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CONFLICT WITHOUT CONTRADICTION: paraconsistency and axiomatizable conflict toleration hierarchies in Evidence Logic

Overview

Evidence Logic (EL) goes beyond Classical Logic (CL) in its primitive expressivity by including both confirmatory and refutatory predications, additionally equipped with evidence level annotations. Previous work has characterized the Boolean Sentence Algebras (BSAs) of the monadic, functional, and undecidable varieties of EL [4], [5]. From the perspective that our knowledge of the world is often less-than-certain, that is to say "evidential", application-wise EL is conceptually antecedent to CL and provides a broad foundational framework wherein axiomatizable extensions reach out to a number of the more domain-specific recent constructions of logics for the representation and processing of uncertainty in Artificial Intelligence (AI). In this paper we analyze EL from this point of view in sections 1 and 2. In Section 3 the relationship between this work and issues in paraconsistency is briefly explored.

For any $n > 1$ let $\mathrm{E}_n = \{e_i = \frac{i}{n-1} : i = 1, \ldots, n-1\}$, with smallest evidence value and evidence increment $\varepsilon = e_1$, be an Evidence Space of evidence annotations. Focusing on notions related to the degree of evidential conflict a theory may permit, we construct three hierarchies of axiomatizable extensions of EL and characterize the BSAs of the theories they entail. Let d, $d \in \{1, \ldots, n\}$, denote the "degree of conflict toleration of confirmatory and refutatory evidence". In the paper we characterize the monadic, func-

tional, and undecidable BSAs across all three hierarchies (see [9]). Here, let us informally state the results for monadic similarity types μ stipulating p proposition symbols, k constant symbols, an u unary predicate symbols. Then each such theory, it turns out, has ordered basis of order type

$$\omega^{m^u} \cdot m^p \cdot \sum_{i=1}^{k} s_{ki} \cdot m^{ui}$$

where the s_{ki} are Sterling Numbers of the second kind, and where m depends on which of the three conflict toleration schemes is chosen, the size $n-1$ of the Evidence Space E_n, and the degree d of conflict toleration permitted in the theory.

Roughly, the d^{th} logic in the first hierarchy will tolerate (and hence represent and process) conflict of evidence only so long as the minimum of the confirmatory and refutatory evidence is at most $(d-1)\varepsilon$. This hierarchy of increasingly permissive logics begins with a logic allowing no conflict at all and ends with the maximally permissive logic EL. In fact, these logics can be viewed as logics generalizing ideas of Aristotle concerning the nature of privatives. The d^{th} monadic logic here has BSA with ordered basis as above with $m = d(2n-d)$.

The d^{th} logic in the second hierarchy will tolerate conflict only so long as the sum of the confirmatory and refutatory evidence is at most $1 + (d-1)\varepsilon$. This hierarchy is one of increasingly permissive Dempster-Schafer type logics (see [1], [2], [13] and [15]), beginning with a logic modeling part of the Dempster-Shafer logic itself and ending with EL. The d^{th} monadic logic here has BSA with ordered basis as above with $m = (n(n+1)+(d-1)(2n-d))/2$.

The d^{th} logic in the third hierarchy will tolerate conflict only so long as the sum of the confirmatory and refutatory evidence is at most $(d-1)\varepsilon$. This is a hierarchy of increasingly permissive sub-Dempster-Shafer type logics, beginning with a trivialized logic effectively containing only equality and constants and ending with the logic modeling part of the Dempster-Shafer logic. Here the d^{th} monadic logic has BSA with ordered basis as above with $m = d(d+1)/2$.

For a brief description of EL the reader is referred to the abstract [5]. For the broader context of EL and for the precise construction of EL and the characterization of the various Boolean Sentence Algebras (BSAs) of EL for monadic, functional, and undecidable languages see [4]. Also, in [8] the concept of negation, in a number of ways the most fundamental and problematic concept in the representation and processing of evidential knowledge, was explored making use of the machinery of EL. The two logics in [6] and

[7] together with the three conflict toleration hierarchies of logics defined and characterized in the present paper, provide some insight into the relation between current work in the knowledge representation and processing problem area of AI and the machinery built into EL. In fact, the characterization of the BSAs of these logics in EL provides, to the extent these logics model some of the mosaic of the constructions in current AI work, some insight into the structure of these AI logics and the relationships between them.

