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DEFINING ONTOLOGICAL CATEGORIES IN AN EXPANSION OF BELIEF DYNAMICS

There have been attempts to get some logic out of belief dynamics, i.e. attempts to define the constants of propositional logic in terms of functions from sets of beliefs to sets of beliefs.[1] It would be interesting to see whether something similar could be done for ontological categories, i.e. ontological constants. The theory presented here will be a (modest) expansion of belief dynamics: it will not only incorporate beliefs, but also parts of beliefs, so called *belief fragments*. On the basis of this we will give a belief-dynamical account of the ontological categories of states of affairs, individuals, properties of arbitrary adicities and properties of arbitrary orders.

1. The background: Belief fragments

The fundamental idea of this paper is that two beliefs can have something in common. In the following the concept of a belief will be taken as primitive. Beliefs are *intensional*, they are *about* something, either about the world or about other beliefs. Beliefs are the things you change when you change your mind. They stand in logical relations: they can imply one another, be consistent or inconsistent with one another. Beliefs are expressed by declarative sentences, but not only by such sentences. If I say to a group of people 'Would everybody who hadn't had lunch yet please raise their hands' and Hannah raises her hand this action of hers expresses her belief that she didn't

[1] See [3], chapter 6.

have lunch yet, as would have done her uttering the declarative 'I didn't have lunch yet'. It is important to note that there is no shared syntactic feature between these two expressions of the same belief.[2] Beliefs must therefore be understood non-syntactically, given that they can be represented in forms which do not share syntactic elements and even in forms where it does not seem to make sense to talk about syntactic elements (as in the case of Hannah's raising her hand).

Now it seems to be clear that beliefs can have something in common. If I believe that Hannah is hungry and you believe that Peter is hungry our two beliefs have something in common. Most obviously, they are both about somebody being hungry. Let us call whatever it is they have in common, which is no entire belief but rather a *part* of a belief a *belief fragment*. Consider the following two beliefs:

1. *The Duke of Edinburgh is over fifty.*
2. *The American President is over fifty.*

What do these two beliefs have in common? They have something in common which isn't itself a belief, but rather a part of a belief, a belief fragment: this is IS OVER FIFTY. It is important to note that IS OVER FIFTY isn't the same as 'is over fifty' — we are not talking about syntax here (otherwise we would be restricted to forms of representing beliefs where it made sense to talk about syntax). To see this consider a third belief:

3. *The Chancellor of Cambridge University is male.*

The syntactic elements which the *sentences* 1 and 3 have in common ('the', 'of', 'is') are not the belief fragments which the two *beliefs* 1 and 3 have in common. What the beliefs have in common is that they are both about a certain person, namely about Prince Phillip. The belief fragment PRINCE PHILLIP which they have in common is not identical any syntactical element which the English formulations of the two beliefs given above have in common. We will want to say that for every object x there is some fragment such that this fragment is part of any belief expressed by a sentence referring to x.

Note that we are not assuming that beliefs have any *particular* structure. They will have *some* structure, but for our purposes it is irrelevant what this is. The sentences given above which express beliefs 1–3 happen to share

[2] A similar problem arises when we try to characterize propositions. For an attempt to deal with this problem by considering the implication relation between properties which shares something of the spirit of this essay see [5].

some syntactic elements. This is due to the way we have chosen to express them. But of course we could have expressed them in all sorts of ways (by formulating them in different languages, by using pictures or diagrams), including ways in which they did not have any syntactic features in common. This, however, would not change the fact that the *beliefs expressed* in the different ways share a common belief fragment.

We are not specifying what kind of entity a fragment is, whether it is linguistic, psychological, material (a brain state, say), functional or whatever. All we claim here is that if beliefs are supposed to be entities of kind X, belief fragments should be considered to be parts of entities of kind X.[3]

There are beliefs which have more than one thing in common. Take for example the following two beliefs:

4. *The apple and the banana are on the table.*
5. *The apple and the banana are both to the left of the vase.*

4 and 5 have two belief fragments in common, namely THE APPLE and THE BANANA. One might be tempted to treat them just as a single fragment THE APPLE AND THE BANANA. But such a procedure cannot be generalized as can be seen from the following example:

6. *Peter washes his white shirt.*
7. *Charles washes his white car.*

Here WASHES and WHITE are fragments common to both 6 and 7 but we would not want to say that there is *one* fragment which somehow contains these two as parts which 6 and 7 have in common. For the sake of simplicity we are therefore going to disregard such cases in our inquiry and confine ourselves to beliefs which have at most a single fragment in common. There will be such beliefs since we assume that at least some belief fragments are atomic.

