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Anna Pietryga

PARACONSISTENT LOGICS, CONVENTIONALISM AND ONTOLOGY

Abstract. Paraconsistent logics may be viewed as one of the last elements in a series of rapid developments in science in the 19th and early 20th c., triggered by the appearance of non-Euclidean geometries. The philosophy of conventionalism, which gave a metatheoretical framework to the basic changes involved, may also help in evaluating the truth import of (paraconsistent) logic and in determining its relation to ontology.

My paper squares with the "package" suggested by Professor Da Costa in his address, as to what he called "agnosticism" concerning the true contradictions. It takes a totally different perspective, though. It may be interesting to see how similar results can be reached in a completely different way.

The purpose of this paper is twofold:

1) to show the appearance of paraconsistent logics against the background of multiple scientific developments.
2) to underline the importance of the conventionalist thought for the interpretation of paraconsistent logics

I

Paraconsistent logic appeared, significantly, after the period of rapid developments in scientific theories in the 19th and early 20th century, triggered by the appearance of non-Euclidean geometries. Other achievements to be mentioned here include philosophy as well as physics and linguistics. The following examples, together with the appendix containing chronology of the most important events, may help notice how quickly the example of rejecting axioms given by geometry spread over other disciplines, rendering new results. (The labels given below are of course only provisional, as the disciplines are sometimes difficult to separate.)

Geometry

Absolute time and space were declared subjective already by Kant in 1770 [22]. This challenged their "absolute value", but on the other hand did not present any alternative to the traditional picture of the world (except, of course, in epistemology). About 50 years later, the first publication on non-Euclidean geometry appeared due to Bolyai[1], and soon Lobachevsky and Riemann presented their own non-Euclidean results. The so-called hyperbolic geometry of Bolyai and Lobachevsky allows an infinite number of "parallels", containing a given point, to be drawn for a given line, while Riemann's elliptic geometry allows no infinite lines at all and no "parallels". These theses are contrary to Euclidean axioms 2 and 5.[2]

The non-Euclidean geometries are often mentioned with reference to non-traditional logics. Łukasiewicz explicitly referred to them e.g. in his foreword to "*O zasadzie sprzeczności u Arystotelesa*" [11], where he stated that the logic of that time was related to Aristotle's work just like Euclidean geometry was related to the *Elements* and that it needed a similar scrutiny. (In 1910, Łukasiewicz suggested that the non-Aristotelian logic to appear would challenge the principle of contradiction.)[3]

Even though the discovery (or creation) of the non-Euclidean geometries is just a part of the scene, it is fairly important for the achievements to follow. In constructing alternative geometries, mathematicians shook off the long-established habits of thought. It is important to notice that the change was really shocking. Some earlier thinkers, unable to prove the 5th axiom, chose rather to disbelieve their own calculations than to admit the result. (Gauss, reportedly, discovered the possibility of constructing a non-Euclidean geom-

[1] Bolyai's father devoted a considerable part of his life to the attempts of proving Euclid's axiom concerning parallel lines (see below); in fact, he warned his son not to deal with that "infernal Dead Sea", which consumed so much of his own life [14, vol. 19].

[2] Axiom 2.: An interval can be prolonged indefinitely.
Axiom 5.: If a line meets two other lines so as to make the interior angles on one side of it together less than two right angles, the two other lines, if prolonged indefinitely, will meet on this side [14, vol. 19].

[3] On the problem of non-Aristotelian vs non-Chrysippean logic, cf. [24].

etry before Bolyai, yet he was reluctant to publish the results) [14, vol. 19]. Next, by showing that opposite axioms can be substituted one for the other, the discovery paved the way for the development of conventionalism; it also encouraged other attempts at like substitutions and changes of perspective.

Philosophy of science – conventionalism

Around the turn of the century, the question of the status of scientific theories was also addressed by philosophers, namely by the French conventionalists — Pierre Duhem, Henri Poincaré and Eduard Le Roy. In a way, they could be considered predecessors of intuitionists in the philosophy of mathematics, as their views included the appreciation of intuition (under the influence of Bergson, perhaps most explicit in the works of Le Roy [20]); besides, they showed some reservations towards the formulation of the theory — which might be compared to the intuitionistic distrust towards language.