1. The construction of EL

To begin, let us briefly review the construction of EL (for details see [4]). For each integer $n > 1$, let the Evidence Space of size $n - 1$ be the linear order E_n as defined above. For each $n > 1$ and each logical similarity type τ, the Evidence Logic $\mathrm{EL}_{n,\tau}$ is equipped with both confirmatory and refutatory predicate symbols R_c and R_r for each $\tau(i)$-ary predicate, as well as an Evidence Space E_n of evidence annotations for atomic formulas, while added to a usual set of logical axioms are axioms which ensure that "stronger evidence strictly entails weaker evidence"; also, models of $\mathrm{EL}_{n,\tau}$ are similarly equipped, providing annotated confirmatory and refutatory relations interpreting each $\tau(i)$-ary predicate. Note that trivializing all refutatory predicates yields an evidence logic which is, like Classical Logic, purely confirmatory; we refer to this logic as Confirmatory Evidence Logic $\mathrm{CEL}_{n,\tau}$. Further, $\mathrm{EL}_{2,\tau}$ may be viewed as an Absolute Evidence Logic AEL_τ, while $\mathrm{CEL}_{2,\tau}$ is both confirmatory and absolute and is exactly the Classical Logic CL_τ.

Let μ be *monadic*, stipulating p propositions, k constants, and u unary predicates; let μ' be *functional*, obtained by adding to μ the stipulation of one unary function; and let ν be *undecidable*, stipulating a finite number of predicates/functions including at least one predicate or function which is at least binary or at least two unary functions. For a theory T, let $\mathrm{BSA}(T)$ be the Boolean Sentence Algebra of T, let $\mathrm{BA}(\alpha)$ be the Boolean Algebra with ordered basis of order type α, and let $\cong$ denote "recursive isomorphism".

In [4], it is proven that for monadic μ as stipulated above,

$$\mathrm{BSA}(\mathrm{EL}_{n,\mu}) \;\cong\; \mathrm{BA}(\omega^{n^{2u}} \cdot n^{2p} \cdot \sum_{i=1}^{k} s_{ki} \cdot n^{2ui}),$$

while the functional varieties of EL are all recursively isomorphic to the functional variety of CL and the undecidable varieties of EL are all recursively isomorphic to the universal Classical Logic $\mathrm{CL}_{\langle 2\rangle}$ with one binary relation. All of these results are proven making heavy use of the substantial machinery developed by Bill Hanf and Dale Myers to study the recursive isomorphism types of languages of Classical Logic (see [3]). For example, in proving in [4] that the undecidable variety $\mathrm{EL}_{n,\nu}$ of EL is recursively isomorphic to $\mathrm{CL}_{\langle 2\rangle}$, a CL language with many (in fact, exactly $2(n-1)$) binary predicates is used to interpret $\mathrm{EL}_{n,\nu}$ in CL, and then this larger language is interpreted in $\mathrm{CL}_{\langle 2\rangle}$; the smoothness with which all these interpretations proceed results from a heavy use of the Hanf-Myers machinery which allows one to argue comfortably about BSAs using the dual topological spaces of models of the BSAs and, for example, Ehrenfeucht-Fraisse games to show elementary equivalence. Of course, since recursive BSA isomorphism preserves soundness and completeness, these results yield immediately and effectively the soundness and completeness of EL.

As elaborated in [4] (see also [6]), EL is constructed as a foundational logical framework which is conceptually antecedent to Classical Logic, and which may be appropriate as a logical framework for *evidential knowledge* in the same sense that Classical Logic provides such a framework for *absolute knowledge*. In that case, it will be among the various extensions of EL that many of the AI frameworks for the representation and processing of varieties of uncertain knowledge in various specialized AI domains will be found. So, by looking at families of such extensions we are both analyzing EL itself to understand its structure better and illuminating in a generic fashion varieties of evidential machinery potentially important to a number of classes of specialized AI domains. Further, EL provides an explication of the concept of negation which goes beyond that provided by Classical Logic, an explication that approaches nearer to the common sense uses of negation in the weighing of confirmatory and refutatory evidence, in the weighing of evidence pro and con, in the common distinction between "absence of evidence" and "evidence of absence".

