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THE IDENTITY OF STRONG INDISCERNIBILITY

CONTENTS

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1. Nontraditional Theory of Predication*

The following considerations are to be seen in the framework of nontraditional theory of predication (NTP), which stems in its basic features from Sinowjew (cf. Sinowjew 1970, Sinowjew/Wessel 1975), and which is described in detail in Wessel 1989. In NTP we distinguish between external and internal negation. External negation $\sim$ is the usual negation of classical sentential logic. Classical sentential logic is extended by the following rules concerning internal negation. For that purpose syntax is chosen in a way that internal negation $\neg$ can only occur immediately in front of a predicate variable:

(1) Predicate formulas $P(s_1, \ldots, s_n)$ and $\neg P(s_1, \ldots, s_n)$ are ascribed truth values v and f in the same way as sentential variables are. Two predicate formulas are different if and only if they are graphically different.

(2) If A has the value v, $\neg A$ has the value f.

(3) If $\neg A$ has the value v, A has the value f.

(4) If A has the value f, the value of $\neg A$ does not depend on the value of A, that is, $\neg A$ may have the value v as well as the value f.

(5) If $\neg A$ has the value f, the value of A does not depend on the value of $\neg A$.

In short, two contrary formulas A and $\neg A$ cannot both be true, but they can both be false. Instead of a formula of the form $\sim A \wedge \sim \neg A$ I write $?A$.

2. Rules for Existential Presuppositions

In NTP elementary predicative sentences of the form $P(s_1, \ldots, s_n)$ and $\neg P(s_1, \ldots, s_n)$ have existential import in the following sense: They can only be true, if their subjects $s_1, \ldots, s_n$ exist. Concerning the predicate of existence E the following applies in NTP:

E1. $P(s_1, \ldots, s_n) \supset E(s_1) \wedge \ldots \wedge E(s_n)$

E2. $\neg P(s_1, \ldots, s_n) \supset E(s_1) \wedge \ldots \wedge E(s_n)$

Conversely: if s_1 or $\ldots$ or s_n does not exist, then $?P(s_1, \ldots, s_n)$, i.e., $\sim P(s_1, \ldots, s_n) \wedge \sim \neg P(s_1, \ldots, s_n)$.

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E3. $\sim E(s_1) \vee \ldots \vee \sim E(s_n) \supset ?P(s_1, \ldots, s_n)$.

Further, the following holds because $\neg E(s)$ is logically false:

E4. $\sim \neg E(s)$.

K. H. Krampitz sets in his dissertation B (Krampitz 1990) the task of formulating rules according to which one can find out existential presuppositions of compounded sentences. I follow Krampitz and state a version of these rules which is more precise.

It is reasonable to speak of the existential import of a sentence only if that sentence is true. Hence, I introduce for the sentence 'If A is true, then A has existential import' the abbreviation 'A has the characteristic e', and for the sentence 'If A is true, then A does not have existential import' the abbreviation 'A has the characteristic n'. Concerning e und n, the following rules apply:

R1. All elementary predicative sentences have the characteristic e.

R2. If A has the characteristic e, then $\sim A$ has the characteristic n.

R3. If A has the characteristic n, then $\sim A$ has the characteristic e.

R4. $A \vee B$ has the characteristic e if and only if A has the characteristic e and B has the characteristic e.

The three following rules are derived ones.

R5. $A \wedge B$ has the characteristic e if and only if A or B has the characteristic e.

R6. $A \supset B$ has the characteristic e if and only if A has the characteristic n and B has the characteristic e.

R7. $A \equiv B$ has the characteristic n if and only if A and B either both have the characteristic e, or both the characteristic n.

R8. $\forall iA$ und $\exists iA$ have the characteristic e if and only if A has the characteristic e.

We postulate an additional rule for definitions of the form $A \equiv_{Def} B$. Because definitions cannot be true or false it is senseless to speak of the existential import of definitions. But since one gets sentences of the form $A \equiv B$ from definitions of the form $A \equiv_{Def} B$, we state the following definition rule.

