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THE MASTER ARGUMENT
AND BRANCHING TIME

Abstract. It is argued that reconstructions of the so-called 'Master Argument' of Dideros Cronos to the effect that possibility should be understood as present or future truth, essentially relies on two axioms: i) that every true proposition concerning the past is necessary, and ii) that it follows necessarily from a proposition being true that it always has been the case that it would be true. It is furthermore argued that these two axioms are inconsistent in the sense that any tense/modal semantics which incorporates both collapses either modally (fails to distinguish between truth simpliciter and modalised truth) or temporally (fails to offer a plausible semantical account for propositions about the future). This finding is, furthermore, taken as indicator for the more generel claim that there are principled difficulties involved in construing semantics for combined tense/modal logical systems.

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1. Introduction

The Master Argument was developed by the Megarean philosopher Diodorus Cronos in support of his concept of possibility as 'what either is or will be the case' — a concept of modality which, as noted by Aristotle (see [Aristotle 1908], Vol. XIII, Book VIII, Sec. III.), is in best accordance with the general Eleatic viewpoint that motion and change is impossible. In what follows, two reconstructions of The Master Argument will be explored and discussed in the light of different views on the nature of time and, *a fortiori*, which semantics is preferred when dealing with temporal expressions. Particular attention will be paid to reconstructions based on a branching concept of time on which the outcome of the Master Argument — the Diodorean concept of modality — seems most plausible. However, the claim will be that no plausible reconstruction has hitherto been reached. Neither on linear nor on branched tense semantics. More generally it will be argued that reconstructing the Master Argument is not a feasible project until a satisfactory combined tense-modal semantic has been developed. It is furthermore suggested, rather pessimistically, that such a semantic never will be developed, not due to lack of ability among those who have so far tried, but for *a priori* reasons. Motion and change are certainly not impossible. What *is* impossible is to capture such dynamic aspects of the world in a model. And prevented from making such aspects manifest in a model the prospects for a combined tense-modal semantics are meagre.

2. The Master Argument

It is a bit of an exaggeration actually to call The Master Argument 'an argument'. What it refers to is, at best, a small fragment of an argument consisting of three apparently independent statements. From Diodorus' contemporaries we know that he argued for the inconsistency of this triad and that he, on this basis, rejected the third statement. Any demonstration of the alleged inconsistency and reasons why he gave up exactly the third statement seem, however, to be completely lost to posterity. The three statements are:

(D1) Every proposition true about the past is necessary.

(D2) An impossible proposition cannot follow from a possible one.

(D3) There is a proposition which is possible, but which neither is nor will be true.

Two questions ought to be addressed before any suggestion is put forward as to how to build up an argument from these fragments. Firstly what is meant by 'follow' in (D2) and secondly in which sense is the phrase 'proposition' used in (D1)–(D3)?

As regards the former question there seem to be three possible answers. 'Follow' might be meant in some temporal sense. This interpretation is, among others, found in Rescher's reconstruction of The Master Argument (see [Rescher 1966]) and gives (D2) the content that the impossible does not follow after the possible, i.e., that what is now possible cannot subsequently be impossible. The plausibility of this reading depends very much on the answer to the second question, i.e. on the right interpretation of the phrase 'proposition'. Is 'proposition' taken e.g. to be some kind of propositional function such that propositions depend for their reference on the time of utterance; if so, then this temporal interpretation of 'follow' as 'follow after' seems highly unlikely. For, take e.g. the proposition that 'it is possible that the Principal decides to give increasing financial support to Faculty of Arts and decreasing support to Faculty of Science'. When read as a propositional function this proposition is certainly true as uttered today and tomorrow and the day after tomorrow and in general for each day in the future in which he remains Principal of The University. But, the day he retires this proposition will not be possible any more (unless, of course the Principal decides to spend his private savings on The Faculty of Arts) and something impossible would thus follow after something possible.

