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Jaśkowski's Discovery and Various Ways to Master Contradictions

Abstract. This paper presents the history of the world's first school of paraconsistency: the Torunian School of Paraconsistent Logic. Its founding father is the Polish logician Stanisław Jaśkowski, who first formulated a system of paraconsistent logic. Both the approach he presented and the work of subsequent generations of logicians allow us to speak of the entire school. In this article, we present the characteristics of this school, comparing them with those of other national, but later, schools of paraconsistency. In addition to extensive factual information concerning Jaskowski's discovery and subsequent development of this topic, we also refer to and comment on the latest works presented in this volume.

Keywords: ex falso/contradictione quodlibet; explosion; paraconistency; paraconsistent logic; American-Australian school; Canadian school; Belgian school; Brazilian school; Torunian School of Paraconsistent Logic; TSPL

1. Introduction and overview

Consider the language For built over an infinite countable set Var of variables, one unary connective $\neg$, four binary connectives $\wedge, \vee, \to, \leftrightarrow$ and brackets (,). This is the language in which classical propositional logic CPL is usually defined.

Among the distinctive inference rules of CPL, *ex falso/contradictione quodlibet* (*ECQ*, for short) is surely among the most debated ones. It can be presented as either a rule or a law: $\{p, \neg p\} \models q$, $\models p \wedge \neg p \to q$, $\models \neg p \wedge p \to q$, $\models p \to (\neg p \to q)$, $\models \neg p \to (p \to q)$, $\bot \models q$, $\models \bot \to q$ etc., where $\models$ is the consequence relation of CPL while $\bot$ is an arbitrary contradiction.

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As the reader may notice, EFQ encodes in the object language the fact that contradictions are *never* satisfiable. Consequently, according to classical logic, contradictions cannot be coherently reasoned about.

As is usually accepted, the main aim of paraconsistent logic is to avoid the above problematic properties by providing the non-trivial tools in the case of premises, theses, data, etc. that are inconsistent. The term "paraconsistent logic" was coined in 1976 by the Peruvian philosopher Francisco Miró Quesada Cantuarias, under the request of Newton da Costa, a founder of the Brazilian school of paraconsistent logic (see Priest, Tanaka and Weber, 2025).

In this paper, we present the following topics. In the second section, we briefly describe the basic characteristics of the various recognized schools of paraconsistency. In the next section, we discuss the founder of the Polish school of paraconsistency and his discovery. The essence of this section is the unequivocal assertion that the first system of paraconsistent logic in history (called logic D_2) was created by the Torunian logician Stanisław Jaśkowski. The dates of his publications on the topic leave no doubt that he was the first logician to accomplish this.

In Section 3, we also discuss the main issues, problems, and achievements of the Torunian School of Paraconsistent Logic (TSPL).[1] The important conclusion of this section is to outline the characteristics of TSPL with distinguishing features of the whole school and the approach to paraconsistent logic. Therefore, the distinction of this school is not merely historical in nature but is clearly substantively justified.

In the last section we describe those results of WCP6 and 2nd Stanisław Jaśkowski Memorial Symposium that have been published as papers in this volume.

The entire article justifies the following theses:

1. Jaśkowski's work did not only have an impact on various subfields of logic, but also gave the beginning to a school that originated in Toruń and has been oriented around paraconsistency.
2. Jaśkowski's approach was the first formal and logical approach to the problem of inconsistent theories
3. The publication of Jaśkowski's articles established the world's first school of paraconsistency, which differs from later schools in its unique features; this school, because of its origins, should be called the *Torunian School of Paraconsistent Logic*.

[1] For another presentation of the Polish School, see also (Nicolás-Francisco, 2022).

2. Schools of paraconsistent logic

We have several schools of paraconsistent logic that offer different solutions to the problem. Of course, the division today becomes just conventional, since various features interact. Taking into account the alphabetical order, we will briefly discuss their important features.

American-Australian school. This school can be characterized by pointing out the following people and the issues they address:

- Alan Ross Anderson and Nuel D. Belnap in 1950's (1959; 1960) proposed their systems to avoid paradoxes of material and strict implication, among them forms of *ex contradictione quodlibet* where unwanted, so in this case paraconsistent effect is being achieved 'by the way'.
- Richard Routley and Robert K. Meyer (1973) gave Kripke style semantics by means of ternary relation. And this 'angle' can be seen as more interested in semantics that is, it is leaning towards issues such as paradoxes and ontological status of inconsistencies.
- These semantical interests lead also to the development of syntactical formulations (Logic of Paradox by Graham Priest (1979)).
- There is also an algebraic/many-valued approach by Dunn (1966) and Belnap (1976, 1977a).

Belgian school. The school was initiated (Batens, 1980, 1986) and developed by Diderik Batens (1995, 1997, 1998, 1999, 2000,b, 2003) with his collaborators (Batens and Meheus, 2000; Verdée, 2009), including interest in discussive logic (Meheus, 1999, 2006) (see below). It is worth pointing out the following features of the Belgian school:

- the basic idea is to use two consequence relations: weaker (lower limit logic (LLL)) and a stronger one (upper limit logic (ULL)),
- LLL can be used always, while ULL determines conclusions only in a normal situation, were it is assumed that all formulas behave normally unless and until proven otherwise,
- what normality is depends on particular adaptive logic, for the case inconsistency-adaptive logic, abnormalities are inconsistencies,
- The proofs are dynamic in the sense that formulas inferred at some stage may not be obtained at a later stage.

Brazilian school. The founder of this school was da Costa. It seems the leading idea is that classical inferences are valid in the consistent cases, where the notion of consistency — usually handled meta-theoretically — is included into the object language, and expressed in particular formulas (Asenjo, 1966; Asenjo and Tamburino, 1975; Carnielli, Coniglio and Marcos, 2007; da Costa, 1963, 1974; Da Costa and Arruda, 1963).

Canadian school. Raymond Jennings and Peter Schotch introduced the concept of the level of inconsistency of a set of premises. This measure is determined by the smallest number of subsets into which a given set of premises must be divided to make each element of the partition consistent. The minimal logical cover of the set of premises determined in this way determines forcing, where a formula follows from a set of premises when it follows in the sense of an initial consequence relation from some element of this cover of the set of premises, and this is the case for any minimal logical cover of the set of premises (Schotch and Jennings, 1980, 1989; Schotch, Brown and Jenning, 2009).

The last school that can be described alphabetically is the *Polish/Torunian school of paraconsistent logic*. Its general characteristic is: *putting inconsistent data in a consistent framework by translations, mainly into modal languages*. We will devote more attention to it in the following sections of the article.

3. Jaśkowski and his discovery

Since Jaśkowski is an important figure in TSPL, as its founder, it is worth starting with his scientific biography.

