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HEAPS AND GLUTS: Paraconsistent Logic Applied to Vagueness

Abstract. This paper is an attempt to show that the subvaluation theory is not a good theory of vagueness. It begins with a short review of supervaluation and subvaluation theories and procedes to evaluate the subvaluation theory. Subvaluationism shares all the main short-comings of supervaluationism. Moreover, the solution to the sorites paradox proposed by subvaluationists is not satisfactory. There is another solution which subvaluationists could avail themselves of, but it destroys the whole motivation for using a paraconsistent logic and is not different from the one offered by supervaluationism.

1. Introduction

D. Hyde in his 1997 paper "From Heaps and Gaps to Heaps of Gluts" proposed a new theory of vagueness, namely the subvaluation theory. This theory is a dual of the familiar supervaluation theory. The logic proposed by supervaluationists is an incomplete logic in a sense that it treats some vague statements as neither true nor false. In contrast, the logic underlying the subvaluation theory is a paraconsistent logic, for according to subvaluationists there are statements which are both true and false.

It was Jaśkowski who suggested that paraconsistent logic may be used as an analysis of vagueness. He proposed to call a deductive system which includes theses containing some vague terms *a discursive system*. Such a system contains theses that express propositions which may not agree with each other. He wrote ([6], p. 149):

> To bring out the nature of the theses of such a system it would be proper to precede each thesis by the reservation: [...] "for a certain admissible meaning of a term used". Hence, the joining of a thesis to a discursive system has a different intuitive meaning than has the assertion in an ordinary system.

The subvaluation theory of vagueness is an attempt explain just how discursive systems so defined can combain assertions which contain vague expressions and which may not agree with each other.

As it is well known one of the most characteristic features of vague expressions, the feature that distinguishes them from all other expressions, is that they admit of borderline cases, i.e. cases of which it is doubtful whether the expression can be ascribed to them or not. Vague words do not introduce a sharp boundary between things to which they apply and things to which they do not. In borderline cases it is unclear whether a given thing possesses the relevant property. A man who is 1,90 m in height is considered tall, one who is 1,65 m is considered not tall, but it is unclear whether one who is 1,75 m is tall or not.

Both supervaluation theory and subvaluation theory treat vagueness as a semantic phenomenon. The question of application of vague words to borderline cases is inquiry resistant. Some cases are left undecided and there is nothing one can do about it. One may measure certain man's height to a millimetre, but if this man is a borderline case of the expression "tall man", then even such a precise mesurement would not tell one whether the man is tall or not. Vagueness is not a matter of the lack of knowledge of the borderline cases and whatever one could learn about them would not help one.

Nevertheless speakers do sometimes apply vague terms to borderline cases. No matter what the speaker decides he may be said to have *precisified* a vague word in a certain way. Therefore, it may be argued that vague terms allow different admissible ways of precisification. A precisification counts as *admissible* if it conforms with the positive and negative extensions; i.e. if it draws a boundary somewhere in the penumbra.

This is the starting point of both super- and subvaluationism. It is worth noticing that the assumption that vague terms admit different precisifications is not tantamount to the claim that precisifications could be substituted for vague terms. Neither does supervaluationism nor subvaluationism propose to eliminate vagueness from the language. Precisifications should rather be regarded as ways of analysis. There is no one way of making a vague term precise (this would be a mere stipulation), but there are a few ways that could, in principle, be admissible.

On both accounts each vague term has its positive cases (i.e. those that come up true in all admissible precisifications), negative cases (i.e. those that come up false in all admissible precisifications) and borderline cases, which come up true in some admissible precisifications and false in some others. What differentiates supervaluationism and subvaluationism are the concepts of truth they employ.

As has been mentioned, the subvaluation theory is a dual of the supervaluation theory, so it is useful to outline briefly the main features of supervaluationism first, and then consider the dual theory.