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R9. A definition $A \equiv_{Def} B$ must be constructed in a way that the sentence $A \equiv B$ has the characteristic n.

We can represent our rules clearly by the following tables similar to truth tables:

A	$\sim A$	$\forall iA$	$\exists iA$
e	n	e	e
n	e	n	n

A	B	$A \wedge B$	$A \vee B$	$A \supset B$	$A \equiv B$	$A \equiv_{Def} B$
e	e	e	e	n	n	n
n	e	e	n	e	e	e
e	n	e	n	n	e	e
n	n	n	n	n	n	n

If one now ascribes the characteristic e to sentence variables and elementary predicate formulas, then one can find out regarding each formula, whether it has the characteristic e or n. If schemes of formulas are used, then the characteristics e and n have to be ascribed to the metavariables A, B, C etc. So the following metatheorem applies:

MT1. Every formula of classical quantification logic without identity has one and only one of the characteristics e or n.

Let us consider the following axiomatization of classical quantification logic without identity:

A1. $A \supset (B \supset A)$

A2. $A \supset (B \supset C) \supset (A \supset B \supset (B \supset C))$

A3. $\sim A \supset \sim B \supset (B \supset A)$

A4. $\forall i(A \supset B) \supset (A \supset \forall iB)$, where i is not free in A.

A5. $\forall iA \supset A\{i/j\}$, where i is free for j in A.

R1. If $\vdash A$ and $\vdash A \supset B$, then $\vdash B$.

R2. If $\vdash A$, then $\vdash \forall iA$.

It is easy to demonstrate that all schemes of axioms for all ascriptions of e respectively n to occurring metavariables have the characteristic n, and that the rules only lead from formulas with the characteristic n to formulas with the characteristic n. So the following metatheorem applies:

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MT2. All theorems (tautologies) of classical sentential and quantification logic without identity have the characteristic n, i.e., they do not have existential import.

To be more philosophical, this means that all tautologies of this logic do not state anything about reality outside of languages. MT2 answers precisely a philosophical question which is extensively discussed in literature (cf. Wittgenstein 1967, 1969, Müller 1967, Lazerowitz 1977). From MT2 one gets with R3:

MT3. All contradictions of classical sentential and quantification logic without identity have the characteristic e.

The characteristic e, however, can not be interpreted here as "having existential import", since only true sentences can have existential import. But contradictions are false by logical reasons. To express it philosophically this means that also all contradictions do not state anything about reality outside of languages.

3. The Case of Identity

Let us now turn to quantification logic with identity. Identity is a binary relation. A sentence of the form $x = y$ has existential import.

One gets quantification logic with identity by the following complements to quantification logic without identity.

(1) Besides the rule of replacement concerning identities, $x = x$ is stated as an axiom.

But $x = x$ has existential import, and that is why it is not logically true. A sentence $x = x$ is true, if only the term x is not empty. This recognition is already to be found in Russell's papers. He writes:

> Apart from them [from logical propositions; H. W.] there are many that can be expressed in logical terms, but cannot be proved from logic, and are certainly not propositions that form part of logic. Suppose you take such proposition as: 'There is at least one thing in the world.' That is a proposition that you can express in logical terms. It will mean, if you like, that the propositional function '$x = x$' is a possible one. That is a proposition, therefore, that you can express in logical terms; but you cannot know from logic whether it is true ore false. So far as you do know it, you know it empirically, because there might happen not to be a universe, and then it would not be true. (Russell 1988, p. 107)

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(2) Church adds the following (Church 1956):

1. $x = x$
2. $x = y \wedge P(x) \supset P(y)$.

In difference to the first one the second formula does not have existential import.

(3) Hao Wang gives the following complement (Quine 1980, p.13):

$$P(y) \equiv \exists x(x = y \wedge P(x)).$$

This formula does not have existential import. But in doing proofs in this system mistakes may occur by which formulas which have existential import may quickly infiltrate. For instance, $y = y$ is proved the following way. In the axiom the predicate $\sim (... = y)$ is substituted for $P(...)$, and one gets:

$$\sim (y = y) \equiv \exists x(x = y \wedge \sim x = y).$$

Because $\sim \exists x(x = y \wedge \sim x = y)$ is provable, one concludes $y = y$, which has existential import.