For our account we need two primitive notions. The first is the meet operation $\cap_B$. This takes two beliefs and returns the largest fragment which they have in common, else it returns $\emptyset$. Meeting can be iterated: $\cap_B$ can also return the ('smaller') single largest fragment which two fragments have in common, if there is one. And finally the meet operation can take a belief and a belief fragment and return the smaller fragment which the belief and the belief fragment have in common. We will write B_i, B_j, ... for beliefs and $B_i^\sharp$, $B_j^\sharp$, ... for belief fragments.

[3] Cf. [4], 7–9.

The second primitive notion is the two-place $\boxplus$ operation which allows us to expand fragments and thus to build complexes out of fragments. The expansion operation takes two belief fragments $B^\sharp_i$, $B^\sharp_j$ and returns either

(a) a belief B_n or

(b) another belief fragment $B^\sharp_m$ or

(c) the set $\{B^\sharp_i, B^\sharp_j\}$ of the two belief fragments,

depending on the nature of the fragments taken as arguments.

Let us consider an example for each possibility in turn. For the first case, let $B^\sharp_i$ be PIERO and $B^\sharp_j$ MARRIED LUCREZIA, then $B^\sharp_i \boxplus B^\sharp_j = B_n =$ PIERO MARRIED LUCREZIA.

For the second possibility, let $B^\sharp_i$ be IS MARRIED TO and $B^\sharp_j$ LORENZO, then $B^\sharp_i \boxplus B^\sharp_j = B_m =$ LORENZO IS MARRIED TO (or IS MARRIED TO LORENZO — see below).

For the final option let $B^\sharp_i$ again be IS MARRIED TO but $B^\sharp_j$ IS THE BROTHER OF. Clearly in this case there is no belief or belief fragment which can be constructed out of $B^\sharp_i$ and $B^\sharp_j$ so $\boxplus$ will return the set of the two as a default value.[4]

We suppose that expansion is commutative and associative, i.e.

(COMM) $$B^\sharp_i \boxplus B^\sharp_j = B^\sharp_j \boxplus B^\sharp_i$$ [5]

(ASS) $$(B^\sharp_i \boxplus B^\sharp_j) \boxplus B^\sharp_k = B^\sharp_i \boxplus (B^\sharp_j \boxplus B^\sharp_k)$$ [6]

As reader will have already noted from the example of $(B \boxplus B^\sharp_1)$, $\boxplus$ behaves quite differently from the expansion operator in standard belief dynamics.

Firstly, $(B^\sharp_i \boxplus B^\sharp_j) \boxplus B^\sharp_j$ will not be the same as $(B^\sharp_i \boxplus B^\sharp_j)$, i.e. expanding a fragment twice by another fragment will not give the same result as doing it only once (as is true for standard expansion where $(A + \alpha) + \alpha = A + \alpha$).

[4] We could plausibly extend $\boxplus$ by allowing it to take a belief fragment and a belief and return a belief fragment. For example, it could take the belief PIERO IS MARRIED TO LUCREZIA and expand it by the fragment ENTAILS THAT. This would then return the belief fragment ENTAILS THAT PIERO IS MARRIED TO LUCREZIA (or PIERO IS MARRIED TO LUCREZIA ENTAILS THAT).

[5] This does not conflict with the fact that LORENZO IS MARRRIED $\neq$ IS MARRIED TO LORENZO. The function γ (see below) will always select one of these, but which is selected is independent of the order in which the arguments are put toghether.

[6] In general iteration of $\boxplus$ will presuppose that the value of the previous application is a belief fragment as well, and not a belief or a set of belief fragments.

For example, expanding the above fragment IS MARRIED TO *twice* by the fragment LORENZO gives LORENZO IS MARRIED TO LORENZO.

