3. *Suppose that* $F = \{a, b, c, d\}$ *and let* **R4** *be a defeasible rule* $\{d\} \Rightarrow \neg q$ *instead of the defeater* $\{d\} \rightsquigarrow \neg q$. *Then, we have* $T \vdash -\partial q$ *and* $T \vdash -\partial\neg q$ *by the inference condition* $-\partial.\mathbf{2.3}$. *By the translation rule* **TR5**, *the defeasible rule* **R4** *is translated into*

$$d:(1,0) \wedge \sim q:(0,1) \rightarrow q:(1,0). \tag{4}$$

Then, the defeasible theory T *is translated into a VALPSN* $tr(T) = \{a : (2,0),\ b : (2,0),\ c : (2,0),\ d : (2,0),\ (1), (2), (3), (4)\}$, *which has two stable models,*

$$I_1 = \{a:(2,0),\ \ b:(2,0),\ \ c:(2,0),\ \ d:(2,0),\ \ q:(1,0)\},$$
$$I_2 = \{a:(2,0),\ \ b:(2,0),\ \ c:(2,0),\ \ d:(2,0),\ \ q:(0,1)\}.$$

Neither $q:(1,0)$ *nor* $q:(0,1)$ *can be satisfied by both the stable models* I_1 *and* I_2.

4. *Suppose that* $F = \{a, b\}$ *and there is no superiority relation between* **R1** *and* **R2**. *Then, we have* $T \vdash -\partial q$ *and* $T \vdash -\partial\neg q$ *by the inference condition* $-\partial.\mathbf{2.3}$. *By the translation rule* **TR5**, *the defeasible rules* **R1** *and* **R2** *are respectively translated into*

$$a:(1,0) \wedge \sim q:(0,1) \rightarrow q:(1,0) \tag{5}$$
$$b:(1,0) \wedge \sim q:(1,0) \rightarrow q:(0,1). \tag{6}$$

Then, the defeasible theory T *is translated into a VALPSN* $tr(T) = \{a : (2,0),\ b:(2,0),\ (3), (5), (6)\}$, *which has two stable models,*

$$I_1 = \{a:(2,0),\ \ b:(2,0),\ \ c:(0,0),\ \ d:(0,0),\ \ q:(1,0)\},$$
$$I_2 = \{a:(2,0),\ \ b:(2,0),\ \ c:(0,0),\ \ d:(0,0),\ \ q:(0,1)\}.$$

Neither $q:(1,0)$ *nor* $q:(0,1)$ *can be satisfied by both the stable models* I_1 *and* I_2.

Let us take one more example. We describe how the defeasible theory in Example 3.1 is translated into a VALPSN.

EXAMPLE 4.3. *Let us remind Genetically Altered Penguin (GAP).*
F1 $gap(o)$ *is translated by the translation rule* **TR1** *into*

$$gap(o):(2,0). \tag{1}$$

R1 *$gap(o) \rightarrow p(o)$ is translated by the translation rules* **TR2** *and* **TR3** *into*

$$gap(o):(2,0) \rightarrow p(o):(2,0), \tag{2}$$

$$gap(o):(1,0) \wedge \sim p(o):(0,1) \rightarrow p(o):(1,0). \tag{3}$$

R2 *$p(o) \rightarrow b(o)$ is translated by the translation rules* **TR2** *and* **TR3** *into*

$$p(o):(2,0) \rightarrow b(o):(2,0), \tag{4}$$

$$p(o):(1,0) \wedge \sim b(o):(0,1) \rightarrow b(o):(1,0). \tag{5}$$

R3 *$b(o) \Rightarrow f(o)$ is translated by the translation rule* **TR6** *into*

$$b(o):(1,0) \wedge \sim p(o):(1,0) \wedge \sim f(o):(0,1) \rightarrow f(o):(1,0). \tag{6}$$

R4 *$p(o) \Rightarrow \neg f(o)$ is translated by the translation rule* **TR6** *into*

$$p(o):(1,0) \wedge \sim gap(o):(1,0) \wedge \sim f(o):(1,0) \rightarrow f(o):(0,1). \tag{7}$$

Then, we have a VALPSN $P = \{(1), \dots, (7)\}$ as the VALPSN-translation of GAP, which has only one stable model

$$I_1 = \{\, gap(o):(2,0),\ p(o):(2,0),\ b(o):(2,0),\ f(o):(0,0)\,\}.$$

Since $I_1 \not\models f(o):(1,0)$ and $I_1 \not\models f(o):(0,1)$, I_1 shows that $+\Delta\, gap(o)$, $+\Delta p(o)$, $+\Delta b(o)$ $-\partial f(o)$ and $-\partial\neg f(o)$.