Another answer to the first question could be that 'follow' is simply intended to mean material implication. Finally the answer might be that 'follow' is used as some stronger relation such as entailment. From what we know about Megarian philosophy in general it seems most likely that it is this third reading that was intended by Diodorus (see [Kneales], Chap. III.3.). But even within the scope of an intentional relation such as entailment, we are still left with a wide range of options to choose between. Among these, one in which 'R follows from Q' is taken to mean that Q *always* materially implies R. We shall see later on that such a reading might cause circularity in The Master Argument. However, for now it will do to fix the meaning of 'follow' in (D2) as some kind of intensional entailment relation.

As regards the second question we shall pursue two different readings of 'proposition', one I will call temporal *de re* and one I will call temporal *de dicto*. A temporal *de re* reading means that the reference of a proposition q is fixed once and for all independently of the time of its utterance. On a *de re* reading the picture is that the entire world — the past and future

history — is spread out in front of us. We are thus observing the world from an eternal point of view independent of any particular temporal outlook. Every event in the world is temporally ordered as before or after some other event but is not ordered relative to us i.e. there is no privileged moment in the world we can pick out as the present and hence no ordering in past and future. This might sound a bit queer. For how can e.g. a proposition such as 'It is raining' ever be attributed any content if abstracted into this eternal context? Is it the strong claim that it always has been and always will continue to be raining, or does it mean that it is raining at some particular time, and in that case, *which* time? This is of course only a problem for propositions which are not time indexed. Such non- indexed propositions seem to be ambiguous, referring either to a class of instances, all the rainy days in the same way as a predicate such as 'red' refers to the class of all red objects, or they refer to a truth value, true if it is rainy the day in question just as 'this apple is red' refers to a truth value. Under the *de re* reading they should be understood in this latter sense. But the difficulty is of course to sort out which day has to be rainy for the claim to be satisfied. How can such a particular content possibly be put into a *de re* proposition? And further: how is it possible to put different contents into qualitatively identical *de re* tokens of that proposition type?

This dilemma is very similar to the more familiar dilemma of giving content to modal propositions in their *de re* senses. How can we e.g. make sense of a sentence such as "Necessarily the number of planets is greater than seven" in its *de re* sense i.e. when refraining from any particular possible world outlook? In its *de dicto* sense the matter is fairly simple. The sentence can easily be evaluated in every possible world by checking in all these worlds whether the sentence "The number of planets is greater than seven" (or an adequate translation of that sentence into their language) is true. But in its *de re* sense we are supposed to pick out a certain number, the number seven, and investigate whether it is greater than seven in every particular world. The question is, how do we initially pick out *that* number. And the answer is that we cheat. We do take a standpoint in one particular world, viz. the actual, fix the reference of 'number of planets', and *then* abstract ourselves out to the wordless perspective. In the case of temporal *de re* propositions we do something similar. We take a standpoint at one particular time, the present time, fix the reference of 'it is now raining', and *then* abstract ourselves out into eternity. So, when reading 'It is raining now' in its *de re* sense we pretend to read it from outside any temporal context, but, in doing so, we still keep in mind one particular temporal context in

which the reference of 'now' was fixed — hooked on to the temporal world, as it were — and preserve *this* time in a small corner of our huge timeless space. The only difference from the modal case is that the modal runway remains constant, i.e. as the actual world, whereas the temporal runway moves (literally) as time goes by. The referent of a predicate such as 'number of planets' in *de re* modal statements is fixed from the same possible world outlook on every occasion, viz. from the actual world, whereas the referent of 'now' in *de re* temporal statements is fixed from a different temporal outlook from case to case. A *de re* statement in itself bears no witness of which temporal outlook initially has served as reference-fixing. Sometimes it can be traced from the broader context. But in formal representations of *de re* propositions no such context is available.

The alternative is to read 'proposition' in the temporal *de dicto* sense. In that case they are always read within a temporal context; not some fixed context, but the arbitrary context provided by the time of utterance. Past, present and future tenses occurring in such *de dicto* propositions are thus absolute in the sense that something described in past tense statements is always prior to something described in future tense propositions. In contrast, something uttered in a *de re* past tense proposition might very well be posterior to something described in a *de re* future tense proposition. One immediate consequence of this is that tense logical rules such as $q \rightarrow \mathsf{PF}q$ only are valid when the wffs in question are read as *de dicto* propositions.

