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Srećko Kovač

Form, Content, Extensions: Some Perspectives on the Early Development of Modern Logic

Abstract. In contrast to Kant's metatheoretical ("transcendental") approach grounded in the operations of an abstract logical subject, Herbart and Bolzano aimed to avoid subject-dependent foundations of logic. They instead proposed formal logics founded, respectively, on the form of conceptual content (Herbart) and on the extensions of logical forms (Bolzano). Among the traditions originating from Herbart and Bolzano, we consider the role of Bolzano's informal disciple Robert Zimmermann, who subsequently adopted Herbartianism, and Zimmermann's own disciple, the mathematical logician and philosopher Albino Nagy. Special attention is given to Nagy's analysis of expressive completeness and his treatment of the Jevons-Clifford problem: how many and which types and representatives of types there are of Boolean propositional functions. We transform this latter problem into the question of types of sequent refutations and examine the corresponding admissibility of the cut rule.

Keywords: logical form; content of a concept; extension; Robert von Zimmermann; Albino Nagy; completeness; Jevons-Clifford problem; sequents type; cut admissibility

1. Introduction

After Leibniz's work in algebraic logic, together with his ideas on a universal language for science, and following Kant's refocusing on the formal character of logic and on metatheoretical questions, several interconnected traditions in logic and philosophy emerged. Notably, traditions originating from Herbart and Bolzano contributed significantly to the development of modern logic. Within these traditions, we examine in particular the role of Bolzano's informal disciple, Robert von Zimmer-

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mann, who later adopted Herbartianism, and Zimmermann's own disciple, the mathematical logician and philosopher Albino Nagy. We argue that these lines of development are important alongside, and in connection with, the commonly acknowledged Boolean and Frege-Russellian traditions, thus providing a fuller understanding of the origins and early development of modern symbolic logic.

Both Leibniz's and Kant's approaches are clearly opposed to psychologistic foundations of logic — that is, logic grounded in, or concerned with, empirical features of human thought. Moreover, Kant particularly emphasized the notion of formal logic (which he typically referred to as "general logic"). Unlike special logics of particular sciences, Kant's formal logic should abstract from the specific characteristics of different object domains and is intended to universally hold for our cognition and thought (see, e.g., B IX, 76, 79, 80).[1]

On the other hand, Kant clearly distinguished a metatheoretical approach ("transcendental logic") to the foundations of logic and other theoretical knowledge[2] from Leibniz's ultimately metaphysically founded doctrine of the "truths of reason".[3] Kant understood his metatheory in an abstract-subjective sense, explicitly non-psychological and non-metaphysical. He defined logical forms in terms of the operations performed by the abstract subject (agent) — "I" ("transcendental unity of self-consciousness"[4]) and analyzed their validity in application to the real, sensibly given data.[5] Kant believed Leibniz did not clearly dis-

[1] We follow the standard citation format in Kant literature, with "B" denoting the second edition of Kant's *Critique of Pure Reason* (1787), and Roman or Arabic numerals for the original pagination.

[2] "Transcendental logic" deals, not with objects themselves, but with our cognition of objects, setting aside psychological conditions. In Kant's terms, it focuses solely on our "pure," non-empirical thought and the origin of our cognitions independently of objects [B 79–80].

[3] "Truths of reason" ("necessary truths") are innate in our "reasonable soul or spirit" (*esprit*) [29, § 29] and in harmony with reality [28, p. 70, cf. p. 76]; see also [27, p. 484].

[4] This unity can be understood as a *possible* logical awareness accompanying all our thoughts and intuitions ("The *I think* must be able to accompany all my representations," cf. B 131–132). For Kant, transcendental unity of consciousness is purely formal and logical, potentially related to any of our representations regardless of their content (cf. B 406–407, 413).

[5] Although Kant claimed formal logic was already complete as a science (cf. B VIII–IX), he left room for further advances in metatheoretical research on the foun-

tinguish between logical (formal) and ontological (real) aspects, and, for instance, criticized Leibniz's ontological proof of God's existence for failing to achieve anything beyond formal logic.[6] Additionally, he emphasized a clear distinction between the logical and real (causal) versions of the principle of sufficient reason.[7]

Based on his abstract-subjective approach, Kant establishes his proof ("transcendental deduction") of the adequacy of categories for our sensible experience (the "constitutive use" of categories for sensibly "given" objects). He also defines an overall conceptual "model" (as we might call it today) within which our theoretical knowledge develops ("regulative use"). This model consists of three members: (1) the absolute totality of our thought — the idea of the absolute unity of the subject of our thoughts; (2) the absolute totality of the series of conditions for appearances — the idea of the world; and (3) the absolute totality of possible positive predicates of things — the idea of God. It could be understood as a conceptual outline of a set-theoretically defined model $\langle S \neq \emptyset, R_1, \ldots, R_n, V\rangle$, consisting of a non-empty set S (of all thinkable objects), relations $R_1, \ldots, R_n$ on S (for Kant these are relations between states of affairs of objects), and the valuation V of predicates over D.

Herbart and Bolzano sought to avoid subject-dependent foundations of logic (as respected, for instance, in W. T. Krug [24]). Instead, they proposed a formal logic grounded directly on concepts and on the extensions of logical forms, independent of the transcendental subject. Thus, for Herbart, logic does not deal with the activity of a thinking subject (even abstract one), but rather with *what* is thought (conceived) — namely, concepts [14, p. 81] ("concepts [...] thought only by their content" [15, pp. 126–127])[8]. "That what is thought" ("content," see Zimmermann 60, pp. 12–13) is independent of the actual psychological act of thinking and is not a real object either. Instead, it functions as a "prescription" (*Vorschrift*) and an "ideal" for our thinking [15, pp. 59–60, 126–127] (cf. "Norm," Drobisch 8, p. 2; "Ideal," Zimmermann 60, p. 18). One concept should be presupposed (hypothetically as a subject,

dations of logic — for example, concerning the completeness of his table of judgments (see the seminal work by Klaus Reich [49]).

[6] See B 630; 22, pp. 1527–1529.

[7] For example, Kant [18, vol. VIII, p. 193–195].

[8] Only content in general is intended here, in its formal interrelationships, rather than particular content and its concrete properties.

das Vorausgesetzte), and another concept should be connected to it (*das Angeknüpfte*) [14, pp. 97, 105]. A judgment as a whole is a (positive or negative) decision regarding the connection between a subject and a predicate [14, p. 99]. Thus, in a syllogisms, concepts (subject S, predicate P, and a middle term M) are *posited* or *removed* (*Setzen*, *Aufheben*) according to specific inference forms. For example, the positing of S is connected with the positing of M, and further, with the positing of P if M is its subject (see Herbart 14, p. 116). As a refinement within the Herbartian tradition, we encounter mathematical representations of logical forms (e.g., Drobisch 8, "Logisch-mathematischer Anhang," pp. 196–235, referred to by Herbart 14, p. 129, fn. 1).

For Herbart, the activity of "thinking" (questioning, deciding, presupposing, connecting, positing, removing), as involved in a judgment or inference, plays only a secondary role — as a "means" ("vehicle") for combining concepts. Concepts themselves are primary: "it is up to concepts themselves whether they will fit with one another or not" [14, p. 96].[9] In contrast, Kant's approach gives fundamental importance to the operational aspect of concepts, grounded in the analytic unity of abstract, logical consciousness (cf. B 134, fn.).

Bolzano is even farther than Herbart from any subjectivist conception of logic. According to Bolzano, logic deals with truths, ideas, propositions, and inferences "in themselves," as objective, but lacking positive ontological status. Objective propositions, objective truths, and objective representations (cf. expressions 'Satz an sich,' 'Wahrheit an sich,' 'Vorstellung an sich' — *objektiver Satz*, *objektive Wahrheit*, *objektive Vorstellung*) constitute the independent "subject-matter" ("material") and "content" of our knowledge and thought. "There are" (*es gibt*) propositions — not in an ontological or abstract-subjective sense, nor merely in a normative one — but as non-actual (non-existent, non-being) referents. On this basis, Bolzano offers his well-known extensional definition of derivability (*Ableitbarkeit*) using (model-theoretical) "variations" of propositions.[10]

We focus on a line of development in logic, traced through Robert von Zimmermann, an influential professor of philosophy in Vienna during the second half of the 19th century, to the algebraic logic of his student

[9] Cf. also "[...] forms of the possible connection of what is thought [...] which are allowed by what is thought itself according to its nature" [14, p. 82, our transl.]

[10] See [2, e.g., I §§ 19, 25, 48; II §§ 122, 162].