4.2. The relation between defeasible theory and VALPSN

We provide proofs for the relation described in Figure 2, i.e., a literal q is definitely or defeasibly provable in a defeasible theory T *if and only if* for any stable model of the VALPSN $tr(T)$ satisfies vector annotated literals $q:(2,0)$ or $q:(1,0)$, respectively.

Let $T = (\,F, \mathcal{C}, R, >\,)$ be a defeasible theory, $P = tr(T)$ the VALPSN-translation of the defeasible theory T, P^I the Gelfond-Lifschitz transformation of the VALPSN P based on an interpretation I, q a literal, and M_P the set of all stable models of the VALPSN P. First, we define an interpretation $T_{P^I} \uparrow 0$ to be a special interpretation that assigns the truth value $(0,0)$ to all members of B_{P^I}, and an interpretation $T_{P^I} \uparrow 1$ to be a special interpretation that assigns the truth value $(0,0)$ to all members of $B_{P^I} \setminus \{q\}$ and the truth value $(2,0)$ to any literal $q \in F$. Moreover, let

$$T_{P^I} \uparrow n = T_{P^I}(T_{P^I} \uparrow (n-1))$$

for any integer $n \geq 2$ (refer to Definition 2.9 of T_{P^I}). Then, we have the following theorems.

THEOREM 4.1. $T \vdash +\Delta q \iff \forall I \in M_P; T_{PI} \uparrow \omega \models q:(2,0)$.

PROOF. By induction on the number of lines in a proof of $+\Delta q$ and an integer i ($i \geq 1$) such that $T_{PI} \uparrow i \models q:(2,0)$.

Basis. ($\Rightarrow$ part) Suppose that $\mathrm{P}(1) = +\Delta q$. From the inference condition $+\Delta.1$, there exists a fact $q \in F$. Then, there also exists a vasn-clause $q:(2,0) \in P$ as the translation of the fact q by the translation rule **TR1**. Moreover, for any stable model I of the VALPSN P, $q:(2,0) \in P^I$. Thus, by the definition of the interpretation $T_{PI} \uparrow 1$,

$$T_{PI} \uparrow 1 \models q:(2,0).$$

($\Leftarrow$ part) Conversely, suppose that for any stable model I of the VALPSN P, $T_{PI} \uparrow 1 \models q:(2,0)$. Then, by the definition of $T_{PI} \uparrow 1$, there exists a unit vasn-clause $q:(2,0) \in P$. Then, there exists a literal $q \in F$ as the inverse translation of the unit vasn-clause $q:(2,0)$. Thus, by the inference condition $+\Delta.\mathbf{1}$,

$$\mathrm{P}(1) = +\Delta q$$

Induction Hypothesis. There is an integer $i \geq 1$ such that, for any literal q, for any stable model I of the VALPSN P,

$$+\Delta q \in \mathrm{P}(1..i) \iff \forall I \in M_P; T_{PI} \uparrow i \models q:(2,0),$$

Induction Step. ($\Rightarrow$ part) Suppose that $\mathrm{P}(i{+}1) = +\Delta q \quad (i > 1)$. We consider the following two cases.

Case 1. If $+\Delta q$ is derived by the inference condition $+\Delta.\mathbf{1}$, it has been proved in the Basis.