3. *De re* and *de dicto* reconstruction of The Master Argument

It is now time to consider particular attempts at reconstructing The Master Argument. The first one is due to F. S. Michael (see [Michael 1976]) according to whom the first and third premises, respectively, can be formulated in the following way:

(D1′) $$(\mathsf{T}(q, t') \wedge t' \prec t) \rightarrow \Box \mathsf{T}(q, t).$$

(If a proposition q is true at some time t' before t, then the truth of q is necessary at t.)

(D3′) $$\Diamond \mathsf{T}(q, n) \wedge \mathsf{T}(\neg q, n) \wedge \forall_t \, (t \succ n \rightarrow \mathsf{T}(\neg q, t)).$$

(There is a proposition q which is possible, but false now and also at any future time.)

Now the argument proceeds by taking (D3′) and pointing out that q must have been false also before n, since if $\mathsf{T}(q, t')$ for some t' before n then we

would have $\mathsf{T}(q,t')$ and $t' \prec n$ and hence, by (D1$'$), that $\Box\mathsf{T}(q,n)$ and hence also $\mathsf{T}(q,n)$ which contradicts (D3$'$). Therefore $\forall_t (t \prec n \rightarrow \mathsf{T}(\neg q,t))$. But then, by (D1$'$), $\Box\mathsf{T}(\neg q,n)$ which by $\Box/\Diamond$ — interdefinability is equivalent to $\neg\Diamond\mathsf{T}(q,n)$ which, again, contradicts (D3$'$)!

It is striking that this conclusion, on Michael's account, can be reached without even introducing (D2). Even more striking, perhaps, that it can be reached only on the assumption from (D3$'$) that $\Diamond\mathsf{T}(q,n)$ and $\mathsf{T}(\neg q,n)$ (that q will not be the case in the future seems to be a superfluous fact).

The reason is, however, fairly simple: Michael takes propositions to be *de re* in the sense explained above. This means that a proposition, if true once, always will remain so. In particular it will always have been true in the past. So if something which *was* true is necessarily true now (D1$'$), it follows, without too much ado, that something once true, is necessarily true now. And, by the same token, once false, always false, and thus impossible now. (D1), when understood *de re*, is thus equivalent to the thesis that every modality collapses into actuality ($\Diamond\mathsf{T}(q,n) = \Box\mathsf{T}(q,n) = \mathsf{T}(q,n)$). Given (D1), it is therefore not surprising that Michael can produce a paradox only out of the assumption that $\Diamond\mathsf{T}(q,n)$ and $\mathsf{T}(\neg q,n)$.

What *is* surprising, however, is that Michael has taken (D1) in its *de re* sense. For although his reasoning, thus understood, is indisputable, (D1) seems plainly false in its *de re* interpretation. The intuition underlying (D1) is that it is in no one's power to change the past and, hence, that truths about the past, now, are necessary. But truths can only be 'about the past' if they are considered from within some temporal context. The *de re* proposition q, if it has any content at all, must have been loaded with this content in a temporal context in which this content is expressible by means of temporal indices. q refers e.g. to the fact which at a certain time t is expressible as "Today the Principal will close down the Department of Chemistry and thus raise money for twenty new chairs in the Department of Philosophy". This is the way q has been coded, as it were. Now, for q to be something past, it must be possible to decode it again in another temporal context later than t where it has to be expressible — this time by means of other temporal indices e.g. "x days ago the Principal decided to shoot down the Department of Chemistry etc.". Only from this later perspective is q about the past. But to decode q from within a particular temporal context is exactly to conceive of q as a *de dicto* proposition whose *prima facie* value can be read off directly relative to a certain context. For (D1) to make any sense at all, it must thus use 'proposition' in the *de dicto* sense. So (D1$'$) is simply false.

The next reconstruction we will consider, the one due to A. Prior (see [Prior 1967] p. 32–33), does take (D1) in its *de dicto* sense. Prior makes use of a minimal modal/tense-logic in his reconstruction:

(D1″) $\mathsf{P}q \to \Box \mathsf{P}q\,.$

(Every true proposition concerning the past is necessary.)

(D2″) $\Box(q \to r) \to (\neg\Diamond r \to \neg\Diamond q)\,.$

(What is impossible cannot follow from what is possible.)

