



Wojciech Suchoń

## VASIĹIEV: WHAT DID HE EXACTLY DO?

0. Some ninety years ago a young Russian physician travelled through the Western Europe. He was preparing himself for entrance upon a regular professor position at Kazan University. His main interest was psychology and most of his journey time he spent in Germany. This was the time of great changes both in psychology and in logic: at the beginning of the twentieth century Husserlian criticism on confusing their problems won, but not in Germany where Brentano school still treated logic as a part of psychology.

Logic of those days had two main interests. The first, declining, was in some problems of syllogistics, the other, raising, in modern logical calculi — mainly in propositional ones. A problem vividly discussed in syllogistics was the genuine sense of particular propositions. The question was: what does “some” mean — *some and perhaps all* or *some and not all*? For the modern logic it was characteristic to dispute the role of traditional Fundamental Laws of Thought, namely the law of excluded middle and the law of contradiction.

Our hero — you have surely guessed his name, N. A. VasiĹiev — undertook the problem and strongly argued for interpretation of particular propositions of the kind *Some S is P* as *Only some of S are P*. Being conscious of difficulties an interpretation like this brings for Aristotelian syllogistics, he developed a new syllogistics. The form in which he exposed it suggested that the new calculus would be breaking the logical laws mentioned above. This feature of his papers was the main reason for growing interest in VasiĹiev’s work in our time.

As it usually happens with the texts prophetic in some sense, papers by VasiĹiev are very ambiguous. They may be read in different manners and



each reading method claims to be the only legitimate. One can, however, think that Vasiliev's original idea, with all its vagueness, is lost for us forever. We are doomed to conceive it through the achievements of posterior logicians.

In last decades some logicians claimed that N. A. Vasiliev was the "founding father" of at least three different — very interesting and rapidly developing — branches of logical calculi. In the following text we will try to answer the question: Taking into account Vasiliev's own papers, are such claims legitimate?

\* \* \*

At first, we will give a short survey of N. A. Vasiliev's logical output:

1. What are the main achievements in Vasiliev's logical heritage? The author thinks that the central position among writings left by Vasiliev is occupied by two non-Aristotelian syllogistics (the so-called *logic of notions* and *imaginary logic*). The first was conceived in 1910, the latter — one year later (in 1911). Vasiliev presented a sketch of the *logic of notions* in print in [12]. This booklet is a record of a lecture given by Vasiliev in May of the same year to acquire *venia legendi*. It contains an attempt to construct a theory of syllogistic argumentation in which particular premises perceived in a nontraditional manner occur. Vasiliev called propositions of this kind *accidental*. The truth of accidental proposition, unlike that of a standard particular proposition, excludes truthfulness of the correspondent general proposition. Accidental propositions are formed with the phrases *Only some \_are\_ / Only some \_are not\_*. Since those expressions are equivalent, only one *copula* is needed as their formal counterpart.

Vasiliev strongly stressed the importance of studying the role played in argumentation by accidental premises. He was convinced that in everyday talk, also in the language of science, particular propositions are used in the first place in "accidental sense". Moreover, the set composed from general propositions and an accidental one allows for description of the basic relation that can arise between extensions of subject and predicate.

With accidental propositions there appears — beside the well-known categories of quality (*affirmative* and *negative*) — a new one: *indifference*, earlier unknown. Truth-functional characteristic of these propositions forbids to attribute them any of the traditionally specified qualities.

The quantity to be assigned for accidental propositions also causes some difficulties. Language suggestion is that they are particular. Theory of op-

position shows, however, that they are not contradictories to general propositions, but only contraries. The last stated reason was a guideline for Vasiliev's final conclusion: he declared them general.

Vasiliev, taking into account interrelations among categorical propositions of his syllogistics, formulated the *rule of excluded fourth* and promulgated the break of the *law of excluded middle*. It gave him a proud conviction that he was the first to overrun the limitation of Aristotelian logic. At the same time he must have been disappointed by the trifling number of valid moods revealed in the *logic of notions*.

