that every tautology (a sentence universally valid) is a theorem (is provable in predicate calculus), we have no general decision method for theoremhood. It shows that, beyond propositional calculus, semantics is somehow prior to syntax. The priority of semantics over syntax was commonly recognized as a general consequence of limitative theorems of Gödel (incompletenesss of arithmetic) and Tarski (undefinability of arithmetical truth in arithmetic). However, this phenomenon was rather related to arithmetic and its oversystems. The foregoing considerations show that the distinguished role of semantics appears just in the case of predicate logic, in spite of the completeness theorem, which suggests that syntax and semantics are al pari in elementary logic. In particular, semantic properties of the whole first-order logic are not fully expressible in its syntax. As an outcome of this observation we have that a purely syntactic characterization of logic is proper only in the case of propositional calculus.

Now taking incompleteness into account, we can identify further 'degrees of ...'. Although arithmetic shares undecidability with predicate logic, it is also incomplete and not finitary axiomatizable. In order to have its semantic completeness, we must supplement its inference rules by the ω–rule which operates on infinite sets of premises. This fact is at odds with the axiom (C5) for Cn, which implies that inference rules are finitary in the sense that they apply to inferences from a finite set of assumptions. The dots after the last occurrence of 'degree of' are introduced quite intentionally, because I do not regard arithmetic (or formal systems strong enough to express it) as a part of logic (see Woleński 1995; Woleński 1995a). In particular, I claim that

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logic ends where the parity, even not perfect, of syntax and semantics loses its validity. However, we can embed this new application of 'degree of' in a more general scheme. Let us agree that particular collections of metamathematical properties are associated with degrees of 'being formal'. Traditionally, logic is considered as a formal science, and mathematics is another member of this family. We can think about being logical as a special case of being formal. We can think about a hierarchy, which starts with propositional calculus, proceeds through predicate logic and reaches arithmetic (and the rest of pure mathematics). Two first points comprise being logical, although not in the same amount, and the third place is occupied by something, which is formal and extralogical. It is possible to diversify the third realm by other hierarchies, for instance, arithmetical or analytic. There is a clear connection between the traditional understanding of 'formal' as opposed to 'having content', and that proposed here. If we proceed from being more logical to being less logical, and the degree from logical to non-logical, the amount of being formal is lesser. It means that the contentual parameter becomes greater. I do not think that even propositional calculus is completely free of content, that is, purely formal in the traditional sense, because a certain amount of content, for example coming from the metatheory, appears in definitions of logical constants. However, propositional calculus is more formal than predicate logic in the sense that semantic properties of the former are fully explicable in its syntax. Thus the degree of being formal is measured according to the relation between syntax and semantics in a given theory. In a sense, if the degree of being formal increases, the amount of content decreases. And this is what we should expect.

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