He was born in 22.06.1906 in Warsaw (died 16.11.1965 in Warsaw), in a noble family. Jaśkowski was a student of Jan Łukasiewicz, promoted by Łukasiewicz in 1932. During the II WW he defended Poland as a volunteer and was imprisoned by Germans. After the II WW Jaśkowski worked in Łodź (only temporally) and finally in Toruń. In 1959 Jaśkowski became the rector of NCL in Toruń (1959–1962) Jaśkowski was a member of Lvow-Warsaw School. Jaśkowski contributed to development — among others — proof-theory and semantics. He proposed the first system of natural deduction, very close to the way how proofs are made in practice (it was his topic of the dissertation he defended in 1932 under the supervision of Jan Łukasiewicz, later published as *On the rules of suppositions in formal logic* (1934) — in so called *number 0 of Studia Logica*. This was the first article on natural deduction before

G. Gentzen published his own article. In 1948 Jaśkowski proposed the world's first formal system of paraconsistent logic.

Let us say more on Jaśkowski's discovery.

In 1948 Jaśkowski published "Rachunek zdań dla systemów dedukcyjnych sprzecznych" (Jaśkowski, 1948, 1999a). In the work, the author introduced the discussive logic D_2 to address the problem of providing a calculus for non-trivial inconsistent systems that were rich enough in terms of practical inferences and that had intuitive justification. In 1949 Jaśkowski published "O koniunkcji dyskusyjnej w rachunku zdań dla systemó w dedukcyjnych sprzecznych" ("On the discussive conjunction in the propositional calculus for inconsistent deductive systems") introducing the right discussive conjunction in the discussive logic.

In 2022 we had the 75th anniversary of the presentation given during a meeting of the Section of Mathematical and Natural Sciences Scientific Society of Toruń. The lecture was just published as the first Jaśkowski's paper on discussive logic: "Rachunek zdań dla systemów dedukcyjnych sprzecznych", *Studia Societatis Scientiarum Torunensis*, Sect. A, vol. I, no. 5, 1948: 57–77. Therefore, The Second Stanisław Jaśkowski Memorial Symposium was also organized as part of The Sixth World Congress of Paraconsistency (5–8 September 2022, Toruń, Poland).

Jaśkowski's (1948) aim was to formulate a logic,[2] which could be used as a basis of inconsistent systems, which, being inconsistent, would not be overcomplete. So, even if the system contains contradictory statements, it is not equal to the set of all formulas. Jaśkowski presented a method of generating theses of discussive systems, as he called them, by referring to a situation of discussion. During a discussion, inconsistent statements may arise, but we are not ready to derive all possible conclusions from them.

Opinions that appear explicitly in the discussion can be treated as preceded by the proviso: "according to the opinion of one of participants of the discussion" which can be formally expressed by preceding the statement with the words: "it is possible that". Therefore, if we take the position of an external observer, all opinions expressed in the discussion are *only possible*. Furthermore, conclusions drawn from explicitly formulated views in the discussion are also *only possible*. In Jaśkowski's approach, explicit opinions, as well as the conclusions drawn from them, are treated as theses of the discussion system. The approach proposed

[2] We also follow the analysis presented in (Nasieniewski and Pietruszczak, 2013),

by Jaśkowski requires the use of modal language. Jaśkowski originally used logic S5.

Jaśkowski noted that if an implication in a discussion system is to be closed under *modus ponens*, it cannot be a material implication (Jaśkowski, 1948, p. 66):

> [...] out of the two theses one of which is
>
> $$\mathfrak{P} \to \mathfrak{Q}\,,$$
>
> and thus states: "it is possible that if $\mathfrak{P}$, then $\mathfrak{Q}$", and the other is
>
> $$\mathfrak{P}\,,$$
>
> and thus states: "it is possible that $\mathfrak{P}$", it does not follow that "it is possible that $\mathfrak{Q}$", so that the thesis
>
> $$\mathfrak{Q}\,,$$
>
> does not follow intuitively, as the rule of *modus ponens* requires.

This is due to the fact that:

$$\Diamond(p \to q) \to (\Diamond p \to \Diamond q)$$

is not a thesis of S5.

As an appropriate to be used in the formulation of discussive logic Jaśkowski chooses discussive implication: '$\to^{\mathrm{d}}$'. In the formal language containing possibility operator $\Diamond$ Jaśkowski defines a formula $p \to^{\mathrm{d}} q$ by $\Diamond p \to q$. Jaśkowski gives an intuitive meaning of it: "if anyone states that p, then q" (Jaśkowski, 1948, p. 67). Jaśkowski notes that (see 1999a, p. 44; 1948, p. 67):

> in every discussive system two theses, one of the form:
>
> $$\mathfrak{P} \to^{\mathrm{d}} \mathfrak{Q}\,,$$
>
> and the other of the form:
>
> $$\mathfrak{P}\,,$$
>
> entail the thesis
>
> $$\mathfrak{Q}\,,$$
>
> and that on the strength of the theorem
>
> $$\Diamond(\Diamond p \to q) \to (\Diamond p \to \Diamond q). \tag{3.1}$$

This concept of discussive implication ensures closure of sets of theses of deductive systems on under *modus ponens*. Also some other connectives can be defined as discussive.

A discussive equivalence (notation: '$p \leftrightarrow^{\mathrm{d}} q$') Jaśkowski defines as:

$$(\Diamond p \to q) \wedge (\Diamond q \to \Diamond p).$$

Additionally, in (Jaśkowski, 1949) a discussive conjunction (notation: $p \wedge^{\mathrm{d}} q$) has been introduced:

$$p \wedge \Diamond q.$$

Jaśkowski also made there an update of the definition of discussive equivalence:

$$(p \to^{\mathrm{d}} q) \wedge^{\mathrm{d}} (q \to^{\mathrm{d}} p).$$

In order to define discussion logics, one can use a transformation from the discussion language into the language of modal logic.[3]

Let $\mathsf{For}^{\mathrm{d}}$ be the set of all formulas of the discussive language with constants: $\neg$, $\vee$, $\wedge^{\mathrm{d}}$, $\to^{\mathrm{d}}$, $\leftrightarrow^{\mathrm{d}}$, and let $\mathrm{For}^{\mathrm{M}}$ be the set of all modal formulas. *Jaśkowski's transformation* is a function $-^{\mathtt{d}}$ from $\mathsf{For}^{\mathrm{d}}$ into $\mathrm{For}^{\mathrm{M}}$ such that:

1. $(a)^{\mathtt{d}} = a$, for any variable a,
2. and for any $A, B \in \mathsf{For}^{\mathrm{d}}$:
   (a) $(\neg A)^{\mathtt{d}} = \neg A^{\mathtt{d}}$,
   (b) $(A \vee B)^{\mathtt{d}} = A^{\mathtt{d}} \vee B^{\mathtt{d}}$,
   (c) $(A \wedge^{\mathrm{d}} B)^{\mathtt{d}} = A^{\mathtt{d}} \wedge \Diamond B^{\mathtt{d}}$,
   (d) $(A \to^{\mathrm{d}} B)^{\mathtt{d}} = \Diamond A^{\mathtt{d}} \to B^{\mathtt{d}}$
   (e) $(A \leftrightarrow^{\mathrm{d}} B)^{\mathtt{d}} = (\Diamond A^{\mathtt{d}} \to B^{\mathtt{d}}) \wedge \Diamond(\Diamond B^{\mathtt{d}} \to A^{\mathtt{d}})$.