Secondly, in the standard case the expansion of a set of beliefs will always deliver a unique object of the same kind, namely a (now expanded) set of beliefs. But with our expansion operator it will not always be the case that an expansion of some fragment will yield some unique new belief fragment (or belief). Suppose some fragment corresponds to the dyadic property 'adores'. If we want to expand this by the belief fragment corresponding to 'Dante', it is clear that we get another belief fragment, namely one corresponding to a monadic property. But it is not clear what this property is; it may be either 'Dante adores' or 'adores Dante'. This point generalizes: for every fragment corresponding to an n-adic property (where $n \geqslant 2$) which is expanded by a belief fragment there are n different possible results.

In order to give a systematic account of our notion of expansion, there are now two possibilities. We could either say that expanding a fragment corresponding to an n-adic property yields a set Γ containing the different possibilities, so that in the above case the result of expanding ADORES by DANTE would be {ADORES DANTE, DANTE ADORES}. Alternatively we could define the expansion in terms of a function $\gamma(\Gamma)$ which selects exactly one member of Γ.

The problem with the first construction is that it brings in an unwelcome asymmetry between e.g. fragments corresponding to 'naturally' monadic properties and those corresponding to monadic properties which were produced from n-adic properties by 'filling in' $n-1$ places. Obviously they will behave differently under expansion: the expansion of the naturally monadic predicates by the appropriate fragments will yield a belief, while the expansion of the formerly n-adic properties will yield a *set* of beliefs. The expansion of IS DEAD by BEATRICE will just give the belief BEATRICE IS DEAD, while expanding the result of expanding ADORES by DANTE by BEATRICE gives the set {DANTE ADORES BEATRICE, BEATRICE ADORES DANTE}.

We will therefore use the second construction. We assume that there is a function γ which selects exactly one of the possible outcomes of the expansion in Γ (if there is more than one), else γ just returns the only member of Γ.

2. The program: Which fragment is which?

As we saw above, $\cap_B$ will supply us with various kinds of belief fragments. What we want to try to do is to group the fragments according to the ontological categories they correspond to. There is a fairly natural way in which beliefs correspond to states of affairs so that in a similar way fragments of beliefs should correspond to fragments of states of affairs (i.e. properties and individuals). Is there a way of telling which groups of fragments correspond to which categories?

We will look at an attempt of grouping fragments by their behaviour under expansion. Clearly not every fragment can be used to expand every other fragment: if $B^\sharp_i$ is IS YELLOW and $B^\sharp_j$ is IS BIGGER THAN expanding the one by the other will not be a belief, nor will it be a belief fragment, i.e. there is no belief B or fragment $B^\sharp$ such that $B^\sharp_i \boxplus B^\sharp_j = B$ or $B^\sharp_i \boxplus B^\sharp_j = B^\sharp$.

2.1. Adicity

One way in which this can be exploited is in defining different adicities of properties. Suppose we have two properties $B^\sharp_i$ and $B^\sharp_j$ such that $B^\sharp_i \boxplus B^\sharp_j = B^\sharp_k$ (i.e. expanding the one by the other gives another belief fragment) while $B^\sharp_k \boxplus B^\sharp_i = B$ (i.e. expanding the result by the first fragment gives a belief). In this case the belief fragment $B^\sharp_j$ must correspond to a dyadic property. (Note that this doesn't entail that $B^\sharp_i$ must correspond to an individual — $B^\sharp_j$ could correspond to a property of order n and $B^\sharp_i$ to one of order $n-1$.) For example, $B^\sharp_i$ could be PETER and $B^\sharp_j$ IS MARRIED TO so that expanding $B^\sharp_j$ by $B^\sharp_i$ gives the belief fragment PETER IS MARRIED TO (or IS MARRIED TO PETER) (thus a fragment corresponding to a monadic property), while another expansion by $B^\sharp_i$ gives the belief (corresponding to a state of affairs) PETER IS MARRIED TO PETER). In another possible interpretation $B^\sharp_i$ could be BIGGER THAN and $B^\sharp_j$ IS THE INVERSE OF, so that $B^\sharp_i$ would not correspond to an individual.

We can generalize this by introducing the notion of *n-copy-saturability*. We say that some belief fragment $B^\sharp_i$ is n-copy-saturable ($n$cs) by some belief fragment $B^\sharp_j$ iff there is some belief B_k such that $(B^\sharp_j \boxplus \cdots \boxplus (B^\sharp_j \boxplus (B^\sharp_j \boxplus B^\sharp_i))) = B_k$, where $\boxplus$ occurs n times on the left-hand-side of the equation.