2. The Hierarchies and their characterization

In this paper we characterize three hierarchies of extensions of $\mathrm{EL}_{n,\tau}$ which essentially reflect different levels d, $d = 1, \ldots, n$, of conflict toleration in regard to confirmatory and refutatory evidence. The first hierarchy we consider is related to ideas of Aristotle concerning privatives and has to do with the level d of '*absolute* conflict' tolerated: the d^{th} logic will tolerate conflict between the confirmatory and the refutatory only up to the point where the minimum of the confirmatory and the refutatory evidence levels is less than or equal to $(d-1)\varepsilon$. The second hierarchy generalizes part of the Dempster-

Shafer framework and has to do with a measure of the 'sum of conflicting evidence' tolerated: the d^{th} logic in this case will tolerate such conflict only so long as the sum of the confirmatory and refutatory evidence is at most $1 + (d-1)\varepsilon$. The third hierarchy is in a sense a sub-Dempster-Shafer hierarchy even less tolerant of conflict than the second hierarchy: the d^{th} logic in this case will tolerate conflict only so long as the sum of the confirmatory and refutatory evidence is a most $(d-1)\varepsilon$.

Let us turn now to the precise construction and characterization of these three hierarchies of logics. Fix n and let d range from 1 through n.

The *Aristotelian Evidence Logic of degree d*, denoted $\mathrm{AL}_{n,\tau}(d)$, is the logic

$$\mathrm{AL}_{n,\tau}(d) \stackrel{\text{def}}{=} \mathrm{EL}_{n,\tau}(\Psi_{1,d})$$

where $\Psi_{1,d}$ is the axiom given by the conjunction of the following sentences:

$\neg(Q_c : e \text{ AND } Q_r : e')$ for all cases where $\min\{e, e'\} > (d-1)\varepsilon$, for each proposition symbol Q stipulated by τ,

$\forall x_1 \ldots x_t\, \neg(R_c x_1 \ldots x_t : e \text{ AND } R_r x_1 \ldots x_t : e')$ for all cases where $\min\{e, e'\} > (d-1)\varepsilon$, for each t-ary predicate symbol R stipulated by τ.

Then the following proposition characterizes the Boolean Sentence Algebras of the hierarchy of monadic logics $\mathrm{AL}_{n,\mu}(d)$.

PROPOSITION 1. *For each* $d = 1, \ldots, n$,

$$\mathrm{BSA}(\mathrm{AL}_{n,\mu}(d)) \cong \mathrm{BA}(\omega^{m^u} \cdot m^p \cdot \sum_{i-1}^{k} s_{ki} \cdot m^{ui})$$

where $m = d(2n-d)$.

The *Dempster-Shafer Logic of degree d*, denoted $\mathrm{DSL}_{n,\tau}(d)$, is the logic

$$\mathrm{DSL}_{n,\tau}(d) \stackrel{\text{def}}{=} \mathrm{EL}_{n,\tau}(\Psi_{2,d})$$

where $\Psi_{2,d}$ is the axiom given by the conjunction of the following sentences:

$\neg(Q_c : e \text{ AND } Q_r : e')$ for all cases where $e + e' > 1 + (d-1)\varepsilon$, for each proposition symbol Q stipulated by τ,

$\forall\, x_1 \ldots x_t\, \neg(R_c x_1 \ldots x_t : e \text{ AND } R_r x_1 \ldots x_t : e$ for all cases where $e + e' > 1 + (d-1)\varepsilon$, for each t-ary predicate symbol R stipulated by τ.

Then the following proposition characterizes the Boolean Sentence Algebras of the hierarchy of monadic logics $\mathrm{DSL}_{n,\mu}(d)$.

PROPOSITION 2. *For each* $d = 1, \ldots, n$,

$$\mathrm{BSA}(\mathrm{DSL}_{n,\mu}(d)) \;\cong\; \mathrm{BA}(\omega^{m^u} \cdot m^p \cdot \sum_{i-1}^{k} s_{ki} \cdot m^{ui})$$

where $m = (n(n+1) + (d-1)(2n-d))/2$.