The logic D_2 (as a set of formulas) was formulated with the help of the modal logic $\mathsf{S5}$ (Jaśkowski, 1948, 1949). In his paper first paper Jaśkowski meant also a consequence relation of $\vdash_{\mathsf{D}_2}$, which can be formulated in the following way (Nasieniewski and Pietruszczak, 2012, 2013):

$$A_1, \ldots, A_n \vdash_{\mathsf{D}_2} B \text{ iff } \Diamond A_1^{\mathtt{d}} \to (\cdots \to (\Diamond A_n^{\mathtt{d}} \to \Diamond B^{\mathtt{d}}) \ldots) \in \mathsf{S5}.$$

A natural question is why Jaśkowski named his system D_2. The answer is simple. The capital letter D refers to the Polish adjective "dyskusyjny" (English "discussive"), while the number 2 stands for two-values. Jaśkowski called his system a *two-valued propositional calculus*, which seems to be a complemenation of the explanation of the meaning of used dentation (Jaśkowski, 1948, p. 67).

[3] Jaśkowski used an enriched modal-discussive logic and then selected the aimed discussive theses.

4. Torunian School of Paraconsistent Logic: past, now, and future

In the previous section, we presented the founder of TSPL and his basic idea. Here, we concentrate on the Torunian School of Paraconsistent Logic in the following aspects: people, basic facts, and the possible future of TSPL.[4]

Many issues were considered by Jaśkowski's pupils, including Jerzy Kotas and Lech Dubikajtis, who cooperated with da Costa (see 1968). These were, among others, the syntactic formulation of D_2 (Kotas, 1974a) but also extensions of D_2 on the predicative case (see da Costa, 1975; Da Costa and Dubikajtis, 1968, 1977).

Another issue was the modal logics connected with D_2 by the fact that the definition of D_2 is based on the notion of M-counterpart of $\mathsf{S5}$. This notion was further generalised and studied. In this context, one should recall works (Błaszczuk and Dziobiak, 1975, 1977), but also (Furmanowski, 1975; Perzanowski, 1975), and more recently (Nasieniewski and Pietruszczak, 2008). There are also works on more philosophical issues connected with Jaśkowski logic, including an interpretation of discussive logic. Here one should stress works (Urchs, 1993), but also (Mruczek-Nasieniewska, Nasieniewski, and Pietruszczak, 2019; Nasieniewski and Pietruszczak, 2013).

And finally, there are also investigations on variants of discussive logic, including a discussive consequence relation. Here, one should among others recall works by Max Urchs (who introduces the notion of *discussive* Jaśkowski-*systems* (see Ciuciura, 2006, 2008a,b; Mruczek-Nasieniewska, Nasieniewski, and Pietruszczak, 2019, 2025; Urchs, 1995), as well as (Nicolás-Francisco, 2022).

We will now tell this story in more detail, including dates.

Dubikajtis was a student of Jaśkowski. In 1967 he met da Costa in Paris and started to work together on the discussive logic. In 1968, Dubikajtis and da Costa published the paper "Sur la logique discursive de Jaśkowski" ("On Jaśkowski's discussive logic") (Da Costa and Dubikajtis, 1968) where they discussed the first axiomatization of the discussive logic. In 1977 da Costa and Dubikajtis published "On Jaśkowski discussive logic". In the work, it is considered the left discussive

[4] For an interesting perspective on TSPL and its presentation, please consult (Urchs, 2025).

conjunction replacing the right discussive conjunction (Da Costa and Dubikajtis, 1977).

To sum up, their contribution to the development of TSPL is:

- They sketched the way in which an axiomatisation of D_2 can be conducted, but also gave a complete axiomatisation of a variant of D_2 with left discussive conjunction.
- They also constructed a discussive predicate calculus of the first and higher orders.

In 1969 Kotas, also Jaśkowski's student, prepared "On the algebra of classes of formulae of Jaśkowski's discussive system" (1971), where it is investigated the algebra that underlies the system D_2. In 197 Kotas published the paper "The axiomatization of Jaśkowski discussive system" (1974a), showing that the discussive system is finitely axiomatizable. Also in 1974, Kotas published "On quantity of logical values in the discussive D_2 system and in modular logic" (1974b) showing that the discussive logic is characterized by an infinite quantity of values. In 1979 Kotas and da Costa published "A new formulation of discussive logic" (1979). In the work, the authors introduced a natural deduction system for the discussive logic with the left discussive conjunction.

Kotas's contribution to TSPL is:

- He considered properties of algebras arising from investigations on classes determined by D_2 inferential equivalence, he also gave a detailed procedure of axiomatizing D_2 with the right discussive conjunction.
- He considered another formulation of discussive predicate calculus with equality.

It is also worth mentioning that in 1970 Lafayette de Moraes wrote, under the supervision of da Costa, the first thesis on discussive logic under the title "Sobre a Lógica Discursiva de Jaśkowski" (de Moraes, 1970). This thesis presented a first-order discussive logic different from the one introduced by da Costa and Dubikajtis in 1968.

Another representative of this school was Tomasz Furmanowski. In 1975 Tomasz Furmanowski published "Remarks on Discussive Propositional Calculus" (Furmanowski, 1975), where it is shown that one can use any modal system M intermediate between the modal systems $\mathsf{S4}$ and $\mathsf{S5}$ to define the discussive logic. His contribution is as follows:

- Furmanowski has shown that any normal modal logic between $\mathsf{S4}$ and $\mathsf{S5}$ (including $\mathsf{S4}$) can be used to obtain a discussive logic that is equal to D_2.

- He also considered other translation (where both arguments of $\to$ and $\wedge$ are 'modalized').

An extremely important contribution to the development of TSPL was made by Jerzy Perzanowski, Jerzy Błaszczuk, and Wiesław Dziobiak. Perzanowski (1975) published "On M-fragments and L-fragments of normal modal propositional logics" . In the work, it is discussed the $\Box^n$-counterparts and $\Diamond^n$-counterparts of various modal systems and strengthened the result of defined different systems of discussive logic. In 1976 Błaszczuk and Dziobiak published "An axiomatization of M^n-counterparts for some modal logics". In the work, it is investigated the problem of the axiomatization of $\Diamond^n$-counterparts of different modal systems. In 1984 Błaszczuk published "Some Paraconsistent Sentential Calculi", where investigated the modal-counterparts of different modal systems, generalizing the investigation of $\Box^n$-counterparts and $\Diamond^n$-counterparts of modal systems to define systems of discussive logic[5] Their contribution can be summarized as follows:

- They considered weaker then *Siv* normal modal logics that define D_2, in particular they gave axiomatic characterisations of the weakest normal modal logic defining D_2.
- Besides they made investigations on generalizations of the notion of the M-counterpart of $\mathsf{S5}$, that is the set of formulas that while preceded by $\Diamond$, become theses of $\mathsf{S5}$.

Again, it's also worth mentioning that in 1985 Lafayette de Moraes published "On discussive set theory". In the work, the author introduced a discussive set theory based on a first-order logic with equality using the modal system $\mathsf{S5}$.