Among their theses one can find the following:

1) geometrical axioms are conventions; they are true by definition (it's a nonsense to ask whether a geometry is true or not); they *are* hidden *definitions*. They are chosen to fit our convenience; the only condition is that they should not cause contradiction [16];
2) physical laws are either *dogmatic definitions* which cannot be tested for their truth and falsity or *practical instructions* which are rather *effective* than *true* [10];
3) no experiment can ever destroy a single hypothesis, but only the whole theoretical set. Theories form systems, in which various components may be modified to fit the data: *experimentum crucis* does not exist in physics [6].

The famous Poincaré's story describes a world, contained within a sphere, in which the temperature is highest in the middle and gets lower towards the boundaries. Every object changes its size with temperature, and all bodies do so in the same proportion. Poincaré concludes that people in such world would not know it was limited, because getting smaller and smaller and walking more and more slowly they would never reach it borders. Another example he refers to is that of two-dimensional creatures, unable to accept Euclidean geometry, which would go beyond their experience [16]. In both of these worlds, their describe the phenomena they encountered, using their own theories, and never understanding our description of the same facts.

Physics

The theory of relativity, prepared by the work of many authors[4] and finally articulated by Einstein (1905), suggested that space and time — the two Kantian *a priori* forms — were not independent quantities. Another important idea put forward by Einstein was that light, under certain conditions, could be treated as a stream of particles, and not as a wave. In doing so, he questioned another traditional distinction. Soon, a reverse thesis was put forward by de Broglie, who claimed that the particles of matter could be treated as waves. The thesis formed the foundation for Schroedinger's equation, describing the phenomena of quantum mechanics (and supported by laboratory evidence one year after its formulation). Thus, both the opposition of space and time and the opposition of wave and particle (or energy and matter) were no longer considered as absolutely valid.

A perhaps still more surprising result was presented by Heisenberg: in relation to a given direction, we can only know either the position or the momentum of an object (e.g. of a particle), but never both. Moreover, this limitation cannot be changed by inventing better measuring instruments.

The last — and much discussed[5] — puzzling thesis to be mentioned here was put forward by Schroedinger, suggesting that facts come into being only when experienced. Before that, they exist together with other possible facts — all of them mutually exclusive — in the so-called superposition. Schroedinger's thesis, surprisingly enough, brought back to the current European thought the *esse* = *percipi* principle found in subjective idealism and also admitted the simultaneous existence of contradictory states.

In this way, by 1927 physics had not only shown that traditional oppositions may be questioned and modified (if not refuted); it had also challenged the commonsensical view concerning the relation between the truth and the cognizant subject: its results suggested that our access to knowledge is limited not only by the imperfections of man-made instruments but also by inherent properties of the object of study; furthermore, reality may be ambivalent, and therefore a subject seeking definite answers may be seeking his/her own mistake.

[4] Like Lorentz and Poincaré. The experiment of Michelson and Morley had no impact on the formulation of Einstein's theory [18].

[5] On the discussion concerning the problem, cf. e.g. chapter 34 of [18].

Philosophy of mathematics

In mathematics, the changes led to what is sometimes called the second crisis and compared to the one that occurred when Pythagorean tradition faced the problem of irrational magnitudes [1]. The changes involved the departure from the Kantian view of mathematics as rendering synthetic results determined by the two distinct *a priori* forms of time and space. The three main solutions which eventually appeared differed as to their attitude towards the Kantian problems of the *a priori* as well as to their views upon the language of mathematics. All of them had appeared by 1930.

The first of them to appear is known as logicism. Concentrating on the reduction of mathematics to logic, it considered mathematical truths as analytic (mathematical truths can be derived from pure, i.e. formal logic, a thesis known as derivational logicism) [3]. This anti-Kantian view soon faced its own difficulties: the antinomies of set theory. The theory of types, which was supposed to solve the problem, was criticised for its ad hoc axioms.