Albino Nagy. Nagy, in particular, contributed to the foundations of logic and to the so-called Jevons-Clifford problem (concerning the species and number of types and of representatives of types of Boolean propositional functions). Among later logicians, notably Gödel and Bernays were acquainted with the Jevons-Clifford problem and Nagy's contribution to its resolution (see Gödel [57, pp. 105, 187] and Bernays [1]; cf. Urquhart [56, on Nagy pp. 294, 295]). However, as it seems [56, pp. 293–294], with the rise of the "Frege-Peano-Russellian" approach to logic, the problem was "largely forgotten."[11] Interest in the problem was revived later, particularly in the mid-20th century, within electro engineering and computer science (in the analysis and minimization of circuits and networks). However, our aim is to show, using Nagy's approach as an example, that the Jevons-Clifford problem also contains implicit or latent aspects relevant from the perspective of the subsequent development of formal logic (e.g., in sequent calculus and tableaus).

2. Extensions and content in Robert von Zimmermann

Robert von Zimmermann (1824–1898) played a significant role in transmitting and further developing the ideas of Bolzano and Herbart, as well as in facilitating the subsequent development of logic.[12] Figure 1 outlines a genealogical fragment centered on Zimmermann's role (see [23, p. 500]; cf. Marotti 34, p. 97, focusing on Brentano).[13] Notably, Zimmermann supervised Twardowski — the founder of the Lvov-Warsaw school —

[11] Nevertheless, Nagy references Peano in the context of Jevons-Clifford problem, though only at a general level. See [42, p. 345], [40, p. 100], and [45, p. X].

[12] For philosophy in general, see, for example, [53] and [11].

[13] Zimmermann was a direct disciple of Bernard Bolzano. He received his PhD in Prague under Franz Serafin Exner (1802–1853), who was particularly influenced by Bolzano and Herbart; he later completed his habilitation in 1846 in Vienna under Franz Karl Lott (1807–1874), a disciple of Herbart (for logic, cf. [9, 31]). Zimmermann taught philosophy at the universities of Olomouc (1849–1852), Prague (1852–1861), and Vienna (1861–1896), where he was the supervisor, for example, of Josef Dastich (1835–1870) (in Prague), and of Mathias (Matjáš) Amos Drbal (1829–1885), Franjo pl. Marković (1845–1914), Kazimierz Twardowski (1866–1938) and Albino Nagy (1866–1901) (in Vienna). All of the Zimmermann's aforementioned disciples authored logic textbooks or lecture notes [5, 7, 33, 40, 55], but only Nagy's textbook, taken as a whole, is an introduction to mathematical logic. Besides Zimmermann, Gustav Adolf Lindner (1828–1887) was another student of Exner and the author of an influential textbook on "formal" logic (in a Herbartian sense) [30].

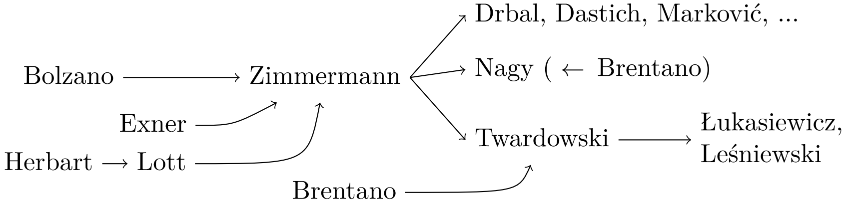

Figure 1: Zimmermann's tradition within the development of logic

after Brentano, Twardowski's main teacher, relinquished his professorship status at the University of Vienna. Other students of Brentano, such as Benno Kerry and Edmund Husserl, also studied under Zimmermann. Given his positive view of the use of mathematical methods in philosophy, Zimmermann may have contributed to creating a favorable background for Albino Nagy as both a philosophical and a mathematical logician (capable of producing strictly technical results in logic). Zimmermann himself considered the mathematical theory of probability a possible branch of logic [61] (cf. 52, pp. 18–19, footnote 24).

Zimmermann's early view of logic was largely *set-theoretically extensional* [59], combined with intuitively extensional (graphical) and intuitively intensional (exemplifying) approaches. The extension of a representation (*Vorstellung*) is generally conceived, as in Bolzano, as the sum of objects (*Gegenstände*) to which the representation refers [59, pp. 12, 9](cf. [2, I, pp. 297–298]). The influence of Bolzano is particularly evident in concepts such as "objective proposition," "objective concept," the categorical form "A has b," "compatibility" (*Verträglichkeit*), and "derivability" (*Ableitbarkeit*, consequence). Zimmermann employed Bolzano's method of variation and adopted his classification of logical forms. For instance, Zimmermann understands inference (*Schluß*) in light of Bolzano's notion of "derivability" [59, p. 45] and usually explains special forms of inference extensionally — via set-theoretical analysis of the relationship of the extensions involved and through Euler diagrams. His classification of inferences is based on complexity: the number of premises and of "common" ("middle") concepts.

We provide two of Zimmermann's *examples* of inference: one (1) with *existential* import, in the form "What has a, has b," or "A is B," involving operators 'all,' 'some,' 'exists' (cf. $\exists x$) [59, p. 64]; and another

(2) *without* clear *existential* import, in the form “Where there are [...], there are also [...]” [59, p. 81] (cf. $\forall x(\phi(x) \to \psi(x))$). ‘U(A),’ ‘U(B),’ etc. denote the extension (*Umfang*) of A, B, etc., respectively

The first example (1) is as follows:

What has a, has b. (A is B.)
What has c, has non-a. (C is non-A.)

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1. What has a, has non-c. (No A is C.)
2. What has a, has both b and non-c. (Each A is B and non-C.)
3. The representation of something that has b but non-c, has objecthood*. (Some B are not C.) (IV. Fesapo.)

*In German: „hat Gegenständlichkeit” (cf. $\neq \varnothing$; see [59, pp. 46–47, 39]; not necessarily an actually existing object is meant [2, II, §137 pp. 52–53]).

Zimmermann’s explanation of the first conclusion is extensional — almost as in a 20th century logic textbook (cf. Figure 2):

Remark. U(A) and U(non-A), and — because U(C) either $\simeq$ [or] $<$ U(non-A) — U(A) and U(C), too, are mutually exclusive. Since, due to the first premise, a B is connected to each A, so, in case U(A) $\simeq$ U(B), also U(C) and U(B) are mutually exclusive; or, if U(A) $<$ U(B), so there should be at least one or several B which are A, and consequently, since U(C) and U(A) are mutually exclusive, they are non-C.

Compared to Bolzano [2, II, 413], we observe in Zimmermann explicit set-theoretical presentation and argumentation (supported by examples and graphic representations).

Here is another example (2), this time without common terms:

What has a, has b. (A is B.)
What has c, has d. (C is D.)
What has e, has f. (E is F.)

Where the properties a, c, e occur, the properties b, d, and f also occur. (Where there are A, C, E, there are also B, D, F.)

Zimmermann’s justification is extensional.

Remark. U(B) $\simeq >$ U(A); U(D) $\simeq >$ U(C); U(F) $\simeq >$ U(E). If A, C, and E have some, no matter how small, part of their extension

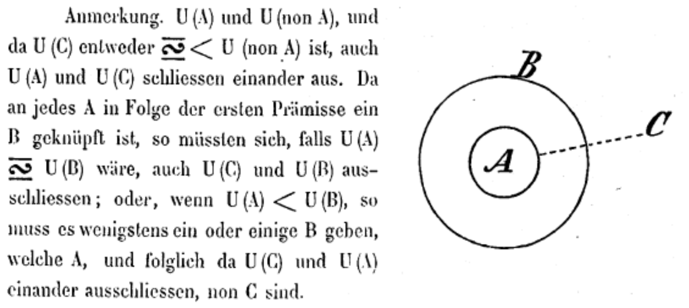
Anmerkung. U (A) und U (non A), und da U (C) entweder ≂ < U (non A) ist, auch U (A) und U (C) schliessen einander aus. Da an jedes A in Folge der ersten Prämisse ein B geknüpft ist, so müssten sich, falls U (A) ≂ U (B) wäre, auch U (C) und U (B) ausschliessen; oder, wenn U (A) < U (B), so muss es wenigstens ein oder einige B geben, welche A, und folglich da U (C) und U (A) einander ausschliessen, non C sind.

Figure 2: Zimmermann's justification of an inference with existential import [59, p. 64]

in common, then B, D, and F must also have some, no matter how small, part of their extension in common; which is precisely what the conclusion states.

(See Figure 3; cf. Bolzano's explication of such a conclusion in terms of universal quantification over objects, in 2, II, pp. 440–441).