A more developed presentation of the *logic of notions*, adapted to modern standard, can be found in [10]. Here we recall only the summary: the list of formulas analogous to that given by C. A. Meredith for Aristotle's syllogistics.

premises	$n$
terms	$n + 1$
figures	$2^n$
conclusive figures	$n + 2$
figures of types <b>1</b> , <b>3</b> and <b>4</b>	1
figures of type <b>2</b>	$n - 1$
moods in a single figure	$3^{n+1}$
valid moods in a single figure of types <b>1</b> and <b>2</b>	2
valid moods in a single figure of types <b>3</b> and <b>4</b>	1
moods in all	$2^n \cdot 3^{n+1}$
valid moods in all	$2n + 2$

**2.** For the first time the sketch of the second of Vasiliev's syllogistics, which he called *imaginary logic*, was presented in January 1911 during the session of Physico-Mathematical Society, and widely discussed on several meetings. The mature version of the ideas exposed then was published in two subsequent articles: [14] and [15]. Those writings included also polemics with reviews of [12] and commentaries to voices of debaters from the discussion mentioned above.

It was a natural development of *logic of notions*, because the main idea was to recover traditional particular propositions and to add another new kind of proposition: contradictory to accidental ones. Those newly introduced propositions were included in the category of indifferent ones.

Strictly syllogistic consequences of changes mentioned earlier remained only sketched (mainly in [14]). Like in the case of the *logic of notions*, we give here a summary of a “modern style” presentation of the *imaginary logic*. A more thorough exposition of its reconstruction may be found in [11].

premises	$n$
conclusive figures	$\frac{n^2+n+2}{2}$
figures of types <b>1</b> and <b>4</b>	1
figures of types <b>2</b> and <b>3</b>	$n - 1$
figures of type <b>5</b>	$\frac{n^2-3n+2}{2}$
valid moods in a single figure of types <b>2</b> , <b>3</b> (normal) and <b>5</b>	10
valid moods in a single figure of types <b>1</b> and <b>3</b> (M1)	11
valid moods in a figure of type <b>4</b>	$3n + 3$
valid moods in all	$5n^2 + 8n + 5$

**3.** Vasiliev’s further investigations were dominated by an endeavour to elucidate the role played in his logic by the law of contradiction. In the author’s opinion, the problem arose, in some sense accidentally.

We must remember that Vasiliev, making attempts to clearly expose the truth-functional characteristic of universal indifferent propositions explained that they were true if and only if S is P and (simultaneously) non-P. There were also other explanations in his texts: such a proposition is true when S is P or non-P or S may be P, but it may also turn out non-P. This collection of various formulations allows us to comprehend what Vasiliev really had in mind describing the sense of the circumstances in which the universal indifferent propositions were true. Roughly speaking he wanted to say that the extensions of the subject S and the predicate P overlap or that S contains P (it means they are in relations  $\delta$  and  $\gamma$  — see Keynes [4], chapter V).

His contemporaries were fascinated only by the literally taken first formulation. They suggested that for some propositions in Vasiliev’s logic the condition of their truth was to accept a contradiction. They emphasised that this was an evident rupture with logical tradition, an open rejection of fundamental law of thought. As it was said, Vasiliev had a strong feeling of novelty of his conception, so he was bewitched by the perspective of becoming a follower of Lobatschevsky in logic. He exploited the similitude, and in the likeness of Lobatschevsky’s name for non-Euclidean geometry, called his non-Aristotelian logic *imaginary*, too.

Clearly, the resemblance between the two systems was of no significance for establishing the truth of a contradiction. Vasiliev undertook the task in a very specific and genuinely syllogistic way. He considered not the functor of negation but negative propositions, so he had two forms of denying: *No\_is* and *Some\_is not*. This is the source of the idea of two negations and of the method of explanation of the difference between “absolute” and “simple” negation. If, instead of “simple” negation [*Some\_are not*], we take the “new” one [*Only some\_are not*], we will see that the conjunction of the sentence *Only some S is P* and its negation *Only some S is not P* may be true (which is rather trivial).