The substitution mentioned above is not logically correct, because it leads from a formula which does not have existential import to a formula which has existential import. The substitution rule for predicate variables has to be restricted so that e-formulas are only substituted for e-formulas, and n-formulas only for n-formulas.

The philosophical difficulties of the definition of identity by Leibniz are discussed in detail in Lorenz 1982, Schirn 1975, 1976, and Griffin 1977. Many of these problems can be solved within the framework of the conception developed here. Leibniz defines identity as follows:

> Definition 1. Those terms are 'the same' of which one can be substituted for the other without loss of truth. Thus, suppose that there are A and B; that A is an ingredient of some true proposition, and that on substituting B for A in some occurrence of A there a new proposition is formed, which is also true. If this always holds good in the case of any such proposition, A and B are said to be 'the same'; conversely, if A and B are the same, the substitution which I have mentioned will hold good. The same terms are also called 'coincident'; sometimes, however, A and A are called 'the same', whereas A and B, if they are the same, are called 'coincident'.
>
> Definition 2. Those terms are 'different' which are not the same, i.e., in which a substitution sometimes does not hold good. (Leibniz 1966, p. 122)

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Later this definition got the following symbolical version:

$$x = y \ \equiv_{Def} \ \forall P(P(x) \equiv P(y)).$$

This definition is not correct, since the bisubjunction

$$x = y \ \equiv \ \forall P(P(x) \equiv P(y))$$

has existential import and therefore is not logically true. Here Peirce is right, when he writes: "Leibniz's 'principle of indiscernibles' is all nonsense. No doubt, all things differ; but there is no logical necessity for it." (C.S. Peirce, Collected Papers I-VI, Cambridge/Mass. 1931–1935, 4.311) But this does not prevent him from setting at another place the formula

$$x = y \ \equiv \ \forall P(P(x) \wedge P(y) \vee \sim P(x) \wedge \sim P(y))$$

(Ibid., 3.398), which has existential import as well and therefore is not logically true.

Usually one to distinguishes between the principle of identity of indiscernibles

$$\forall x \forall y \forall P((P(x) \equiv P(y)) \supset x = y)$$

and the principle of the indiscernibility of identicals (also called *the principle of substitution*)

$$\forall x \forall y (x = y \supset \forall P(P(x) \equiv P(y)))$$

The first one has existential import and therefore is not logically true. The second does not have existential import and is acceptable. In Wessel 1987, 1988, in dealing with the problem of vague predicates in the framework of NTP, I saw that the principle of Leibniz concerning the identity of indiscernibles does not apply, since in NTP the formulas $P(x) \equiv P(y)$ and $\neg P(x) \equiv \neg P(y)$ are not equivalent. At that time I meant that in the framework of NTP it would not be sufficient if x and y have to be only affirmed the same predicates to classify them as identicals, but also have to be denied the same predicates. That was why I defined strong identity as follows:

$$x = y \ \equiv_{Def} \ \forall P((P(x) \equiv P(y)) \wedge (\neg P(x) \equiv \neg P(y))).$$

Nowadays I do not maintain this definition any longer since the corresponding bisubjunction has existential import.

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4. Discernibility vs. Indiscernibility: Some Formal Notions

Now I introduce in the framework of NTP some terms which serve to clarify the problematics of identity. I presume that the terms I will suggest can also contribute to clarify the problem of identity which are discussed in the framework of the philosophy of quantum mechanics (cf. Dalla Chiara 1991, Wessel 1994).

Weak discernibility:

D1. $x \parallel y \equiv_{Def} \exists P(P(x) \wedge \sim P(y) \vee \sim P(x) \wedge P(y))$

Please note that $\sim P(y) \equiv \neg P(y) \vee ?P(y)$ and $\sim P(x) \equiv \neg P(x) \vee ?P(x)$. This means that a sentence $a \parallel b$ is true, if for instance $E(a)$ and $\sim E(b)$. It is sufficient for the truth of a sentence $a \parallel b$ that only one of the two terms a and b is not empty. With D1 and all the following definitions rule R9 was taken notice of, i.e., the corresponding bisubjunctions do not have existential import.