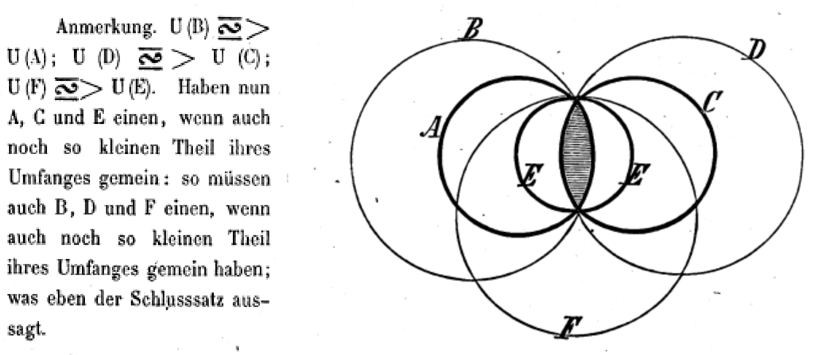
Anmerkung. U (B) ≂ > U (A); U (D) ≂ > U (C); U (F) ≂ > U (E). Haben nun A, C und E einen, wenn auch noch so kleinen Theil ihres Umfanges gemein: so müssen auch B, D und F einen, wenn auch noch so kleinen Theil ihres Umfanges gemein haben; was eben der Schlusssatz aussagt.

Figure 3: Zimmermann's justification of an inference without common terms [59, p. 81] (graphics, ibid.)

In the 1860 and 1867 editions of his textbook, Zimmermann adopted a Herbartian, content-focused approach. For him, the general formula for the content (*Inhalt*) of a composed concept is:

$$F(a, b, c \ldots),$$

where F is the form of the connection (*Verknüpfung*), and $a, b, c, \ldots$ are the material (*Stoff*) of the content. According to Zimmermann, simple concepts possess no form; in such cases, the material coincides with the concept's content. Using the notation

$$F(a, b)[1, F(c), F(d, e), F(f, g, h), \ldots]$$

Zimmermann expresses that the content $F(a, b)$ remains constant across the contents $F(a, b), F(a, b, c), F(a, b, d, e), F(a, b, f, g, h), \ldots$ [60, p. 24]. The extension (*Umfang*) of a concept is, following Herbart, "conceptualized": the extension of a concept with the content $F(a, b)$ consists of the concepts with the contents $F(a, b, c), F(a, b, d, e), F(a, b, f, g, h), \ldots$ [60, pp. 27, 29–30]. Concepts are posited or excluded based on their connectibility with one another, either each time or sometimes. As for Herbart, a judgment is a decision concerning whether there is such a connection between S (subject) and P (predicate) [60, p. 42 §54]. Logical form of a syllogism consists in a special configuration of positing and excluding concepts, which leads to a decision about the connectibility of S and P, as, for example, is illustrated in the mode *Ferio* [60, p. 79]:

As often as M is posited, P is excluded.
Sometimes, if S is posited, M is posited.

Therefore sometimes, if S is posited, P is excluded.

The general form $F(a, b, c \ldots)$ of a concept's content (which can also be found earlier in Lotze) allows any formally (mathematically) expressible interdependency of notes within a content. The Herbartian distinction between the decision and positing (or excluding, removing) of concepts likewise anticipated a clear formal distinction between assertion and proposition (*Satz*), i.e., between $\vdash \phi$ and ϕ, even though "decision" was ascribed to the "quality" of a judgment.[14]

In particular, the occasional use of algebraic expressions, extensionality, and the generally positive stance on the application of mathematical methods in logic (as well as in other sciences, such as psychology), together with the recognition of the important mediating role of mathematics for ontology and theory of knowledge (on this, see Nagy 36),

[14] Kant ascribed assertion to the modality of judgment (rather than to its quality): assertoric modality means that the content of a judgment — including its quality — is considered true [B 100].

facilitated the introduction of mathematical logic and, in particular, adoption of algebraic methods in logic, as can be seen in the work of Zimmermann's disciple Albino Nagy.[15]

3. Algebraic methods and foundations of logic in Albino Nagy

Albino (Bjeloslav) Nagy, a student of Robert Zimmermann in Vienna, developed an interest in applying mathematics to logic already during his gymnasium studies in Zadar (independently of Peano, as he notes [35, p. 1]).[16] His university studies under Zimmermann in Vienna clearly

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15 In general, Zimmermann continues the tradition of logic as a *formal* discipline, established primarily by Kant, but with significant alterations along Bolzanoan and Herbartian lines and with the occasional introduction of mathematical tools. In [59], Zimmermann even agreed with Kant's view that formal logic is essentially completed already in Aristotle (p. 1). However, according to [59], the laws of logic should be grounded in "objective truth" (p. 12), in contrast to Kant's notion of objectivity, which is based on the a priori synthetic unity of apperception. "Pure," "objective" logic should consider the "forms of . . . the truth itself," even including the "ontological factor of knowledge," whereas only applied objective logic studies human limitations in dealing with truth — and thus (as for Kant) incorporates the "psychological factor of knowledge" [59, pp. 5–6].

In the 1860 and 1867 editions of his logic, the Herbartian shift from objects to concepts and their content in the foundations of logic led, as noted above, to defining the extension of a concept in terms of its subordinated concepts (whereas, for Kant in [20], the extension is defined by the objects denoted by the concept — similarly in [59]). Specific Kantian features — such as the four most general aspects of judgment (quantity, quality, relation, and modality) and the division of judgments and inferences into categorical, hypothetical, and disjunctive forms — are more explicitly present in Zimmermann [59] (though partially transformed) than in the later editions.

16 Albino Nagy was born in 1866 in Trogir (Croatia, then part of the Habsburg Monarchy) and died 1901 in Rome. He was active in Zadar (Croatia, then in the Austro-Hungarian Monarchy, Dalmatia), Vienna, and Italy (Velletri, Rome, and Taranto). According to C. Burali-Forti [3, p. V], Nagy is regarded, alongside Peano, as co-founder of mathematical logic in Italy, where he arrived in 1889. Nagy also refers to his earlier essay written in Zadar in 1884 [35, p. 26, fn. 2], which included the application of coordinate systems and n-dimensional space to logic—predating both his appointments in Italy and the publication of Peano [45]. Nagy published in Italian and German in Italy, Austria, Germany, and Croatia (Dalmatia) on algebraic logic, foundations and philosophy of logic, and the history of philosophy. A monograph on Albino Nagy by Ivan Macut, in Croatian, includes an extensive bio-bibliography and a significant number of translated papers by Nagy. Otherwise, see, e.g., [10, 44] and [48, pp. 150–191].

supported and deepened these interests.[17] In an early published paper [35, p. 1 and fn. 3 on p. 26], Nagy refers to Zimmermann's view (in accordance with Kant) of mathematizing psychology through the quantification of quality (degree), which prompted Nagy to consider degrees of possessing a note [35, p. 18 and Fig 12]. In a later paper [36], dedicated to and inspired by Zimmermann's philosophy, Nagy analyzes the mediating role of mathematics in Plato — as a bridge between ideas and appearances — and, in a modern context, between external objects and the cognitive subject. While a student in Vienna, Nagy particularly read literature on mathematical logic, including Leibniz, British and American logicians (Boole, Jevons, Peirce, and others), as well as continental European mathematicians and logicians (including Grassmann, Schröder, and Peano). He was also familiar with Weierstrass's mathematical investigations.

3.1. Logic in relation to ontology, psychology, and language

Nagy continues the tradition of logic as a formal discipline (Kant, Herbart, Zimmermann, and the algebraic logicians), advocating the application of mathematics to logic.[18] The neighboring disciplines of ontology, psychology, and linguistics, serve as auxiliary sciences, providing initial presuppositions (*primi dati*) of logic. In particular, Nagy maintains that concepts are presupposed in their psychological, metaphysical, and linguistic aspects [40, p. 20–21], concerning their mental representation, referred objects, and linguistic expressions, respectively.[19] What is logically primitive (in contrast to psychology, metaphysics, and linguistics),

[17] He studied the history of philosophy with Zimmermann and descriptive psychology with Brentano.

[18] The application of mathematics to logic marks a departure of Nagy's approach from that of Kant. Kant strictly distinguishes demonstrative proofs (grounded in exact, concrete evidence through the construction of concepts in intuition, including "symbolic construction") from discursive proofs (proceeding by means of concepts *in abstracto*) [B 762, 745]. Nonetheless, Kant occasionally makes at least partial use of symbolic (and graphic) representations of logical forms — e.g., "in a categorical judgment, x that is contained under b is also under a" [20, §9]. Nagy also quotes Zimmermann's reflections on Kant's rapprochement with "mathematical psychology" in considering degrees (magnitude) of our sensation [35, p. 33 fn. 32].

[19] This is a contrast with Kant, who explicitly excludes metaphysical, psychological, and anthropological presuppositions from the foundations of formal logic, grounding it solely in abstract, logical apperception (B VIII–IX, 134 note; see, for instance, [49, 58] and [21], which is based on a 1992 book in Croatian).

for Nagy, are the formal relations between concepts: I. subordination, II. interference (partial inclusion and partial exclusion), and III. disjunction (total exclusion) [40, pp. 24–25]. It is on these relations that the forms of judgments and inferences are based.

