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Relating Semantics as Compatibility Semantics

Abstract. The question I address here is whether the relating semantics currently available for connexive logics provide a good account of what connexivity is, or at least as good as some of the proposals already in circulation. I will argue that relating semantics for Boolean connexive logics, in its current state, does not give a satisfactory account of connexivity, as it is merely designed to validate the connexive schemas. Nonetheless, following the initial ideas of Jarmużek and Malinowski, I will suggest that the relation in such a semantics can be understood as a sort of compatibility, and that the relating semantics can be naturally enriched so as to make it virtually indistinguishable from the Incompatibility Approach. To put it boldly, my claim is that, if relating semantics for Boolean connexive logics is going to be more than a means to validate the connexive schemas, and if the relating relation to model connexive logics is to be understood as connectedness, as Jarmużek and Malinowski want, the relating relation must have more properties than those currently allowed. Further arguments for those additional properties come from considering highly desirable properties of an implication.

Keywords: connexive logic; Boolean connexive logic; connectedness; relating semantics; compatibility

1. Introduction

Following McCall [24, p. 109], a logic $\boldsymbol{L}$, with a negation N and an implication $>$, is *connexive* iff some of the following schemas are valid in it:[1]

$$\mathsf{N}(A > \mathsf{N}\,A) \qquad \text{(Aristotle's Thesis)}$$

$$\mathsf{N}(\mathsf{N}\,A > A) \qquad \text{(Variant of Aristotle's Thesis)}$$

[1] Useful overviews of connexive logic include [12, 27, 44].

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$$(A > B) > \mathsf{N}(A > \mathsf{N}\,B) \qquad \text{(Boethius' Thesis)}$$
$$(A > \mathsf{N}\,B) > \mathsf{N}(A > B) \qquad \text{(Variant of Boethius' Thesis)}$$

but $(A > B) > (B > A)$, the symmetry of implication, is nonetheless not valid in $\boldsymbol{L}$. In what follows, I will refer to both Aristotle's and Boethius' Theses and their variants collectively as "the connexive schemas", although they are not the only ones, but this will not affect the discussion in the paper.

At least prima facie, the mere satisfaction of these conditions says little to nothing about what connexivity is, what kind of property of logics, implications or negations it is. The question I address here is whether the relating semantics for connexive logics currently available provide a good account of what connexivity is, or at least as good as some of the proposals already in circulation. A good account of connexivity includes

- the description of a property C, distinct from the mere validation of the connexive schemas, such that a claim of the form "The n-ary connective © (or the logic $\boldsymbol{L}$) is C" is true;
- that the n-ary connective © (or the logic $\boldsymbol{L}$) is C entails that the connexive schemas get validated in $\boldsymbol{L}$.

Consider two well-known accounts of connexivity: the Incompatibility Approach and the Bochum Plan. According to the Incompatibility Approach, an implication should be understood in terms of incompatibility in the following manner:

(IA0) $\quad A > B =_{\text{def.}} \mathsf{N}(A \circ \mathsf{N}\,B)$

where '$\circ$' stands for a compatibility connective such that '$A \circ B$' is read 'A and B are compatible'.[2] What is the subject of such (in)compatibility? In most versions, it is the contents of antecedent and consequent, and the question about the properties of this relation of (in)compatibility between contents is, to some extent, orthogonal to the question of what exactly a content is.

Let A and B formulas of some formal language $\mathcal{L}$; and N, $>$ and © be a negation, an implication and any n-ary connective, respectively, of that same language. Suppose now, for the sake of quick presentation,

[2] Compatibility was originally called 'consistency' by Nelson [30], and then 'cotenability' by Kielkopf [18] and Pizzi [32]. Nonetheless, I stick here to McCall's [25] 'compatibility'.

that the following hold in the Incompatibility Approach, for a logic $\boldsymbol{L}$ based on $\mathcal{L}$:

(IA1) $\models_{\boldsymbol{L}} A \circ A$,
(IA2) $A > B \models_{\boldsymbol{L}} A \circ B$,
(IA3) $©(\dots A \dots) \dashv_{\boldsymbol{L}}\models ©(\dots \mathsf{N}\,\mathsf{N}\, A \dots)$,
(IA4) if $\Gamma, A \models_{\boldsymbol{L}} B$ then $\Gamma \models_{\boldsymbol{L}} A > B$.

where '$A \dashv_{\boldsymbol{L}}\models B$' is a short for '$A \models_{\boldsymbol{L}} B$ and $A \models_{\boldsymbol{L}} B$'. In this picture, connexivity is the property of an implication of being true if and only if its antecedent is incompatible with a negation of its consequent, and where compatibility, negation and implication have some extra conditions to ensure so. Thus, (IA0)–(IA4) are our C in the case of the Incompatibility Approach. Then, Aristotle's Theses, by (IA0) and (IA1), and Boethius' Theses, by (IA0)–(IA4), are validated.

So much for the Incompatibility Approach. According to the initial version of the Bochum Plan introduced in [43], connexivity is obtained through special falsity conditions for implication. Thus, suppose that negation and implication have the following Dunn-style truth and falsity conditions:

- $1 \in \sigma(\sim A, i)$ iff $0 \in \sigma(A, i)$,
- $0 \in \sigma(\sim A, i)$ iff $1 \in \sigma(A, i)$,
- $1 \in \sigma(A \to B, i)$ iff for all j, if $i \leq j$, $1 \notin \sigma(A, j)$ or $1 \in \sigma(B, j)$,
- $0 \in \sigma(A \to B, i)$ iff for all j, if $i \leq j$, $1 \notin \sigma(A, j)$ or $0 \in \sigma(B, j)$.

where '1' and '0' stand for truth and falsity; i and j are indexes of evaluation; σ is an assignment of truth values to formula-index pairs, and $\leq$ is a partial ordering among indexes of evaluation. Then one can easily verify that Aristotle's and Boethius' Theses, together with their variants, are true in all interpretations, and that there is some interpretation in which $(A \to B) \to (B \to A)$ is not true. Note that, according to this account, an implication is false if and only if the antecedent is untrue or the consequent is false, instead of the more familiar condition that it is false if and only if the antecedent is true and the consequent is false.[3] Connexive logics encode theories about the necessary conditions to negate an implication. Wansing's falsity conditions for implication give us both necessary and sufficient conditions to negate an implication.