Thus the notion of n-copy-saturability, which says that n expansions by some other belief fragment make the expanded belief fragment into a belief allows us to group belief fragments into classes. The class of dyadic properties will correspond to the class of fragments which are 2cs by some $B^\sharp_m$, the class of triadic properties to the class of fragments which is 3cs by some $B^\sharp_k$ and in general the class of i-adic properties will correspond to those fragments which are $i$cs by some $B^\sharp_l$.

While the notion of copy-saturability does the trick for all adicities greater than two, it fails for the monadic case. In this case the asymmetry between the expanded and the expander will break down since $B_i^\sharp$ will be 1cs by $B_j^\sharp$ and also $B_j^\sharp$ will be 1cs by $B_i^\sharp$. So the class of fragments which are 1cs by some $B_i^\sharp$ will *not* correspond to the class of monadic properties since it will e.g. also contain individuals.

The above procedure also fails to give us a way of distinguishing between different orders of properties of one adicity. If some $B_i^\sharp$ is 2cs by some $B_j^\sharp$ we can be sure that $B_i^\sharp$ is dyadic, but we cannot know what the order of either $B_i^\sharp$ or of $B_j^\sharp$ is, since it could be the case that $B_i^\sharp$ corresponds to a first-order dyadic property (which we will in the future abbreviate as P_2^1) and $B_j^\sharp$ an individual (abbreviated as i), or that $B_i^\sharp = P_2^2$ and $B_j^\sharp = P_n^1$ or $B_i^\sharp = P_2^3$ and $B_j^\sharp = P_m^2$ and so on.

So we need some way of distinguishing first-order properties from individuals and also a method for distinguishing orders of properties. Once we have a notion of an individual, the definition of orders of properties is easy.

Before proceeding to this discussion, however, let us discuss a couple of properties of belief fragments which can be characterized in this framework. The above introduction of the selection function γ gives us a quite natural way of characterizing symmetry.

We can say that a belief fragment $B_i^\sharp$ corresponding to a dyadic property corresponds to a *symmetric* property iff for any two distinct fragments $B_j^\sharp$, $B_k^\sharp$ such that repeatedly expanding $B_i^\sharp$ by them gives some belief B, what B is does not in any way depend on γ.

This corresponds to the idea that what we mean by saying that some property (such as 'married') is symmetric is that 'Romeo married Juliet', 'Juliet married Romeo', 'Juliet and Romeo are married', 'Romeo and Juliet are married', 'A marriage took place between Romeo and Juliet' ... are all the same belief. So iff some fragment corresponds to a symmetric property it makes no difference which of the different possibilities (i.e. members of Γ) γ picks, since these possibilities really just all amount to the same.

We will say that a fragment $B_i^\sharp$ corresponding to a dyadic property corresponds to a *reflexive* property iff for some fragments $B_j^\sharp$, $B_k^\sharp$ and some set of beliefs $\mathfrak{B}$, $((B_i^\sharp \boxplus B_j^\sharp) \boxplus B_k^\sharp) \cup \mathfrak{B} = ((((B_i^\sharp \boxplus B_j^\sharp) \boxplus B_k^\sharp) \boxplus ((B_i^\sharp \boxplus B_j^\sharp) \boxplus B_j^\sharp)) \boxplus ((B_i^\sharp \boxplus B_k^\sharp) \boxplus B_k^\sharp)) \cup \mathfrak{B}$. This characterization depends on the fact that $\mathfrak{B}$ is logically closed so that given we expand it by some belief aPb, when P

is reflexive we get the very same result if we afterwards *also* expand by aPa and bPb.