In Nagy's view, modern logic, through an algebraic approach, overcomes the linguistic and psychological limits of traditional logic and significantly enhances the precision and expressive capacity of logic: (1) the imprecision and indeterminacy inherent in natural language are overcome by the use of a formalized language, and (2) psychological limitations in executing logical acts (a single act of thought can combine only two concepts) is overcome by algebraic operations (a single algebraic operation can combine any finite number of concepts) [39, 40].[20]

In what follows, we focus on two of Nagy's principal results in logic: a special kind of expressive completeness of the algebra of classes and his contribution to the solution of the Jevons-Clifford problem.

3.2. Foundations of logic and conceptual completeness

In his early work, Nagy formulates and emphasizes a fundamental question concerning the expressive completeness of the Boolean calculus of classes (Nagy 35[21] and 40). He first describes the Boolean algebra of classes and then raises the question of whether algebraic forms encompass "all logical forms," or, more precisely, whether they are valid for the entire "domain of the thinkable" (the whole "logical domain," "manifold"). Here are two Nagy's formulations of the completeness problem [35, p. 7]:

> <...> are <...> all logical forms comprised by the used symbols or not? [<...> sono <...> tutte le forme logiche comprese dai simboli che s'usano, o no?],

> <...> are the established forms valid for the whole domain of the thinkable, or they are valid only for special systems of beings that should observe certain conditions. [<...> le forme stabilite valgono per tutto

[20] This should be compared with Kant's understanding of a judgment, not merely as a relation ("objective unity") between *two* concepts, but also, e.g., as an equally primordial relation between *several* judgments within a single disjunctive judgment [B 98, 140–141].

[21] According to the note at the end of the paper, Nagy completed this work in Zadar in May 1889 (at the age of 22).

il campo del pensabile, oppure valgono soltanto per sistemi speciali di enti, i quali devono obbedire a certe condizioni.].

More generally, the problem is how far the application of mathematics to logic can be legitimately extended [35, p. 7]. Nagy reduces this question to whether Boolean algebra is capable of completely capturing concepts in terms of their content (notes, properties, attributes), and, by means of that, of expressing logical entities and logical classes. Evidently, Nagy's problem is strongly determined by his preconception of logic (and of entities and classes in a corresponding logical sense).

In a sense, Nagy bridges the Herbartian content-centered approach and the Bolzanoan extensional approach (also connecting the later and earlier Zimmermann's understanding of logic). The Herbartian content-centered aspect is traceable, for example, in Nagy's declared focus on a concept's content (notes contained in concepts [35, p. 7]), in his approach to the definition of individuals in terms of conceptual content [35, p. 13] (individual concepts), and in his conceptual division of a chosen universe of discourse into a hierarchy of subspecies. On the other hand, a distinctly extensional approach is particularly manifest in his mostly set-theoretical treatment of classes using algebraic operations (unions, intersections, and complements).

Nagy intends to implement a rigorous approach. He generally takes "empirical facts," "observations" about the relations between concepts as his starting point.[22] He then demonstrates that these relations and their laws can be described in terms of algebraic operations. Based on this, he argues that it is possible to define the concepts of "logical entity" and "logical class" algebraically, thereby, he assumes, capturing in principle the entire "domain of the thinkable".

Nagy begins with conceptual content (notes), on the basis of which he defines entities (beings, elements), and then defines classes on the basis of those entities. A logical entity (element) x is defined as the limit product of notes a_i, approximating the identity with x: $lim(\prod_{i=1}^{n} a_i) = x$, with sufficiently big n. This is based on the proof that it is always possible to

[22] Nagy inherits Kant's metatheoretical rigor, as in Kant's program to "exhaustively show and strictly prove formal rules of all thinking" [B IX]. However, whereas Kant's metatheoretical starting point is the a priori, formal "unity of consciousness," Nagy takes the "unity of mind" (*unità della mente*) as a psychological empirical presupposition [35, p. 8]. Heuristically, Kant also uses the experience of a common discourse as basis from which purely formal principles of logic are derived by abstraction.

find a_{n+1} such that $\prod_{i=1}^{n} a_i - \prod_{i=1}^{n+1} a_i > 0$ [35, 14–16].[23] The product of entities yields zero (0, "no entity,', "empty class"), except when the product is of an entity e with itself, which yields e again. The sum of entities forms a class in the broad sense, except when the sum is of one and the same entity e, which yields e.

A logical class a in the broad sense is defined as the sum of entities e_k, $a = \sum_{k=1}^{n} e_k$ [35, p. 23], where the "empty class" (0, e.g., [35, pp. 3], [40, pp. 30–31]) should be added. The product and sum of classes also result in a class (possibly 0; the definition of an individual is treated separately). For Nagy, a class in the *strict* sense contains elements that vary within defined limits, in k "dimensions," that is k of $a_1, \ldots, a_n$ in $(a_1 \ldots a_n)$ are considered variable [35, p. 25].[24]

With these definitions of entity and class, Nagy arrives at the compositionality of his logical language. He makes compositionality explicit when speaking about the definition of entity in the broader sense. Such a definition includes only a finite number of *essential* properties (excluding the accidental ones):[25] in certain cases, $\prod_{i=1}^{n} a_i$ and the so defined e can be mutually substituted, with full, not just limit, equality [35, p. 16]. This contrasts with the position of other students of Zimmermann who adopted (influenced in part by J. S. Mill) a kind of rigid designators, even with essential notes changeable but developing toward an "ideal" concept.[26] Nagy's view differs from the non-compositionality of Russell's

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[23] On "individual concepts," see in [40, pp. 42–43]. This definition predates Schröder's (different) definition of an "individual" (as a special case of a class) in 1891 [51, II/1, pp. 320–321]. Nagy [39, 40] had access to the first volume of Schröder's *Vorlesungen* (1890), and reviewed it in [37]. He mentions the priority of his own definition in a review of a booklet by Schröder's student Voigt [38]. — In their correspondence, Schröder wrote to Nagy: "da es noch gilt der neuen Disziplin die Anerkennung der Mathematiker sowo[h]l als der Philosophen erst zu erobern, so freue ich mich ganz besonders in Ihnen einen unvermutheten thätigen Mitarbeiter gefunden zu haben ..." [40, p. 7]. Schröder [51, II/2 (1905), p. 602], in his bibliography, lists six works by Albino Nagy. On the relationship between Nagy and Schröder, see, e.g., [10, p. 95].

[24] Nagy's idea of generating a class by varying k notes (e.g., varying shades of green or degrees of sound intensity) is reminiscent of Bolzano's method of variation. However, he may have arrived at this idea under the influence of Zimmermann's already mentioned reflections on Kant's categories of intensity [35, p. 33 note 3].

[25] According to Nagy, the distinction between essential and secondary properties is extralogical — it has psychological, ontological or linguistic character [40, p. 41].

[26] E.g., [33]. See on developing (perfectioning) an "empirical concept" in [60, pp. 18–19, in 2nd ed. 1860 pp. 19–20]. However, it has sometimes been argued that an individual concept should still possess relative universality, as it "represents"

elimination of definite descriptions through a construction of existential sentences [50].

3.3. Jevons-Clifford problem

Nagy is known for his work on the problem of determining how many, and which, are the *types* — unchangeable under mutual substitutions of literals[27] — and *representatives* of types (obtained from a type via such substitutions) of Boolean propositions with:

- n arguments ("classes" or "concepts"): $a_1, \ldots, a_n$,[28]
- m constituents (products of n arguments or their negations), e.g., $(a_1 \ldots \bar{a}_k \ldots a_n)$,

where a combination of constituents is expressed by a formula ϕ in complete disjunctive normal form (CDNF), serving as a matrix for an "elementary" proposition $\phi = 0$ or $\phi > 0$. This is the so-called Jevons' or Jevons-Clifford problem (Jevons 17 (1874); Clifford 4 (1877); Ladd 25; Schröder 51, I (1990), Anhang, pp. 647–699; Nagy 40, 41, 42). It is a reductive and combinatorial problem concerning the structure of the internal diversity of a proposition. However, its implications have a broader, proof-theoretical and model-theoretical, significance.

We will now consider only matrices of Boolean elementary formulas and adapt them, using a propositional symbolism, to the framework of propositional sequent calculus, introduced into logic only after Nagy, in the 1930s by Gentzen. "Arguments" ("elements," not in the sense of "beings") will be treated as atoms and represented by sentence letters. We first define propositional type in Nagy's sense.

Definition 3.1 (Propositional type; cf. 42, p. 333, 25, pp. 63–64). Formulas in CDNF (with m disjuncts, and n letters) are of the same propositional type iff they can be transformed into one another by substituting literals for one another.

For example, the following two propositions:

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what is common to all representations of a thing [7, pp. 5, 16]. On proper names, in Zimmermann's student Franjo Marković, as developing from connotative to denotative terms, see Dožudić [6].

[27] I.e., argument letters or their negations.

[28] Nagy also mentions, in general, that arguments are "logical quantities"; however, he clearly excludes "beings" (entities, also called "elements") from arguments.

