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AGAIN ON RELATIVISTIC SEMANTICS

CONTENTS

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1. Introduction

This paper* has two parts: Part I is a continuation of the work [10] and as well as this it deals mainly with the logic of an auxiliary (semantical) theory, $\mathcal{ST}$, in effect generally considered (by textbooks) within the semantics for a typical theory $\mathcal{T}$ of general relativity; and especially the modal features of this auxiliary theory are studied.[1]

Part II deals with the influence had by relativistic theories and especially by the new notion of space-time on pragmatic languages (in Carnap's sense), so that they include, e.g., „now" and „here".[2] By means of some examples, in effect concerning the nearest star Σ, one emphasizes that, in various cases the causal present (causal past, or causal future) cannot be used as the present (past, or future respectively) is ordinarily used in framing pragmatic semantical rules (based on classical physics): this use of the above causal notions would cause big mistakes, or more precisely big discrepancies with the meanings of ordinary pragmatic sentences about times and lengths. In order to frame pragmatic rules based on, e.g., general relativity one suggests the use of certain geodesic manifolds also depending on the 4-velocity of the speaker, or better on a reasonable 4-velocity of the human community being considered. Thus, in case also classical physics can be regarded as a good physical basis, no discrepancy arises.

I think the study made in Part I is interesting for the reasons mentioned in [10, sect. 1]. Let us briefly remember them. First, some primitive notions of $\mathcal{T}$, such as *space-time metric*, are more complex than those of ordinary theories belonging to classical physics or special relativity. For instance, various authors give, in $\mathcal{T}$'s semantics, (different) explanations for *space--time metric*, which are true definitions in terms of notions partly outside the object theory $\mathcal{T}$; in fact these explanations are often explicitly followed by a uniqueness theorem. In addition

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[1] The present Part I, presented at the 2^{nd} International Meeting of the Pittsburgh Center Fellowships Athens 1992, in effect presupposes the knowledge of paper [10] presented at the Symposium „Semantical Aspects of Space-time Theories", Bielefeld 1991, or at least the one of its sects 1–3 without proofs.

[2] Part II was hinted at the Bielefeld symposium mentioned in footnote 1.

(a) *this theorem is proved by using the possibility of certain observations made by means of moving clocks.*

Thus

(b) *in $\mathcal{T}$'s semantical language we have an auxiliary theory $\mathcal{ST}$* — see [12], [15], [10, sects. 2, 3] — *which is of the Mach-Painlevé type, that is of the same type as the theories of classical physics where, e.g., mass and force are defined, sometimes together with the structure of space-time,* — see [14], [1], or footnote 1 in [10].

Consequently (as [1] strongly suggests) in our case the use of semantical modalities and hence the one of *semantical (possible) worlds* (connected with the auxiliary theory $\mathcal{ST}$), briefly *SPWs*, are practically necessary.

Second, if the object theory $\mathcal{T}$ deals with, e.g., materials with memory or is itself a theory $\mathcal{T}_{MP}$ of the Mach-Painlevé type (to some extent)[3], the objective modalities and (hence) *objective possible worlds*, briefly *OPWs*, also are relevant — see [7], [10, sect. 4].[4] Furthermore in any case, roughly speaking, semantical worlds must include objective worlds.

As a preliminary for the third reason let us note that, in order to treat a version of $\mathcal{T}_{MP}$ within classical physics rigorously it is important to know

(α) the transworld identity relations for *mass points* or *matter portions*, and also

(β) those for *event points* in case one wants to define the space-time structure too.

This is equivalent to knowing the natural (*modally*) *absolute notions* of mass points, matter portions and event points respectively.[5]

However, in general relativity phenomena affect the space-time metric (to various extents)[6]; and consequently physicists generally ignore any (natural) absolute notion of event points in general relativity[7], so that for simplicity reasons

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[3] A relativistic theory of this kind is the one of continuous media (possibly with memory) considered (axiomatically) in [5, Chap. 9] and called $\mathcal{T}_{[5]}$ in [10].

[4] The possibility notion used here is in effect the one of *physical possibility* explained in [7] mainly within the framework of classical space-time.

[5] E.g., the property F is said to be (*modally*) *absolute* ($F \in \mathrm{Abs}$), if it is both *modally constant* (MConst) and *modally separated* (MSep) where$F \in M\mathrm{Const} \equiv_D \forall x.\Diamond x \in F \supset \Box x \in F$, $F \in M\mathrm{Sep} \equiv_D (\forall x, y \in F).\Diamond x = y \supset \Box x = y$.

[6] This gives rise to the problem TWIEP of transworld identity of event points; see [3].