T1. $x \parallel y \equiv \exists P(P(x) \wedge \sim P(y) \vee \sim P(x) \wedge P(y))$ D1

T2. $\sim (x \parallel y) \equiv \sim \exists P(P(x) \wedge \sim P(y) \vee \sim P(x) \wedge P(y))$ T1, sentential logic (SL)

T3. $\sim (x \parallel y) \equiv \forall P(P(x) \supset P(y))$ T2, quantification logic (QL)

T4. $\sim (x \parallel x)$

Proof: We substitute y by x in T1 and get:

1. $x \parallel x \equiv \exists P(P(x) \wedge \sim P(x) \vee \sim P(x) \wedge P(x))$
2. $\sim \exists P(P(x) \wedge \sim P(x) \vee \sim P(x) \wedge P(x))$ QL
3. $\sim (x \parallel x)$ 1,SL

T5. $x \parallel y \supset y \parallel x$

1. $x \parallel y$ assumption of the proof (a)
2. $\exists P(P(x) \wedge \sim P(y) \vee \sim P(x) \wedge P(y))$ 1, T1
3. $P'(x) \wedge \sim P'(y) \vee \sim P'(x) \wedge P'(y)$ B$\exists$, 2 (P' is a constant predicate)

1.1. $P'(x) \wedge \sim P'(y)$ 3, ramified proof
1.2. $\sim P'(y) \wedge P'(x)$ SL, 1.1.
1.3. $P'(y) \wedge \sim P'(x) \vee \sim P'(y) \wedge P'(x)$ SL, 1.2.
2.1. $\sim P'(x) \wedge P'(y)$ 3, ramified proof
2.2. $P'(y) \wedge \sim P'(x)$ SL, 2.1.

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2.3.	$P'(y)\wedge \sim P'(x)\vee \sim P'(y) \wedge P'(x)$	SL, 2.2.
4.	$P'(y)\wedge \sim P'(x)\vee \sim P'(y) \wedge P'(x)$	
5.	$\exists P(P(y)\wedge \sim P(x)\vee \sim P(y) \wedge P(x))$	E∃, 4
6.	$y \parallel x$	5, 4, T1

T6. $\sim (x \parallel y) \supset\sim (y \parallel x)$ T5, SL

T7. $\sim (x \parallel y)\wedge \sim (y \parallel z) \supset\sim (x \parallel z)$

1.	$\sim (x \parallel y)$	a
2.	$\sim (y \parallel z)$	a
3.	$\forall P(\ P(x) \supset P(y))$	1, T3
4.	$\forall P(\ P(y) \supset P(z))$	2, T3
5.	$P(x) \supset P(y)$	B∀, 3
6.	$P(y) \supset P(z)$	B∀, 4
7.	$P(x) \supset P(z)$	5, 6, SL
8.	$\forall P(P(x) \supset P(z))$	E∀, 7
9.	$\sim (x \parallel z)$	8, T3

It is easy to see that the relation $\parallel$ is not a transitive one. If we substitute in $x \parallel y \wedge y \parallel z \supset x \parallel z$ the variable x for z then we get $x \parallel y \wedge y \parallel x \supset x \parallel x$. Here the antecedence can be true, but the consequence cannot. The relation of weak discernibility is so a non-reflexive, symmetrical and non-transitive one.