(a) $A_1A_2\bar{A}_3\bar{A}_4 \vee A_1\bar{A}_2A_3A_4$,

(b) $A_1\bar{A}_2A_3A_4 \vee \bar{A}_1\bar{A}_2\bar{A}_3\bar{A}_4$,

are of the same type because (a) can be transformed into (b) by substitutions $A_1/\bar{A}_2, A_2/A_1, \bar{A}_2/\bar{A}_1, A_3/\bar{A}_3, \bar{A}_3/A_3, A_4/\bar{A}_4$, $\bar{A}_4/A_4$ ($\bar{A}_2$ for A_1, etc.), and (b) into (a) by the inverse substitutions (for A_3, A_4 and their negations, the inverses are already included in the first case). Thus, propositions (c) and (d):

(c) $A_1A_2 \vee A_1\bar{A}_2$,

(d) $A_1\bar{A}_2 \vee \bar{A}_1A_2$,

are not of the same type (there is no consistent substitution for A_1 in (c)). Cf. [42, p. 333].

Each type belongs to some form $T^m_{(n)}$, where m is the number of disjuncts ("constituents") and n is the number of atoms ("arguments", $a_1, \ldots, a_n$ in Nagy's notation).

Nagy gives a procedure for the solution of the problem of

1. the number and species of representatives for the types of the form $T^2_{(n)}$,
2. the number and species of types and representatives of the forms $T^3_{(n)}$ and $T^4_{(n)}$,

(cf. 42, p. 335 and 57, p. 105), and introduces into the general problem of $T^m_{(n)}$ [42]. A general solution for the number of types for $T^m_{(n)}$ was given only in 1937 and 1940 by György Pólya (see Urquhart 56 and Pólya 47). In addressing the problem, Nagy also uses Clifford's graphical method.[29]

We illustrate types in Table 1 using the example with two atoms, $T^m_{(2)}$, in Nagy's symbolism. Sixteen *species* of sums (disjunctions), with $m = 0, \ldots, 4$ disjuncts (levels, *Aushebungen*, *rilievi*), fall under six types (I–VI, * indicates the type IV and its species). Each type has a characteristic distance (*Abstand*) structure regarding the number of differences between the disjuncts taken pairwise (three pairs for type V; four triples, each with three pairs, for type VI); cf. [40, pp. 96–97], [41, p. 152]).

Let us note that the number of species with respect to *level* is $\binom{2^n}{m}$, for example, $\binom{2^2}{2} = 6$, and with respect to *type*, for example, for $m = 2$, the number of species is $2^{n-1}\binom{n}{k}$ (k is "distance," the number of differences

[29] In this method, m is the number of vertices (disjuncts), n the number of dimensions (atoms); for example, $T^4_{(2)}$ corresponds to a square, and $T^4_{(3)}$ to a tetrahedron. To be of the same type means that the shapes or solids can be made to overlap by translation and rotation.

m	species	types, distance structures
0	0	I. 0
1	$ab, a\bar{b}, \bar{a}b, \bar{a}\bar{b}$	II. ab
2	$ab + a\bar{b}, ab + \bar{a}b, ab + \bar{a}\bar{b}*$, $a\bar{b} + \bar{a}b*, a\bar{b} + \bar{a}\bar{b}, \bar{a}b + \bar{a}\bar{b}$	III. $ab + a\bar{b}$ (*i.e.*, a), 1 IV. $ab + \bar{a}\bar{b}*$, 2
3	$ab + a\bar{b} + \bar{a}b, ab + a\bar{b} + \bar{a}\bar{b}$, $ab + \bar{a}b + \bar{a}\bar{b}, a\bar{b} + \bar{a}b + \bar{a}\bar{b}$	V. $ab + a\bar{b} + \bar{a}b$ (*i.e.*, $a + b$), (112)
4	$ab + a\bar{b} + \bar{a}b + \bar{a}\bar{b}$	VI. $ab + a\bar{b} + \bar{a}b + \bar{a}\bar{b}$ (*i.e.*, 1), (112)(112)(112)(112)

Table 1: Types of propositional functions for $T^m_{(2)}$

among disjuncts; 42, p. 335). — The number of types grows rapidly, e.g., for $n = 3$ it is 22, for $n = 4$ it is 402, for $n = 5$ it is 1.228.158 [56, p. 285].

Corresponding types of Boolean propositions are presented in Table 2 in Nagy's symbolism, with arguments interpreted as classes (concepts) (in propositional format, '$= 0$' means 'always false,' '> 0' means 'not always false,' and '$= 1$' is 'always true'; see [40, pp. 97–98, 71–72]).

level	universal	particular
0	$0 = 0$ principle of identity	$0 > 0$ absurd proposition
1	$ab = 0$ All a are $\bar{b}$	$ab > 0$ Some a are b
2	$a = 0$ There is no a	$a > 0$ There is an a
2	$ab + \bar{a}\bar{b} = 0$ a is equal to $\bar{b}$, equipollence	$ab + \bar{a}\bar{b} > 0$ a is not equal to $\bar{b}$, non-equipollence
3	$a + b = 0$ There are neither a nor b	$a + b > 0$ There are either a, or b, or both
4	$1 = 0$ absurd proposition	$1 > 0$ principle of contradiction

Table 2: Types of propositions for $T^m_{(2)}$

From the contemporary standpoint, properties of formal systems can be examined and proved with respect to single types or collections of types. We show this through examples of sequent calculus.

Remark 3.1. We give a further illustration of propositional types. In Gödel's compactness proof in [13], the following construction of instantiations of a formula A is used (for the case of weak completeness): $A(\vec{x}_1, \vec{y}_1), A(\vec{x}_1, \vec{y}_1) \wedge A(\vec{x}_2, \vec{y}_2), A(\vec{x}_1, \vec{y}_1) \wedge A(\vec{x}_2, \vec{y}_2) \wedge A(\vec{x}_3, \vec{y}_3), \ldots$ ($\vec{x}_{i>1}$ is a new sequence of already used variables, where sequences are ordered by the sum of indices; $\vec{y}_i$ is a sequence of new variables). If we transform these formulas into complete disjunctive normal form (CDNF), we can observe that the procedure progresses toward increasingly complex propositional types in a specific way. If m is the number of disjuncts of the basic formula, m' the number of disjuncts of the formula constructed at the previous step, and, at the next step, h new atoms are added to the previous n' atoms, the previous type $T_{n'}^{m'}$ extends to $T_{n'+h}^{m' \times m}$, because with each new step the old disjuncts are being multiplied with each new disjunct.[30]

3.4. Distance types and formal systems

We now analyze Nagy's propositional types from the perspective of the sequent calculus, a later-established standard part of contemporary logical methodology. It can be shown that propositional types correspond to a natural classification of sequent rules and determine relevant structural features of sequent proofs. They also affect the conditions for cut admissibility in specific ways.

Propositional types can be transformed into tableau format in an obvious way: the disjuncts of a type correspond to tableau branches, while the conjuncts in each disjunct correspond to the literals on that branch. Propositional types can thus be transformed into corresponding types of refutation sequents and sequent derivations by applying the standard tableau-to-sequent transformation method (by placing the t-formulas of a tableau into the antecedent of a sequent and the f-formulas into the succedent).

We begin with some preliminaries. By the sequent calculus, we mean the system K (cf. Indrzejczak 16, p. 9), omitting $\rightarrow$ and its rules. We call this system K^-. Accordingly, the sequents are of the form $\mathit{\Gamma} \Longrightarrow \mathit{\Delta}$, where $\mathit{\Gamma}, \mathit{\Delta}$ are sets (not multisets). Thus, there are in the system left

[30] As another example, consider the propositional logic of change by K. Świętorzecka [54], where the language gradually expands through the introduction of new atomic sentences. Sets of valuations, developing through histories, may be understood as belonging to distinct types.

and right negation rules (L¬, R¬), left and right conjunction rules (L∧, R∧), and left and right disjunction rules (L∨, R∨), as well as axioms of the form $\Gamma \Longrightarrow \Delta$, where $\Gamma \cap \Delta \neq \varnothing$. Left and right Weakening, and Cut are admissible (see Indrzejczak 16, p. 22, 39). $\bot$ is shorthand for $A \wedge \neg A$ for any sentence letter A, and $\top = \neg\bot$ (cf. 16, p. 3). Also, by convention, an empty antecedent is equivalent to $\top$ and an empty succedent to $\bot$. The formulas that are not introduced or applied to by a sequent rule (principal and side formulas, respectively, or active formulas) are called parametric formulas (context) (cf. 16, p. 9).

In extending Nagy's theory to the sequent calculus, we introduce the following definition of sequent types:

Definition 3.2 (Sequents type). Sets S and S' of sequents are of the same sequents type if the active formulas in S can be transformed into the active formulas in S' and vice versa in 1–1 manner by using ¬ rules and substituting literals for one another.