[7] G. Zampieri determined a natural (modally) absolute notion $\mathcal{EP}^Z$ of event points in general relativity in connection with *really possible worlds*, i.e. possible worlds diverging from the real one (e.g., because of some possible experiments) — see e.g. A_4 to A_5 in [10] and footnote 10 there.

(c) *it is natural (and practically mandatory) to use the extensional notion EP of event points in connection with the typical (object) theory $\mathcal{T}$ of general relativity.*

Furthermore

(d) *The afore-mentioned experiments, to be considered within $\mathcal{ST}$ and capable to determine the structure of any possible space-time* S_4 *(to be regarded as the actual one) do not affect* S_4*'s structure.*[8] Therefore in connection with the *SPWs* related to S_4 an absolute notion $\mathcal{EP}_{\mathrm{S}_4}$ of event points is obvious — just as in classical physics or special relativity.

Thus the natural semantical notion $\mathcal{EP}$ of event points, to be considered for $\mathcal{ST}$ (which has a richer semantical content than $\mathcal{T}$) is partly absolute and partly extensional, in a way specified in sect. 4. These peculiar features of $\mathcal{ST}$ afford the third (and main) reason for working out [10] and the present Part I.

Let us add that from [10] and Part I it appears that the above peculiar notion $\mathcal{EP}$ (for $\mathcal{ST}$) has the same role as the absolute classical notion $\mathcal{EP}_c$ of event points, in determining the structure of space-time.

The afore-mentioned modal features of $\mathcal{ST}$ — see (c) and (d) — can (somehow) be taken into account (in formalizing $\mathcal{ST}$) either on the basis (of a suitable generalization) of the unusual (but simple) extensional language introduced in [1], or (in a deeper way) by using a (suitably powerful) theory of modal logic that involves intensional predication and is thus capable to define notions such as *modally absolute concept*. Such are theories with types introduced in [13], [2], [6], or [11] (related to [2]), as well as the typeless theories presented in [4] and [6] (these works, except [13] consider explicitly notions such as *modally absolute concept*).

In [10] $\mathcal{ST}$ is formalized according to the former alternative (based on [1]). In the present Part I $\mathcal{ST}$'s formalization is performed according to the latter alternative, and more precisely on the basis of [2] or [4] because these works seem to be the fittest to treat $\mathcal{ST}$'s peculiar modal features, inside a modal theory, from a general point of view. Among other things $\mathcal{ST}$'s second formalization has this advantage. Remembering from [10, sect. 4] that the class OPW of objective possible worlds (for $\mathcal{T}$) can be regarded as a partition of SPW, now it can be defined in terms of the notion $\mathcal{EP}$ of event points in effect used in connection with $\mathcal{ST}$ — see sect. 4. Instead the primitives of $\mathcal{ST}$'s first formalization include both OPW and an analogue, $\mathcal{EP}(\mathcal{W})$, of $\mathcal{EP}$ — see [10, sect. 4].[9]

[8] These experiments are supposed to be implementable by using arbitrarily small amounts of mass and energy.

[9] If $\mathcal{T}$ is a relativistic theory $\mathcal{T}_{MP}$ of the Mach-Painlevè type, its other primitives

Lastly, since the present Part I is tightly connected with the foundational point of view of [1] (and [2]), which in particular involves possibility postulates explicitly, let us remember that below $A7$ in [10, sect. 1] some papers of physics or biology by M. Pitteri, A. Zanardo & M. Rizzotti, and especially A. Montanaro are mentioned, which show that certain scientific reviews (not devoted to foundations) are becoming interested in the above point of view (admittedly indirectly).

Part I
On certain notions, partly extensional and partly modal, relevant for the semantics of general relativity

2. On a postulate of general relativity to which an auxiliary semantical theory is in effect associated

In stating (axiomatically) a theory $\mathcal{T}$ of general relativity one can use, among other things, a postulate of this form — see Post. 1 in [10].

Postulate 2.1. *There is an (admissible) space-time frame*

$$x = \phi(\mathcal{E}) \qquad \text{for } \mathcal{E} \in \mathrm{S}_4, \text{ the (actual) space-time.} \tag{2.1}$$

Furthermore there is a space-time metric-tensor field, and hence a space--time metric, expressed in ϕ by

$$g_{\alpha\beta} = \hat{g}_{\alpha\beta}(x) \quad (\alpha, \beta = 0, \dots, 3); \qquad ds^2 = -g_{\alpha\beta}(x)dx^\alpha dx^\beta \tag{2.2}$$

where $\hat{g}_{\alpha\beta}(\,\cdot\,)$ has certain mathematical properties — see (2) *to* (3) *in* [10].