2. The supervaluation theory of vagueness

According to supervaluationists vague words are vague because of *deficiency* of meaning.[1] It is obvious that when a speaker states of a case belonging to the positive extension of "tall" (e.g. a man who is 1,90 m) that he is tall, he speaks truly, and when he says that a case from the negative extension is tall, he speaks falsely, but it is not obvious what truth-value his statement that a borderline case is tall has. The supervaluation theory proposes to consider such statements as neither true nor false. Thus the under-determination of meaning of vague terms results in the existence of truth-value gaps.

Precisifications themselves are not vague; they have sharp boundaries. Within a given precisification each sentence is either true or false. A corresponding assignment of truth-values to statements within each precisification is called an *admissible valuation*. Each admissible valuation is classical, for it assigns one of two classical truth-values to each statement. The *supervaluation* is a function that assigns supertruth exactly to those statements that are assigned truth by all admissible valuations, superfalsity exactly to those statements that are assigned falsehood by all admissible valuations and neither to the rest. So, a supervaluation has truth-value gaps. Borderline statements, which are assigned different truth-values within different precisifications, are not assigned any value by a supervaluation. The general idea of the theory of supervaluation is that truth should be identified with supertruth and falsity with superfalsity.

In the supervaluation theory "validity" is defined as a preservation of (super)truth. Since it is not the case that each statement is either (super)true or (super)false, the Principle of Bivalence fails. The Law of Excluded Middle holds however, for in each precisification one of its disjuncts is true. Consider,

[1] The classical paper on the supervaluation theory of vagueness is [7].

for instance, a disjunction of borderline sentences:

Either a pile of k grains makes a heap or it does not make a heap.

In each precisification a pile of k grains is either on the heap- or on the non-heap-side. Therefore, this disjunction comes up true in each admissible precisification and therefore it is (super)true. Nevertheless, its disjuncts are neither (super)true nor (super)false. Hence, disjunction may be (super)true despite the fact that neither of its disjuncts is. However this is not always the case. "Either a pile of k grains makes a heap or a pile of k grains makes a heap" is neither (super)true nor (super)false, for it is equivalent to the borderline statement "A pile of k grains makes a heap" which is — according to our assumption — neither (super)true nor (super)false. Since the truth-values of the compound statements depend not only on the truth-values of the components but also on their content, the logical connectives cease to be truth-functional.

The idea of the possibility of making a vague word more precise in several, equally acceptable ways, allows supervaluationists to resolve the sorites paradox. Consider the following formulation of the sorites reasoning:

(A)
A pile of 100,000 grains is a heap.
For any n, if a pile of n grains is a heap, then a pile of $n-1$ grains is a heap.
———
A pile of 1 grain is a heap.

The major premise:

(1) For any n, if a pile of n grains is a heap, then a pile of $n-1$ grains is a heap,

is (super)false on the supervaluationists' account. It is (super)false, because in each precisification there is a different counterexample to it (see [1], p. 312). Thus, it is (super)true that:

(2) There is an n, such that a pile of n grains is a heap, but a pile of $n-1$ grains is not a heap.

This does not mean however, that there is a determinate answer to the question *which* n it is. The statement (2) comes up (super)true, but in each precisification it is true in virtue of different n. Hence, despite the fact that the major premise is (super)false, the vague word in question remains vague, for it is impossible to specify which n falsifies this premise.

Thus, according to the supervaluation theory the sorites argument is valid but unsound. In other words, although the reasoning is valid, the conclusion reached by means of it is superfalse for one of the premises is superfalse.

3. The subvaluation theory of vagueness

Recall that on the supervaluation approach the phenomenon of vagueness has been regarded as a result of the *under*-determination of meaning. On the present approach the explanation is exactly opposite. The existence of borderline cases is a result of the *over*-determination of meaning. Thus, the subvaluation theory does not regard borderline cases as devoid of truth-value. On the contrary, it regards them as being *both* true *and* false. Therefore, the logic proposed by subvaluationists has to be a paraconsistent logic.

The suggestion that borderline cases are such that the vague term both applies and does not apply to them has been made quite often. To follow such a suggestion is to regard borderline statements as both true and false at the same time. The problem is that such a conception of borderline statements seems to lead immediately to the denial of the law of non-contradiction (LNC). And this is precisely a step that most people are not willing to take.