Transitivity is a bit more complicated to characterize. Here we have to make our characterization relative to the structure of γ. Suppose γ selects the expansion which fills in the first place (i.e. iff the dyadic P is expanded by some a, γ selects aP rather than Pa from the set Γ of possibilities.). We can then say that a fragment $B_i^\sharp$ corresponding to a dyadic property corresponds to a transitive property iff for some fragments $B_j^\sharp$, $B_k^\sharp$ and $B_l^\sharp$ and some set of beliefs $\mathfrak{B}$ $(((B_i^\sharp \boxplus B_j^\sharp) \boxplus B_k^\sharp) \boxplus ((B_i^\sharp \boxplus B_k^\sharp) \boxplus B_l^\sharp)) \boxplus \mathfrak{B} = (((B_i^\sharp \boxplus B_j^\sharp) \boxplus B_k^\sharp) \boxplus ((B_i^\sharp \boxplus B_k^\sharp) \boxplus B_l^\sharp) \boxplus ((B_i^\sharp \boxplus B_j^\sharp) \boxplus B_l^\sharp)) \boxplus \mathfrak{B}$. Again this characterization depends on the fact that $\mathfrak{B}$ is logically closed so that given we expand it by some beliefs aPb and bPc when P is transitive we get the very same result if we afterwards *also* expand by aPc.

2.2. Order

To give an account of the different orders of properties different belief fragments correspond to we start from the class of fragments which are 1cs by some $B_i^\sharp$, i.e. the class $\mathcal{G}$ containing fragments corresponding to i, P_1^1, P_1^2, ...

We then split $\mathcal{G}$ into partitions $G_1, \ldots, G_n$ which fulfill the following properties:

- For no two members from any G_i will the expansion of the one by the other yield a belief.
- If for some $B_i^\sharp$ in some G_j and some some $B_k^\sharp$ in some distinct G_l it is the case that $B_i^\sharp \boxplus B_k^\sharp$ yields a belief this is also true for all other members of G_j and G_l. (Call G_j and G_l *corresponding classes* in this case). For every G_i there is exactly one corresponding class.

Such a partitioning ensures that every order of properties is contained wholly and purely in some partition of $\mathcal{G}$. We just don't know which partition contains which.

We will now proceed by considering the following construction: Take some $B_i^\sharp \in \mathcal{G}$ and select some distinct $B_j^\sharp \in \mathcal{G}$ such that $B_i^\sharp \boxplus B_j^\sharp$ yields a belief. Now take $B_j^\sharp$ and select another $B_k^\sharp$ (if possible from a partition different from the one of which $B_i^\sharp$ is a member) such that $B_j^\sharp \boxplus B_k^\sharp$ yields a belief. Then consider $B_k^\sharp$ and so on. We call such a sequence an *expansion sequence* which can be written in the following way:

$$\begin{array}{llllll}
B_i^\sharp \overset{\boxplus}{\hookleftarrow} B_j^\sharp & = B_m & & & & \\
\quad\quad\;\; | & & & & & \\
\quad\quad\; B_j^\sharp \overset{\boxplus}{\hookleftarrow} B_k^\sharp & & = B_n & & & \\
\quad\quad\quad\quad\quad\;\; | & & & & & \\
\quad\quad\quad\quad\quad B_k^\sharp \overset{\boxplus}{\hookleftarrow} B_l^\sharp & & & = B_o & & \\
\quad\quad\quad\quad\quad\quad\quad\vdots & & & & &
\end{array}$$

(Here the arrow indicates that the fragment on the left of the arrow is expanded by the fragment on the right in the respective step and yields the belief to the right of the equation sign.)

If we take a concrete example and let $B_i^\sharp$ be P_1^2 the expansion sequence looks like this:

$$\begin{array}{llllll}
P_1^2 \overset{\boxplus}{\hookleftarrow} P_1^1 & = B_m & & & & \\
\quad\quad\;\; | & & & & & \\
\quad\quad\; P_1^1 \overset{\boxplus}{\hookleftarrow} i & & = B_n & & & \\
\quad\quad\quad\quad\quad\;\; | & & & & & \\
\quad\quad\quad\quad\quad i \overset{\boxplus}{\hookleftarrow} P_1^1 & & & = B_o & & \\
\quad\quad\quad\quad\quad\quad\quad\vdots & & & & &
\end{array}$$

Remember that what we want to do is finding the partition containing those and only those belief fragments corresponding to individuals. Since the expansion sequences do not end, we cannot in any way identify i as the expansion in the last step of such a sequence. But if we look at the above example we see that the expansion which corresponds to i is *the only expansion both immediately preceded and immediately followed by expansions which come from the same partition*. In the third step in this sequence it wasn't possible to choose the fragment which expands from a *new* partition (as we demanded above, where $B_i^\sharp$ and $B_k^\sharp$ were supposed to come from different partitions). In fact we had to select the fragment which expands in step three (namely P_1^1) from *the very same partition* from which we selected the fragment which expanded in step one.