Two next programmes that appeared are known as formalism and intuitionism, and associated with the names of Hilbert and Bernays versus Brouwer and Heyting, respectively. Hilbert's formalism aimed at consistency proofs for infinitary mathematics via consistency proofs for finitary mathematics, by showing that the former are reducible to the latter, with the ultimate ideal of constructing a formal system without contradiction. The formal bias in Hilbert's programme makes this approach distinct from the third answer to the crisis in mathematics, i.e. Brouwer's intuitionism.

The intuitionists paid hardly any attention to the language of mathematics, concentrating upon mental constructions instead. This led to rejecting many theses of traditional logic. One reason for the explicit disregard for language in intuitionism was that, according to Brouwer, mathematical truths do not involve language, being based on the *intuition of time* as flowing. Besides, the intuitionistic view was that "we can never be sure that the formal system expresses correctly our thoughts". This is due to the "fundamental ambiguousness of language" [8]. The system of intuitionistic logic was, of course, formalised (first by Heyting in 1930), but still the reservations concerning language form a part of the intuitionistic tradition.

Linguistics

In 1921, Edward Sapir published his conclusions concerning the relation between language and culture. According to the so-called Sapir-Whorf hypothesis, which Sapir put forward together with his student, Benjamin Lee Whorf, language influences the perception of the world (which rings the Humboldian bell). Thus, different languages correspond to different views of the world. The famous examples were taken from the Hopi language, which — according to Whorf — did not involve the categories of space and time. Instead of that, Hopi uses the *manifested* vs *manifesting* dichotomy: the *manifested* contains all measurables, including distance as well as time (which is a relation between events which have already happened); the *manifesting*, on the other hand, contains the future and other hidden phenomena, like thoughts, feelings etc. Consequently, Hopi grammatical system, according to Whorf, did not distinguish the past and present tenses.

The hypothesis was illustrated with an explicit comparison of different languages to alternate geometries: "to pass from one language to another is psychologically parallel to passing from one geometrical system of reference to another". (Whorf also formulated the problem of an European and a Hopi discussing geometry.) Our description of the world is not, according to the language relativists, a mere result of our experience: it is based on "the language habits of the group"[6].

Whorf was criticised for his translation techniques as well as for his line of argumentation, supposedly containing a vicious circle. Also, it was rejected for commonsensical reasons: the Hopi "could not" live without the time/space distinction [5]. More recently, Donald Davidson criticized the Sapir-Whorf hypothesis as based on the false "dualism of scheme and content, of organizing system and something waiting to be organized", which he called the "third dogma of empiricism". (Davidson put the Sapir-Whorf hypothesis on a par with Feyerabend and Kuhn's idea of scientific revolution and Quine's views on science [4] — all of these are, in his opinion, based on the same fallacy.)

Correct or not, the Sapir — Whorf hypothesis introduced the general revisionist tendency into linguistics. Its authors followed the example of Bolyai, Lobachevsky and Riemann just like Łukasiewicz and other logicians.

(Paraconsistent) logic

Generally speaking, by 1930 mathematicians had questioned Euclidean geometry and started playing substitution at its axioms; physicists rejected some most well-established dichotomies and expressed their epistemological doubts; linguists claimed that our view of the world depends upon the

[6] Quotations from Whorf are given after [5]; cf. [23]. On the relation between language and ontology, cf. [12].

grammar of our language; philosophers of mathematics attacked basic logical principles and expressed their distrust towards language. These events were accompanied by the philosophical conception of science as a man-made convention, which encouraged the introduction of further changes in the hitherto unquestioned parts of human knowledge.

At the same time, logic itself had undergone important developments, too. It was only in 1879 that axiomatizations for both the sentential and predicate calculus were available, which made it possible to start experimenting *more geometrico* with the system. In 1908 Brouwer started publishing a series of papers showing that the principle of excluded middle was not necessary. In 1921, Łukasiewicz's three-valued logic supported that view. In 1910 Łukasiewicz published *O zasadzie sprzeczności u Arystotelesa*, suggesting a like rejection of the principle of contradiction. In this situation, it seems that the next and perhaps final step to be taken in the series of changes was to admit the possibility and show the way of creating a logical system that would not involve the principle of contradiction. The ground for Stanisław Jaśkowski was prepared.