The subvaluation theory is a theory which does not reject LNC, but which nevertheless claims that borderline cases are both true and false. These apparently inconsistent claims are not inconsistent on the subvaluationists' account thanks to their definition of truth. While supervaluationists define truth as truth in all admissible precisifications, the subvaluationists' truth is *truth in some admissible precisification*. A given statement is considered true if there is at least one precisification in which it comes out true. Similarly false is a statement that is false in at least one precisification. Since there are both such precisifications in which borderline statements are true and such precisifications in which they are false, all borderline statements are regarded as both true and false. In addition, the statements which are true in all admissible precisifications are considered determinately true and the statements which are false in all admissible precisifications are determinately false. So, borderline cases are neither determinately true nor determinately false.

"True in some precisification" is a primary notion, "determinately true" is secondary and is defined by means of that former notion. Such a definition of truth requires that the logic underlying the theory of subvaluation is paraconsistent; i.e. it is inconsistent (for some statements A, both A and $\neg A$ belong to the theory) but it is not trivial (the spread-principle:

$$A, \neg A \vDash_{\mathrm{SbV}} B$$

is not valid). The logic adopted by supervaluationism is in fact a modal paraconsistent logic invented by Jaśkowski. On the present approach each thesis should be preceded by a reservation "for a certain admissible precisification of the terms used".

It should be noted that the subvaluation logic does not preserve classical consequence. Hence,

$$\neg(A_1, \ldots, A_n \vDash_{\mathrm{Cl}} B \implies A_1, \ldots, A_n \vDash_{\mathrm{SbV}} B).$$

The following is valid however:

$$A_1, \ldots, A_n \vDash_{\mathrm{Cl}} B \implies A_1 \,\&\, \cdots \,\&\, A_n \vDash_{\mathrm{SbV}} B.$$

There are serious doubts whether or not conjunction (i.e. "&") used in this logic can still be called "conjunction", for the principle of adjunction fails:

$$A, B \nvDash_{\mathrm{SbV}} A \,\&\, B\,,$$

and it may seem that if a connective fails adjunction, then it is not conjunction (cf. [7], p. 159). The failure of adjunction explains the validity of LNC.

$$A, \neg A \nvDash_{\mathrm{SbV}} A \,\&\, \neg A\,.$$

The explanation of this fact is that although there is a precisification in which A is true and there is a precisification in which $\neg A$ is true, there is no precisification in which both A and $\neg A$ are true.

In the subvaluation theory "validity" is defined as preservation of truth (as opposed to preservation of determinate truth). An argument is SbV-valid iff whenever the premises are true in some admissible precisification the conclusion is true in some admissible precisification. The precisification in which the premise is true need not be the same precisification in which the conclusion is true. It appears that all the classical tautologies are preserved in the subvaluation logic. On this account it is the principle of the exclusivity of truth-values that must fail. There are sentences which have two opposite truth-values. As has been mentioned LNC is still valid, however.

Unlike supervaluationism, subvaluationism claims that the sorites paradox is not valid (however, see below). It claims that in the sorites argument the paradoxical conclusion does not follow from the premises because the mode of inference, i.e. *modus ponendo ponens* (*MPP*) is not unrestrictedly valid. Consider the sorites paradox again:

(A)
A pile of 100,000 grains is a heap.
For any n, if a pile of n grains is a heap, then a pile of $n-1$ grains is a heap.
———
A pile of 1 grain is a heap.

The subvaluationists' argument to the effect that this reasoning is invalid goes as follows ([4], p. 648):

The sentence "A pile of n grains is a heap", where a pile of n grains counts as a borderline case for "heap", is both true and false; so it is true. Since it is also false, the material conditional "If a pile of n grains is a heap, then a pile of $n-1$ grains is a heap" is true by virtue of the falsity of its antecedent. Nonetheless, pile of $n-1$ grains might be determinately not a heap, thus making the sentence "A pile of $n-1$ grains is a heap" false. So, the conclusion is that *MPP* is not valid for material implication.