Vasiliev did not offer any further essential development of this idea — first he was hindered by the beginning of the I World War, then by his insanity.

The reconstruction of Vasiliev inquiries into new syllogistics explains — the author thinks — the presence of a number of staggering phrases in his works. They are unexpected for modern readers, especially for those who are not acquainted with the history of syllogistics. There is, however, an article (by A. Korcik [6]) in which logical achievements of Vasiliev are described as having strictly syllogistic character. Alas, no matter what its great value for the sterling appreciation of Vasiliev’s work was, this article is rarely quoted in literature of the subject. We can conjecture that the proper reason of illegitimate recognition of Vasiliev as a harbinger of investigations leading to some nonclassical propositional calculi issues from reading of his papers without taking into account the historical context in which his key-notions emerged.

Now we pass to a cursory inspection of contemporary trends in logical investigations which hold N. A. Vasiliev for their “spiritual father”<sup>1</sup>. We will mention only those utterances which are most often cited as relevant to establish the role Vasiliev plays in the twentieth century logic. Let us recall that Vasiliev was suggested as a precursor of three-valued logic (and even wider: many-valued ones), but also of intuitionistic and paraconsistent logics. In the author’s opinion all these conjectures are highly doubtful.

4. The idea of three-valued logic is attributed to Vasiliev by L. Chwistek, G. Kline and A. Małcev.

Chwistek (in [3]) presents Vasiliev’s inventions as the oldest system of many-valued calculus; the only source for such presentation was the text [16] inserted in proceedings of Neapolitan congress. He avers that Vasiliev con-

---

<sup>1</sup> Obviously Vasiliev can hardly be treated as such, taking into account exiguous reception of his inventions, at times when those clues of investigations arose.

structed a noncontradictory logical system starting from an analysis of three kinds of sentences:  $S$  is  $P$ ,  $S$  is not  $P$  and  $S$  is  $P$  and non- $P$ ; the latest are stated when the extension of  $P$  is vague. Chwistek adds that investigation of such statements is very interesting, because utterances of this kind are often used in everyday language. Moreover, they exceed the limitations of strict, scientific thinking. He admits that final estimation of Vasiliev's logic is difficult.

We must clearly say: Vasiliev did not have in mind any conception of the third logical value — on the contrary, while formulating the *law of non-selfcontradiction* he decidedly stated that there were only two logical values (truth and falsity) which could never be assigned together to any sentence.

Vasiliev never tried to prove that his logic was not contradictory, and no one did this in the thirties, at the time when Chwistek wrote his book.

Vasiliev indeed used phrases  $S$  is  $P$ ,  $S$  is not  $P$  and  $S$  is  $P$  and non- $P$  while explaining the sense of categorical sentences coming into play in the *logic of notions*. It is, after all, easy to decipher<sup>2</sup> the awkward description of relations (between extensions of subject and predicate) ensuring the truth of propositions of different kinds (accordingly: universal affirmative, universal negative, accidental). Nothing in Vasiliev's writings refers to vagueness of extension of a name; Vasiliev never even mentioned extensions.

Reading the following fragment of the chapter we are compelled to presume that the only reason for Chwistek's interest in Vasiliev logic was the possibility to shake the primacy of Łukasiewicz in creation of many-valued logic (he hated Łukasiewicz as a reactionary philosopher). A similar in some sense attack on Łukasiewicz's role (but without giving explicit reasons for it) one can find in the article by Kline [5] much better known among western logicians.

G. Kline states that Vasiliev is the very originator of the general idea of three-valued logic set forth in [12, 14, 15, 17] during the period 1910–1913. Kline's first argument is that Vasiliev (in [14]) claimed that his logical system is build on a set of axioms rejecting the law of non-contradiction. Vasiliev's remarks about the parallel between Lobatshevsky's geometry and his logic are adduced to support this claim.