E.g. the two primitive notions of (*admissible*) *space-time frame* and *space-time metric* have to be explained in $\mathcal{T}$'s semantics; and to attain this goal, first, by means of some usual explanations the notions of *event points*, *test particles, clocks,* and *free photons* can be rendered intuitively clear. Consequently other notions, incidentally the notions (5)–(7) in [10, sect. 2] and the above two primitives can be defined in $\mathcal{T}$'s semantics, say in the auxiliary theory $\mathcal{ST}$ — see Definition 1 in [10, sect. 3].

(different from $\mathcal{EP}$), such as *matter portion*, are determined like their classical analogues; and they have the same roles as these.

Furthermore some postulates can be stated in $\mathcal{ST}$ again, for instance Post. 2 to Post. 5 in [10], that allow us to prove, e.g., the uniqueness of the actual space-time metric — see Theorem 2 of $\mathcal{ST}$ in [10, sect. 3]. The existence of the same metric is asserted just by Post. 1 of the object theory $\mathcal{T}$ in [10, sect. 2]. Incidentally the auxiliary theory $\mathcal{ST}$, which contains more notions than $\mathcal{T}$, is also supposed to include all postulates of $\mathcal{T}$ (and some additional postulates).

For the present aims it is not necessary to consider the details of the afore-mentioned definitions and postulates. It suffices to note that in order to state, for instance, the existence and uniqueness of the space-time metric, one can in effect use a necessity assumption and a possibility one. In more details, in [10] the following pattern is implemented in this connection; and incidentally the same pattern is used for defining, e.g., mass in classical particle mechanics — see footnote 1 in [10].[10] First, in the afore-mentioned Definition 1 of space-time metric one requires that, if certain experiments take place, then necessarily certain results are obtained.

Second, the existence of this metric is postulated (Post. 1 in [10]) and thus the afore-mentioned necessity assumption is in effect stated.

Third, the afore-mentioned possibility assumption is stated just as a postulate — see Post. 5 in [10] — and incidentally it is essential, as well as the necessity assumption, in order to prove the afore-mentioned Theorem 2 (of uniqueness) in [10].

3. Some preliminaries for $\mathcal{ST}$'s 2nd formalization

Since $\mathcal{T}$'s 2nd formalization is based on [2], let us note that within the logical modal calculus MC^ν introduced there one defines an analogue for the set of possible worlds: the set $\mathcal{E}\ell$ of *absolute elementary ranges* — see [2, Definition 48.2], p. 204 — as well as the assertion $|_u$ which contains only the variable „u" free and in effect means *the possible world* $u (\in \mathcal{E}\ell)$ *occurs.* Then $\Diamond.|_u \wedge p$ says that p occurs in u. Let us remember the logical theorems

$$(3.1)\qquad \begin{array}{ll} \vdash \mathcal{E}\ell \in \mathrm{Abs}, & \vdash \Diamond|_u \supset u \in \mathcal{E}\ell, \\ \vdash |_u \wedge^{\cup} p \equiv |_u \supset^{\cap} p, & \vdash \Box p \equiv (\forall u \in \mathcal{E}\ell).|_u \wedge^{\cup} p\,. \end{array}$$

where, e.g.

$$(3.2)\qquad \begin{array}{ll} p \supset^{\cap} q \equiv_D \Box.p \supset q, & x =^{\cap} y \equiv_D \Box x = y, \\ p \wedge^{\cup} q \equiv_D \Diamond.p \wedge q, & x \in {}^{\cap}F \equiv_D \Box x \in F\,. \end{array}$$

[10] The same pattern is used to define force, besides mass, within a non-mandatory part of my notes for students of Rational Mechanics.

In case $\mathcal{W} \subseteq \mathcal{E}\ell$, it is useful to define p *is* $\begin{cases} \textit{possible} \\ \textit{necessary} \end{cases}$ *on* $\mathcal{W}$ and, e.g., *F holds for x on $\mathcal{W}$* by

$$(3.3) \qquad \begin{aligned} &\begin{cases} \Diamond_{\mathcal{W}} p \\ \Box_{\mathcal{W}} p \end{cases} \equiv_D \ \mathcal{W} \subseteq \mathcal{E}\ell \wedge \begin{cases} (\exists u \in \mathcal{W}) \\ (\forall u \in \mathcal{W}) \end{cases} .|_u \wedge^{\cup} p, \\ &x \in {}_{\mathcal{W}} F \ \equiv_D \ \Box_{\mathcal{W}} x \in F \,. \end{aligned}$$