Subvaluationists treat vagueness as a species of *ambiguity*. Different precisifications are regarded as different meanings. The statement "n grains can make a heap" is true on some meanings of "heap" and false on some other meanings. The sorites reasoning is invalid because it equivocates between different meanings. One meaning of "heap" is assumed in the minor premise and another meaning is taken into account in the major premise. As Hyde puts it ([4], p. 650):

> *Modus ponens* applied to equivocal premises fails to be truth-preserving, but this is hardly news.

The conclusion Hyde draws is that the virtues and vices of the supervaluation theory are mirrored in the subvaluation theory. They use different concepts of truth, but as yet there are no conclusive arguments that would favour one of these concepts over the other. Hence, the same arguments that are used in defence of the supervaluation theory may be used to defend subvaluationism. The former is tenable only if the latter is.

4. Evaluation of the subvaluation theory of vagueness

It is worth remembering that supervaluationism itself faces several serious problems. Subvaluationism is not any better off in addressing those problems. It shares supervaluationism's vices, but in addition it does not share some of supervaluationism's virtues. It has some of its own vices as well.

The main problems connected with supervaluationism are:

(A) The fact that it is not able to handle higher-order vagueness.

(B) The claim that vague statements are true prior to precisification is contentious.

(C) Its solution to the sorites paradox is extremely counterintuitive.

Let us consider these objections in turn and check whether or not subvaluationism can handle them better than supervaluationism does.

(A) Higher-order vagueness

The supervaluation theory is not able to accommodate vagueness within its framework. It proposes to analyse vague expressions in terms of admissible precisifications. Now, if "admissible precisification" is itself precise, then we end up with precise boundaries after all: one between positive cases and borderline cases and the other between borderline cases and negative cases. Hence, although there is no cut-off point between persons who are bald and ones who are not bald, there exists nevertheless a sharp boundary between clearly bald persons and those that are not clearly bald. This seems implausible, for instead of one sharp cut-off point one is merely offering two such points. Thus, one may argue that the supervaluation did not solve the problem but merely shifted it to another boundary (cf. [2], p. 196).

On the other hand, if "admissible precisification" is itself vague, then supervaluationism must give up the desire for a precise metalanguage. If it is unclear which precisifications are admissible, then it is also unclear which statements are true in all admissible precisifications. Therefore there will be statements which will be neither clearly (super)true nor clearly (super)false.

It is clear that **the subvaluation theory** is equally incapable of handling higher-order vagueness, because this incapability is strictly connected with the whole idea of precisifications. Hence, the higher-order vagueness objection formulated against supervaluationists applies here too.

Recall that the difference between borderline and other cases is defined in terms of precisifications; borderline statements are those that are true in some admissible precisifications and false in some other admissible precisifications, whereas, for instance, positive cases are those that are true in all admissible precisifications. Given that there is a limited number of admissible precisifications and assuming that each precisification is precise one may take the «extreme» precisification and obtain a clear division between borderline and positive cases. In the sorites argument concerning "heap", in which the consecutive steps differ by a single grain, there must be a last statement which is both true and false, and whose immediate neighbour is determinately true. In a precise meta-language vagueness dissapears.

On the other hand the analysis proposed by subvaluationism accords ill with a vague meta-language. In such a language borderline statements would be not only those that are both true and false but also those that are neither clearly determinately true nor clearly borderline. In other words borderline cases would be not only those that come up true and false in different precisifications, but also those about which it is unclear whether they come up true and false. But then, the whole idea of precisifications is lost.

(B) Ambiguity

In order to make their solution to the sorites paradox more plausible **supervaluationists** use an analogy with ambiguity (see [3], p. 284). They argue that ambiguity provides examples of disjunctions of the form $A \vee \neg A$ which are true despite the fact that neither of the disjuncts is true. According to them, an ambiguous sentence is true if each of its disambiguations is true. Consider the ambiguous statement J:

(J) John went to the bank.