Urchs also made an interesting and significant contribution to the history of the School. In 1986, Urchs published "On two systems of Stanisław Jaśkowski" (Urchs, 1986), where he defined a system for a discussive logic that can also be used to represent causal relations. In 1993 Urchs in the paper "Powerful paraconsistent logic" (Urchs, 1993) introduced the notion of discussive Jaśkowski systems. Urchs's contribution can be summarized:

- Urchs considered a system combining in one system causality with discussive approach.

[5] Some basic concepts constituting discussive logic have been presented in (Nasieniewski and Nicolás-Francisco, 2025).

- He also proposed a defense against some critiques of discussive logic as well as effort as regards intuitive presentation of Jaśkowski's works.

In the meantime, in 1998 it was celebrated in Toruń the Memorial Symposium Paraconsistent Logic, Logical Philosophy, Informatics and Mathematics on the occasion of Jaśkowski's seminal paper — it was the first Memorial Symposium. There was published a corrected translation of Jaśkowski's seminal paper, edited by Jerzy Perzanowski and Andrzej Pietruszczak (Jaśkowski, 1999a). As regards the paper from 1948, it was the translation of Olgierd Wojtasiewicz, corrected by Perzanowski and edited by Pietruszczak. The paper from 1949 was translated for the first time. This translation was prepared by Perzanowski (Jaśkowski, 1999b).

The next stage of development of the idea introduced by TSPL took place after 2000. In 2005 Janusz Ciuciura published "On the da Costa, Dubikajtis and Kotas' System of Discursive Logic, D_2^*" (see Ciuciura, 2005). In the work, it is introduced another axiomatization for da Costa and Dubikajtis' system of 1977. In 2005, Jean-Yves Béziau published "The paraconsistent logic Z. A possible solution to Jaśkowski's problem" (see Béziau, 2006). In the work, the author introduced the logic Z as a mean to solve the problem of providing a quite rich paraconsistent logic with intuitive justification. In 2006, Ciuciura (2006) considered a variant of the discussive logic in which a discussive negation $\sim_d$ is introduced. In 2006, Joke Meheus (2006) introduced systems of adaptive discussive logics, combining thus Flemish and Polish approaches to paraconsistency. In 2018 Hitoshi Omori and Jesse Alama published "Axiomatizing Jaśkowski's Discussive Logic D_2" (Omori and Alama, 2018). They presented an axiomatization of D_2 in the language Jaśkowski suggested in his second paper on discussive logic. They also examined a variant of D_2, introduced by Ciuciura following a suggestion given by Perzanowski (see Jaśkowski, 1999b, p. 59), in which negation is also taken to be discussive (see Ciuciura, 2006). Collectively, their contributions are as follows:

- Ciuciura recalled that axiomatisation by da Costa and Kotas does not refer to the original formulation of D_2. It is connected to the fact that at some point this axiomatization started to be treated as an axiomatisation of the original Jaśkowski D_2.
- He gave a proposal of an axiomatization of the real D_2.
- This axiomatisation was later improved by Omori and Alama.
- Ciuciura proposed also a discussive system with modalised negation.
- Béziau proposed a system (with modalised negation) that was meant to be a solution to so-called Jaśkowski problem: to find a system rich

enough to be used in some practise, intuitive in its formulation and, of course, non-explosive.

- Meheus proposed a system combining features of adaptive logics and discussive one, where the possibility operators referring intuitively to the external, impartial arbiter were introduced.

Other researchers whose contribution to the development of TSPL should be mentioned are Marek Nasieniewski and Krystyna Mruczek-Nasieniewska. In 2005 Mruczek-Nasieniewska and Nasieniewski published "Syntactical and semantical characterization of a class of paraconsistent logics" (see Mruczek-Nasieniewska and Nasieniewski, 2005), where they built upon Béziau's work (2006) to present different paraconsistent logics using normal modal logics other than $\mathsf{S5}$. In the same year, similar observations were made by Marcos (2005). Let us add in the context of logic Z that a proposal of a tableau system for this logic was given in (Jarmużek, Nasieniewski, and Wąsiak, 2025).

In 2017 Mruczek-Nasieniewska and Nasieniewski published "Logics with impossibility as the negation and regular extensions of the deontic logic D_2" (see Mruczek-Nasieniewska and Nasieniewski, 2017). In this work, Mruczek-Nasieniewska and Nasieniewski built again upon (Béziau, 2006) and considered negation to be defined as impossibility, instead of unnecessity to obtain expressibility of analogous logics to logic Z using regular logics, being extensions of the smallest regular deontic logic. In "A characterization of some Z-like logics" (Mruczek-Nasieniewska and Nasieniewski, 2018), the authors made yet another try to obtain an extension of Béziau's results taking two negations to be understood as unnecessity and as impossibility. In 2019, Mruczek-Nasieniewska and Nasieniewski published "A Kotas-style characterization of minimal discussive logic" (see Mruczek-Nasieniewska and Nasieniewski, 2019). In this work, Mruczek-Nasieniewska and Nasieniewski considered syntactical characterization of a minimal variant of discussive logic D_2. Instead of considering that each participant has access to the assertions of all other participants in the discussion, they exploited the idea that a participant must have access to the assertions of at least another participant in the discussion. In 2020 Mruczek-Nasieniewska and Nasieniewski published "On correspondence of standard modalities and negative ones on the basis of regular and quasi-regular logics" (see Mruczek-Nasieniewska and Nasieniewski, 2020), where they tried to strengthen results obtained in (Mruczek-Nasieniewska and Nasieniewski, 2018). Their collective contribution can be summarized as follows:

- They proposed ways to generalize Béziau's approach for cases of normal, regular and partially quasi-regular modal logics, obtaining completeness results for Z-like logics.
- They also worked on variants of discussive logics, in particular, constructed a minimal variant of discussive logic.

Nasieniewski and Pietruszczak published the following papers:

- "The weakest regular modal logic defining Jaśkowski's logic D_2", "New axiomatizations of the weakest regular modal logic defining Jaśkowski's logic D_2", and "Semantics for regular logics connected with Jaśkowski's discussive logic D_2" (2008; 2009a; 2009b), where considered the smallest regular modal logic, which enables defining the discussive logic D_2;
- "A method of generating modal logics defining Jaśkowski's discussive logic D_2" (2011), where provided a method to obtain various modal logics that can be used to define the discussive logic D_2.
- "On the weakest modal logics defining Jaśkowski's logic D_2 and the D_2-consequence" and "On modal logics defining Jaśkowski's D_2-consequence" (2012; 2013), where studied normal and regular modal logics that can be used to define the discussive logic in terms of logical consequence;
- "Axiomatization of minimal modal logics defining Jaśkowski's-like discussive logics" (2014), where presented axiomatizations of the minimal modal logics that can be used to define some variants of discussive logics.