Case 2. If $+\Delta q$ is derived by the inference condition $+\Delta.\mathbf{2}$, there exists a strict rule $\{a_1, \ldots, a_k\} \to q \in R$ such that for each integer j ($1 \leq j \leq k$), $+\Delta a_j \in \mathrm{P}(1..i)$. Then, there exists a vasn-clause $a_1:(2,0) \wedge \cdots \wedge a_k:(2,0) \to q:(2,0) \in P$ as the translation of the strict rule $\{a_1, \ldots, a_k\} \to q \in R$ by the translation rule **TR2**. By the Induction Hypothesis, for each integer j ($1 \leq j \leq k$), $T_{PI} \uparrow i \models a_j:(2,0)$. Thus, by the definition of $T_{PI} \uparrow (i+1)$,

$$T_{PI} \uparrow (i+1) \models q:(2,0).$$

($\Leftarrow$ part) Conversely, suppose that $T_{PI} \uparrow (i+1) \models q:(2,0)$. Then, there exists a vasn-clause $a_1:(2,0) \wedge \cdots \wedge a_k:(2,0) \to q:(2,0) \in P^I$ as the translation of a strict rule $\{a_1, \ldots, a_k\} \to q \in R$, and for each integer j ($1 \leq j \leq k$), $T_{PI} \uparrow i \models a_j:(2,0)$. Then, by the Induction Hypothesis, for each j ($1 \leq j \leq k$), $+\Delta a_j$. Thus, we have $\mathrm{P}(i{+}1) = +\Delta q$. □

Theorem 4.2. $T \vdash -\Delta q \iff \exists I \in M_P; T_{PI} \uparrow \omega \models \sim q:(2,0)$.

Proof. First of all, we refer to Theorem 3.3 in Billington [2]: *Let* $T_1 = (F, \mathcal{C}, R, >)$ *be a defeasible theory. Suppose* $d \in \{\Delta, \partial\}$. *If* q *is a literal, then it is not the case that both* $T_1 \vdash +dq$ *and* $T_1 \vdash -dq$.

($\Rightarrow$ part) Suppose that $T \vdash -\Delta q$. From Theorem 3.3, we have $T \nvdash +\Delta q$ and from Theorem 4.1, we have $T \nvdash +\Delta q \iff \exists I \in M_P; T_{PI} \uparrow \omega \not\models q:(2,0)$. Thus, we have there exists a stable model I such that $T_{PI} \uparrow \omega \not\models q:(2,0)$.

($\Leftarrow$ part) Conversely, suppose that there exists a stable model I of the VALPSN P such that $T_{PI} \uparrow \omega \not\models q:(2,0)$. Then, we can trace back easily the above ($\Rightarrow$ part) proof to the conclusion $T \vdash -\Delta q$. □

Theorem 4.3. $T \vdash +\partial q \iff \forall I \in M_P; T_{PI} \uparrow \omega \models q:(1,0)$.

Proof. By induction on the number of lines in a proof of $+\partial q$ and an integer $i (i \geq 1)$ such that $T_{PI} \uparrow i \models q:(1,0)$.

Basis. ($\Rightarrow$ part) It is apparent that there can not exist $\mathrm{P}(1) = +\partial q$ by the inference conditions $+\partial.\mathbf{1}$ and $+\partial.\mathbf{2}$. Therefore, we suppose that $\mathrm{P}(2) = +\partial q$. Then, only the inference condition $+\partial.\mathbf{1}$ can be applied to the derivation of $+\partial q$ and there exists $+\Delta q \in \mathrm{P}(1)$. From the Basis in Theorem 4.1, for any stable model I of the VALPSN P, $T_{PI} \uparrow 1 \models q:(2,0)$. Since $(1,0) \preceq (2,0)$, $T_{PI} \uparrow 1 \models q:(2,0)$ implies $T_{PI} \uparrow 2 \models q:(1,0)$. Thus, we have

$$T_{PI} \uparrow 2 \models q:(1,0).$$

($\Leftarrow$ part) Conversely, suppose that for any interpretation I that can be a stable model of the VALPSN P, $T_{PI} \uparrow 2 \models q:(1,0)$. Then, by the definition of $T_{PI} \uparrow 1$ and the translation rule **TR1**, there exists a unit vasn-clause $q:(2,0) \in P^I$. Therefore, $q \in F$, $\mathrm{P}(1) = +\Delta q$ and $\mathrm{P}(2) = +\partial q$. Hence,

$$\mathrm{P}(2) = +\partial q \iff T_{PI} \uparrow 1 \models q:(1,0).$$

Induction Hypothesis. There is an integer $i \geq 2$ such that, for any literal q,

$$+\partial q \in \mathrm{P}(1..i) \iff T_{PI} \uparrow i \models q:(1,0).$$

Induction Step. We have to take the following three cases Case 1–3 into account to prove this induction step with respect to the inference conditions $+\partial$ and the translation rules **TR1–6**. Suppose that $\mathrm{P}(i{+}1){=}{+}\partial q$.