The *Sub-Dempster-Shafer Logic of degree* d, denoted $\mathrm{SDSL}_{n,\tau}(d)$, is the logic

$$\mathrm{SDSL}_{n,\tau}(d) \stackrel{\text{def}}{=} \mathrm{EL}_{n,\tau}(\Psi_{3,d})$$

where $\Psi_{3,d}$ is the axiom given by the conjunction of the following sentences:

$\neg(Q_c : e \text{ AND } Q_r : e')$ for all cases where $e + e' > (d-1)\varepsilon$, for each proposition symbol Q stipulated by τ

$\forall x_1 \ldots x_t\ \neg(R_c x_1 \ldots x_t : e \text{ AND } R_r x_1 \ldots x_t : e')$ for all cases where $e + e' > (d-1)\varepsilon$, for each t-ary predicate symbol R stipulated by τ.

Then the following proposition characterizes the Boolean Sentence Algebras of the hierarchy of monadic logics $\mathrm{SDSL}_{n,\mu}(d)$.

PROPOSITION 3. *For each* $d = 1, \ldots, n$,

$$\mathrm{BSA}(\mathrm{SDSL}_{n,\mu}(d)) \;\cong\; \mathrm{BA}(\omega^{m^u} \cdot m^p \cdot \sum_{i-1}^{k} s_{ki} \cdot m^{ui})$$

where $m = d(d+1)/2$.

PROOF OF PROPOSITIONS 1, 2 AND 3. Fix $n > 1$ and d in the range 1 through n. In view of the more detailed arguments given in [4], it is sufficient here to consider just the case where μ stipulates only proposition symbols and the equality predicate is not present. So consider the theory of the language containing only proposition symbols, let us say the p proposition symbols $\mathrm{Q}(i)$, $i = 1, \ldots, p$. The atoms of the Boolean Sentence Algebra of this theory are the n^{2p} p-ary conjunctions with i^{th} conjunct any one of the n^2 conjunctive pairings formed over the two collections of sentences for each $i = 1, \ldots, p$:

$$\alpha_{c,j}(i) \stackrel{\text{def}}{=} \begin{cases} \mathrm{Q}(i)_c : 1 & \text{if } j = n-1 \\ \neg\,\mathrm{Q}(i)_c : e_{j+1} \wedge \mathrm{Q}(i)_c : e_j & \text{if } j = n-2, \ldots, 1, \\ \neg\,\mathrm{Q}(i)_c : e_1 & \text{if } j = 0 \end{cases}$$

and

$$\alpha_{r,j}(i) \stackrel{\text{def}}{=} \begin{cases} \mathrm{Q}(i)_r : 1 & \text{if } j = n-1 \\ \neg\,\mathrm{Q}(i)_r : e_{j+1} \wedge \mathrm{Q}(i)_r : e_j & \text{if } j = n-2, \ldots, 1, \\ \neg\,\mathrm{Q}(i)_r : e_1 & \text{if } j = 0 \end{cases}$$

Let us, naturally enough, refer to any of these conjunctive pairings of the $\alpha_{c,j}(i)$ and the $\alpha_{r,j'}(i)$ as a *preatom*. We will, in turn, prove each of propositions 1 through 3 by analyzing the ways in which each of the three axioms $\Psi_{1,d}$ through $\Psi_{3,d}$ modify the above Boolean algebra.

First, let us consider $\Psi_{1,d}$. $\alpha_{c,j}(i)$ cojoins with *all* $\alpha_{r,j'}(i)$ to form a preatom just in case $j = 0, \ldots, d-1$, since it is in precisely these cases that the level of conflict cannot exceed $(d-1)\varepsilon$ no matter what evidence level $j'\varepsilon$ is asserted by $\alpha_{r,j'}(i)$; this yields exactly dn preatoms. On the other hand, for $j = d, \ldots, n-1$, $\alpha_{c,j}(i)$ similarly conjoins with exactly the $\alpha_{r,j'}(i)$ $(0 \leqslant j' \leqslant d-1)$ to form preatoms; the yield in this case is thus $(n-d)d$ preatoms. Altogether then, we conclude that $\mathrm{BSA}(\mathrm{AL}_{n,\mu}(d))$ is as stated in Proposition 1 with

$$dn + (n-d)d = d(2n-d) = m$$

and the proof of Proposition 1 is completed.