Their main contributions are:

- They strengthened the results by Perzanowski, Furmanowski and Dziobiak and Błaszczuk by generating modal logics minimal in various classes of modal logics (including the smallest such logic in the family of all modal logics), that allow do define discussive logic and discussive consequence relation.
- They also considered similar task for discussive-like logics.

LeLet us summery this fragment by mentioning that, in (Jarmużek, Nasieniewski, 2025), a proposal for tableau systems was given for logics considered in (Nasieniewski and Pietruszczak, 2014).

In the following years, the team of TSPL expanded. In particular, Luis Estrada-González, Marek Nasieniewski and Ricardo Arturo Nicolás-Francisco indicated a project of combining discussive approach with the idea of connexivity. This way, a paper "Discussive connexivity" arose

(Estrada-González, Nasieniewski, and Nicolás-Francisco, 2025), where the authors introduced a connexive logic based on the 'adjunctive' characterization of discussive logic. In the paper a tableaux system for the proposed logic is provided. The motivation for such a system is an attempt to invalidate the following schemas:

$$p, \neg p \models p \to \neg p \qquad \text{and} \qquad p, \neg p \models \neg p \to p,$$

where negation $\neg$ and conditional $\to$ are used in a connexive discussion. The system introduced is named the system **DisCo** (Discussive-Connexive). This system is based on an expansion of the logic **BD** of Belnap-Dunn using some intentional conditional. This research has opened a new space for future work. In particular, we expect them to develop the following problems:

- Extend the system **DisCo** to a first-order logic.
- Investigate the possibility of using **DisCo** to develop a theory of probability.
- Investigate the structure of lattices in which **DisCo** and other connexive logics are considered.
- Combine other non-classical, in particular anti-classical logics, with discussive logic.

As was already mentioned, there are works that, on the one hand, consider some discussive logics weaker than D_2 (Grigoriev, Nasieniewski, Mruczek-Nasieniewska et al., 2023; Mruczek-Nasieniewska and Nasieniewski, 2019), but, on the other hand, strengthen D_2 by means of modal operators that are present in the object language (Mruczek-Nasieniewska and Nasieniewski, 2025; Mruczek-Nasieniewska, Nasieniewski, and Pietruszczak, 2019, 2025).

To summarize this section, it is important to note the characteristics and main features of TSPL:

- translations into modal languages and back
- explanation of inconsistency in a consistent framework
- considering various ways of expressing inconsistent logic
- considering variants of other logics in which discussive-like logics can be defined.

5. WCP6 and Stanisław Jaśkowski Memorial Symposium

As we wrote, in 2022, we had the 75th anniversary of the first discussive logic — the first formal approach to paraconsistency. Therefore, the Second Stanisław Jaśkowski Memorial Symposium was also organized as part of The Sixth World Congress of Paraconsistency (5–8 September 2022, Toruń, Poland) to commemorate this fact (https://wcp6.umk.pl/pages/main_page/).

Representatives of all schools of paraconsistency were present at the congress. Dozens of papers were also presented, including those devoted to the founder of TSPL and its historical discovery. Some of the best papers, whose topics range from epistemology to the philosophy and history of (paraconsistent) logic, as well as formal logical investigations, are published in this volume. We provide an overview thereof in the sequel.

The paper "Can we test inconsistent empirical theories?" by Luis Felipe Bartolo Alegre (2025) discusses the logical possibility of testing inconsistent but nontrivial empirical theories. The main thesis is that, despite the logical challenges associated with their inconsistent consequences, such theories can indeed be subjected to empirical testing. Motivated by the observation that science is not free of inconsistencies, the author argues in favor of this possibility by proposing a small but essential amendment to Popper's falsifiability criterion. This extension results in a rejectability criterion that can accommodate some inconsistent empirical theories.

In the paper "Is p and $\neg p$ a contradiction?" (2025), Jean-Yves Béziau discusses how to formulate and understand a contradiction. Indeed, the author presents a variety of formulations of the proposition corresponding to "p and $\neg p$". Moreover, a relationship between the concept of a contradiction and the concept of negation involved therein is established. As an output, the author provides a definition of a contradiction as a pair of propositions that can neither be true, nor false together, therefore "freeing" the concept of a contradiction from its usual formulation. As a consequence of such a position, one has that, for example, "p and $\neg p$" is not a contradiction, if $\neg$ is meant as a paraconsistent negation.

"Literal and controllable paraconsistency" by Janusz Ciuciura (2025) explores several calculi yielding a paraconsistent logic at the level of literals in the following sense. A contradiction of the form "A and not-A" is not explosive, namely, it does not entail any formula if and only if A is either a propositional variable or an iterated negation thereof. A system of logic $\mathsf{P}_{\mathcal{LI}}$ enjoying such a property is yielded upon considering a subsystem of Sette's calculus obtained upon removing an axiom ensuring the explosiveness of contradictions involving negated formulas. Furthermore, the author investigates a "gently paraconsistent" subsystem $\mathsf{GP}_{\mathcal{LI}}$ of $\mathsf{P}_{\mathcal{LI}}$ obtained upon weakening the definition of explosiveness for non-literal formulas. Finally, a further logic of controllable paraconsistency (in the sense that non-explosive contradictions are allowed, though in a controllable manner) $\mathsf{P}_{\mathcal{AB}}$ and some of its extensions are outlined.

In "What is the principle of (non-)contradiction, precisely? The struggle at the dawn of formal logic", Adam Trybus (2025) investigates the history of logical inquiries into the principle of non-contradiction in modern formal logic. Particular attention is paid to proposals made by Christine Ladd-Franklin, that the author puts into relationship with other attempts made at the beginning of the twentieth century.

In "The beginnings of Toruń logic", Max Urchs (2025) provides an analysis of early developments of Toruńian logic upon outlining the history of the two main traditions which played a role in its ultimate establishment, and of their initiators: Tadeusz Czeżowski and Stanisław Jaśkowski.

The paper "Paradoxes versus contradictions in logic of sentential operators" by Michał Walicki (2025) concerns with extensions of (first or higher-order) Classical logic by means of sentential operators and quantifiers, interpreted substitutionally over unrestricted substitution class. Specifically, the language of first-order logic is extended with free sentential variables, and bound s-variables which are s-quantified in the usual manner. To express properties of sentences, the language is further extended with operators. The language introduced allows to define an extension LSO of first-order LK with new rules capable of handling the left/right-introduction of universal quantifiers over sentences. A semantics for LSO is provided by means of *language graphs*, i.e., directed graphs of a special sort. The intended aim is to provide a formal treatment of paradoxes without indulging into paraconsistency while still avoiding explosion.

In "Constructive logic is connexive and contradictory", Heinrich Wansing (2025) deals with the constructiveness of the contra-classical, connexive, paraconsistent, and negation-inconsistent non-trivial first-order logic QC — i.e., the first-order counterpart of Wansing's connexive logic C — which, in turn, is a connexive variant of Almukdad and Nelson's logic QN4. In particular, the author shows that, although both QC and QN4 avoid Smullyan's "Drinker's principle" and its dual, unlike the former, the latter does not allow us to prove principles which, instead, seem rather intuitive, namely the so-called *Drinker's truism* and its dual. Moreover, QC turns out to be in some sense "more constructive" than QN4. Indeed, it has a connexive proof/disproof (López-Escobar) interpretation enjoying proof/disproof parity (under the assumption that proof/disproof parity holds for literals). This feature is not shared by QN4.