D2. We call the external negation of the relation of weak discernibility $\sim (\ldots \parallel \ldots)$ *virtual identity.*

The relation of virtual identity is totally reflexive (T4), symmetrical (T6) and transitive (T7). Often virtual identity is confused with the relation of identity, respectively identity is defined by using virtual identity. Such a definition is therefore not acceptable since a sentence of the form $x = y$ has existential import whereas a sentence of the form $\sim (x \parallel y)$ does not. Hence, quantification logic with identity should correctly be called *quantification logic with virtual identity.*

Strong indiscernibility:

D3. $\neg(x \parallel y) \equiv_{Def} E(x) \wedge E(y)\wedge \sim (x \parallel y)$

T8. $\neg(x \parallel y) \equiv E(x) \wedge E(y)\wedge \sim (x \parallel y)$ D3

T9. $\sim \neg(x \parallel y) \equiv\sim E(x)\vee \sim E(y) \vee (x \parallel y)$ T8, SL

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T10. $E(x) \equiv \neg(x \parallel x)$ T8, T4, SL

T11. $E(x) \supset \neg(x \parallel x)$ (E-reflexivity of the relation $\neg(... \parallel ...)$)T10

T12. $\neg(x \parallel y) \supset \neg(y \parallel x)$

1.	$\neg(x \parallel y)$	a
2.	$E(x)$	
3.	$E(y)$	1, T8, SL
4.	$\sim (x \parallel y)$	
5.	$\sim (y \parallel x)$	4, T6
6.	$\neg(y \parallel x)$	3, 2, 5, T8

T13. $\neg(x \parallel y) \wedge \neg(y \parallel x) \supset \neg(x \parallel z)$

1.	$\neg(x \parallel y)$	a
2.	$\neg(y \parallel x)$	a
3.	$E(x)$	1, T8
4.	$E(y)$	1,T8
5.	$E(z)$	2, T8
6.	$\sim (x \parallel y)$	1, T8
7.	$\sim (y \parallel z)$	2, T8
8.	$\sim (x \parallel z)$	6, 7, T7
9.	$\neg(x \parallel z)$	3, 5, 8, T8

So the relation of strong indiscernibility is E-reflexive, symmetrical and transitive.

Strong discernibility:

D4. $x \mid y \equiv_{Def} \exists P(P(x) \wedge \neg P(y) \vee \neg P(x) \wedge P(y))$

T14. $x \mid y \equiv \exists P(P(x) \wedge \neg P(y) \vee \neg P(x) \wedge P(y))$ D4

T15. $\sim (x \mid y) \equiv \sim \exists P(P(x) \wedge \neg P(y) \vee \neg P(x) \wedge P(y))$ T14, SL

T16. $\sim (x \mid y) \equiv \forall P((\sim P(x) \vee \sim \neg P(y)) \wedge (\sim \neg P(x) \vee \sim P(y))$ T15, QL

T17. $\sim (x \mid x)$ T15, NTP, QL

T18. $x \mid y \supset y \mid x$

1.	$x \mid y$	a
2.	$\exists P(P(x) \wedge \neg P(y) \vee \neg P(x) \wedge P(y))$	1, T14
3.	$P'(x) \wedge \neg P'(y) \vee \neg P'(x) \wedge P'(y)$	2, B$\exists$
4.	$P'(y) \wedge \neg P'(x) \vee \neg P'(y) \wedge P'(x)$	3, SL
5.	$\exists P(P(y) \wedge \neg P(x) \vee \neg P(y) \wedge P(x))$	4, E$\exists$
6.	$y \mid x$	5, T14

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T19. $\sim(x \mid y) \supset \sim(y \mid x)$ T18

The relation of strong discernibility is not transitive, because if we substitute in $x \mid y \wedge y \mid z \supset x \mid z$ the variable x for z, then we get $x \mid y \wedge y \mid x \supset x \mid x$. Here the antecedence can be true, but the consequence can not (T17). Also the external negation of the relation of strong discernibility $\sim(\ldots \mid \ldots)$ is not transitive, i.e., $\sim(x \mid y) \wedge \sim(y \mid z) \supset \sim(x \mid z)$ does not apply. The three formulas $\sim(x \mid y)$, $\sim(y \mid z)$ and $(x \mid z)$ are together satisfiable. In accordance with the definitions these three formulas are equivalent with:

$$\forall P((\sim P(x) \vee \sim \neg P(y)) \wedge (\sim \neg P(x) \vee \sim P(y))),$$
$$\forall P((\sim P(y) \vee \sim \neg P(z)) \wedge (\sim \neg P(y) \vee \sim P(z))),$$
$$\exists P((P(x) \wedge \neg P(z)) \vee (\neg P(x) \wedge P(z))).$$