Case 1. In this case, $+\Delta q \in \mathrm{P}(1..i)$ and $\mathrm{P}(i{+}1) = +\partial q$. Then, by Theorem 4.1, for some integer m $(1 \leq m \leq i)$, $T_{PI} \uparrow m \models q:(2,0)$, and it implies,

$T_{P^I} \uparrow (i+1) \models q:(2,0)$ and $T_{P^I} \uparrow (i+1) \models q:(1,0)$, by the monotonicity of the operator T_{P^I}.

Case 2. In this case, there are a rule $\boldsymbol{A} \circ q$ that has no superiority relation; or a superiority relation $\boldsymbol{A} \circ q > \boldsymbol{B} \bullet \bar{q}$ and no defeater superior to the rule $\boldsymbol{A} \circ q$, where $\circ, \bullet \in \{\rightarrow, \Rightarrow\}$. Suppose that $+\partial q$ is derived based on the rule $\boldsymbol{A} \circ q$, where $\boldsymbol{A} = \{a_1, \ldots, a_k\}$. Then, the antecedent $\boldsymbol{A}$ is defeasibly provable and $+\partial \bar{q} \notin \mathrm{P}(1..i)$. Therefore, by the translation rules **TR3** or **TR5**, there is a vasn-clause clause

$$a_1:(1,0) \wedge \ldots \wedge a_k:(1,0) \wedge \sim q:(0,1) \rightarrow q:(1,0) \in P.$$

Moreover, there is a vasn-clause

$$a_1:(1,0) \wedge \ldots \wedge a_k:(1,0) \rightarrow q:(1,0) \in P^I$$

such that for each j $(1 \leq j \leq k)$, $T_{P^I} \uparrow i \models a_j:(1,0)$ and $T_{P^I} \uparrow i \not\models q:(0,1)$ by the Induction Hypothesis. Thus, by the definition of the operator T_{P^I}, we have

$$T_{P^I} \uparrow (i+1) \models q:(1,0)$$

Case 3. In this case, there is a superiority relation $\boldsymbol{B} \bullet \bar{q} > \boldsymbol{A} \circ q$, where $\circ, \bullet \in \{\rightarrow, \Rightarrow\}$; $\boldsymbol{A} = \{a_1, \ldots, a_k\}$ and $\boldsymbol{B} = \{b_1, \ldots, b_l\}$. Moreover, $+\partial q$ is derived based on the rule $\boldsymbol{A} \circ q$. Then, the antecedent $\boldsymbol{A}$ is defeasibly provable and the literal $\bar{q}$ cannot be derived by other rules, i.e., the literal $\bar{q}$ is neither definitely nor defeasibly provable and the antecedent $\boldsymbol{B}$ is not defeasibly provable. Therefore, by the translation rules **TR4** or **TR6**, there are vasn-clauses

$$\begin{gathered}
a_1:(1,0) \wedge \ldots \wedge a_k:(1,0) \wedge \sim b_1:(1,0) \wedge \sim q:(0,1) \rightarrow q:(1,0), \\
a_1:(1,0) \wedge \ldots \wedge a_k:(1,0) \wedge \sim b_2:(1,0) \wedge \sim q:(0,1) \rightarrow q:(1,0), \\
\vdots \\
a_1:(1,0) \wedge \ldots \wedge a_k:(1,0) \wedge \sim b_l:(1,0) \wedge \sim q:(0,1) \rightarrow q:(1,0)
\end{gathered}$$

in the VALPSN P. Moreover, there exists a vasn-clause

$$a_1:(1,0) \wedge \ldots \wedge a_k:(1,0) \rightarrow q:(1,0) \in P^I$$

such that for each j $(1 \leq j \leq k)$, $T_{P^I} \uparrow i \models a_j:(1,0)$, for some integer $m \in \{1, \ldots, l\}$, $T_{P^I} \uparrow i \not\models b_m:(1,0)$, and $T_{P^I} \uparrow i \not\models q:(0,1)$ by the Induction Hypothesis. Thus, by the definition of the operator T_{P^I}, we have

$$T_{P^I} \uparrow (i+1) \models q:(1,0)$$

Conversely, suppose that for any stable model I of the VALPSN P,

$$T_{P^I} \uparrow (i+1) \models q:(1,0).$$

We consider the following three cases Case 4–6.

