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OPENING ADDRESS: PARACONSISTENT LOGIC

1. Introduction

I am honoured with and touched by the invitation of delivering the opening address of this Congress. Firstly, to see paraconsistent logic flourishing and growing, as we can readily see by simply glacing over the programme of this conference, is among one of my greatest joys. Secondly, and equally important, because this congress takes place in the University of Toruń. I am honoured for having lectured here, a most congenial and stimulating place, and could not think of a better place for a conference dedicated to the memory of Stanisław Jaśkowski. In particular, I am delighted for having had a correspondence with him, and although I was deprived of the pleasure of meeting him personally, I was fortunate enough for having colaborated with some of his disciples, such as L. Dubikajtis and T. Kotas. All and all, Toruń in particular and Poland in general are for me a second home, for all the kindness and care everyone has shown to me over several years, since my very first visit to this country.

What I would like to discuss today are some general points about paraconsistency. I shall start by considering what I take to be one of the most compelling motivations for the construction of paraconsistent logic: the possibilities it opens up in the foundations of set theory. Secondly, I would like to take a brief look at the history of paraconsistent logic. And finally, I intend to make some comments of a more philosophical nature.

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Motivation: paraconsistency and set theory

In 1901, in his paper on mathematical problems, Hilbert noticed that: 'The mathematician will have to take account not only of those theories that come near to reality but also, as in geometry, *of all logically possible theories*, and he must always be careful to obtain a complete survey of the consequences implied by the system of axioms laid down'. To some extent, it is possible to say that paraconsistent logic has appeared as the result of applying this Hilbertian norm to the axiomatisation of set theory. In order to see why this is the case, let me recall a few things about set theory.

We can say that Cantor's naive theory was based on two fundamental principles: the postulate of extensionality (if the sets x and y have the same elements, then they are equal), and the postulate of separation or comprehension (every property determines a set, composed of the objects that have this property). The latter postulate, in the standard language of set theory, becomes the following formula (or scheme of formulas):

$$\exists y \forall x (x \in y \leftrightarrow F(x)) \tag{1}$$

Now, as is well-know, it is enough to replace the formula $F(x)$, in (1), for $x \notin x$ to derive Russell's paradox. That is, the principle of separation (1) is inconsistent. Thus, if one adds (1) to first-order logic, conceived as the logic of set theoretic language, a trivial theory is obtained.

There are also other paradoxes, such as Curry's and Moh Schaw-Kwei's, that indicate that (1) is trivial or, more precisely, trivialises the language of set theory, if the underlying logic is classical — even if we ignore negation. In other words, classical positive logic is incompatible with (1); the same holds also for several other logics, such as the intuitionistic one.

Now, classical set theories are distinguished by the restrictions that are imposed on (1) so that paradoxes can be avoided. In order that the theory thus obtained does not become too weak, some further axioms, besides extensionality and separation (with due restrictions), are added, depending on the particular case we are considering.

Thus, for instance, in Zermelo-Fraenkel set theory (ZF), separation is formulated in the following way:

$$\exists y \forall x (x \in y \leftrightarrow (F(x) \wedge x \in z)), \tag{2}$$

where the variables are subject to obvious conditions. In ZF, then, $F(x)$ determines the subset of the elements of the set z that have the property F

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(or satisfy the formula F(x)). In the Kelly Morse system, on the other hand, separation is as follows:

$$\exists y \forall x (x \in y \leftrightarrow (F(x) \wedge \exists z (x \in z))) \tag{3}$$

And, finally, in Quine's NF the notion of stratification is employed, and the scheme of separation has the form:

$$\exists y \forall x (x \in y \leftrightarrow F(x)) \tag{4}$$

provided that the formula F(x) be stratifiable (besides the standard conditions with regard to the variables).

However, if we adopt Hilbert's motto, we can ask whether it would be possible to examine the problem from a distinct point of view. In other words, what is needed in order to maintain the scheme (1) *without* restrictions (with no regard to the conditions on the variables)? The answer is immediate: one should change the underlying logic, so that (1) does not inevitably lead to trivialisation. The separation scheme, without 'big' restrictions, leads to contradictions. The consequence is that this logic has to be paraconsistent.

It was slowly verified that there are infinitely many ways to make the classical restrictions to the separation scheme weaker, each of them corresponding to distinct categories of paraconsistent logics. Furthermore, extremely feeble logics have been formulated, and based on them it is possible to employ, without trivialisation, the scheme (1). Some set theories, in which the forms (2), (3) and (4) of separation are either combined or adopted in isolation, have also been constructed.