II

Let us turn now to the second question of this paper, that is, to the importance of the conventionalist thought for the interpretation of paraconsistent logics. The philosophy which is so closely linked to their orgin may be expected to shed some light on the status of their results, even if some points in the conventionalist thought may be exaggerated or false. I would like to pay closer attention to two points made by conventionalists, adding some information on how they were further elaborated by other thinkers. I believe that both of them contain a grain of truth which relates also to the paraconsistent logics.

The first point refers to the revisability of various elements of science. This thesis was formulated by Duhem [6] and later discussed by Tarski [21] (In Quine's writings, the thesis is meant to fight one of the "Two dogmas of empiricism" [19].) The formula known as the Duhem-Quine thesis says that no fact can ever destroy a single hypothesis, or — in its stronger form — that every particular hypothesis can be retained in every circumstances (at the expense of rejecting other elements of the theory). The less visible part of this formula says that every element of a theory, including the principles of logic, is liable to revision. This thesis reminds us that our theories are not finished products or perfect insights. The history of science gives only too many examples of once well-established theses which had to be modified or rejected. (Some of them have been mentioned in the first part of this paper.)

Therefore, even though we may be seeking the scientific truth, we are bound to strive through the imperfections of our knowledge. Duhem claimed that physical laws are neither true or false, but approximate. Still, the truth may be considered the ultimate aim of that process, as it is e.g. in the philosophy of Peirce ("The real [...] is that which, sooner or later, information and reasoning would finally result in [...]") [15], even though our scientific ambitions may be limited, in Popper's words, to the *verisimilitude* of our theories.

The other point refers to the relation between convention and truth. Conventionalists declared that some part of our knowledge of the world depends on conventions — a declaration which may be regarded as a decline from the search of truth. On the other hand, however, they explicitly declared that in the case of a convention it is nonsense to ask whether it is true or not. In doing so, they paid due respect to the truth, which does not depend on our caprice. On the contrary, it is the goal of the scientist to establish a balance between the accepted theses and form a coherent theory of the part of reality he is trying to investigate. It seems that the conventionalists appreciated the difference between *the true* and *the postulated*, corresponding to the relation between *reality* and its (*iconic) model* in the present-day philosophy of science.[7]

In Poincaré's view, we must choose the facts we take into consideration in science; we are choosing them from the shapeless cloud of experience [16]. Knowledge in itself is not worth our pursuit; we direct our attention at some purpose and study only the related facts (the facts we choose to consider related). Within that fragment at least some statements are based on conventions and could possibly be formulated differently.

It is perhaps worth mentioning that Popper, who is considered an opponent of conventionalism, formulated a parallel view. In Popper's view, it is pointless to study every possible subject (like e.g. the correlation between the size of the book and its weight); a scientist should formulate interesting problems and then try to solve them [17]. Moreover, we should not aim at maximum precision for its own sake, as we cannot foresee what *sort* of precision will be needed in future. In Popper's example, it was not possible to know beforehand that the notion of simultaneity should be made more pre-

[7] A theory may be considered as an iconic model of the class of phenomena it refers to and as a mathematical model of its own formulation (and not as a set of sentences). Cf. [7]

cise — before the theory of relativity appeared [18]. (The example is not quite correct, as the problem had been discussed by Poincaré already in 1902).

There are two things, yet, that *are* important in a theory, namely:

1) its logical content (the class of its consequences)
2) the set of sentences that are *not* compatible with the theory. These are called the informational content of the theory.

The problem is we can never quite foresee either of them. In Popper's words: "we never know what we are talking about". The theory may prove to have some consequences we did not wish it to have. (Cf. intuitionists distrust towards language.) Popper's solution was to improve the theory only when we face problems.