Second, let us consider $\Psi_{2,d}$. If $j = 0, \ldots, d-1$, then $\alpha_{c,j}(i)$, because of what the axiom $\Psi_{2,d}$ asserts about a combined evidence level not exceeding $1+(d-1)\varepsilon$, conjoins consistently with *all* the $\alpha_{r,j'}(i)$ to form a preatom; thus in this case we have dn preatoms. In contrast to this, for each $j = d, \ldots, n-1$, $\alpha_{c,j}(i)$ will conjoin consistently with just the $n+d-j-1$ assertions $\alpha_{r,0}(i)$, $\alpha_{r,1}(i)$, $\ldots$, $\alpha_{r,n+d-j-2}(i)$, since $\alpha_{c,j}(i)$ has evidence level $j\varepsilon$ while $\alpha_{r,k}(i)$ has evidence level $k\varepsilon$ so

$$\text{for just those } k \text{ with } 0 \leqslant k \leqslant n+d-j-2$$

the Dempster-Shafer combined evidence level is $j\varepsilon + k\varepsilon \leqslant j\varepsilon + (n+d-j-2)\varepsilon = (n+d-2)\varepsilon = (n-1)\varepsilon + (d-1)\varepsilon = 1 + (d-1)\varepsilon$; thus this case gives rise to precisely

$$\sum_{j=d}^{n-1} n+d-j-1 = \sum_{j=d}^{n-1} j$$

preatoms. Hence, we conclude that $\mathrm{BSA}(\mathrm{DSL}_{n,\mu}(d))$ is as claimed in Proposition 2 with

$$dn+\sum_{j=d}^{n-1} j = dn+(n-1)n/2-(d-1)d/2 = \\ (n(n+1)+(d-1)(2n-d))/2 = m,$$

which concludes the proof of Proposition 2.

Finally, consider $\Psi_{3,d}$. For each $j = 0, \ldots d-1$, $\alpha_{c,j}(i)$ conjoins consistently with $\alpha_{r,k}(i)$ if the combined evidence level $j\varepsilon + k\varepsilon$ is at most $(d-1)\varepsilon$, which implies that $0 \leqslant k \leqslant d-j-1$ if the conjunction is to be a preatom (i.e., consistent with $\Psi_{3,d}$), and so the yield here is

$$\sum_{j=0}^{d-1} d-j = \sum_{j=1}^{d} j = d(d+1)/2\,.$$

And, clearly, none of the $\alpha_{c,j}(i)$ for $j \geqslant d$ give rise, in conjunction with any of the $\alpha_{r,k}(i)$, to any further preatoms. This completes, then, the proof of Proposition 3. □

Turning to the functional and undecidable similarity types, note that with arguments similar to those in [4], for functional similarity types μ' one gets the result that for all d:

$$\begin{aligned} \mathrm{BSA}(\mathrm{AL}_{n,\mu'}(d)) &\cong \mathrm{BSA}(\mathrm{CL}_{\mu'}), \\ \mathrm{BSA}(\mathrm{DSL}_{n,\mu'}(d)) &\cong \mathrm{BSA}(\mathrm{CL}_{\mu'}), \text{ and} \\ \mathrm{BSA}(\mathrm{SDSL}_{n,\mu'}(d)) &\cong \mathrm{BSA}(\mathrm{CL}_{\mu'}), \end{aligned}$$

while for undecidable similarity types ν it is the case that for all d:

$$\begin{aligned} \mathrm{BSA}(\mathrm{AL}_{n,\nu}(d)) &\cong \mathrm{BSA}(\mathrm{CL}_{\langle 2\rangle}), \\ \mathrm{BSA}(\mathrm{DSL}_{n,\nu}(d)) &\cong \mathrm{BSA}(\mathrm{CL}_{\langle 2\rangle}), \text{ and} \\ \mathrm{BSA}(\mathrm{SDSL}_{n,\nu}(d)) &\cong \mathrm{BSA}(\mathrm{CL}_{\langle 2\rangle}), \text{ for } d \neq 1. \end{aligned}$$

(The analysis of $\mathrm{SDSL}_{n,\nu}(1)$ is as follows: if the undecidability of ν is purely relational, then the isomorphism type of $\mathrm{SDSL}_{n,\nu}(1)$ depends in an obvious way on whether or not ν stipulates a unary function; if the undecidability of ν is not purely relational, then the isomorphism type of $\mathrm{SDSL}_{n,\nu}(1)$ is that of $\mathrm{CL}_{\langle 2\rangle}$.)