The paper "True, untrue, valid, invalid, provable, unprovable" by Zach Weber (2025) is motivated by an interest in exploring and extending a radical approach to paraconsistency, particularly where truth-value gluts exist. The author starts from the observation that many paraconsistent logics, such as Priest's (first-order) Logic of paradox LP, although admitting the possibility of true contradictions, are still strongly committed to consistency when it comes to work at a metatheoretical level. In view of this fact, the author evaluates the philosophical and technical tenability of adopting a glut-friendly paraconsistent metatheory (based on the logic BCK expanded with a De Morgan's negation) and so revising usual concepts from semantics and proof-theory, such as the notion of a model, of validity and provability.

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References

Anderson, Alan Ross, and Nuel D. Belnap, Jr., 1959, "Modalities in Ackermann's 'rigorous implication' ", *The Journal of Symbolic Logic*, 24 (2): 107–111. DOI: 10.2307/296475410.2307/2964754

Asenjo, Florencio G., 1966, "A calculus of antinomies", *Notre Dame Journal of Formal Logic*, 7 (1): 103–105. DOI: 10.1305/ndjfl/1093958482

Asenjo, Florencio G., and J. Tamburino, 1975, "Logic of antinomies", *Notre Dame Journal of Formal Logic*, 16 (1): 17–44. DOI: 10.1305/ndjfl/1093891610

Bartolo Alegre, Luis Felipe, 2025, "Can we test inconsistent empirical theories?", *Logic and Logical Philosophy*, 34 (4): 501–545. DOI: 10.12775/LLP.2025.011

Batens, Diderik, 1980, "Paraconsistent extensional propositional logics", *Logique et Analyse*, 23 (90–91): 195–234.

Batens, Diderik, 1986, "Dialectical dynamics within formal logics", *Logique et Analyse*, 114: 161–173.

Batens, Diderik, 1989, "Dynamic dialectical logics", pages 187–217 in Graham Priest, Richard Routley, and Jean Norman (eds), *Paraconsistent Logic. Essays on the Inconsistent*, Philosophia Verlag, München.

Batens, Diderik, 1995, "Blocks. The clue to dynamic aspects of logic", *Logique et Analyse*, 150–152: 285–328.

Batens, Diderik, 1997, "Inconsistencies and beyond. A logical-philosophical discussion", *Revue Internationale de Philosophie*, 200: 259–273.

Batens, Diderik, 1998, "A dynamic semantics for inconsistency-adaptive logics", *Bulletin of the Section of Logic*, 27: 15–18.

Batens, Diderik, 1999, "Zero logic adding up to classical logic", *Logical Studies*, 2. http://hdl.handle.net/1854/LU-109631

Batens, Diderik, 2000a, "A survey of inconsistency-adaptive logics", pages 49–73 in Diderik Batens, Chris Mortensen, Graham Priest, and Jean Paul Van Bendegem (eds.), *Frontiers of Paraconsistent Logic*, Research Studies Press, Baldock.

Batens, Diderik, 2000b, "Towards the unification of inconsistency handling mechanisms", *Logic and Logical Philosophy*, 8: 5–31. DOI: 10.12775/LLP.2000.001

Batens, Diderik, 2003, "Criteria causing inconsistencies. General gluts as opposed to negation gluts", *Logic and Logical Philosophy*, 11/12: 5–37. DOI: 10.12775/LLP.2003.001

Batens, Diderik, and Joke Meheus, 2000, "A tableau method for inconsistency-adaptive logics", pages 127–142 in Roy Dyckhoff (ed.), *Automated Reasoning with Analytic Tableaux and Related Methods*, Lecture Notes in Artificial Intelligence, vol. 1847, Springer, Berlin, Heidelberg. DOI: 10.1007/10722086_13

Belnap, Nuel D., 1960, "Entailment and relevance", *The Journal of Symbolic Logic*, 25 (2): 144–146. DOI: 10.2307/2964210

Belnap, Nuel D., 1976, "How a computer should think", pages 30–56 in Ryle Gilbert (ed.), *Contemporary Aspects of Philosophy*, Oriel Press Ltd., Stocksfield, London, Henley-on-Thames and Boston.

Belnap, Nuel D., 1977a, "A useful four-valued logic", pages 5–37 in J. M. Dunn and G. Epstein (eds), *Modern Uses of Multiple-Valued Logic*, Episteme, vol. 2., Springer, Dordrecht. DOI: 10.1007/978-94-010-1161-7_2

Béziau, Jean-Yves, 2005, "Paraconsistent logic from a modal viewpoint", *Journal of Applied Logic*, 3 (1): 7–14. DOI: 10.1016/j.jal.2004.07.009

Béziau, Jean-Yves, 2006, "The paraconsistent logic Z", *Logic and Logical Philosophy*, 15: 99–111. DOI: 10.12775/LLP.2006.006

Beziau, Jean-Yves, 2025, "Is p and $\neg p$ a contradiction?", *Logic and Logical Philosophy*, 34 (4): 547–560. DOI: 10.12775/LLP.2025.004

Błaszczuk, Jerzy J., 1984, "Some paraconsistent sentential calculi", *Studia Logica*, 43: 51–61. DOI: 10.1007/BF00935739

Błaszczuk, Jerzy J., and Wiesław Dziobiak, 1975, "Remarks on Perzanowski's modal system", *Bulletin of the Section of Logic*, 4 (2): 57–64.

Błaszczuk, Jerzy J., and Wiesław Dziobiak, 1976, "An axiomatization of M^n-counterparts for some modal calculi", *Reports on Mathematical Logic*, 6: 3–6.

Błaszczuk, Jerzy J., and Wiesław Dziobiak, 1977, "Modal logics connected with systems $\mathrm{S4_n}$ of Sobociński", *Studia Logica*, 36: 151–175. DOI: 10.1007/BF02121260

Carnielli, Walter, Marcelo E. Coniglio, and João Marcos, 2007, "Logics of formal inconsistency", pages 1–93 in D. M. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, Springer Netherlands, Dordrecht. DOI: 10.1007/978-1-4020-6324-4_1

Ciuciura, Janusz, 2005, "On the da Costa, Dubikajtis and Kotas' system of the discursive logic, D_2^*", *Logic and Logical Philosophy*, 14: 235–252. DOI: 10.12775/LLP.2005.015

Ciuciura, Janusz, 2006, "A quasi-discursive system ND_2^+", *Notre Dame Journal of Formal Logic*, 47 (3): 371–384. DOI: 10.1305/ndjfl/1163775444

Ciuciura, Janusz, 2008a, "Frontiers of the discursive logic", *Bulletin of the Section of Logic 37*, 2: 81–92.

Ciuciura, Janusz, 2008b, "Negations in the adjunctive discursive logic", *Bulletin of the Section of Logic*, 37 (3–4): 143–160.