Generally speaking, conventionalists teaching is that our knowledge is nothing but provisional, and the statements it contains are, to some extent at least, conventions aimed at organizing fragmentary data. If we refer these points to logic and ontology, we may conclude that logic constructs its own world, just like natural (ethnic) languages construct their own worlds, and like natural sciences (in a different sense of "natural") construct theirs. These worlds are meant as representations of the objective world. A logic and its ontology, and generally speaking any theory and its model of reality, come as a pair, as a instruction and the machine. Logic gives directions on how to move within the so-and-so constructed world of relations without the risk of drawing false conclusions from true premises.[8] This may be compared to telling the way in a(n unknown) city. In another city one needs different directives to walk from a railway station to a marketplace. Traditional logic was adequate for a consistent model of reality; it could not cope with inconsistent models. Paraconsistent logics correspond to models containing some inconsistencies

If the world that fits our logic is paraconsistent, this does not mean to say that the extralinguistic world is paraconsistent — it means that our model is. This is parallel to the fact that asserting a statement does not make it true in the extralinguistic world. (Saying "the coffee is hot" does not make it hot.)[9] It is worth noticing that Jaśkowski's system [9] did not deal

[8] A similar view was very briefly presented by Bocheński in [3]. Bocheński also gives a brief review of various opinions concerning the relation between logic and ontology. Cf. also Żegleń [25] and Muszyński [13].

[9] Even though there are many expressions that actually create the facts they refer to, like: "I take you to be my wife" or "I name this ship the Queen Elizabeth", when uttered

with inconsistent reality: it dealt with inconsistent opinions. The world in which various people believe in various things does not become contradictory because of this. Dialetheism does not follow from the fact of constructing a paraconsistent logic or inconsistent physical theories. (And non-dialetheism does not follow from traditional logic, either.)

This does not mean we should not construct paraconsistent systems. We may follow Hilbert's example and try how far the possibilities of constructing such systems actually go. If the construction of a formal logic is to be judged for its truth import, it may well be: the results do give us some information about the technical possibilities of formal systems, if not about the extralinguistic world.

Appendix: Chronology

about 500 B.C. Pythagorean crisis.

about 300 B.C. Euclid. *Elements*.

1770 Immanuel Kant. *De mundi sensibilis et intelligibilis forma et principiis.* A dissertation presenting time and space as subjective forms.

1823 Janos Bolyai. Non-Euclidean geometry (published 1932).

1826 Nikolay Lobachevsky. Non-Euclidean geometry (published 1829).

1854 Bernhard Riemann. Non-Euclidean geometry.

1879 Gottlob Frege. *Begriffsschrift*. The first axiomatic logical system.

1902 Henri Poincaré. *La science et l'hypotèse*.

1905 Albert Einstein. Special theory of relativity.

1908 Luitzen Egbertus Jan Brouwer. *On the untrustworthiness of the Logical Principles* (presenting a criticism of the principle of the excluded middle).

1910 Jan Łukasiewicz. *O zasadzie sprzeczności u Arystotelesa*.

1910 Nicolas A. Vasil'iev. Non-Aristotelian syllogistics: logic of notions.

1911 Nicolas A. Vasil'iev. Non-Aristotelian syllogistics: imaginary logic.

1918–21 Jan Łukasiewicz. Three-valued logic.

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in appropriate circumstances. They were discussed by J. L. Austin, who gave them the name of *performative utterances* or *performatives*. Cf. [2]

1921 Edward Sapir. *Language*.

1923–24 Louis de Broglie. A thesis on the dual character of matter.

1926 Erwin Schrödinger. Schrödinger's equation.

1927 Werner Heisenberg. Uncertainty principle.

1927 Clinton Davisson and Lester Germer; George Thomson. Evidence for wave qualities of the electrons.

1930 Arent Heyting. First formalised intuitionistic system.

1935 Erwin Schrödinger. *Die gegenwaertige Situation in der Quantummechanik*. (A paper on superposition).

1948 Stanisław Jaśkowski. Discussive logic.

1959 Newton Da Costa. Many-valued paraconsistent logic.

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Anna Pietryga
Instytut Filologii Polskiej
Uniwersytet Opolski
ul. Oleska 48, 45-052 Opole, Poland
apap@zeus.polsl.gliwice.pl