Let J_1 and J_2 be its disambiguations, i.e. "John went to the money-bank" and "John went to the river-bank" respectively, and suppose that only J_1 is true. Then neither J nor $\neg$J is true, for each has a false disambiguation. However, the disjunction $\mathrm{J} \vee \neg \mathrm{J}$ is true, because both its disambiguations, namely $\mathrm{J}_1 \vee \neg \mathrm{J}_1$ and $\mathrm{J}_2 \vee \neg \mathrm{J}_2$ are true.

This conception has been criticised. It has been argued that the assertion on which this reasoning is based, namely the assertion that an ambiguous sentence is true if each of its disambiguations is true, is itself not true. Tye argues ([10], p. 143) that

> [t]he fact that an ambiguous sentence would be true were it to come out true under all its permissible disambiguations is not a good reason to hold that the sentence is, in fact, true, prior to disambiguation.

Tye claims that before the statement "John went to the bank" is disambiguated it is neither true nor false, since prior to disambiguation one cannot know what it means.

The same objection may be raised against the supervaluationists' claim that

(2) There is an n, such that a pile of n grains is a heap, but a pile of $n-1$ grains is not a heap,

is true. Tye argues that the fact that (2) would be true, were "heap" made precise in any acceptable way, is not a reason to assert that (2) is true prior to any precisification of "heap" (see [10], p. 143).

It is worth pointing out however, that supervaluationists are by no means forced to treat vagueness as the same phenomenon as ambiguity (cf. [3], p. 282). The solution to the sorites does not hinge on the treatment of ambiguous statements. The analysis was meant merely to illustrate the solution. Hence, potential failure in supervaluationists' analysis of ambiguity is not tantamount to the incorrectness of their analysis of vagueness.

In contrast, it seems that the motivation for using **paraconsistent logic** as the analysis of the phenomenon of vagueness depends heavily on treating vagueness as a species of ambiguity. The fact that Jaśkowski regarded vagueness as the same phenomenon as ambiguity is made clear by the following remark of his ([6], p. 144, my emphasis):

> Any vagueness of a term a can result in a contradiction of sentences, because with reference to the same object X we may say that "X is a" and also "X is not a", *according to the meaning of the term a adopted for the moment.*

If we interpret vague properties as overlapping each other then we get simple inconsistency. On such an interpretation there just are patches which both are and are not red. Hence, LNC has to go. The claim that borderline statements are both true and false is consistent with LNC only if we interpret them as ambiguous; i.e. true under one meaning and false under another meaning. The claim that vagueness is a kind of ambiguity is extremely counterintuitive, however. Our intuitions are exactly opposite in fact. Vagueness and ambiguity seem to be two different phenomena. An expression is *ambiguous* if it has two (or more) different meanings. Those meanings can either be vague or precise. Before an ambiguous expression is disambiguated we do not know which meaning is relevant to the given occasion of use. Usually it is the context that does the disambiguating. If someone utters "Jones went to the bank. He withdraw 1000 £ from his account", we know that it is the money-bank Jones went to. However, if someone said only "Jones went to the bank" we would not know whether Jones went fishing or went to make some cash transactions. Thus, the statement "Jones went to the bank" is not particularly informative. We learn that Jones went either to the money-bank or to the river-bank but we do not know to which one of them. Had the expression "bank" more meanings, we would learn even less. Therefore, it seems that one does not know what the statement "Jones went to the bank" means, until it is disambiguated. For, although one knows that it is one of the two meanings which is correct, one does not know which one.

In contrast, a *vague* expression does not switch between two or more different meanings. It has only one meaning which is not precise. One does not have to disambiguate the meaning of a vague expression "tall" in order to know what the statement "Jones is tall" means. This statement has only one meaning and it concerns Jones's height. It is not a precise meaning, but nevertheless it is an unambiguous one.