Let us remark briefly how the results of the present paper generalize the earlier results in [6] and [7] concerning the Aristotelian logic and the Dempster-Shafer logic as presented there. First, it is to be noted how the $\mathrm{SDSL}_{n,\mu}(d)$ hierarchy above provides an ordered set of increasingly weaker (less stringently axiomatized) logics $\mathrm{SDSL}_{n,\mu}(d)$, strictly stronger, for $1 \leqslant d \leqslant n-1$, than the logic $\mathrm{DSL}_{n,\mu}$ of [6] and [7], with

$$\mathrm{SDSL}_{n,\mu}(n) = \mathrm{DSL}_{n,\mu}\ .$$

Second, the $\mathrm{DSL}_{n,\mu}(d)$ hierarchy above provides an ordered set of increasingly weaker logics $\mathrm{DSL}_{n,\mu}(d)$, strictly weaker, for $2 \leqslant d \leqslant n$, than $\mathrm{DSL}_{n,\mu}$, with

$$\begin{aligned} \mathrm{DSL}_{n,\mu}(1) &= \mathrm{DSL}_{n,\mu} \\ \mathrm{DSL}_{n,\mu}(n) &= \mathrm{EL}_{n,\mu}\,. \end{aligned}$$

Third, the $\mathrm{AL}_{n,\mu}(d)$ hierarchy provides an ordered set of increasingly weaker logics $\mathrm{AL}_{n,\mu}(d)$, strictly stronger, for $1 \leqslant d \leqslant n-1$, than $\mathrm{EL}_{n,\mu}$, with

$$\mathrm{AL}_{n,\mu}(n) = \mathrm{EL}_{n,\mu}\ .$$

Finally, note that with respect to the Aristotelian logic AL_{μ} in [6] and [7],

$$\mathrm{AL}_{2,\mu}(1) = \mathrm{DSL}_{2,\mu}(1) = \mathrm{AL}_{\mu}\ .$$

Diagrammatically, we have the following where, from the top, theories are progressively weaker, ending with the "pure" theory of monadic Evidence Logic $\mathrm{EL}_{n,\mu}$:

$$\begin{array}{ccccccccc}
\mathrm{SDSL}_{n,\mu}(1) & & & & & & \\
\mathrm{SDSL}_{n,\mu}(2) & & & & & & \\
\vdots & & & & & & \\
\mathrm{SDSL}_{n,\mu}(n) & = & \mathrm{DSL}_{n,\mu} & = & \mathrm{DSL}_{n,\mu}(1) & & \mathrm{AL}_{n,\mu}(1) \\
 & & & & \mathrm{DSL}_{n,\mu}(2) & & \mathrm{AL}_{n,\mu}(2) \\
 & & & & \vdots & & \vdots \\
 & & & & \mathrm{DSL}_{n,\mu}(n-2) & & \mathrm{AL}_{n,\mu}(n-2) \\
 & & & & \mathrm{DSL}_{n,\mu}(n-1) & = & \mathrm{AL}_{n,\mu}(n-1) \\
 & & \mathrm{EL}_{n,\mu} & = & \mathrm{DSL}_{n,\mu}(n) & = & \mathrm{AL}_{n,\mu}(n)
\end{array}$$

Finally, let us note the following perspective in regard to the above results. To better understand some of the recent efforts in AI to construct logics which provide at least partially successful machinery for the representation and processing of uncertainty, the three hierarchies of EL logics studied here illustrate the rich mosaic of axiomatizable extensions of EL which provide logics appropriate to a wide variety of AI domains. In particular one sees more clearly how current AI frameworks for the handling of uncertainty like that of Dempster-Shafer, coming more than two thousand years after Aristotle's preliminary grappling with the problems surrounding negation, are but one more step in the continuing process of the construction of frameworks which help us to better understand and more efficaciously utilize our meagre, and often uncertain, knowledge about our world.