A sequents type is thus invariant under a substitution of literals for one another, exactly as is the case for the propositional types, and under the transposition of literals according to the ¬ rules. For example, the pair of sequents $A, B, \Gamma \Longrightarrow \Delta$ and $A, \Gamma \Longrightarrow \Delta, B$ belongs to the same sequents type as the pair $\neg B, C, \Gamma' \Longrightarrow \Delta'$ and $\neg B, \Gamma' \Longrightarrow \Delta', C$. Also, $A, B, \Gamma \Longrightarrow \Delta$ and $A, \Gamma' \Longrightarrow \Delta', \neg B$ belong to the same sequents type. In addition, we introduce the following definitions:

Definition 3.3 (Lower-level sequents type). We call a sequents type belonging to $T^{m'<m}_{n'\leq n}$ or $T^{m'\leq m}_{n'<n}$ a lower-level sequents type with respect to sequents types belonging to T^{m}_{n} (i.e., where either the number of disjuncts or the number of letters is strictly smaller).

Definition 3.4 (Type of sequent rules). Sequent rules are of the same type iff they are the first rules applicable to the sequents of the same type after the intermediate application (if at all) of the rules applicable to lower-level sequents types.

Definition 3.5 (Derivation type). Derivations D and D' are of the same type iff they begin with the sequents of the same sequents type, along with axioms (not necessarily the same, if at all), allowing initial sequents to occur more than once in D or D', and utilizing derivation types belonging to lower-level sequents types and the rules applicable to the results of these derivations. Derivations beginning with sequents of type T^m_n (possibly along with axioms) will be called derivations of type T^m_n.

Definition 3.6 (Characteristic derivation). A characteristic derivation for a sequents type is a derivation that uses as premises all and only the sequents belonging to this sequents type.

By 'subderivation' of a derivation, we mean each derivation that is a part of this derivation.

Non-axiomatic initial sequents ("non-logical axioms") are referred to as *postulates*.

Type I (i.e., 0) of $T^0_{(2)}$, without disjuncts, is the same for each $T^0_{(n)}$. According to Nagy, it is the vacuous proposition $0 = 0$ (*nichtsagende Aussage* [42, p. 335]). In the continuation, we will not consider this type. (It could also be understood simply as the closure in a tableau, $\times$, and axioms in sequent calculus.)

We note that for $T^m_{(1)}$, there is only one type of elementary propositions (with two species, $A, \neg A$). This type is at the same time a simple tableau format, with the corresponding sequents $A \Longrightarrow$, $\Longrightarrow A$, which are of the same type (because transformable into one another by the negation rules, the only applicable ones, and substitution).

3.4.1. $T^1_{(2)}$, type AB (II)

The tableau formats of the representatives of type AB (type II), with no distance structure (i.e., its distance structure is 0), are as follows:

$$\begin{array}{cccc} \mathrm{t}A & \mathrm{t}A & \mathrm{f}A & \mathrm{f}A \\ \mathrm{t}B & \mathrm{f}B & \mathrm{t}B & \mathrm{f}B \end{array}$$

The corresponding refutation sequents are:

$$A, B \Longrightarrow \qquad A \Longrightarrow B \qquad B \Longrightarrow A \qquad \Longrightarrow A, B$$

All these sequents are reducible to one another by L$\neg$ or R$\neg$ rule (e.g., $A \Longrightarrow B$ to $A, \neg B \Longrightarrow$) and substitution, and thus belong to the *same sequents type*, representable by $A, B \Longrightarrow$ (Definition 3.2). Including the full context (parametric formulas), the sequents type AB (II) amounts to

$$A, B, \Gamma \Longrightarrow \Delta$$

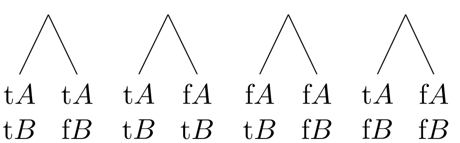
For this sequents type, a *non-branching* rule L$\wedge$ or R$\vee$ is applicable *one* time for one occurrence of the sequent in each of the four cases (after the application of a ¬ rule if needed). No other rules beside the rules for ¬ and non-branching rules for ∧ and ∨ are immediately applicable, except in cases involving repeated sequents (multiset of sequents). However, it is easy to see that multisets of initial sequents do not extend the conclusion beyond equivalent antecedents or succedents: for instance, conclusions from two occurrences of $A, B \Longrightarrow$, by any applicable rules, are sequents such as $(A \vee A), B \Longrightarrow$ or $(A \wedge B) \vee (A \wedge B) \Longrightarrow$ or $A \vee A \Longrightarrow \neg B$ (equivalences with $A \Longrightarrow \neg B$), etc.

The generalization to sequent types $T^1_{(n)}$ is immediate: the application of L∧ or R∨ can be iterated as to include any of n literals in a sequent. Semantically, any finite valuation can be expressed by a sequent (actually, a one-member sequents set) of the form $T^1_{(n)}$, leaving the values of the rest of the infinitely many propositional letters implicit as parameters).

Finally, generalizing A and B of the type AB to ϕ and ψ, respectively, as representing any formulas, the non-branching sequent rules (L∧, R∨) can be said to be of the *generalized* type II.

3.4.2. $T^2_{(2)}$, type $AB \vee A\bar{B}$ (III)

Type III is a branching type with one common constituent of the two branches:



The corresponding pairs of refutation sequents are:

$$\begin{array}{ll} A, B \Longrightarrow, & A \Longrightarrow B, \\ A, B \Longrightarrow, & B \Longrightarrow A, \\ B \Longrightarrow A, & \Longrightarrow A, B, \\ A \Longrightarrow B, & \Longrightarrow A, B. \end{array}$$

Evidently, all pairs belong to the same type, III (Definition 3.2); for example, all pairs are reducible to the first one by appropriate substitution of literals and, where needed, ¬ rules, as in the case of the fourth pair: from $A \Longrightarrow B$ we obtain $A, \neg B \Longrightarrow$, from $\Longrightarrow A, B$ we obtain $\neg A \Longrightarrow B$, and by substitutions $\neg B/B, \neg A/A$, we get the first pair.

Including the full context, the sequents of type III can be presented as:

$$A, B, \Gamma \Longrightarrow \Delta \qquad A, \Gamma \Longrightarrow \Delta, B.$$

This type of sequent pairs, belonging to T^2_2, with the distance structure 1, taken as a pair of postulates, determines a specific type of derivations. Derivations of this type begin with all $m = 2$ initial sequents (along with axioms, if any) and apply a branching rule (L$\vee$ or R$\wedge$) with a non-empty common context of the two initial sequents, along with the application of subderivations of lower types.

Here are two examples of characteristic derivations that begin with the sequent pairs III, and apply rules for lower-level sequent types, except in the last line:

$$\dfrac{\dfrac{A, B, \Gamma' \Longrightarrow \Delta}{A, \Gamma' \Longrightarrow \Delta, \neg B}\,\mathrm{R}\neg \qquad A, \Gamma' \Longrightarrow \Delta, B}{A, \Gamma' \Longrightarrow \Delta, B \wedge \neg B}\,\mathrm{R}\wedge$$

($\Gamma = \Gamma' \cup \{A\}$).

$$\dfrac{\dfrac{\Gamma \Longrightarrow \Delta', B, A}{\neg A, \Gamma \Longrightarrow \Delta', B}\,\mathrm{L}\neg \qquad A, \Gamma \Longrightarrow \Delta', B}{A \vee \neg A, \Gamma \Longrightarrow \Delta', B}\,\mathrm{L}\vee$$

($\Delta = \Delta' \cup \{B\}$).

Corresponding to the distance structure 1, the above initial sequents are characterized by *partial difference* of antecedents or succedents from one another in *one* literal ($d = 1$), and thus by their *partial identity* in $n - d$, i.e, in *one* literal. So, with the level $m = 2$ of the propositional type, an *immediate non-empty* application of the *branching* rules, R$\wedge$ or L$\vee$ — i.e., over the identical parametric literal (A in the first example, B in the second one) — becomes possible.

On the other hand, the partial cross-identity between the antecedent of one sequent and the succedent of the other on one literal means that Cut rule is immediately applicable. In our first example, we can derive $A \Longrightarrow$; in a generalized format:

$$\dfrac{A, \Gamma' \Longrightarrow \Delta, B \qquad B, A, \Pi' \Longrightarrow \Sigma}{A, \Gamma', \Pi' \Longrightarrow \Delta, \Sigma}\,\mathrm{Cut}$$

($\Gamma' \cup \{A\} = \Gamma, \Pi' \cup \{A\} = \Pi$). In the second example, $\Longrightarrow B$ immediately follows by Cut. See below Subsection 3.5 on cut admissibility.