Case 4. In this case, $T_{P^I} \uparrow i \models q:(2,0)$. Then, by Theorem 4.1, $+\Delta q \in$ P$(1..i)$. Therefore, by the inference condition $+\partial.\mathbf{1}$, we have

$$\mathrm{P}(i{+}1) = +\partial q.$$

Case 5. In this case, the vector annotated literal $q:(1,0)$ is the head of a vasn-clause that is translated by the translation rules **TR3** or **TR5**. Then, there is a vasn-clause

$$a_1:(1,0) \wedge \cdots \wedge a_k:(1,0) \rightarrow q:(1,0) \in P^I$$

such that for each integer $j(1 \leq j \leq k)$, $T_{P^I} \uparrow i \models a_j:(1,0)$, and $T_{P^I} \uparrow i \not\models q:(0,1)$. Then, by the Induction Hypothesis, for each integer j $(1 \leq j \leq k)$, $+\partial a_j \in$ P$(1..i)$, and $-\partial\neg q \in$ P$(1..i)$. Therefore, by the inference condition $+\partial.\mathbf{2}$,

$$\mathrm{P}(i{+}1) = +\partial q.$$

Case 6. In this case, the vector annotated literal $q:(1,0)$ is the head of a vasn-clause that is translated by the translation rules **TR4** or **TR6**. Then, there is a vasn-clause

$$a_1:(1,0) \wedge \cdots \wedge a_k:(1,0) \rightarrow q:(1,0) \in P^I$$

such that for each integer $j(1 \leq j \leq k)$, $T_{P^I} \uparrow i \models a_j:(1,0)$, and $T_{P^I} \uparrow i \not\models q:(0,1)$. Then, by the Induction Hypothesis, for each integer j $(1 \leq j \leq k)$, $+\partial a_j \in$ P$(1..i)$ and $-\partial\neg q \in$ P$(1..i)$. Therefore, by the inference condition $+\partial.\mathbf{2}$,

$$\mathrm{P}(i{+}1) = +\partial q. \qquad \square$$

Theorem 4.4. $\quad T \vdash -\partial q \iff T_{P^I} \uparrow \omega \models \sim q : (1,0).$

Proof. Suppose that $T \vdash -\partial q$. From Theorem 3.3, we have $T \nvdash +\partial q$. From Theorem 4.3, we also have $T \nvdash +\Delta q \longleftrightarrow T_{P^I} \uparrow \omega \not\models q : (1,0)$. Thus,

$$T_{P^I} \uparrow \omega \not\models q : (1,0), \quad (\text{i.e., } T_{P^I} \uparrow \omega \models \sim q : (1,0)\ ).$$

Conversely, suppose that $T_{P^I} \uparrow \omega \not\models q : (1,0)$. We can trace the above proof back to $T \vdash -\partial q$. $\square$

5. Conclusion

In this paper, we have proposed VALPSN that can deal with defeasible reasoning. We have clarified the relation between VALPSN and Billington's defeasible logic. First, we proposed the translation from the defeasible logic into VALPSN, and proved that there is a correspondence between the defeasible logic provability and the VALPSN stable model satisfiability based on the translation. This correspondence shows that the defeasible logic derivation can be replaced by the stable model computation of the corresponding VALPSN, and that VALPSN can provide a semantics for the defeasible logic. However, if we replace the defeasible theory derivation by the VALPSN stable model computation, some problems in terms of computational complexity arise. Since each defeasible or strict rule in the defeasible logics are translated into more than one VALPSN clauses, there may be a case which has too many VALPSN clauses in the stable model computation. Basically, the stable model computation takes so long time when the VALPSN contains many clauses, even if it is implemented in a powerful workstation. If we want to implement a VALPSN stable model computing system, some strategies are required to speed up the computation.

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