3. Discussion

Let us call the "Middle Ground" that region of our knowledge where, although no contradictions arise, uncertainty abounds and our knowledge is most usually evidential in character, and conflict arises from the simultaneous presence of both confirmatory and refutatory evidence. Evidence Logic (EL), as presented in the earlier sections of this paper, shows that for this Middle Ground robust knowledge representation frameworks can be constructed which embody no inconsistency but rather get along quite well by simply allowing a generous amount of evidential conflict. Further, EL provides an explication of the concept of negation which extends that of classical logic and shows that at least some of the foundational issues of negation which have given rise to the importance of paraconsistent logics can indeed be captured by logics which are strictly prior to paraconsistency, that is, which, while reaching beyond the negation of classical logic, do not allow any contradictions.

Paraconsistent logics offer a reasonable framework where circumstances involving contradictions are present. On the other hand, while of course classical logics sometimes do offer a reasonable framework where no contradictions are present, there are vast areas of human knowledge requiring something beyond classical logic, while yet something less drastic that paraconsistent logic. In fact, regularly much of our knowledge is not absolute, but only evidential. That is, often our knowledge is confirmatorily or refutatorily evidential, and in fact gradationally so. Further, in many of these commonly occurring circumstances, conflict often arises in the sense that one has simultaneously both some confirmatory and some refutatory evidence in regard to a circumstance.

Note that such conflict can well attain without there being any contradiction. Here we are using the term 'contradiction' in the usual sense in which it involves the classical concept of negation (NOT A meaning "it is not the case that A") and an assertion A AND NOT A. That is, *conflict* may arise because of the presence of both confirmatory and refutatory evidence regarding A, and this is to be carefully distinguished from the *contradiction* which arises if one has both the presence and the absence of evidence of one sort or another with respect to A. In the latter case, where contradictions arise, we hope for a yet reasonable situation and seek to successfully utilize a non-explosive framework, a paraconsistent logic. In the former case, where conflict but no contradictions arise, classical logic fails to offer us sufficient representational breadth to handle the conflict while paraconsistent logics are a departure from classical logic far beyond what is needed.

In such cases of conflict, then, what is needed are systems which go beyond classical logic in allowing for the representation and processing of gradational confirmatory and refutatory evidential knowledge. Schematically, we may represent the three rough knowledge categories mentioned above, and the corresponding three logical frameworks addressed to meeting their needs, as follows:

Three Knowledge Categories		
always absolute and confirmatory, but never contradictory	sometimes evidential and even conflicting, but never contradictory	sometimes contradictory, yet not explosive

Appropriate Logical Frameworks		
classical logics	evidential logics like Evidence Logic (EL)	paraconsistent logics

So our focus here is the midde category above (the Middle Ground), knowledge which is sometimes evidential and even conflicting, but never contradictory. Certainly it is not necessary to argue for the breadth of occurrence of the Middle Ground, and hence for the clear need for logics which deal with such knowledge. Reflection on the nature of the knowledge with which we are most commonly confronted leads easily to the conclusion that this knowledge is in the main evidential in character; this knowledge is rarely absolute and, while conflicting evidence is often involved, contradictions are rarely present. In earlier sections of this paper we have seen how it is that EL deals efficaciously with some of the needs of the Middle Ground. EL provides enough machinery to adequately represent and process the gradational evidential, and often conflicting, knowledge so ubiquitous in the Middle Ground.

In [8] one finds exemplification of how EL can serve for exploration of such further aspects of negation. In that paper, Aristotle's insights in *Prior Analytics* regarding relations between negation as absence and privation are clarified by using EL. Obvious clarification, which we cannot pursue here, concerning the often confused usage of privatives in contemporary elementary logic, becomes possible with such application of the machinery in EL. Briefly, sentential negation addresses *absence* while privation addresses *otherness*, and EL helps to clarify this by using refutatory evidentials to explicate privatives.