Ciuciura, Janusz, 2025, "Literal and controllable paraconsistency", *Logic and Logical Philosophy*, 34 (4): 561–579. DOI: 10.12775/LLP.2024.027

da Costa, Newton C. A., 1963, "Calculs propositionnels pour les systèmes formels inconsistants", *Comptes Rendus Helodomadaires des Seances de l'Academie des Sciences Paris*, Series A, 257: 3790–3792.

da Costa, Newton C. A., 1974, "On the theory of inconsistent formal systems", *Notre Dame Journal of Formal Logic*, 15 (4): 497–510. DOI: 10.1305/ndjfl/1093891487

da Costa, Newton C. A., 1975, "Remarks on Jaśkowski's discussive logic", *Reports on Mathematical Logic*, 4: 7–16.

da Costa, Newton C. A., and Ayda I. Arruda, 1963, "O paradoxo de Curry – Moh Shaw-Kwei" *Boletim da Sociedade de Matemática de S. Paolo*, 18: 83–89.

da Costa, Newton C. A., and Lech Dubikajtis, 1968, "Sur la logique discursive de Jaśkowski", *Bulletin de L'Académie Polonaise des Sciences*, 16 (7)): 551–557.

da Costa, Newton C. A., and Lech Dubikajtis, 1977, "On Jaśkowski discussive logic", pages 37–56 in A. Arruda, N. da Costa, and R. Chuaqui (eds.), *Non-Classical Logics, Model Theory and Computability. Proceedings of the Third Latin-American Symposium on Mathematical Logic, Campinas, Brazil, July 11-17, 1976*, North-Holland Publishing Company.

de Moraes, Lafayette, 1970, "Sobre a lógica discursiva de Jaśkowski", Master Thesis, USP.

Dunn, Jon Michael, 1966, "The algebra of intensional logics", Ph.D. thesis, University of Pittsburgh.

Estrada-González, Luis, Marek Nasieniewski, and Ricardo Arturo Nicolás-Francisco, 2025, "Discussive connexivity" (submitted).

Furmanowski, Tomasz, 1975, "Remarks on discussive propositional calculus", *Studia Logica*, 34: 39–43. DOI: 10.1007/BF02314422

Grigoriev, Oleg, Marek Nasieniewski, Krystyna Mruczek-Nasieniewska, Yaroslav Petrukhin, and Vasily Shangin, 2023, 'Axiomatizing a minimal discussive logic", *Studia Logica*, 111: 855–895. DOI: 10.1007/s11225-023-10042-9

Jarmużek, Tomasz, and Marek Nasieniewski, 2025, "On discussive-like logics obtained by means of certain types of tableau systems" (submitted).

Jarmużek, Tomasz, Marek Nasieniewski, and Kacper Wąsiak, 2025, "A tableau system for Béziau's logic Z" (submitted).

Jaśkowski, Stanisław, 1948, "Rachunek zdań dla systemów dedukcyjnych sprzecznych", *Studia Societatis Scientiarum Torunensis*, Sect. A, I (5): 57–77.

Jaśkowski, Stanisław, 1949, "O koniunkcji dyskusyjnej w rachunku zdań dla systemów dedukcyjnych sprzecznych", *Studia Societatis Scientiarum Torunensis*, Sect. A, I (8): 171–172.

Jaśkowski, Stanisław, 1999a, "A propositional calculus for inconsistent deductive systems", *Logic and Logical Philosophy*, 7: 35–56. The second English translation of (Jaśkowski, 1948). DOI: 10.12775/LLP.1999.003

Jaśkowski, Stanisław, 1999b, "On the discussive conjunction in the propositional calculus for inconsistent deductive systems", *Logic and Logical Philosophy*, 7: 57–59. The English translation of (Jaśkowski, 1949). DOI: 10.12775/LLP.1999.004

Kotas, Jerzy, 1971 "On the algebra of classes of formulae of Jaśkowski's discussive system", *Studia Logica*, 27: 81–90. DOI: 10.1007/BF02282553

Kotas, Jerzy, 1974a, "The axiomatization of S. Jaśkowski's discussive system", *Studia Logica*, 33 (2): 195–200. DOI: 10.1007/BF02120494

Kotas, Jerzy, 1974b, "On quantity of logical values in the discussive D_2 system and in modular logic", *Studia Logica*, 33: 273–275. DOI: 10.1007/BF02123281

Kotas, Jerzy, and Newton C. A. da Costa, 1979, "A new formulation of discussive logic", *Studia Logica*, 38: 429–445. DOI: 10.1007/BF00370480

Marcos, João, 2005, "Nearly every normal modal logic is paranormal", *Logique et Analyse*, 48 (189/192): 279–300.

Meheus, Joke, 1999, "Erotetic arguments from inconsistent premises", *Logique et Analyse*, 165–166: 49–80.

Meheus, Joke, 2006, "An adaptive logic based on Jaśkowski's approach to paraconsistency", *Journal of Philosophical Logic*, 35: 539–567. DOI: 10.1007/s10992-005-9004-9

Mruczek-Nasieniewska, Krystyna, and Marek Nasieniewski, 2004, "A modal formulation of some paraconsistent logics. Part II", presented at International Workshop on Problem Solving in the Sciences: Adaptive and Interrogative Perspectives – 9th VlaPolo, Ghent University.

Mruczek-Nasieniewska, Krystyna, and Marek Nasieniewski, 2005, "Syntactical and semantical characterization of a class of paraconsistent logics", *Bulletin of the Section of Logic*, 34 (4): 229–248.

Mruczek-Nasieniewska, Krystyna, and Marek Nasieniewski, 2008, "Paraconsistent logics obtained by J.-Y. Béziau's method by means of some non-normal modal logics", *Bulletin of the Section of Logic*, 37 (3/4): 185–196.

Mruczek-Nasieniewska, Krystyna, and Marek Nasieniewski, 2017, "Logics with impossibility as the negation and regular extensions of the deontic logic D_2", *Bulletin of the Section of Logic*, 46 (3/4): 263–282. DOI: 10.18778/0138-0680.46.3.4.06

Mruczek-Nasieniewska, Krystyna, and Marek Nasieniewski, 2018, "A characterisation of some-like logics", *Logica Universalis*, 12: 207–219. DOI: 10.1007/s11787-018-0184-9

Mruczek-Nasieniewska, Krystyna, and Marek Nasieniewski, 2019, "A Kotas-style characterisation of minimal discussive logic", *Axioms*, 8 (4), 108: 1–17. DOI: 10.3390/axioms8040108

Mruczek-Nasieniewska, Krystyna, and Marek Nasieniewski, 2020, "On correspondence of standard modalities and negative ones on the basis of regular and quasi-regular logics", *Studia Logica*, 108: 1087–1123. DOI: 10.1007/s11225-019-09890-1

Mruczek-Nasieniewska, Krystyna, and Marek Nasieniewski, "On a modal discussive logic corresponding to an extended model of discussion", in Filip Grgić, Kordula Świętorzecka, Anna Brożek (eds.), *Logic, Knowledge, and Tradition. Essays in Honor of Srećko Kovač* (accepted).