Obviously, e.g., $=_{\mathcal{W}}$ (*equal on* $\mathcal{W}$), $\supset_{\mathcal{W}}$, and $\subset_{\mathcal{W}}$ are defined like $\in {}_{\mathcal{W}}$. By (3.3) we have the logical theorems:

$$(3.4) \qquad \begin{aligned} &\mathcal{W} \subseteq \mathcal{E}\ell \supset .\Box_{\mathcal{W}} p \equiv \sim \Diamond_{\mathcal{W}} \sim p, \\ &\Diamond_{\mathcal{E}\ell}\, p \equiv \Diamond p, \\ &\vdash p \supset_{\mathcal{E}\ell} q. \equiv .p \supset^{\cap} q \,. \end{aligned}$$

To prove $(3.4)_1$ assume q, where $q \equiv_D \mathcal{W} \subseteq \mathcal{E}\ell$. By $(3.3)_1$ and (then) $(3.1)_3$ it easily implies the equivalences $\sim \Diamond_{\mathcal{W}} \sim p \equiv \sim q \vee \sim (\exists u \in \mathcal{W})\,(|_u \wedge^{\cup} \sim p) \equiv q \supset (\forall u \in \mathcal{W})\,(|_u \supset^{\cap} p) \equiv (\forall u \in \mathcal{W})\,(|_u \wedge^{\cup} p) \equiv \Box_{\mathcal{W}} p$ by $(3.3)_1$. We conclude that $(3.4)_1$ holds.

For $\mathcal{W} \subseteq \mathcal{E}\ell$, let us also define F is *modally constant, modally separated*, or (*modally*) *absolute on* $\mathcal{W}$, *$\mathcal{W}$ holds* (briefly $|^{\mathcal{W}}$), and *F is extensional at* $\mathcal{W}$ by

$$(3.5) \qquad \begin{aligned} &F \in \begin{cases} M\mathrm{Const}_{\mathcal{W}} \\ \mathrm{Sep}_{\mathcal{W}} \end{cases} \equiv_D \ \mathcal{W} \subseteq^{\cap} \mathcal{E}\ell \wedge \\ &\qquad \begin{cases} (\forall x).\Diamond_{\mathcal{W}} x \in F \supset x \in_{\mathcal{W}} F, \\ (\forall x, y).\Diamond_{\mathcal{W}} (x \in F \wedge y \in F \wedge x = y) \supset x =_{\mathcal{W}} y, \end{cases} \end{aligned}$$

$$(3.6) \qquad \begin{aligned} &\mathrm{Abs}_{\mathcal{W}} \ =_D \ M\mathrm{Const}_{\mathcal{W}} \cap M\mathrm{Sep}_{\mathcal{W}}, \\ &|^{\mathcal{W}} \ \equiv_D \mathcal{W} \ \subseteq \mathcal{E}\ell \wedge (\exists u \in \mathcal{W})|_u, \end{aligned}$$

and

$$(3.7) \qquad F \in \mathrm{Ext}_{(\mathcal{W})} \ \equiv_D \ (\forall x, y).x =_{\mathcal{W}} y \wedge x \in F \supset_{\mathcal{W}} y \in F \,.$$

It is natural to define $\mathbb{P}$ is an (*absolute*) *partition* of $\mathcal{E}\ell$ by — see footnote 5

$$(3.8) \qquad \begin{aligned} &\mathbb{P} \in \mathrm{AbPrt}_{\mathcal{E}\ell} \ \equiv_D \ \mathbb{P} \in \mathrm{Abs} \wedge \cup\mathbb{P} = \mathcal{E}\ell \ \wedge \\ &\qquad (\forall \mathcal{W}, \mathcal{W}' \in \mathbb{P}).\mathcal{W} \in \mathrm{Abs} \wedge .\mathcal{W} \cap \mathcal{W}' \neq \emptyset \equiv \mathcal{W} = \mathcal{W}' \,. \end{aligned}$$

In connection with such $\mathbb{P}$ it is useful to say that *F is extensional w.r.t.* $\mathbb{P}$, if F is both absolute on and extensional at every member $\mathcal{W}$ of $\mathbb{P}$:

$$F \in \mathrm{Ext}^{(\mathbb{P})} \equiv_D \mathbb{P} \in \mathrm{AbPrt}_{\mathcal{E}\ell} \wedge (\forall \mathcal{W} \in \mathbb{P}).F \in \mathrm{Abs}_{\mathcal{W}} \wedge F \in \mathrm{Ext}_{(\mathcal{W})} \tag{3.9}$$[11]

Briefly it is obvious that

(A) *if $\mathbb{P}$ and Q are partitions of $\mathcal{E}\ell$, Q is finer than $\mathbb{P}$, and the property F is absolute on $\mathbb{P}$'s members, then it is certainly such also on Q's members.*

However, under the reasonable assumption

$$\Box(\exists x, y)x \neq y, \tag{3.10}$$

(B) *if in addition Q is strictly finer than $\mathbb{P}$ and F is extensional w.r.t. Q, then F cannot be absolute on $\mathbb{P}$'s members.*