Penultimately, let us consider the following thesis:

(1) using classical logics whenever possible, only moving to evidential logics in the less fortunate circumstances when our knowledge is indeed evidential, and

(2) using evidential logics whenever possible, only moving to paraconsistent logics in the even less fortunate circumstances when our knowledge is indeed in some cases contradictory.

(1) is certainly uncontroversial. In contrast, (2) brings to the fore the problem-laden boundary between evidential and paraconsistent logics. To have a contradiction, say P AND NOT P, is to have simultaneously the presence of the circumstance P describes and the absence of that circumstance. In some sense, this is as perplexing as it is common and unperplexing to have simultaneously some evidence confirmatory of P and some evidence refutatory of P. Clearly this boundary needs to be explored.

Along the lines of the analysis in the 1993 paired papers by Smiley and Priest [16], which attempt to further penetrate the complexity of the concept of negation, let us raise the following query. In their papers, Smiley and Priest face, and at least partially elucidate, a number of the difficult complexities surrounding the concept of negation. But the terrain is rough and often perplexities seem to overwhelm all attempts to provide sufficiently sharp linguistic codification of distinctions being addressed. For example, consider the following, from p. 20 of the Smiley paper:

> The classical idea links negation to acceptance and rejection through the equivalence between accepting $\sim A$ and rejecting A. Indeed, it takes the equivalence so much for granted that its adherents are liable to overlook or even deny the separate existence of rejection. For Priest, however, while the joint acceptance and rejection of A is impossible, the joint acceptance of A and $\sim A$ is possible or even mandatory. He therefore needs to deny that accepting $\sim A$ implies rejecting A [...].

See also, for example, Section 9 of Priest [12] on 'denial'. Would it not be productive to investigate carrying out this analysis upon a base logic of EL rather than upon the base logic of classical logic?

It seems to this writer that at least some of the unclarity involved in the debate could be overcome if one made use of the machinery of EL, its confirmatory and refutatory predications and the crisp use of classical negation simply for the absence thereof. The debate, as carried on by Smiley and Priest, seems at times a glut of expressivity in which so much undisambiguatable overloading is occurring that crisp analysis is no longer possible. It impresses as a beautiful dance of ideas, a work of "codification art", but the web of conceptual overloading is so complex that one feels, however unjustified, that the debate is violating one of the basic goals we strive constantly for in doing science and philosophy: clarity. But these judgments are clearly off the mark: negation is simply a very difficult problem area and any who make a genuine attempt to gain further ground, as Smiley and Priest have admirably done, not surprisingly uncover further perplexities at the depth of their analyses. Indeed, the conclusion we draw from the papers of Smiley and Priest is not that one or the other is right, but rather simply that good work has been done, progress has been made, but the concept of negation remains problematic.

Finally, we make some remarks in support of the position, respecting paraconsistency, that while pluralism and reformism are clearly tenable positions, dialetheism, is not. For, the fact of the matter is that our ignorance of the real world (hereafter R) is just too great at present for dialetheism to be tenable. While we refer the reader to [10] for discussion of this perspective, we make the following brief remarks.

Dialetheism is based on a greater knowledge of R than we in fact have. When Priest [12, p.3] says "the theoretical object has to fit the real object; and how this behaves is not a matter of choice", the problem is not being faced squarely: for the fact is, we don't yet know the real object. To assert what dialetheism does, that there really are contradictory simultaneous happenings in R, is just simply reaching beyond what we now know about R. As Russell said in the wonderfully lucid paper [14, pp. 91–92]:

> My own belief is that most of the problems of epistemology, in so far as they are genuine, are really problems of physics and physiology [...].

It is for this reason (and possibly others as well, but it is just this reason we are focussing on here) that dialetheism is presently untenable. Scientific theories, including foundational theories like those we are here concerned with, should not be theologies.

The foundational issues which give rise to, and are addressed by, paraconsistency make it clear that we must carefully respect our ignorance of R.

We are simply building models and analyzing their partial efficacy: to assert more than that, to forget our great ignorance of R, is to be merely theological. As Popper [11, p. 59] said,

> Theories are nets to catch what we call 'the world': to rationalize, to explain, and to master it. We endeavor to make the mesh ever finer and finer.

That's where we are now, and this state of our ignorance limits what is assertable: dialetheism goes beyond.

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