(D3″) $(\neg q \wedge \neg \mathsf{F}q) \wedge \Diamond q\,.$

(Something that neither is, nor will be is possible.)

In Prior's reconstruction the following two auxiliaries are added to the original premises:

(P1) $\Box(q \to \mathsf{HF}q)\,.$

(From something's being the case it follows necessarily that it has always been the case that it would be the case.)

(P2) $(\neg q \wedge \neg \mathsf{F}q) \to \mathsf{P}\neg \mathsf{F}q\,.$

(Of whatever is and always will be false, it has already been the case that it will always be false.)

Now the negation of (D3″) can be proved from (D1″), (D2″), (P1) and (P2):

1	(1)	$\neg q \wedge \neg \mathsf{F}q$	A
1	(2)	$\mathsf{P}\neg \mathsf{F}q$	→E (P2), 1
1	(3)	$\Box \mathsf{P}\neg \mathsf{F}q$	→E (D1″), 2
	(4)	$\Box \mathsf{P}\neg \mathsf{F}q \to \neg\Diamond q$	→E (P1), (D2″) ($\neg \mathsf{P}\neg \mathsf{F}q/r$ in (D2″))
1	(5)	$\neg\Diamond q$	→E 3, 4
	(6)	$\Diamond q \to (q \vee \mathsf{F}q)$	→I 1, 5, Inversion

Here the conclusion is reached on behalf of (P2) which is a strong semantic thesis claiming that time must be thought of as discrete. The conclusion can, however, also be reached (for details, see [White 1984]) in a modal/tense-logic similar to Prior's, except that it is characterised by a frame defined by the two further restrictions:

Forward linearity: $(\mathsf{F}p \wedge \mathsf{F}q) \ \to \ (\mathsf{F}(p \wedge q) \vee \mathsf{F}(p \wedge \mathsf{F}q) \vee \mathsf{F}(\mathsf{F}p \wedge q))\,,$

Backward seriality: $\mathsf{H}p \to \mathsf{P}p\,.$

The question about dense contra discrete time which so often has been introduced in debates about The Master Arguments (among others by Prior, cf. (P2)) is thus entirely superfluous.

As it stands, however, there are two premises in the argument which are worth scrutinising: (D1″) and (P1). According to one line of attack, which for (quasi)historical reasons often has been called Ockhamist, (P1) is acceptable, but (D1″) ought to be denied, or at least constrained such that it only applies to genuine past tense propositions i.e. not to past tense statements such as "It was the case yesterday that I would tie my laces two days hence" which really expresses something about the future. According to another line of attack, the Peircean inspired, (D1″) should be accepted, but (P1) denied.

4. Ockhamist, Peircean and other branched tense-modal logics

The two responses agree, however, in their main objection: that time is branched and not, as assumed in Prior's reconstruction, linear. This metaphysical idea can be expressed as a structure consisting of a set of moments, K, and a "before-after" relation, $\prec$, restricted by: for any $\alpha, \beta, \gamma \in K$

Connection: $(\beta \prec \alpha \wedge \gamma \prec \alpha) \rightarrow (\beta = \gamma \vee \beta \prec \gamma \vee \gamma \prec \beta)$
Transitivity: $(\alpha \prec \beta \wedge \beta \prec \gamma) \rightarrow \alpha \prec \gamma.$

Furthermore we define paths, or histories, H, in this branching structure as:

(1) for all $\alpha, \beta \in H$: $\alpha \neq \beta \rightarrow (\alpha \prec \beta \vee \beta \prec \alpha)$;

(2) if X is any subset of K satisfying (1), then $X \subseteq H$.

$\mathcal{H}\alpha$ is the set of all H for which $\alpha \in H$.