More thoroughly, it is easy to check that by (3.10)

(C) *if the property F is extensional w.r.t. a partition $\mathbb{P}$ of $\mathcal{E}\ell$, then this partition is uniquely determined*:

$$\begin{gathered} \vdash F \in \mathrm{Ext}^{(\mathbb{P})} \wedge F \in \mathrm{Ext}^{(Q)} \supset \mathbb{P} = Q, \\ \text{i.e. } (\exists \mathbb{P})F \in \mathrm{Ext}^{(\mathbb{P})} \supset (\exists_1 \mathbb{P})F \in \mathrm{Ext}^{(\mathbb{P})} . \end{gathered} \tag{3.11}$$

By (c) and (d) in sect. 1 the notion $\mathcal{EP}$ of event points, reasonably used by most scientists in connection with the semantics of the theory $\mathcal{T}$ (or with $\mathcal{ST}$), appears to be extensional just w.r.t. the partition OPW of the SPWs, determined by the objective possible worlds. Thus by (3.11) this partition can be defined naturally within $\mathcal{ST}$:

$$\begin{gathered} OPW =_D (\imath\mathbb{P})\mathcal{EP} \in \mathrm{Ext}^{(\mathbb{P})} \\ (\Vdash OPW \neq a^* =_D (\imath F)F \neq F, \text{ the non-existing object}). \end{gathered} \tag{3.12}$$

4. Second formalization of the auxiliarity theory $\mathcal{ST}$, regarded to embody the object theory $\mathcal{T}$

We regard the auxiliary theory $\mathcal{ST}$ to have, as primitives both $\mathcal{T}$'s primitives except *admissible space-time frame* and *space-time metric*, and the notions (1) to (7) below (2.3) in [10]. In addition let $\mathcal{ST}$ include the nonlogical axiom (3.10) and the postulates

[11] Note that the replacement of „$F \in \mathrm{Ext}_{(\mathcal{W})}$" in (3.9) with „$F \in \mathrm{Ext}_{\mathcal{W}}$", which expresses F *is extensional on* $\mathcal{W}$ — i.e. $(\forall x, y).x = y \wedge x \in F \supset_{\mathcal{W}} y \in F$ — would cause $\mathcal{W}$ to be a singleton under the assumption (3.10).

$$(4.1)\qquad (\exists\mathbb{P} \in \mathrm{AbPrt}_{\mathcal{E}\ell})\mathcal{EP} \in \mathrm{Ext}^{(\mathbb{P})} \qquad \text{— see (3.8)–(3.9)}$$
$$\Box.\mathcal{EP} \notin (\mathrm{Ext} \cup \mathrm{Abs})\ .$$

By $(4.1)_1$ and $(3.11)_2$ the condition of exact uniqueness for definition $(3.12)_1$ is satisfied. Hence — see (3.11) and $(3.6)_2$ —

$$(4.2)\qquad \vdash OPW \in \mathrm{AbPrt}_{\mathcal{E}\ell}, \qquad \vdash (\exists_1 \mathcal{W} \in OPW)|^{\mathcal{W}}\ .$$

By $(4.1)_2$ the nontriviality of $\mathcal{E}\ell$'s partition $\mathcal{EP}$ is in effect stated.

Of course we regard $\mathcal{ST}$ to contain $\mathcal{T}$'s postulates and the additional postulates Post. 2 to Post. 5 considered in [10, sects. 2–3]. In order to express these postulates easily, it is convenient to introduce the *actual objective possible world* $\mathcal{A}$ by

$$(4.3)\qquad \mathcal{A} =_D (\imath\mathcal{W} \in OPW)|^{\mathcal{W}}, \qquad \text{hence } \vdash \Box|^{\mathcal{A}} \text{ by } (4.2)_2,$$

as well as the *extensionalization* $F^{e,\mathcal{A}}$ *of the property* F *with respect to* $\mathcal{A}$:

$$(4.4)\qquad F^{e,\mathcal{A}} =_D (\lambda x)\,(\exists y \in F)x =_{\mathcal{A}} y \qquad \text{— see below (3.3).}$$

By $(4.3)_1$, $(4.2)_1$, (3.8), $(3.6)_1$, and (3.5),

$$(4.5)\qquad \vdash \mathcal{A} \notin M\mathrm{Sep}, \quad \vdash \mathcal{A} \notin M\mathrm{Const}, \quad \vdash \mathcal{A} \notin OPW$$

— see footnote 5 — and

$$(4.6)\qquad \vdash \mathcal{A} \subseteq^{\cap} \mathcal{E}\ell, \qquad \vdash \mathcal{A} \in OPW^{(e)},$$

where $F^{(e)} =_D (\lambda x)\,(\exists y \in F)x = y$.