An important point is that several paraconsistent set theories contain the classical one, in Zermelo-Fraenkel's, Kelly-Morse's or Quine's formulations. Hence, paraconsistency goes *beyond* the classical domain, and allows, among other things, the reconstruction of traditional mathematics. It is fair then to claim that paraconsistent theories extend the classical ones, just as Poncelet's imaginary geometry encompasses the standard 'actual' geometry.

Furthermore, we should stress a kind of aporia found in the very foundations of logic. Classical elementary logic (indeed even only part of its positive part) and the separation postulate are both evident. We may well claim that they are equally evident or intuitive. However, *they are mutually incompatible*, and constitute thus a case of incompatible evidences — an aporia that undoubtedly would delight the Eleatic philosophers or the sophists.

In particular, notice that classical theories adopt a particular line of approach, and paraconsistent theories, another. All this is in perfect agreement with our quotation of Hilbert: we should explore *all* possibilities. And we

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stress, this exploration contributes to a better understanding even of the classical position itself: a clearer understanding of negation; the conscience of the possibility of the discourse, even if one partially rejects the principle of non-contradiction; a proof that such a principle is at least partially true and so on.

But paraconsistent logic has received motivations from many other fields, scientific and philosophical. On the one hand, it should be stressed here the relevance of Heraclitus's doctrines, certain aspects of Marxism, Hegelian dialectics, certain views of Wittgenstein's on logic and contradiction, the Kantian antinomies, and also Meinong's theory of objects. On the other hand it is undeniable that the presence of inconsistencies in some scientific theories, the semantic paradoxes, the set-theoretic antinomies as well as the use of incompatible theories in physics (Bohr's atom, plasma theory etc.) have been of considerable heuristic worth. More recently, paraconsistent logic has also been motivated by artificial intelligence and other areas of applied science and technology. We can mention, for instance, the problem of handling inconsistent bits of information in the domain of expert systems, or the question of logical programming with contradictory clauses.

Finally, let me just say a few words about terminology. We say that theory T is *inconsistent* or *contradictory* if it contains contradictory theorems, i.e. theorems such that one is the negation of the other; otherwise, T is said to be *consistent*. If the set of theorems of T coincides with the set of its formulas, then T is called *trivial* or *overcomplete*; otherwise, T is called *non-trivial* or *non-overcomplete*. If L is the underlying logic of T we say that L is *paraconsistent* if it can be the underlying logic of inconsistent but non-trivial theories.

Thus, if L is paraconsistent, it distinguishes the two concepts of triviality and inconsistency. Therefore, classical logic, intuitionistic logic and numerous other logics are not paraconsistent in this sense. In paraconsistent logic, the principle of contradiction, in one form or another, is qualified or limited.

Historical considerations

As is well known, the main forerunners of paraconsistent ideas are Jan Łukasiewicz and Nicolaj Vasil'ev. In 1910/1911, independently of each other, they have stressed the importance of a revision of some laws of Aristotelian logic. In this way, they have opened up the possibility of the development — in an analogy with non-Euclidean geometry — of non-Aristotelian logics, namely those in which the principle of contradiction is somewhat restricted.

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In his 1910 work, *On the Principle of Contradiction in Aristotle*, and in a related paper from the same period, Łukasiewicz presented three different Aristotelian formulations of the principle of contradiction: an ontological, a logical, and a psychological. He then rejected each of these formulations, and argued that this principle is not so basic as is usually supposed. A precedent was therefore created for the beginning of non-classical logic. Unable however to elaborate a particular logical system at this time, the precedent, to a certain extent, was lost.

Similarly, although Vasiľev has not formulated a particular system because of his ideas related to imaginary logic, he is correctly taken as a precursor of paraconsistent theories. It should be noted here the inspiration that he drew from Lobatchewski's work on non-Euclidean geometry. More than its name (at the time, this geometry was known as imaginary geometry), the methods of its construction were also strikingly similar to the ones used by Vasiľev. Moreover, Vasiľev believed that, similarly to Lobatchewski's geometry, his logic could also receive a classical interpretation.

But it was not earlier than 1948 that Stanisław Jaśkowski, under Łukasiewicz's influence, would propose the first paraconsistent propositional calculus. And, to the best of my knowledge, he was probably the first to formulate, with regard to inconsistent theories, the issues connected with non-triviality. One of the basic conditions of his system was that, when applied to contradictory theories, it should not make all the formulas a theorem: that is, as opposed to classical logic, the presence of contradictions should not entail the trivialisation of the system.

Furthermore, Jaśkowski's paraconsistent theories have been developed in order to satisfy three crucial motivations: (1) to provide a conceptual machinery to approach the problem of deductively systematising theories which contain contradictions; in particular, (2) the ones whose contradictions are generated by vagueness; and, finally, (3) to study some empirical theories containing contradictory postulates.