Mruczek-Nasieniewska, Krystyna, Marek Nasieniewski, and Andrzej Pietruszczak, 2019, "A modal extension of Jaśkowski's discussive logic D_2", *Logic Journal of IGPL*, 27: 451–477. DOI: 10.1093/jigpal/jzz014

Mruczek-Nasieniewska, Krystyna, Marek Nasieniewski, and Andrzej Pietruszczak, 2025, "Modal discussive logics", *Journal of Philosophical Logic*, 54: 99–142. DOI: 10.1007/s10992-024-09781-4

Nasieniewski, Marek, 2002, "A comparison of two approaches to parainconsistency: Flemish and Polish", *Logic and Logical Philosophy*, 9: 47–74. DOI: 10.12775/LLP.2001.004

Nasieniewski, Marek, 2003, "A modal formulation of some paraconsistent logics", presented at International Workshop on Problem Solving in the Sciences: Adaptive and Interrogative Perspectives – 8th VlaPolo (Zielona Góra, Poland).

Nasieniewski, Marek, Ricardo Arturo Nicolás-Francisco, 2025, "On some concepts constituting discussive logic", *Studia z Historii Filozofii* (submitted).

Nasieniewski, Marek, and Andrzej Pietruszczak, 2008, "The weakest regular modal logic defining Jaśkowski,'s logic D_2", *Bulletin of the Section of Logic*, 37 (3/4): 197–210.

Nasieniewski, Marek, and Andrzej Pietruszczak, 2009a, "New axiomatizations of the weakest regular modal logic defining Jaśkowski's logic D_2", *Bulletin of the Section of Logic*, 38 (1–2): 45–50.

Nasieniewski, Marek, and Andrzej Pietruszczak, 2009b, "Semantics for regular logics connected with Jaśkowski's discussive logic D_2", *Bulletin of the Section of Logic*, 38 (3/4): 173–187.

Nasieniewski, Marek, and Andrzej Pietruszczak, 2011, "A method of generating modal logics defining Jaśkowski's discussive logic D_2", *Studia Logica*, 97 (1): 161–182. DOI: 10.1007/s11225-010-9302-2

Nasieniewski, Marek, and Andrzej Pietruszczak, 2012, "On the weakest modal logics defining Jaśkowski's logic D_2 and the D_2-consequence", *Bulletin of the Section of Logic*, 41 (3/4): 215–232.

Nasieniewski, Marek, and Andrzej Pietruszczak, 2013, "On modal logics defining Jaśkowski's D_2-consequence", pages 141–161, chapter 9, in K. Tanaka, F. Berto, E. Mares, and F. Paoli (eds.), *Paraconsistency: Logic and Applications*, series "Logic, Epistemology and the Unity of Science", Vol. 26, Springer. DOI: 10.1007/978-94-007-4438-7_9

Nasteniewski, Marek, and Andrzej Pietruszczak, 2014, "Axiomatisations of minimal modal logics Defining Jaśkowski-like Discussive Logics", pages 149–163 in Andrzej Indrzejczak, Janusz Kaczmarek, and Michał Zawidzki (eds.), *Trends in Logic XIII*, Wydawnictwo Uniwersytetu Łódzkiego.

Nicolás-Francisco, Ricardo Arturo, 2022, "On the Polish 'via modalization' approach to paraconsistency", *Edukacja Filozoficzna Edukacja Filozoficzna*, 74: 161–182. DOI: 10.14394/edufil.2022.0021

Omori, Hitoshi, and Jesse Alama, 2018, "Axiomatizing Jaśkowski's discussive logic D_2", *Studia Logica*, 106 (6): 1163–1180. DOI: 10.1007/s11225-017-9780-6

Perzanowski, Jerzy, 1975, "On M-fragments i L-fragments of normal modal propositional logics", *Reports on Mathematical Logic*, 5: 63–72.

Priest, Graham, 1979, "The logic of paradox", *Journal of Philosophical Logic*, 8 (1): 219–241. DOI: 10.1007/BF00258428

Priest, Graham, Koji Tanaka, and Zach Weber, 2025, "Paraconsistent logic", in Edward N. Zalta and Uri Nodelman (eds.), *The Stanford Encyclopedia of Philosophy*, Spring 2025 Edition. https://plato.stanford.edu/archives/spr2025/entries/logic-paraconsistent

Routley, Richard, and Robert K. Meyer, 1973, "The semantics of entailment", pages 199–243 in H. Leblanc (ed.), *Truth, Syntax and Modality*, Studies in Logic and the Foundations of Mathematics, vol. 68, Elsevier. DOI: 10.1016/S0049-237X(08)71541-6

Schotch, Peter K., and Raymond E. Jennings, 1980, "Inference and necessity", *Journal of Philosophical Logic*, 9 (3): 327–340. DOI: 10.1007/BF00248398

Schotch, Peter K., and Raymond E. Jennings, 1989, "On detonating", pages 306–327 in G. Priest, R. Routley, and J. Norman (eds.), *Paraconsistent Logic: Essays on the Inconsistent*, Philosophia Verlag, München – Hamden – Wien.

Schotch, Peter K., Bryson Brown, and Raymond Jennings (eds.), , 2009, *On Preserving: Essays on Preservationism and Paraconsistent Logic*, University of Toronto Press.

Trybus, Adam, 2025, "What is the principle of (non-)nontradiction, precisely? The struggle at the dawn of formal logic", *Logic and Logical Philosophy*, 34 (4): 581–602. DOI: 10.12775/LLP.2024.026

Urchs, Max, 1986, "On two systems of Stanisław Jaśkowski", *The Journal of Non-Classical Logic*, 3 (1): 25–32. https://www.cle.unicamp.br/eprints/index.php/jancl/article/view/1037/862

Urchs, Max, 1993, "Powerful paraconsistent logic", *Acta Universitatis Lodziensis. Folia Philosophica*, 9: 109–114.

Urchs, Max, 1995, "Discursive logic towards a logic of rational discourse", *Studia Logica*, 54: 231–249. DOI: 10.1007/BF01063153

Urchs, Max, 2025, "The beginnings of Toruń logic", *Logic and Logical Philosophy*, 34 (4): 603–630. DOI: 10.12775/LLP.2025.010

Verdée, Peter, 2009, "Adaptive logics using the minimal abnormality strategy are Π^1_1-complex", *Synthese*, 167 (1): 93–104. DOI: 10.1007/s11229-007-9291-5

Walicki, Michał, 2025, "Paradoxes versus contradictions in logic of sentential operators", *Logic and Logical Philosophy*, 34 (4): 631–675. DOI: 10.12775/LLP.2024.002

Wansing, Heinrich, 2025, "Constructive logic is connexive and contradictory", *Logic and Logical Philosophy*, 34 (4): 677–703. DOI: 10.12775/LLP.2024.001

Weber, Zach, 2025, "True, untrue, valid, invalid, provable, unprovable", *Logic and Logical Philosophy*, 34 (4): 705–733. DOI: 10.12775/LLP.2024.008

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