Kline's second argument is that Vasiliev considers two kinds of negation (absolute vs. simple [=relative]). This construction seems to Kline well adjusted to cope with manyvaluedness. To reinforce his argument he stresses similarities between Vasiliev's negations and some explanations given by Post on degrees of falsity.

---

<sup>2</sup> Cf. illustrations given by V. Smirnov in [9].

The final argument is based on the *law of excluded fourth* and some of Vasiliev's deliberations on  $n$ -dimensional space and hypothetical logic of the  $n$ th order.

Let us consider the first argument. Rejection of the law of non-contradiction is not restricted to Łukasiewicz's three-valued logic, and Vasiliev's own explicit declaration about the existence of only and exactly two different logical values cannot be taken as an argument for intended manyvaluedness of Vasiliev logic. The parallel Lobatschevsky-Vasiliev is also not conclusive: Vasiliev strongly emphasises that he broke the dichotomy affirmative-negative and not the true-false one. His argumentation leads to enlargement of quality classifications applicable to categorical propositions. Even Kline himself writes that in Vasiliev's imaginary world abandoning of dichotomy true-false is not allowed by Vasiliev. Moreover, this dichotomy is recognised as constitutional law for conceptual apparatus of cognition (law of nonself-contradiction — закон несамопротиворечия).

The second argument is weak, too. Vasiliev ended the reasoning on different negations with a statement: Всего будет три подразделения суждений по качеству; he means that there are three kinds of propositions: affirmative, "absolutely" negative and "ordinary" negative. Kline himself weakens his argument intimating that the logic based upon truth, relative falsity and absolute falsity conceived as logical values was not elaborated in detail (more accurately: not at all). The coincidence in terminology between Post and Vasiliev is only of incidental character.

Following Vasiliev's own explanation<sup>3</sup>  $n$ -dimensional logics (or logics of  $n$ th order; but certainly not  $n$ -valued) are systems in which only the number of quality categories for categorical sentences increases but the number of logical values does not.

We should also pay some attention to A. Maľcev who maintains in [8] that the idea of many-valued logics issues from eminent logician of Kazan University, N. A. Vasiliev. The basic source for this declaration is a series of articles recapitulating the discussion which followed the lecture at Physico-Mathematical Society published in January of 1911 in the journal *Камско-Волжская речь*.

The crucial point in Maľcev's argumentation for Vasiliev's primacy in construction of many-valued logics is an analogy between rejection of Eu-

---

<sup>3</sup> Cf. [14] where it is — to stress its importance — written in italics: Мы можем мыслить логическую систему с  $n$  видами качественных различий суждения, и такую систему мы будем называть логической системой  $n$ -го порядка или  $n$  измерений.

clidean “fifth postulate” and rejection of a logical axiom (namely the *law of excluded middle*). Moreover, Małcev grants that Vasiliev, introducing affirmative, negative and indifferent sentences, created a variant of three-valued logic.

To restore the balance, Małcev adds that Vasiliev did not elaborate the algebra (semantic?) of his calculus, that the idea of rejection of the *law of excluded middle* may be found in Brouwer’s work published two years earlier and that both Łukasiewicz and Post treated three-valued calculus far more deeply.

The question of interdependence between three qualities of Vasiliev’s categorical sentences and three-valuedness was discussed earlier; it remains to consider the problem of rejection of logical axiom, as an element of foundation of many-valued logical system.

The rejection of the *law of excluded middle* would be essential if it were a start point for constructing a system with clear axioms and deduction rules. Vasiliev, moreover, did not build a new logic based on new set of axioms but rather discovered during the investigations that in his syllogistics some trichotomy appears instead of the well known dichotomy. Since negation was involved, he declared rejection of the only associated law: the *law of excluded middle*.