The main difference between the Ockhamist and the Peircean line of attack lies in their corresponding semantics for such a branching structure. In particular to their treatment of future tense statements. One of these semantic differences is that according to Ockhamists, future tense statements are evaluated relative both to a moment, α, and a certain history, H, such that:

$$v_\alpha^H(\mathsf{F}q) = \mathfrak{t} \quad \text{iff} \quad v_\beta^H(q) = \mathfrak{t} \text{ for some } \beta \in H \text{ such that } \alpha \prec \beta.$$

As opposed to this, Peirceans operate with absolute truth in the sense that its evaluation is only relative to a moment:

$$v_\alpha(\mathsf{F}q) = \mathfrak{t} \quad \text{iff} \quad \text{for every } H \in \mathcal{H}\alpha$$
$$\text{there is a } \beta \in H \text{ such that } \alpha \prec \beta \text{ and } v_\beta(q) = \mathfrak{t},$$

i.e. $\mathsf{F}q$ is Peircean-true relative to α iff it is Ockham-true relative to α and to each $H \in \mathcal{H}\alpha$. This is the reason why the Peircean, but not the Ockamist, accepts (D1″) $\mathsf{P}q \to \Box\mathsf{P}q$. The Ockhamist kind of counterexample to (D1″) is "If it was the case yesterday that I would tie my shoe laces two days hence, it was necessarily the case yesterday that I would tie my shoe laces two days hence", which is just to say that something future, viz. my tying the shoe laces tomorrow is necessary. But such a counter example is not a problem for the Peircean. For the Peircean conceives of future truth as absolute future truth which, arguably, is equivalent to necessary truth.

Another semantic difference between Ockhamists and Peirceans is that the point of evaluation changes as one moves through the layers in nested temporal expressions in a Peircean evaluation whereas it is fixed, as the one indicated by the outermost layer, in an Ockhamist evaluation. In evaluating the 'inner' layer, '$\mathsf{F}_{n+m}q$', in the nested expression '$\mathsf{P}_n\mathsf{F}_{n+m}q$' at α, an Ockhamist would e.g. do it from an α-perspective whereas a Peircean would do it from a β-perspective where β is n days prior to α. This is the reason why the Ockhamist, but not the Peircean, accepts (P1) $\Box(q \to \mathsf{HF}q)$.

But, despite these semantic varieties, Prior's reconstruction fails due to false premises on both the Ockhamist and the Peircean account. However, this finding can not be extended in full generality to every branching semantic. And one would not expect it to be. We have seen how the Ockhamist and Peircean semantics diverge on two central questions (absolute vs. relative future truth and stable vs. unstable evaluating perspective for nested temporal expressions) and how these questions impinge on the validity of (D1) and (P1) respectively. Furthermore we have observed that the Peircean stance on the former question supports (D1″) and that the Ockhamist stance on the latter question verifies (P1). This ought to suggest that a combination of the two semantics, which we could call G, and which agrees with Peirceans in maintaining absolute future truth but agrees with Ockhamists in adhering to a stable temporal point of view when evaluating nested expressions, would be coherent with both (D1″) and (P1) and hence constitute a suitable basis for a Priorian reconstruction of the Master Argument. Or so it seems.

But unfortunately matters are not that simple. For with a semantics which favours stable point of evaluation in nested temporal expression, we are committed to accept expressions such as

(G1) $$\mathsf{F}_m q \to \mathsf{P}_n\mathsf{F}_{n+m}q\,,$$

(G2) $$\Box(\mathsf{P}_n\mathsf{F}_{n+m}q \to \mathsf{F}_m q)$$

as valid. ((G1) and (G2) are actually derivable from (P1)). But with these

formulas to hand in addition to the Diodorian premises, it seems that a principle much stronger than Diodorus' conclusion, viz. the principle that futurity implies necessity, is derivable:

1	(1)	$P_n q \to \Box P_n q$	(D1'')
1	(2)	$P_n F_{n+m} q \to \Box P_n F_{n+m} q$	$F_{n+m}q/q$ in 1
3	(3)	$F_m q \to P_n \; erm F_{n+m} q$	(G1)
1,3	(4)	$F_m q \to \Box P_n F_{n+m} q$	$\to$Trans 2, 3
5	(5)	$\Box(q \to r) \to (\Box q \to \Box r)$	(D2'')
6	(6)	$\Box(P_n F_{n+m} q \to F_m q)$	(G2)
5,6	(7)	$\Box P_n F_{n+m} q \to \Box F_m q$	$\to$E 5, 6 ($P_n F_{n+m}/q$, $F_m q/r$ in 5)
1,3,5,6	(8)	$F_m q \to \Box F_m q$	$\to$Trans 4, 7

If this principle is combined with the original Diodorian conclusion $\Diamond q \to (q \vee Fq)$, we get, firstly, that possibility implies actuality or futurity, and, secondly, that futurity implies necessity. Hence, every possible but presently false statement must be necessary! This conclusion is as radical (if not more so) as in Michael's reconstruction from section 3 in which, as it turned out, modality collapsed into actuality. In any case it is, intuitively, an unacceptable consequence of the Priorian reconstruction.