I was delighted to notice, in the early 1960's, that the work I had developed in Brazil by that time had close connections with Jaśkowski's. I recall, as if it were today, reading the English abstract of one of his papers, and realising that the two of us were independently producing works of a striking similarity. I then sent him a letter, and that is how my long term contact with the Polish community of logicians started.

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Philosophical remarks

The construction of paraconsistent set theories made clear that, at least at the level of mathematics, there are inconsistent but non-trivial theories. In other words, there are theories T in which both a formula A and its negation $\neg A$ are theorems of T, and some formula B of T is *not* a theorem. This fact provides an important support for paraconsistency, since it shows that the attempt at accommodating inconsistencies by devising appropriate inconsistent but non-trivial theories is by no means empty or unrealisable. On the contrary, it provides a distinctive perspective to the issues under consideration. Instead of retaining classical logic, and avoiding the inconsistency by rejecting one or another of the premises which generate it — making more or less *ad hoc* moves — we retain the inconsistency, change the underlying logic to a paraconsistent one, and study the properties of the 'inconsistent object' so 'generated'. The important feature is that these 'inconsistent objects' have certain determined properties and lack others: it is simply *not* the case that everything goes with regard to them. So, as opposed to what happens in the case of classical logic, there is a whole new domain of investigation determined by the formulation of paraconsistent logic: the domain of the inconsistent.

Now, the issue arise as to the status of the resulting theory: is it *true*? Can we say that there are true inconsistencies? The answer depends on several considerations. (1) What is the notion of truth used in this context? (2) What kinds of objects are we considering (mathematical objects or physical objects)? (3) What notion of existence is assumed? And how are ontological commitments to be spelled out? Of course, the examination of these issues involves particular philosophical accounts, and I cannot do better here than consider them in fairly general terms. But I hope to say enough in order to make clear the approach I favour with regard to them.

According to some authors (such as Priest), the answer to the question *Are there true contradictions?* is affirmative. The examples given by Priest are the logical and semantic paradoxes, statements about moving objects (objects subject to change), and certain views in the foundations of mathematics. In order to articulate an 'ontic' or 'realist' view about true contradictions, Priest advocates (i) a strong notion of truth — truth *simpliciter* understood in the correspondence sense — (ii) a classical view about existence (as the range of bound variables), and (iii) an extended claim as to the domain of his theory — which incorporates both mathematical and physical objects. (Of course, in order to avoid triviality, Priest adopts a paraconsistent logic:

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see his system LP.) So Priest's approach countenances classical views about truth and existence, and applies them to a wide ranging domain. In our view, Priest's commitment to several doctrines is by no means fortuitous: in order to be adequately accommodated, inconsistencies require a whole package of logical and philosophical doctrines (indeed, a research programme). Of course, there are stronger and weaker programmes, some are closer, some further from classical proposals.

It seems to me that, in retaining classical notions of truth and existence, Priest's proposal became committed to metaphysical views which are decidedly strong. Given the use of truth in the correspondence sense, and the claim that our claims about the world (be it the 'empirical' or the 'mathematical' world) are to be true, Priest's view is *ipso facto* committed to all objects which are postulated in these claims. In particular, his proposal is committed to 'inconsistent objects' in the physical world: the objects to which our inconsistent but true physical theories refer. But how can their existence be established?

The argument to this effect assumes, of course, the classical account of ontological commitment: we are ontologically committed to whatever our bound variables range over. And in the case of inconsistent theories, this criterion leads us to postulate objects which both have and lack a given property (for instance, the liar sentence is both true and false, Russell's set is both a member of itself and it isn't etc.). And the same goes for theories about the physical world.

But this argument is not so conclusive. First of all, this criterion of ontological commitment is *not* independent of particular philosophical assumptions. It comes as part of a philosophical programme — Quine's view — and it has built into it, as it were, a given logic: *classical* first-order logic. It goes without saying that, as such, it is at odds with Priest's own dialetheic approach, in which a paraconsistent logic is advocated. Moreover, Quine's criterion is *not* independent of logic: if we change the underlying logic of a given theory, we change the entities we are quantifying over. This can be seen in several ways. If we move to second-order logic, we are allowed to quantify over predicates and relations. As a result of its strong expressive power, several mathematical theories can be better formulated (in particular, as is well known, arithmetic and analysis are categorical). Because of this, several nominalist proposals (such as those developed by Field and Hellman) have adopted second-order logic as part of their nominalisation strategies of science and mathematics. The idea is that, by increasing the strength of the logic, we can decrease our ontological commitments. Secondly, using para-

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consistent logic, we are allowed to quantify over certain constructions (such as Russell's set) which are impossible in classical logic, given its identification of inconsistency with triviality.