Now one can easily check that the modal operators (restricted to the actual OPW), in effect used within $\mathcal{ST}$'s additional Posts. 2–5 written in [10, sects. 2–3], can be identified with $\Box_{\mathcal{A}}$ and $\Diamond_{\mathcal{A}}$ (in $\mathcal{ST}$'s 2$^{\text{nd}}$ formalization). In Post. 2.1 the space-time frame ϕ is in effect said to be a function from event points to $\mathbb{R}^4$. In order to specify the properties of this notion — also used in [10] within Post. 4, Definition 1, and Post. 5 — let us now consider the natural absolute notion $\mathcal{R}$ of the real numbers (constructed in the usual way on the basis of the analogous notion $\mathbb{N}$ of natural numbers defined in [2]); furthermore set

$$(4.7)\qquad \mathbb{R} =_D \mathcal{R}^{e,\mathcal{A}}, \qquad \text{hence } \vdash \mathbb{R} \in \mathrm{Abs}_{\mathcal{A}} \text{ and} \vdash \mathbb{R} \notin \mathrm{Abs}\ .$$

Now we can say that we must have

$$(4.8)\qquad \phi \in (\mathcal{EP} \to \mathbb{R}^4) \qquad (\text{obviously } \vdash \Box\mathcal{EP} \in \mathrm{Abs}_{\mathcal{A}} \cap \mathrm{Ext}_{(\mathcal{A})})$$

and that *frame on* S_4 can be regarded as the (extensional) class of these functions. Since $\mathcal{EP}$ is not absolute, one cannot replace $\mathbb{R}$ in (4.8) with $\mathcal{R}$. In fact, for some $\mathcal{E}_1$ and $\mathcal{E}_2$, we have $\Diamond.\mathcal{E}_1 = a^* = \mathcal{E}_2$ where $a^* =_D (\imath x)x \neq x$ (the nonexisting object), $\mathcal{E}_1 \neq \mathcal{E}_2$, and $\mathcal{E}_1, \mathcal{E}_2 \in \mathcal{EP}$. Hence from $\phi \in (S_4 \to \mathcal{R}^4)$ and $\mathcal{R} \in \mathrm{Abs}$ one deduces that $\Diamond.\phi(\mathcal{E}_1) = \phi(\mathcal{E}_2)(= a^*)$, $\phi(\mathcal{E}_1) \neq \phi(\mathcal{E}_2)$, and $\phi(\mathcal{E}_1), \phi(\mathcal{E}_2) \in \mathcal{R}^4$; and this is absurd — see footnote 5.

We can identify S_4 with $\mathcal{EP}$ added, if preferred, with the space-time topology — see Post. 3 in [10]. Now we can define (*continuous*) *line of* S_4 and can regard this notion as satisfying condition $(4.8)_2$ in $\mathcal{EP}$. Thus we can mean ℓ *is a possible world-line of the test particle* $\bar{P}$ as follows:

(4.9) $$\ell \in \text{ (line of } S_4) \wedge \Diamond_{\mathcal{A}} \ell = \text{ the world line of } \bar{P}$$

— see (4) in [10, sect. 1] — where $S_4 \in \mathrm{Abs}_{\mathcal{A}}$ and (line of $\bar{P}$) $\in \mathrm{Abs}_{\mathcal{A}}$. Modalities often occur through such a notion in sects. 2 to 3 — see e.g. Post. 4 (11), Definition 1 (α) to (γ), and the consequent of Post. 5 in [10].

Part II
On the influence of relativistic theories on the semantics of pragmatic languages in Carnap's sense

5. On the relativistic semantics of „now", „past", and „future" for the present human community

Briefly speaking, in working on e.g. the semantics of pragmatic languages in Carnap's sense (so that, e.g., „now" and „here" are involved) it may appear natural to interpret the „now" uttered by a speaker $\mathcal{A}$ (Adam) at the event point $\mathcal{E}$, as „in $\mathcal{E}$'s casual present", i.e. as „in the complement of $\mathcal{E}$'s causal past $\{\, \mathcal{E}' : \mathcal{E}' \prec \mathcal{E} \,\}$ joint with $\mathcal{E}$'s causal future $\{\, \mathcal{E}' : \mathcal{E} \prec \mathcal{E}' \,\}$. This interpretation is quite possible in speaking of facts happening in a region $\mathcal{R}$ near $\mathcal{E}$, e.g. on the earth, so that the causal present is a 4-dimensional space-time region intersecting $\mathcal{R}$ very near some hypersurface. Otherwise it seems to me important to note the following. Adam has, at $\mathcal{E}$, a 4-velocity α, which determines the 3-dimensional manifold $\mathcal{M}_{\mathcal{E},u}$ formed by the spatial geodesics through $\mathcal{E}$, orthogonal to u there; in special relativity $\mathcal{M}_{\mathcal{E},u}$ is $\mathcal{A}$'s (locally) rest inertial-space at $\mathcal{E}$; and one can show that