We can thus define the partition of belief fragments corresponding to individuals in the following way:

> it is the class of all those fragments $B_n^\sharp$ which are used in expansion sequences such that $B_{n-1}^\sharp$ (the expansion immediately preceding it) and $B_{n+1}^\sharp$ (the expansion immediately following it) are elements of the same $G_k \in \mathcal{G}$ if there is such a $B_n^\sharp$
>
> or, if there is none in some expansion sequence it is the first member of that sequence (in this case the belief to be expanded corresponds to a first-order property).

Thus given that we know which partition contains the belief fragments corresponding to individuals, it is easy to give a recursive way of telling which partitions contain fragments corresponding to properties of which order. Suppose $B_n^\sharp$ corresponds to an individual. Then some $G_k \in \mathcal{G}$ contains all and only the first-order properties iff for every $B_m^\sharp \in G_k$ there is some belief B_1 such that $B_n^\sharp \boxplus B_m^\sharp = B_1$. Some $G_l \in \mathcal{G}$ contains all and only the second-order properties iff for every $B_o^\sharp \in G_l$ there is some belief B_2 such that $B_m^\sharp \boxplus B_o^\sharp = B_2$ and so on. It is then obvious how to define fragments corresponding to higher-order properties of adicities > 1.

3. Philosophical reflections

It should be noted that although the above characterization relies on differences in the possibilities of putting together fragments to form beliefs this does not mean that we have to assume that some class of fragments is somehow 'complete' while the other is incomplete and cannot stand on its own. This line is taken by Strawson who argues that

> A subject-expression is one which, in a sense, presents a fact in its own right and is to that extent complete. A predicate expression is one which in no sense presents a fact in its own right and is to that extent incomplete. [...] The predicate-expression, on the new criterion is one that can be completed only by explicit coupling with another. [...] We find an additional depth in Frege's metaphor of saturated and the unsaturated constituents. [6, p. 187–188]

On our account none of the fragments is able to represent a state of affairs (not even in 'a certain sense') — this is something which only beliefs can do. And although we consider belief fragments to be incomplete (to the extent to which they are incomplete *beliefs*) this does not mean that they cannot represent as they are; as we have seen above we can make sense of some of them representing individuals while others represent properties of different kinds.

The ontological categories we considered above — states of affairs, individuals, properties of arbitrary orders and adicities — are certainly not all the ontological categories there are. Just think of categories as abstract object, event, mathematical structure, trope or material object. All of these are categories which are as legitimate objects of ontological discussion as the ones we considered. Nevertheless, the latter seem to be distinguished by their *universality* as well as by their *coherence*.

They are universal because relative to every other ontological category there seem to be states of affairs, individuals and properties. The set containing the number seven (an abstract object) is an *individual*, amongst others it has the *property* of having just one member and it is a *state of affairs* that it has this property. The Seven Years' War (an event) is an individual, it has the property of having lasted for seven years and it is a state of affairs that it has lasted for seven years, and so on.

These ontological categories have a particular coherence because they can all be constructed from a rather confined basis. It is possible to account for all of them just in terms of states of affairs and individuals. This is done by employing a procedure due to Ajduckiewizc [1] (which was later developed in [2]) for giving a theory of grammatical categories. We use a primitive functor (intuitively interpreted as 'takes __ and returns ...') and write the complex ontological categories as fractions. Monadic first order properties are defined as $\frac{i}{s}$ (because they take an individual and return a state of affairs), while dyadic first order properties are $\frac{i,i}{s}$ and so forth. Monadic second order properties are taken to be of the form $\frac{\frac{i}{s}}{s}$; and in a similar way all the ontological categories of the different adicities and orders can be constructed. This constructional coherence does not seem to be possessed by other sets of ontological categories. The ensemble of states of affairs, individuals and properties of different adicities and orders which can be defined in a belief-dynamical framework therefore seems to occupy a special place in the system of ontological categories.

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