The point here is that Quine's slogan — to be is to be the value of a variable — can only have any force once a particular logic is admitted. Quine knows that, of course. The problem is that his view assumes a logic (classical fist-order logic) which is not the most adequate to deal with inconsistencies in a heuristically fruitful way.

I suggest to address the inconsistency issue differently. We may well explore the rich representational devices allowed by the use of paraconsistency in inconsistent domains, but withholding any claim to the effect that there are 'inconsistent objects' in reality. Whether the world is indeed 'inconsistent' — assuming that there is a sensible formulation of this claim — is something we would rather be *agnostic* about. Just as empiricists (such as van Fraassen) are agnostic about (the existence of) unobservable entities in science, I am agnostic about the existence of true contradictions in nature. And one of the reasons in support of this claim is an *underdetermination argument*. Given the hierarchy of paraconsistent systems presented in my **C**-systems, there are always infinitely many paraconsistent logics which can be used to accommodate a given 'phenomenon' — whether it is an 'inconsistent' reasoning or an 'inconsistent' theory. Which of these paraconsistent logics reflects *the* logic of the world? There is no argument based on purely observational terms that could establish this in general. We can select, of course, one of these logics on *pragmatic* grounds, but these grounds are certainly not enough to establish a substantive claim about the world. For instance, if one of these logics make the modelling of the inconsistency in question easier, why should this be taken as a reason for this logic to be *true*? Simplicity may well be a sensible criterion to adopt on pragmatic grounds, but the claim that a logic selected on this basis is (likely to be) true, is to confuse pragmatic and epistemic issues. Why should the world conform to our cognitive limitations? Of course, it might well do. But to establish this demands an argument that goes beyond what we observe: it requires a metaphysical claim of the simplicity of reality. However to a certain extent, this is as strong as the claim that there are true contradictions, in the sense that both make substantial assertions about the world that transcend empirical observation. So both are metaphysical claims.

In my view, an alternative programme of interpretation of inconsistencies can be devised in which no commitment to this kind of metaphysics is required. The idea is first to avoid the claim that inconsistent theories

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are *true*; they are *quasi-true* at best. (I have explored this alternative with Steven French and Otávio Bueno in several papers.) The notion of quasi-truth receives a straightforward formal treatment (which I presented with I. Mikenberg and F. Chuaqui in 1986 in *Journal of Symbolic Logic*). But for my present needs it suffices to say that a sentence α is quasi-true if it models adequately only part of a given domain D, leaving open the 'complete' description of the latter. (Of course, in a precise sense, α is consistent with a true description of D.) In claiming that, with regard to inconsistent theories, all we need is to determine their quasi-truth, I am in position to provide a 'formal underpinning' to my agnosticism with regard to true contradictions. The idea is to change the notion of truth to the weaker notion of quasi-truth, withholding then the commitment to 'inconsistent objects', given that there are several distinct structures which describe the domain under consideration, and such objects are not countenanced in all of them. Moreover, I also suggest to revise Quine's slogan about ontological commitment, making explicit its dependence on the underlying logic. In this way, it becomes clear that it is *not* the only criterion to adjudicate between alternative logics.

So, the 'package' suggested here to accommodate inconsistencies is characterised by (1) the claim that inconsistent theories are *quasi-true* at best; (2) an agnosticism with regard to the existence of true contradictions; and (3) a re-evaluation of Quine's view about ontological commitment, emphasising its dependence on the underlying logic.

The striking feature of this 'package' is its logical pluralism, on the one hand, and the fact that it allows us to make sense of the several uses and applications of paraconsistent logic with no commitment to actual 'inconsistent objects'. The logical pluralism derives from the claim (3) above. Depending on the domain we are considering, different kinds of logic may be appropriate. For instance, if we want to model constructive features in mathematical reasoning, an intuitionistic logic is the best alternative; if we are concerned with inconsistent bits o information, the use of a paraconsistent logic is the natural option. In particular, we do *not* reject classical logic: it has its own domains and applications. To this extent, while dealing with distinct domains, paraconsistent logic and classical logic are complementary rather than rivals. (They become rivals only when applied to the *same* domain. The rivalry derives from the fact that they provide different accounts of the logical connectives.)

But in applying paraconsistent logic, we do not have to be committed to the existence of 'inconsistent entities' — this is the point of our claims (1) and (2) above. We can always use the resources provided by this logic

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only to help us in drawing consequences from inconsistent theories without triviality, but with no commitment to the *truth* of the theory in question; it can be at best quasi-true.

In this way, a non-committal (agnostic) interpretation of paraconsistency can be presented, making sense of the applications of paraconsistent logic which have been so important for its motivation. 'Inconsistent objects' can be accommodated without requiring an ontology which includes them. We can use paraconsistent logic without believing in 'actual contradictions'.

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