I choose a constant predicate P' with the following ascriptions of values $P'(x) = v$, $\neg P'(z) = v$, $\neg P'(y) = f$ and $P'(y) = f$, in the case of which all the following formulas have the value v:

$$(\sim P'(x) \vee \sim \neg P'(y)) \wedge (\sim \neg P'(x) \vee \sim P'(y)),$$
$$(\sim P'(y) \vee \sim \neg P'(z)) \wedge (\sim \neg P'(y) \vee \sim P'(z)),$$
$$((P'(x) \wedge \neg P'(z)) \vee (\neg P'(x) \wedge P'(z)).$$

Weak indiscernibility:

D5. $\neg(x \mid y) \equiv_{Def} E(x) \wedge E(y) \wedge \sim(x \mid y)$

T20. $\neg(x \mid y) \equiv E(x) \wedge E(y) \wedge \sim(x \mid y)$ D5

T21. $\sim \neg(x \mid y) \equiv \sim E(x) \vee \sim E(y) \vee (x \mid y)$ T20, SL

T22. $E(x) \equiv \neg(x \mid x)$ T20, T17, SL

T23. $\neg(x \mid y) \supset \neg(y \mid x)$ T20, T19, SL

The relation of weak indiscernibility is not transitive, i.e., $\neg(x \mid y) \wedge \neg(y \mid z) \supset \neg(x \mid z)$ does not apply. This can be recognized from D5 and from the fact that external negation of the relation of strong discernibility is not transitive.

T24. $x \mid y \supset x \parallel y$ T14, T1, QL, NTP

The reverse of T24 does not apply.

T25. $\sim(x \parallel y) \supset \sim(x \mid y)$ T24, SL

T26. $\neg(x \parallel y) \supset \neg(x \mid y)$ T8, T22

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The reverse of T26 does not apply.

T27. $\neg(x \parallel y) \supset \sim (x \parallel y)$ NTP

T28. $\neg(x \mid y) \supset \sim (x \mid y)$ NTP

T29. $\sim \neg(x \mid y) \supset \sim \neg(x \parallel y)$ T26

T30. $(x \parallel y) \supset \sim \neg(x \parallel y)$ T27

T31. $(x \mid y) \supset \sim \neg(x \mid y)$ T28

5. Identity in NTP

To define identity I need some terms of the theory of terms. t should be a term-forming operator which forms from a term x a name of this term tx.

I define the relation of inclusion by meaning:

D6. A term x includes by meaning the term y (symbolically: $tx \rightharpoonup ty$) if and only if every object which shall be denoted by x shall also be denoted by y.

Definition of equality by meaning:

D7. $tx \rightleftharpoons ty \equiv_{Def} (tx \rightharpoonup ty) \wedge (ty \rightharpoonup tx)$

(cf. Wessel 1995).

Sentences concerning inclusion by meaning of terms are in accordance with their logical form predicative sentences and therefore have existential import. Such sentences have the form $\rightharpoonup (tx, ty)$. In the case of sentences of this form, however, the existential import can already be satisfied alone by the syntactical rules of construction. Therefore we accept the following rule:

R10. Concerning $\rightharpoonup (tx, ty)$ existential import is satisfied if and only if x and y are forms of terms (respectively terms) in accordance with the rules of construction of terms which were chosen.

In addition, I introduce the relation of denoting S. $S(x, ty)$ has to be read as: "x is denoted by term y." Concerning the relation of denoting I set the following axioms:

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A1. $E(x) \equiv S(x, tx)$

A2. $\exists P(P(x) \vee \neg P(x)) \supset S(x, tx)$

A3. $E(x) \wedge \sim S(x, ty) \supset \neg S(x, ty)$

A4. $\sim S(tx, ty) \supset \neg S(tx, ty)$

A5. $S(x, ty) \wedge S(y, tz) \supset S(x, tz)$

A6. $S(x, ty) \supset S(y, tx)$, where x and y are singular subject-terms.