3.4.3. $T^2_{(2)}$, type $AB \vee \bar{A}\bar{B}$ (IV)

Type IV, in the tableau format, is a branching type with no common constituents in its two branches. Its two representatives are:

$$
\begin{array}{cc}
\begin{array}{cc} \mathrm{t}A & \mathrm{f}A \\ \mathrm{t}B & \mathrm{f}B \end{array}
&
\begin{array}{cc} \mathrm{t}A & \mathrm{f}A \\ \mathrm{f}B & \mathrm{t}B \end{array}
\end{array}
$$

Sequents

$$A, B \Longrightarrow\ , \qquad \Longrightarrow A, B$$

correspond to the left tableau representative above, and the other pair of sequents,

$$A \Longrightarrow B, \qquad B \Longrightarrow A$$

(corresponding to the right tableau), can be shown to belong to the same type as the first pair by the application of L¬, R¬, and the substitution $\neg B/B$.

In this type of sequent pairs, there is a full identity between the antecedent of one branch and the succedent of another branch — corresponding to *distance structure* 2 of type IV. Thus, type IV of characteristic derivations, without a multiset of initial sequents, allows for an *empty* application of the *branching* rules L∨ or R∧, that is, with no common antecedents (empty Γ), along with lower level derivation steps, as in the following example:

$$
\dfrac{\dfrac{\Gamma \Longrightarrow \Delta, A, B}{\Gamma \Longrightarrow \Delta, A \vee B}\,\mathrm{R}\vee \qquad \dfrac{\dfrac{A, B, \Gamma \Longrightarrow \Delta}{\Gamma \Longrightarrow \Delta, \neg A, \neg B}\,\mathrm{R}\neg\ (2\times)}{\Gamma \Longrightarrow \Delta, \neg A \vee \neg B}\,\mathrm{R}\vee}{\Gamma \Longrightarrow \Delta, (A \vee B) \wedge (\neg A \vee \neg B)}\,\mathrm{R}\wedge
$$

Of course, alternatively, literals not on the left side of the sequents can be moved (by ¬ rules) to the left side, and the empty unification can be performed (by L∨) on the right side, yielding: $(A \wedge B) \vee (\neg A \wedge \neg B), \Gamma \Longrightarrow \Delta$.

Due to the full identity between the antecedent of one initial sequent and the succedent of another, the Cut rule applies, resulting in an axiom, e.g., from the premises of the previous example, the conclusions $\Gamma, A \Longrightarrow \Delta, A$ or $\Gamma, B \Longrightarrow \Delta, B$ follow.[31]

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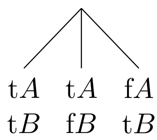
[31] Thanks to Andrzej Indrzejczak for a correction.

3.4.4. $T^3_{(2)}$, type $AB \vee A\bar{B} \vee \bar{A}B$ (V)

Type V has three tableau branches and $\binom{4}{3} = 4$ possible representatives (i.e., four ways to choose three branches). For example:



In the characteristic derivation format for type V, two of the initial sequent pairs are structured for non-empty application and one pair for empty application of a branching rule — corresponding to the *distance structure* (112). Here are two examples, omitting broader context sets for simplicity. In the first one, we begin with one of the three possible pairs of initial sequents and apply the remaining initial sequent to a result of the subderivations for lower-level sequents (III or IV) from the initial pair.

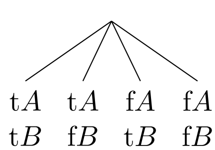
$$\dfrac{\dfrac{\dfrac{A, B \Longrightarrow}{A \Longrightarrow \neg B}\,\mathrm{R}\neg \qquad A \Longrightarrow B}{A \Longrightarrow \bot}\,\mathrm{R}\wedge \qquad \dfrac{\dfrac{B \Longrightarrow A}{B, \neg A \Longrightarrow \bot}\,\mathrm{L}\neg}{B \wedge \neg A \Longrightarrow \bot}\,\mathrm{L}\wedge}{A \vee (B \wedge \neg A) \Longrightarrow \bot}\,\mathrm{L}\vee$$

Otherwise, we start with a multiset of sequents, as in the following example:

$$\dfrac{\dfrac{\dfrac{A, B \Longrightarrow}{A \Longrightarrow \neg B}\,\mathrm{R}\neg \qquad A \Longrightarrow B}{A \Longrightarrow \bot}\,\mathrm{R}\wedge \qquad \dfrac{\dfrac{A, B \Longrightarrow}{B \Longrightarrow \neg A}\,\mathrm{R}\neg \qquad B \Longrightarrow A}{B \Longrightarrow \bot}\,\mathrm{R}\wedge}{A \vee B \Longrightarrow \bot}\,\mathrm{L}\vee$$

3.4.5. $T^4_{(2)}$, type $AB \vee A\bar{B} \vee \bar{A}B \vee \bar{A}\bar{B}$ (VI)

Type VI has one tableau representative containing all four possible branches:



For a corresponding characteristic derivation, in its subderivations, *two* distinct empty applications of branching rules become possible (unlike type V, which allows only one). In general, there are $\binom{4}{2} = 6$ ways to choose pairs of sequents. Besides, there are $\binom{4}{3} = 4$ ways to choose triples of sequents, corresponding to the *distance structure* (112)(112)(112)(112): each of the triples having only one empty application of a branching rule (as for type V). Here is an example, deriving a contradiction:

$$\dfrac{\dfrac{\dfrac{\dfrac{A, B \Longrightarrow}{\Longrightarrow \neg A, \neg B}\,\mathrm{R}\neg \quad \dfrac{A \Longrightarrow B}{\Longrightarrow \neg A, B}\,\mathrm{R}\neg}{\Longrightarrow \neg A, \bot[= B \wedge \neg B]}\,\mathrm{R}\wedge \quad \dfrac{\Longrightarrow A, B \quad \dfrac{B \Longrightarrow A}{\Longrightarrow A, \neg B}\,\mathrm{R}\neg}{\Longrightarrow A, \bot[= B \wedge \neg B]}\,\mathrm{R}\wedge}{\Longrightarrow \bot[= A \wedge \neg A = B \wedge \neg B]}\,\mathrm{R}\wedge}{\top \Longrightarrow \bot}\,\text{by convention}$$

Remark 3.2. From the propositional type $AB \vee A\bar{B} \vee \bar{A}B$, $\top \Longrightarrow \bot$ is not derivable because, for that purpose, the fourth sequent, i.e., the propositional type $AB \vee A\bar{B} \vee \bar{A}B \vee \bar{A}\bar{B}$, is needed. We show this by building a proof of $A \vee \neg A \Longrightarrow B \wedge \neg B$ from the bottom up:

$$\dfrac{\dfrac{A \Longrightarrow B \quad \dfrac{A, B \Longrightarrow}{A \Longrightarrow \neg B}\,\mathrm{R}\neg}{A \Longrightarrow B \wedge \neg B}\,\mathrm{R}\wedge \quad \dfrac{\dfrac{\Longrightarrow A, B}{\neg A \Longrightarrow B}\,\mathrm{L}\neg \quad \dfrac{B \Longrightarrow A}{\neg A \Longrightarrow \neg B}\,\mathrm{R/L}\neg}{\neg A \Longrightarrow B \wedge \neg B}\,\mathrm{R}\wedge}{A \vee \neg A \Longrightarrow B \wedge \neg B \quad [= \top \Longrightarrow \bot]}\,\mathrm{L}\vee$$

3.5. Cut admissibility

We consider extensions of the basic system K^- by postulates of specific types (Definition 3.2) and, consequently, by derivations of the corresponding types. We analyze some of these extensions ($T^m_{(2)}$ and some generalizations) and the conditions under which the Cut rule is addmissible. The proof of Cut admissibility presented in Indrzejczak [16, subsection 1.8.2, pp. 38–41] is adopted as the basis, while, in addition, special cases involving derivations with newly introduced postulates are examined. Specific extensions of derivation types are introduced.

PROPOSITION 3.1. *Cut rule is admissible for the derivations of type $T^m_{(2)}$ with parametric cut formula in a postulate.*[32]

[32] Negri and von Plato [43, chapter 6] describe a procedure for cut elimination by conversion of non-logical axioms into rules. For a recent critical overview of the problem and a new proposal for a solution, see [46].

Proof. The proof of Cut admissibility in Indrzejczak [16, Theorem 1.7, pp. 39–41] can be applied. We extend the subcase (ii) of Case 1 by considering premises which are postulates containing parametric cut formulas. Suppose the left premise is a postulate. Then the conclusion is a postulate of the same shape. For example:

$$\frac{A, \Gamma' \Longrightarrow \Delta', B, \phi \qquad \phi, \Pi \Longrightarrow \Sigma}{A, \Gamma', \Pi \Longrightarrow \Delta', \Sigma, B}\,\mathrm{Cut}$$

The proof is similar if the *right* premise is a postulate (whether the cut formula is active or parametric).