(a) *in some cases, e.g. in speaking of a star three light years far, say* Σ, *it may be incorrect to regard „now" as an equivalent of „in the causal present";* and that

(b) *it is better to interpret „now" as „on* $\mathcal{M}_{\mathcal{E},u}$*" or as „near* $\mathcal{M}_{\mathcal{E},u}$*" for some* $\mathcal{E}$ *in the space-time region of utterance.*

In fact, roughly speaking, assume that (i) $\mathcal{A}$ is correct in asserting (on the earth), at his proper time s when he occupies $\mathcal{E}$:

(α) *tomorrow (at* s_1*) I shall observe the value of the magnitude* $\mathfrak{m}$ *— e.g. radio-activity — on the star* Σ.

Then $\mathcal{A}$ can correctly assert, at s,

(β) *thus I shall know a causally present value of* $\mathfrak{m}$ *on* Σ;

and in case $\mathfrak{m}$'s value is practically constant for some days, the same holds with $\mathcal{A}$'s possible assertion at s:

(γ) *thus I shall not know the value that* $\mathfrak{m}$ *now has on* Σ*, but the value taken there by* $\mathfrak{m}$ *three years (minus one day) ago.*

From (β) and (γ) we deduce (a). Let us add that (a) also holds because, in connection with (γ), $\mathfrak{m}$*'s value on* Σ *now* is intuitively unique, while $\mathfrak{m}$ has many values on Σ in the causal present; in fact, if Σ's distance from the earth is regarded as constant, Σ's causal present for $\mathcal{A}$ lasts six years instead of being instantaneous.

Now we note that, since the utterance time of an assertion is appreciably longer than an instant (even for every day life), all assertions involving „now" have to be treated in an approximate way. Furthermore (b) appears reasonable, e.g., when the every-day life assertion (γ) is considered in special relativity.

Note that *past* („*ago*")is in effect used within (γ) as (*causally*) *before* $\mathcal{M}_{\mathcal{E},u}$; the analogue holds in every-day speech also for *future*, within special or general relativity.

Remark that the non-constancy of the speaker's velocity w.r.t. the earth may be troublesome in (γ) especially in connection with the past assertion involved by (γ). Roughly speaking, this defect is not avoided by replacing u with the earth's 4-velocity u_e, because neither u_e is constant; and for the present human community it is better to replace u with the sun's 4-velocity u_σ: as far as u_σ (unlike u_e) can be regarded as (nearly) constant, *the replacement of* u *with* u_σ *in* (b) renders the semantical rules for „now" independent of the month of utterance.

Being now interested in assertions referring to stars — like (γ) — or to galaxies, we cannot regard space-time as stationary. Instead, also looking forward to more precise conventions for the relativistic semantics of „now", it is convenient to associate to $\mathcal{E}$ (within general relativity) the point $\mathcal{E}_\sigma$ of σ's world line such that $\mathcal{E} \in \mathcal{M}_{\mathcal{E}_\sigma,u_\sigma}$, and to set

$$\mathcal{M}^\sigma_{\mathcal{E}} =_D \mathcal{M}_{\mathcal{E}_\sigma,u_\sigma}, \qquad \text{where } \sigma \text{ denotes the sun.}$$

Now (a) can be improved as follows.

(c) *For the present human community, within general relativity, „now" can be (satisfactorily) interpreted, in any case, as „on* $\mathcal{M}^{\sigma}_{\mathcal{E}}$*" or as „near* $\mathcal{M}^{\sigma}_{\mathcal{E}}$*" for some* $\mathcal{E}$ *in the space-time region of utterance.*

Some analogues of the above examples on „now" hold for „past" (or „future"). In fact assume that no other observations of $\mathfrak{m}$ on Σ were performed before the one mentioned in (α); and that, one year before s, $\mathcal{A}$ uttered the 1^{st} [2^{nd}] of the sentences below.

(δ) *Some values taken by* $\mathfrak{m}$ *on* Σ *in the causal past will be observed by me.*

(ε) *Some of the values that* $\mathfrak{m}$ *has taken on* Σ *will be observed by me.*

Then $\mathcal{A}$ was wrong [correct] in this utterance. We conclude that the phrase „in the present" (or „now") [„in the past"] in effect involved by (γ) [(ε)] cannot be (equivalently) replaced by „in the causal present" [„in the causal past"] (which in effect gives rise to (β) [(δ)]).