These difficulties in reconstructing a branched version of the Master Argument are rather surprising as $\Diamond q$ generally is understood in branched theories as truth in at least one future branch. This understanding seems to be in best accordance with the Diodorian conclusion, and it would thus be expected that it is a feasible project to establish this conclusion without being caught in some severe predicament as the one described above. If not on a linear view, then at least on the basis of some branched account.

But this project is not feasible. Not with the Diodorian premises — nor with any other premises coherent with a branched concept of time. The reason is, that any such reconstruction will have to adopt some combined tense-modal formulae apart from the one given by (D1). But there does not exist any semantics for such a combined language. On the basis of a linear model, it is possible to give a semantics for temporal expressions and hence to develop a tense logic. But an exhaustive semantics for the modal expressions can impossible be given by such a model. Necessity can, to some extent be determined, as in (D1), as past, and hence inevitable, truth. But this does not exhaust the meaning of 'Necessary'. There is also a metaphysical sense of necessary which traditionally has been captured as 'truth in every possible world'. But on a linear model only one world, the actual, can be represented. Likewise it can not provide any meaning of

possibility. Possibility can, of course, be defined as 'not necessary not', but when the only sense attached to necessity is past truth, this leaves us with a rather meagre (and odd) sense of possibility. On a branched view, on the other hand, the meaning of the modal expressions can be captured. Both in the sense as now inevitable and in the metaphysical sense described above as truth in some/all possible worlds. This is usually established by likening possible worlds to future branches in the model. Also the past tense can be given a satisfactory semantic on such a view. But the problem crops up when it comes to a semantic for future tense expressions. In the Peircean and Ockhamist semantics we were given two different suggestions for such expressions. The former equated future truth with truth on at least one future branch, the latter with truth on every future branch. Neither of them is, however, particularly successful in revealing the true meaning of future truth: A claim such as it is going to rain tomorrow might very well be true even though it is not raining tomorrow on every future branch and, likewise, it might very well be false although it is true on one future branch. So neither Ockhamist nor Peircean semantics for future tense expressions are intuitively appealing. That it is going to rain tomorrow can impossible be read off a branched model. It can be derived from the fact that it is raining tomorrow on every future branch — but it is not synonymous with it. Rather than evaluate it relative to some manifested features of the model, it has to be evaluated relative to the dynamic process of branch attrition. It is possible to picture *that* in a model, e.g. a model which represent the result of this branch attrition. But this would only lead us back to a linear theory in which we will have to refrain from giving semantic accounts for the modal expressions.

5. Conclusion

By way of summary: It has been argued that the plausibility of (D1) hinges on what I have called a *de dicto* interpretation of propositions. Whether such a *de dicto* reconstruction, as e.g. Prior's, is successful, depends on which temporal metaphysics one is inclined to adopt and hence on which semantic one favours when dealing with tensed expressions. It has been demonstrated that Prior's premises do not hold according to neither Ockhamist nor Peircean semantics. According to the Ockhamist, (D1) $\mathsf{P}q \rightarrow \Box\mathsf{P}q$ fails whereas (P1) $\Box(q \rightarrow \mathsf{HF}q)$ is accepted. On the Peircean view, it is the other way round. However, there are no principled reason why every branched semantic should

encounter similar difficulties. Indeed, a branched semantics, G, has been presented which is coherent with all Prior's premises. G does, though, face problems on its own. In particular, modality seems, as on Michael's *de re* reconstruction, to collapse into actuality. It has been claimed that every reconstruction, be it branched or linear, will suffer from similar fatal defects. It has furthermore been argued that the source for these difficulties is to be found in the fact that there does not exist any satisfactory semantics for a tense-modal logic.

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