5. We have already mentioned that starting from the set of laws rejected in various places by Vasiliev, one can, with the same probability, suggest that he is precursor not only of many valued logics, but also of some other nonclassical systems. In particular, the rejection of the *law of excluded middle* evokes temptation to connect Vasiliev with firstlings of intuitionism. Such a thought can be found in N. Luzin’s text [7]<sup>4</sup>. Luzin ties Vasiliev ideas directly with current trends in contemporary mathematics. He explains at first the sense of investigations in foundations of mathematics referring to western logicians (Brouwer, Hilbert, Weyl, Borel). Next, he states that rejection of the *law of excluded middle* is characteristic for intuitionism, and that works of Vasiliev were to construct such logic. He concludes that Vasiliev’s ideas agree amazingly well with newest efforts undertaken by mathematicians though they are a decade prior to the latter.

Besides recalling the title of the article [12], which indirectly rejects the *law of excluded middle*, N. Luzin gives no more arguments for his claim. His

---

<sup>4</sup> It is unpublished review of scholar production of N. Vasiliev prepared by professor of Moscow University N. Luzin in January 1927.

statement is untenable considering the rejection (openly declared in [14]) of the *law of noncontradiction* which is accepted by intuitionists.

6. On the other hand, rejection of the *law of noncontradiction* incited to use Vasiliev ideas to surmount difficulties caused by contradiction in the frame of the so called *dialectical logic*. Those attempts were soon abandoned and the clue of rejection of the *law of noncontradiction* was taken up to claim N. Vasiliev the harbinger of paraconsistent logics. In particular, A. Arruda (cf. [1]) did so while presenting three paraconsistent calculi conceived by her self.

At the beginning she did not justify this claim giving, instead, a short rough<sup>5</sup> review of Vasiliev's achievements. Only in the final summary she declared the she saw the kernel of Vasiliev's conception in his admittance of possible existence of logics tolerating contradiction. She mentioned also that Vasiliev examined diverse negations some of which allowed contradiction.

The last remark is perhaps the most valuable: as a matter of fact, in the commentary to the *law of nonselfcontradiction*, Vasiliev remarked that the rule of contradiction was enunciated in the form "nothing may be affirmed and negated about the same thing". Such a formulation is wrong — he states — because it may happen that with the use of another method of proving the truth<sup>6</sup> of negative propositions it would be possible to admit the truth of some sentence and its negation<sup>7</sup>. We can suppose that Vasiliev anticipated, at least in the case of negation, creation of logical calculi meant to investigate functors having more or less arbitrary given truthfunctional characteristic. Moreover, this is so general approach to the problem that it is hard to take it for the prototype for any particular calculus from among those traditionally associated with Vasiliev's name. It would be better to take him for an (unintentional) founder of a method used nowadays in logical investigations.

With the first statement — that acceptance of a possible existence of a logic tolerating contradiction is an important contribution in evolution of paraconsistent calculi — it is hard to dispute.

---

<sup>5</sup> It somewhat confusing that she used not Vasiliev original works but their coverages (e.g. [9] and [5]), and even coverages of coverages (like the review of [9] written for THE JOURNAL OF SYMBOLIC LOGIC by D. D. Comey).

<sup>6</sup> Perhaps it will be better to say: *when the truthfulness characteristic of negation changes*.

<sup>7</sup> It may be taken as the reason to refuse the claim for acknowledge Vasiliev as the precursor of three-valued Łukasiewicz's logic and intuitionism — in both these calculi the formula **KNpp** never can receive value **1**.



7. Now we pass to presentation of our own view on the sense of Vasiliev's achievements and the role played by him in the history of contemporary logic.

Both systems created by Vasiliev may be located inside the trend — characteristic for the turn of the nineteenth and twentieth centuries — exploring those argumentations on categorical sentences in which some interesting copulas, earlier neglected, appear. Syllogistics of this type were constructed, with diverse motivation, by, e.g., W. Hamilton and A. de Morgan. This was the trend founding the base for radical transformation of the twentieth century logic.

Vasiliev was the first logician who — consciously and in a consistent way — built syllogistics transcending Aristotelian limitations. Nowadays, it allows to have a new look on the whole scope of syllogistic reasonings, to indicate the right place for Aristotle's syllogistics and all others, among them those created by Vasiliev.