Tye's attack has been directed towards the supervaluation treatment of vague sentences. But its relevance to subvaluationism is even more evident. If an ambiguous statement cannot be considered true prior to its disambiguation even if *all* its disambiguations are true, then it cannot be deemed true prior to its disambiguation if *one* of its disambiguations is true. The claim that no matter what the speaker's intentions are the utterance "Jones went to the bank" is true prior to its disambiguation in the case in which Jones went to the money-bank and did not go to the river-bank, seems counterintuitive.

Besides, "truth in a precisification" is a very weak notion. It obviously does not accord with the intuitions attached to the common notion of truth. It does not suffice to know that Jones went fishing in order to claim that someone's utterance "Jones went to the bank" is true. One has to know in addition what that person meant by "bank"; which of its meanings he had in mind.

(C) Solution to sorites

The supervaluationists' solution to the sorites paradox has been heavily criticised for its counterintuitiveness. It seems implausible to accept an existential claim knowing that no answer to the question "What is the witness to it?" is available (see [5], p. 259). The theory of supervaluation forces one to accept both that the existential claim "there is such-and-such n" is true, and that "this is such-and-such n" is false about every n ([8], p. 51).

It is not entirely clear what the solution to the sorites paradox proposed by subvaluationists is. On the one hand, Hyde claims that the sorites paradox is not a paradox in fact, for the reasoning used in it is invalid. On the other hand, he argues that the paradox is a mere fallacy of equivocation. Moreover, this latter ex-planation is taken to be a qualification of the former.

As we have seen, Hyde claims that the sorites paradox does not threaten the subvaluation theory, because the subvaluation logic invalidates it. According to him *MPP* is not unrestrictedly valid in this logic and that is why the sorites conclusion does not follow.

It might seem, that the rejection of *MPP* depends on the assumption that there is no higher-order vagueness. The invalid step has the following shape:

$$\begin{array}{ccccc} p \rightarrow q, & p & \vDash_{\text{SbV}} & q \\ \text{F T F} & \text{T} & & \text{F} \end{array}$$

where: p – a pile of n grains is a heap; q – a pile of $n-1$ grains is a heap. p in the first premise is taken to be false and in the second premise — true.

In order to get invalidity two conditions have to be fulfilled:

(a) p has to be a borderline statement and hence be both true and false;

(b) q has to be a determinately false statement (for if it were borderline, it would be both true and false; hence true).

Notice that the heaps described in p and q differ merely by one grain. Hence the conditions (a) and (b) are tantamount to the claim that there is a sharp boundary between borderline and negative cases of "heap". n grains is the last borderline case and $n-1$ grains is the first negative case. Hence, it seems that the phenomenon of higher-order vagueness does not exist. In order to avoid that conclusion the proponents of suvaluationism have to argue that although the boundary between borderline and negative cases exists, it is not boundary lies between n and $n-1$.

Secondly, it has been argued that there are different formulations of the sorites reasoning possible, and the use of *MPP* is not always necessary (see [9], p. 468). Even in the formulation cited by Hyde one uses induction, rather than *MPP*. And in the formulation:

10000 grains make a heap.
1 grain does not make a heap.
―――――――――――――――――
$\exists x$(x grains make a heap & $x-1$ grains do not make a heap).

neither *MPP* nor induction is used.

If this were the case the claim that *MPP* is invalid would not suffice to solve the sorites paradox and subvaluationists would still owe an account of how to deal with those other formulations. It is unclear however, whether there are any formulations that do not use *MPP* at all.[2] Mathematical induction is not different from *MPP* in finite cases and it seems that even in the above formulation *MPP* has to be used in order to derive the conclusion.

[2] Another well-known formulation of the sorites paradox is the following: 1 grain does not make a heap, Not: 1 grain does not make a heap and 2 grains make a heap, [...]. Hence 10000 grains do not make a heap. This formulation does not use *MPP*, but it uses *modus tollendo ponens* instead. Hence, subvaluationists' solution could be applied here as well.