Definition of identity:

D8. $x = y \ \equiv_{Def} \ S(x, ty)$, where x and y are singular subject-terms.

Definition of difference:

D9. $\neg(x = y) \ \equiv_{Def} \ E(x) \wedge E(y) \wedge \sim (x = y)$,
where x and y are singular subject-terms.

In the theory of terms the following principle of replacement (PR) applies concerning terms which are equal by meaning:

$$\frac{\begin{array}{c} tx \rightleftharpoons ty \\ A \end{array}}{A[x/y]}$$

Here $A\,[x/y]$ is the formula (sentence) which one gets from the formula (sentence) A, if one replaces zero or more occurrences of x as a term by y.

A7. $x = y \supset tx \rightleftharpoons ty$

The reverse of A7 does not apply. From A7 and PR one gets the principle of replacement for identities (PRI):

$$\frac{\begin{array}{c} x = y \\ A \end{array}}{A[x/y]}$$

Some theorems:

T32. $\sim S(x, tx) \supset \forall P?P(x)$ A2, NTP, QL

T33. $\sim \neg S(x, ty) \supset \sim E(x) \vee S(x, ty)$ A3, SL

T34. $\sim \neg S(tx, ty) \supset S(tx, ty)$ A4, SL

T35. $P(x) \vee \neg P(x) \supset x = x$ D8, A1, A2

The reverse of T35 does not apply.

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T36.	$\sim (x = x) \supset ?P(x)$	T35, SL
T37.	$E(x) \equiv x = x$	D8, A1, A2
T38.	$x = y \supset y = x$	D8, A6, PRI
T39.	$x = y \wedge y = z \supset x = z$	D8, A5, PRI
T40.	$x = y \supset (P(x) \equiv P(y))$	PRI
T41.	$x = y \supset (\neg P(x) \equiv \neg P(y))$	PRI
T42.	$x = y \supset (?P(x) \equiv ?P(y))$	PRI
T43.	$x = y \supset (P(x) \equiv P(y)) \wedge (\neg P(x) \equiv \neg P(y)) \wedge (?P(x) \equiv ?P(y))$	

The reverses of T40–T43 do not apply.

T44.	$\sim (x = y) \equiv \sim S(x, ty) \vee \sim S(x, tx)$	D8, SL
T45.	$\neg(x = y) \equiv \neg(y = x)$	D9, T38
T46.	$\neg(x = x) \supset E(x)$	D9
T47.	$\sim (x = y) \supset \sim (y = x)$	T38, SL
T48.	$\sim E(x) \supset \sim \neg(x = x)$	T46, SL
T49.	$x = y \supset E(x)$	D8, A1
T50.	$x = y \supset E(y)$	D8, A1, T38
T51.	$x = y \supset S(x, ty)$	D8
T52.	$x = y \supset S(x, tx)$	D8
T53.	$x = y \supset S(y, tx)$	D8, T38
T54.	$x = y \supset \neg(x \parallel y)$	D8, QL, T3, D3, PRI
T55.	$x = y \supset \neg(x \mid y)$	D8, QL, T20, T16
T56.	$x = y \supset \sim (x \parallel y)$	T54, NTP

T56 means that virtual identity follows from identity. The reverse does not apply.