6. On the relativistic semantics for „now" and „here" possibly used by special human communities

Now let us consider a (special) human community that is travelling on a rocket $\mathcal{R}$; and assume that (1) they left Σ six years ago, (ii) they were always travelling along a geodesic of S_4 at about the speed $c/2$ w.r.t. the earth (or σ), where c the speed of light in vacuum, (iii) now they are at an event point $\mathcal{E}$, near the earth, and (iv) they are not interested in stopping at or communicating with the earth. Then

(d) *in order to interpret „now" in any pragmatic sentence uttered at* $\mathcal{E}$ *by a member* $\mathcal{B}$ *(Bernard) of the community travelling in* $\mathcal{R}$*, it is convenient to use the semantical rule proposed in* (c) *with the manifold* $\mathcal{M}^{\sigma}_{u_\sigma}$ *relative to the sun* σ*, replaced by its analogue* $\mathcal{M}_{\mathcal{E}_\mathcal{R},u_\mathcal{R}}$ *for* $\mathcal{R}$ *(so that practically* $\mathcal{E}_\mathcal{R} = \mathcal{E}$*).*

Thus, in particular, $\mathcal{B}$ is practically correct in uttering at $\mathcal{E}$:

(η) *now we are* $3\sqrt{3}/2$ *light years far from* Σ*, and the rocket* $\mathcal{R}$ *was there* $3\sqrt{3}$ *years ago,*

while $\mathcal{A}$, with 4-velocity $u = u_\sigma$, would obviously be correct in uttering at $\mathcal{E}$:

(ι) *now we are* 3 *light years far from* Σ*, and the rocket* $\mathcal{R}$ *was there* 6 *years ago.*

In fact, since we are interested in avoiding (only) big mistakes, we can consider Σ and σ as steady in an inertial space $\mathcal{I}$ of special relativity. Then lengths [times] have the contraction [dilatation] factor $\sqrt{1-\beta^2} = \sqrt{3}/2$ $[2/\sqrt{3}]$ ($\beta = 1/2$). Furthermore, by using Römer units ($c = 1$) and the year as time unit, the proper length of the segment (Σ, σ) is 3, so that $\sqrt{1-\beta^2}\,3$ is its length for $\mathcal{B}$; and the time τ that $\mathcal{R}$'s trip from Σ to σ lasted according to $\mathcal{B}$ is $\mathcal{R}$'s proper time elapsed during this trip, so that $\tau 2/\sqrt{3} = 6$. Now (η) too appears true.

Note that, by e.g. interchanging u_σ and $u_{\mathcal{R}}$ in connection with (η) and (ι), we would obtain assertions including big mistakes about both times and — unlike what happens with the preceding examples — also lengths.

Note that (d), (η), and (ι) refer to a special (human) community supposed not to be feeling itself as a part of the earth community; furthermore $\mathcal{R}$'s intrinsic acceleration $a_{\mathcal{R}}$ is supposed to vanish. Otherwise, and especially if $a_{\mathcal{R}} \neq 0$ appreciably, I think people travelling in $\mathcal{R}$ (and passing through $\mathcal{E}$) would naturally refer to $\mathcal{M}^g_{\mathcal{E}}$ when they are using „now", „past", „future", and „here", as well as when they are evaluating times or distances.[12]

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[12] One can try and define geometrical points of $\mathcal{M}_{\mathcal{E}_\sigma, u_\sigma}$ (and hence those of $\mathcal{M}^g_{\mathcal{E}} =_D \mathcal{M}_{\mathcal{E}_\sigma, u_\sigma}$) in effect as follows. Let $\mathcal{F}_{\mathcal{E}_\sigma}$ be a Fermi (-Walker) frame with the origin at $\mathcal{E}_\sigma \in \ell_\sigma$, where ℓ_σ is σ's world line. For $(\vartheta, \varphi, s) \in [0, \pi] \times [0, 2\pi) \times \mathbb{R}^+$ call $\mathcal{E}_{\vartheta,\varphi,s}(\mathcal{E}_\sigma)$ the event point $\mathcal{E}$ such that (i) the (spatial) geodesics ℓ of end points $\mathcal{E}_\sigma$ and $\mathcal{E}$ (supposed unique) has the length s and (ii) ℓ's unit vector at $\mathcal{E}_\sigma$ has the spherical coordinates ϑ and φ w.r.t. $\mathcal{F}_{\mathcal{E}_\sigma}$. We say that P is a *geometric point of* $\mathcal{M}_{\mathcal{E}_\sigma, u_\sigma}$ if $P = \mathcal{E}_{\vartheta,\varphi,s}(\mathcal{E}_\sigma)$ for some $(\vartheta, \varphi, s) \in [0, \pi] \times [0, 2\pi) \times \mathbb{R}^+$.

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