Thirdly, it seems that even in the formulation (A) it is not *MPP* that is the culprit. Given the definition of validity formulated above, MPP is invalid, because it does not preserve subvaluationists' truth. Because of the existence of statements that are both true and false, one can start with true premises and arrive at a false conclusion. However, this explanation seems at least insufficient. It is true that there are in this theory statements which are both true and false, but they are true and false in their different meanings. The use of *MPP* in the sorites reasoning equivocates between different meanings of p. To prove that a given form of reasoning is not valid in the case of equivocation is not to prove that it is not valid *simplicite*. Blaming *MPP* for the invalidity of reasoning: "Jones went to the money-bank"; and "If Jones went to the river-bank, then Jones withdrew some money"; hence "Jones withdrew some money" seems utterly unjustified. It is not the mode of reasoning which is to be blamed. The reasoning contains a defect: it commits the fallacy of equivocation.

If the reasoning $p \to q, p \vDash_{\mathrm{SbV}} q$ equivocates between two different meanings of p, then it has the following form in fact: $p \to q, r \vDash_{\mathrm{SbV}} q$. No wonder that it does not work. On the face of it *MPP* is invalid, but after closer investigation it appears that it has been simply misapplied. Thus, in the subvaluation theory the rejection of *MPP* lacks motivation.

There is another way in which **subvaluationism could provide a solution** to the sorites paradox, however. Consider the formulation (A) again. The minor premise is true in all admissible precisifications. So, it is determinately true. In contrast, the conclusion is false in all admissible precisifications. Hence, it is determinately false. What about the major premise? It seems that it is also false in all precisifications. Recall that in each precisification there is an n such that although n grains do make a heap, $n-1$ grains do not. It follows then, that the premise "For any n, if a pile of n grains is a heap, then a pile of $n-1$ grains is a heap" is determinately false on the subvaluationists' account. Hence, the subvaluation proposes the same sort of solution to the sorites paradox as supervaluation did: it claims that the major premise is false. Moreover, in order to justify this claim subvaluationists are forced to use the same arguments. For in order to claim that the major premise is determinately false, they have to argue that the statement:

$$\exists n(n \text{ grains make a heap } \& \ n-1 \text{ grains do not make a heap})$$

is true. This statement is in fact true in all precisifications, so it is determinately true. Now, subvaluationists have to repeat supervaluationists' argument to the effect that although there exists such n, the actual witness cannot be produced. Hence, they are faced with the counterintuitiveness objection immediately.

In the above reasoning the notion of truth is barely used. It is "determinate truth" that plays the major role. Notice, moreover, that subvaluationists' defence of LNC also involves determinate truth-values. LNC is valid, because there is no such precisification in which "$A \& \neg A$" is true. The claim that it is not true is tantamount to the claim that it is determinately false. It suggests that subvaluationists should take this latter notion, rather than the former one, to be *the* notion of truth. If they do so however, they will have to abandon paraconsistent logic and use the incomplete logic instead, for borderline statements are neither determinately true nor determinately false. But then, subvaluationism would collapse into supervaluationism.

5. Conclusion

The differences between the supervaluation and subvaluation theories speak to the disadvantage of the latter. There is nothing in subvaluationism that would help in replying to the objections raised against supervaluationism. As has been mentioned the same objections can be raised against the subvaluationism itself. Moreover, both theories employ notions of truth that are by no means common notions. However, supervaluationists' (super)truth seems to have more in common with the ordinary notion of truth than the notion of "truth in a precisification". The use of paraconsistent logic has been motivated by this latter notion. It seems though, that subvaluationists are not able to solve the sorites paradox unless they appeal to determinate truth, which is a counterpart of (super)truth. Moreover, paraconsistent logic can be applied to vagueness only if we make vagueness the same phenomenon as ambiguity. As has been argued there are strong arguments against such an identification.

What is even more important, subvaluationism, as well as supervaluationism, proposes to analyse vagueness in a precise metalanguage. Applying paraconsistent logic to precisifications results in the claim that although there is no sharp boundary between determinately true statements and determinately false statements, there are *two* boundaries instead: one separates determinately true statements from those that are both true and false, and the other separates true and false statements from determinately false ones. Someone who finds this implausible had better look for some other theory of vagueness.

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