Inductive step is carried out by decreasing the complexity of the cut formula or by decreasing the height of the application of Cut. The cut formula may or may not be principal in both premises [16, p. 40–41]. ⊣

Proposition 3.2. *Cut rule is admissible for derivations of type IV with postulates.*

Proof. In the proof of Cut admissibility in [16, pp. 39–41], we extend the subcase (i) (in which the cut formula is active) of Case 1, by premises of type IV as postulates. Consider, first, the following example:

$$\frac{A, \Gamma' \Longrightarrow \Delta, B \qquad B, \Pi \Longrightarrow \Sigma', A}{A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma', A}\,\mathrm{Cut}$$

($\Sigma = \Sigma' \cup \{A\}$). The result of the application of Cut is an axiom, independently of whether the left or the right sequent is a postulate. Thus, the preceding part of the derivation is dispensable. Similarly for the other instance of the characteristic derivations of this type: see here the example mentioned at the end of the subsection 3.4.3 with an axiom resulting by Cut. The proof is similar for the subcase (ii) of [16] (with the cut formula parametric). ⊣

Corollary 3.3. *Cut is admissible for any* m *and* n *of* $T^{m}_{(n)}$ *in the following additional basic subcases* (see Proposition 3.1), *where the left or right premise is a postulate and either* (a) *the cut formula is parametric, or* (b) *both premises are postulates that overlap in at least one inner propositional letter or in at least one outer propositional letter, aside from the active cut formula:*

$A_1, ..., A_k, \Gamma \Longrightarrow \Delta, A_{k+1}, ..., A_n \quad A_m, ..., A_n, \Pi \Longrightarrow \Sigma, A_1, ..., A_{m-1}$
$(m \leqslant n-1)$.

PROOF. (a) The proof is similar to the proof of Proposition 3.1. In the example for the proof of Proposition 3.1, replace A with $A_1, \ldots, A_k$ and B with $A_{k+1}, \ldots, A_{m-1}$. (b) Cut results with an axiom. ⊣

COROLLARY 3.4. *If the cut formula is active in a postulate, Cut can be dismissed if the other premise is an axiom.*

PROOF. The proof corresponds to Case 1 of [16, p. 39], with the slightly simplified subcase (i) (where the cut formula is active in an *axiom*): no Weakening by the cut formula is needed, because the conclusion itself is a postulate. ⊣

To achieve, at least indirectly, full admissibility of Cut, we introduce a convention related to branching rules:

DEFINITION 3.7 (Derivation type $T(\bot)$). If $\neg\bot, \Gamma, \Pi \Longrightarrow \Delta, \Sigma$ or $\Gamma, \Pi \Longrightarrow \Delta, \Sigma, \bot$ are derivable from initial sequents in a derivation type T, then $\Gamma, \Pi \Longrightarrow \Delta, \Sigma$ is an initial sequent. This extends derivation type T to derivation type $T(\bot)$.[33]

PROPOSITION 3.5. *The Cut rule is admissible in the derivation type* $T^m_{(2)}(\bot)$.

PROOF. Again, we follow and appropriately extend the proof of Cut admissibility in [16, pp. 39–41]. For our purposes, Case 1 of the proof should be complemented with the instances that include postulates as premises. Assume the *left* premise is a postulate (the proof for the case with the *right* premise as a postulate is similar). In subcase (i), the cut formula of the *left* premise is *active*. Since the cut formula appears in a postulate rather than in an axiom, it does not necessarily occur on both sides of the left sequent ($\Gamma = \Gamma' \cup \{A\}$):

$$\frac{A, \Gamma' \Longrightarrow \Delta, B \qquad B, \Pi \Longrightarrow \Sigma}{A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma}\,\text{Cut}$$

After applying L¬ to B in the left sequent and unifying contexts (by Weakening), in Figure 4, with possibly empty context, we derive $B \vee \neg B, A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma$ by L∨ (it together with $\Longrightarrow B \vee \neg B$, from $B \Longrightarrow B$, gives by Cut the intended conclusion, where $B \vee \neg B$ is omitted).

For convenience, we provide the continuation of the proof for this example, completely analogous to the proof in [46, Theorem 1]. (a) If

[33] Compare this idea with the use of Cut closure in Piazza and Tesi [46].

$$\dfrac{\dfrac{\dfrac{A, \Gamma' \Longrightarrow \Delta, B}{\neg B, A, \Gamma' \Longrightarrow \Delta}\,\mathrm{L}\neg}{\neg B, A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma}\,\mathrm{W} \qquad \dfrac{\dfrac{B, \Pi \Longrightarrow \Sigma}{B, A, \Pi \Longrightarrow \Sigma}\,\mathrm{W}}{B, A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma}\,\mathrm{W}}{B \vee \neg B, A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma}\,\mathrm{L}\vee$$

Figure 4: The derivation of $B \vee \neg B, A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma$

the right premise is an axiom, and $\Sigma = \Sigma' \cup \{B\}$, then the conclusion obtainable by Cut ($A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma$) is a weakened left premise, i.e., a postulate. If other formulas in Π and Σ are active in this axiom, then the conclusion follows from this axiom by W. (b) If the right premise is a postulate with B active, then the conclusion ($A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma$) is an initial sequent according to Definition 3.7 (from $B \vee \neg B, A, \Gamma', \Pi \Longrightarrow \Delta, \Sigma$). And with B parametric, the conclusion is a postulate omitting parameter B. (c) If the right premise is not an initial sequent but a conclusion, we reason by inductive step on the height of a sequent proof. Eventually, the right premise is derived from an initial sequent (or sequents) that contain B in the antecedent (because B itself, as atomic, cannot be a principal formula of the application of a rule). The application of Cut is thus moved upwards, finally up to the initial sequents, and disappears (cf. (a) and (b)).

In the following example (again, the left premise is a postulate with the active cut formula),

$$\dfrac{\Gamma \Longrightarrow \Delta', A, B \qquad B, \Pi \Longrightarrow \Sigma}{\Gamma, \Pi \Longrightarrow A, \Delta', \Sigma}\,\mathrm{Cut}$$

($\Delta = \Delta' \cup \{A\}$), we can apply R¬ to B in the right sequent and unify the contexts by Weakening to get $\Gamma, \Pi \Longrightarrow A, \Delta', \Sigma, B \wedge \neg B$ after R∧ (with possibly empty context). Similarly as in the previous example, using Definition 3.7, it follows that $\Gamma, \Pi \Longrightarrow A, \Delta', \Sigma$ is an initial sequent of the corresponding derivation type $T^m_{(2)}(\bot)$. ⊣

Corollary 3.6. *Proposition 3.5 can be generalized for types with any number n of letters in the premises.*

Proof. Here is a generalization of the last example from the proof of Proposition 3.5, allowing for any number n of atomic propositions, with A_k as the cut formula:

$$\dfrac{\mathrm{W}\,\dfrac{\{A\}_1^{k-1}, \varGamma \Longrightarrow \varDelta, \{A\}_k^n}{\{A\}_1^{k-1}, \varGamma, \varPi \Longrightarrow \varDelta, \varSigma, \{A\}_k^n} \qquad \dfrac{\dfrac{A_k, \varPi \Longrightarrow \varSigma}{\varPi \Longrightarrow \varSigma, \neg A_k}\,\mathrm{R}\neg}{\{A\}_1^{k-1}, \varGamma, \varPi \Longrightarrow \varDelta, \varSigma, \neg A_k, \{A\}_{k+1}^n}\,\mathrm{W}}{\{A\}_1^{k-1}, \varGamma, \varPi \Longrightarrow \varDelta, \varSigma, A_k \wedge \neg A_k, \{A\}_{k+1}^n}\,\mathrm{R}\wedge$$

$\{A\}_1^{k-1}, \varGamma, \varPi \Longrightarrow \varDelta, \varSigma, \{A\}_{k+1}^n$ is to be included in the appropriate T($\bot$) derivation type as an initial sequent (Definition 3.7). The proof of Proposition 3.5 holds regardless of the number of letters involved. ⊣

As this initial study aims to show, distinguishing Jevons-Clifford types provides a criterion for a meaningful classification and postulative extensions of sequent derivations, and can be a helpful tool for examining and determining the scope of Cut admissibility (with or without extension to $T(\bot)$).

4. Conclusion

In the historical examples we have analyzed, an intrinsically logical formal rigor and an extensional approach gradually emerged after Kant's metalogical ("transcendental") turn. In addition, such approaches (according to Nagy's completeness) have their corresponding conceptual content, while opening the way for further logical (proof- and model-theoretical) perspectives and questions — that would only be systematically explored later in the history of logic. For instance, distinguishing propositional types, in the sense of Jevons-Clifford problem, helps us clarify under which conditions Cut is admissible in the sequent calculus with postulates. Additionally, the historical fragments considered here confirm the significant role played by Central European logical-philosophical traditions — especially those stemming from Herbart and Bolzano — in shaping modern logic and logically based philosophy.

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Srećko Kovač
emeritus of the Institute of Philosophy
public institute of the Republic of Croatia
Zagreb, Croatia
skovac@ifzg.hr