### References

- [1] A. Арруда, *Воображаемая логика Васильева* (Imaginary logic of Vasiliev), Избранные труды Н. А. Васильева (187–208), Москва 1989.
- [2] В. А. Бажанов, *Николай Александрович Васильев: жизнь и творчество* (Nicolas Vasiliev: Live and Work), Избранные труды Н. А. Васильева (209–228), Москва 1989.
- [3] L. Chwistek, *Granice nauki. Zarys logiki i metodologii nauk ścisłych* (Limits of science. Outline of logic and methodology of sciences), Lwów b.r. (? 1935).
- [4] J. N. Keynes, *Studies and Exercises in Formal Logic*, London 1906.
- [5] G. L. Kline, *N. A. Vasiliev and the Development of many-valued Logics*, Contributions to Logic and Methodology in Honor of J. M. Bocheński (315–326), Amsterdam 1965.
- [6] A. Korcik, *Przyczynek do historii klasycznej teorii opozycji zdań asertorycznych* (Contribution to the history of classical theory of opposition), Roczniki Filozoficzne IV (33–49), 1955.
- [7] Н. Лузин, *Отзыв о работах Н. А. Васильева по математической логике* (About works of N. A. Vasiliev on mathematical logic), Избранные труды Н. А. Васильева (184–185), Москва 1989.
- [8] А. И. Мальцев, *Отрывок из статьи и "К истории алгебры в СССР за первые 25 лет"* (Fragment of article of "On the history of algebra in Soviet Union during first 25 years"), Избранные труды Н. А. Васильева (185–187), Москва 1989.



- [9] В. А. Смирнов, *Логические взгляды Н. А. Васильева* (N. A. Vasiliev's opinions on logic), *Очерки по истории логики в России* (242–257), Москва 1962.
- [10] W. Suchoń, *Sylogistyki Wasiljewa: Logika pojęć* (Vasiliev's sylogistic: Logic of notions), *Ruch Filozoficzny LV* (35–51), 1998.
- [11] W. Suchoń, *Sylogistyki Wasiljewa: Logika "urojona"* (Vasiliev's sylogistic: Imaginary logic), *Ruch Filozoficzny LV* (53–64), 1998.
- [12] Н. А. Васильев, *О частных суждениях, о треугольнике противоположностей, о законе исключенного четвертого* (On particular propositions, triangle of oppositions and law of excluded fourth), Казань 1910 (*Ученые записки Казанского Университета*).
- [13] Н. А. Васильев, *Воображаемая логика (конспект лекции)* (Imaginary logic (conspectus of lectures)), *Избранные труды Н. А. Васильева* (126–130), Москва 1989.
- [14] Н. А. Васильев, *Воображаемая (неаристотелева) логика* (Imaginary (non-Aristotelian) logic), *Журнал Министерства Народного Просвещения ШЛ/8* (207–246), 1912.
- [15] Н. А. Васильев, *Логика и металогика* (Logic and metalogic), *Логос* 1–2 (53–81), 1912/3.
- [16] N. Vasiliev, *Imaginary (non-Aristotelian) Logic*, *Atti dei Quinto Congresso Internazionale di Filosofia*, Napoli 1925.
- [17] Н. А. Васильев, *Отчет приват-доцента по кафедре философии Императорского Казанского Университета Н. А. Васильева о ходе его научных занятий за время с 1 июля 1911 г. по 1 июля 1912 г.* (Account of course of scientific research from 1 July 1911 to 1 July 1912 by N. A. Vasiliev, privatdozent of Imperial University of Kazan), *Избранные труды Н. А. Васильева* (149–169), Москва 1989.

WOJCIECH SUCHOŃ  
Department of Logic  
Jagiellonian University  
ul. Grodzka 52  
31-044 Kraków, Poland  
suchon@grodzki.phil.s.uj.edu.pl