T57.	$x = y \supset \sim (x \mid y)$	T55, NTP
T58.	$\neg(x \parallel y) \supset x = y$	T8, T3, QL, D8
T59.	$x = y \equiv \neg(x \parallel y)$ (Identity of the strong indiscernibility)	T55, T58
T60.	$\neg(x = y) \supset x \parallel y$	D9, A6, T1, QL
T61.	$\neg(x = y) \supset E(x)$	D9

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T62.	$\neg(x = y) \supset E(y)$	D9
T63.	$\neg(x = y) \supset \neg S(x, ty)$	T61, T62, A3, D9
T64.	$\neg(x = y) \supset \neg S(y, tx)$	T63, D8, D9, A4
T65.	$?(x \mid y) \supset \sim E(x) \vee \sim E(y)$	T21, NTP
T66.	$?(x \parallel y) \supset \sim E(x) \vee \sim E(y)$	T9, NTP
T67.	$?(x = y) \supset \sim E(x) \vee \sim E(y)$	D9, NTP
T68.	$E(x) \wedge E(y) \supset (x \mid y) \vee \neg(x \mid y)$	T65
T69.	$E(x) \wedge E(y) \supset (x \parallel y) \vee \neg(x \parallel y)$	T66
T70.	$E(x) \wedge E(y) \supset (x = y) \vee \neg(x = y)$	T67
T71.	$E(x) \wedge E(y) \supset \sim ?(x \mid y)$	T68, NTP
T72.	$E(x) \wedge E(y) \supset \sim ?(x \parallel y)$	T69, NTP
T73.	$E(x) \wedge E(y) \supset \sim ?(x = y)$	T70, NTP
T74.	$E(x) \wedge E(y) \supset (\sim (x \mid y) \equiv \neg(x \mid y))$	T71, NTP
T75.	$E(x) \wedge E(y) \supset (\sim (x \parallel y) \equiv \neg(x \parallel y))$	T72, NTP
T76.	$E(x) \wedge E(y) \supset (\sim (x = y) \equiv \neg(x = y))$	T73, NTP
T77.	$P(x) \supset x = x$	T37, E1
T78.	$\neg P(x) \supset x = x$	T37, E2
T79.	$\sim \neg(x = x)$	D9, T37

6. Applications

With the help of the terms just proposed the so-called paradox of Poincaré can be solved. He writes in his paper *Science and hypothesis*:

> It has been observed, for example, that a weight A of 10 grams and a weight B of 11 grams produce identical sensations, that the weight B is just as indistinguishable from a weight C of 12 grams, but that the weight A is easily distinguished from the weight C. Thus the raw results of experience may be expressed by the following relations: $A = B$, $B = C$, $A < C$, which may be regarded as the formula of the physical continuum. But here is an intolerable discord with the principle of contradiction, and the need of stopping remove this has compelled us to invent the mathematical continuum. (Poincaré 1946, p. 46)

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In his paper “The value of science” he repeats the same argumentation and adds:

> Doubtless if we measured the weights with a good balance instead of judging them by hand, we could distinguish the weight of 11 grams from those of 10 and 12 grams, and our formula would become $A < B$, $B < C$, $A < C$. But we should always find between A and B and between B and C new elements D and E, such that $A = D$, $D = B$, $A < B$; $B = E$, $E = C$, $B < C$, and the difficulty would only be receded and the nebula would always remain unresolved; the mind alone can resolve it and the mathematical continuum it is which is the nebula resolved into stars. (Poincaré 1946, p. 240f.)

It is evident that there is no contradiction, if one uses the terminology which I propose, and if one distinguishes between identity and indiscernibility. Poincaré’s term ‘physical continuum’ is not appropriate for the situation described. In Weyl 1918 the author distinguishes between mathematical and perceptible (anschaulichem) continuum and already gives hints concerning the problematics of quantum mechanics. Čapek speaks of a continuum of perceptions or of a qualitative continuum and mentions as an example the psychical simultaneity which was described by Russell in 1915:

> Suppose, for example, the sounds A, B, C, D, E occur in succession, and three of them can be experienced together. The C will belong to a total experience containing A, B, C, to one containing B, C, D, and to one containing C, D, E. . . . In the above instance, C is at the end of the specious present of A, B, C, in the middle of that of B, C, D, and at the beginning of that of C, D, E. (Russell 1915, p.218)

And at another place:

> Suppose that I see a given object A continuously while I am hearing two successive sounds B and C. The B is simultaneous with A and A with C, but B is not simultaneous with C. (Russell 1915, p.228)

All the situations described above can perfectly be represented with the terminology I proposed.

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