[3] These are by no means the only understandings of connexivity. There is the negation-as-cancellation approach; see [38, Ch. 2, Sect. 4] and [33] for some developments (without endorsing it), and [45] for a critical study. See [41] for a sympathetic view.

Connexive schemas are validated not by chance nor by hand, but by a principled theory on what is to negate an implication.

Does relating semantics for Boolean connexive logics, in its current state, give us a similar account of connexivity? My suggested answer is 'No'. Relating semantics, in its current state, is merely designed to validate the connexive schemas, i.e., it is a "merely formal semantics" in the terminology of [4].[4] In the Boolean approach, the evaluation condition of the connexive implication is the evaluation condition of a material implication plus the requirement that antecedent and consequent are related. (See the next section for details.) The properties of such a relation between antecedent and consequent are not settled, with the only exception of those that make the connexive schemas true. In that sense, such relating approach is ad hoc, merely designed to validate the connexive schemas, with no prior (or further) information about what the relation is and what else, if anything, it should validate to be a relation appropriate to evaluate an implication.

Nonetheless, I will suggest that the relating semantics for Boolean connexive logics can be understood as a version of the Incompatibility Approach, and that such a relating semantics can be naturally enriched so as to make it indistinguishable from the Incompatibility Approach. Boldly put, my claim is that, if relating semantics for Boolean connexive logics is going to be more than a means to validate the connexive schemas — "an illuminating and philosophically significant semantics", again in the terminology of [4] — the relating relation must have more properties than those currently allowed; if implication in Boolean connexive logics is going to be truly an implication, more properties than those currently allowed for the relating relation are needed, in fact, those needed to make the relating semantics a competitive account of connexivity.

The plan of the paper is as follows. In Section 2, I give the essentials of (non-modal) Boolean connexive logics at tutorial speed. In Section 3, I start the discussion of the nature of the relating relation $\mathcal{R}$ employed in Boolean connexive logics, and referred to as "connectedness" by Jarmużek and Malinowski. My main claim there is that connectedness induces more properties than those initially demanded by Jarmużek and Malinowski. In Section 4, I investigate whether $\mathcal{R}$ can be understood as compatibility, which again would induce more properties, and partic-

[4] But see the Epilogue.

ularly the reflexivity of $\mathcal{R}$. In Section 5, I give another argument for those additional properties of $\mathcal{R}$, but this time having implications as a starting point of the discussion. Finally, in Section 6, I show that $\mathcal{R}$ can be understood as compatibility, at least in Pizzi's sense.

Before starting, two disclaimers are in order here. Although I am sympathetic to the Incompatibility Approach, I will not argue for it here. I merely try to make a case for understanding relating semantics (for Boolean connexive logics) as a species of the Incompatibility Approach, without any claims regarding its superiority or otherwise. Second, I am also sympathetic to other connexive principles, such as the following (where $\otimes$ is a generic conjunction):

$$\mathsf{N}((A > B) \otimes (\mathsf{N}\,A > B)) \qquad \text{(Aristotle's Second Thesis)}$$
$$\mathsf{N}((A > B) \otimes (A > \mathsf{N}\,B)) \qquad \text{(Abelard's Principle)}$$
$$\mathsf{N}(A > \mathsf{N}\,B) > (A > B) \qquad \text{(Converse of Boethius' Thesis)}$$
$$\mathsf{N}(A > B) > (A > \mathsf{N}\,B) \qquad \text{(Converse of the Variant of Boethius' Thesis)}$$

I make no claims about their correctness, neither simpliciter nor *qua* connexive principles, and I say no word about how to validate them in a relating setting, either. Work on the latter is done in [20], and the former requires too many separate works, although in [3, 31] there is interesting work in that direction.

2. Boolean connexive logics

Boolean connexive logics expand the language of classical zero-order logic using conjunction, disjunction and negation, by a relating implication, $\to^{\mathrm{w}}$, the semantics of which is constrained by a binary relation $\mathcal{R}$ on the set of all formulas, $\mathcal{L}$. A (relating) *model* then is a pair $\langle v, \mathcal{R}\rangle$, where v is a classical valuation function. '$\mathbf{M}_{\mathcal{L}}$' will denote the set of all such models.[5]

Formulas receive the following evaluation conditions, for all $\langle v, R\rangle \in \mathbf{M}_{\mathcal{L}}$ and all A, $B \in \mathcal{L}$:

- $\langle v, \mathcal{R}\rangle \models A$ if and only if $v(A) = 1$, if $A \in \mathrm{Var}$,

[5] There are also modal connexive logics, see, e.g., [16], but they can be safely left aside for the present discussion.

- $\langle v, \mathcal{R}\rangle \models \neg A$ if and only if $\langle v, \mathcal{R}\rangle \not\models A$,
- $\langle v, \mathcal{R}\rangle \models A \wedge B$ if and only if $\langle v, \mathcal{R}\rangle \models A$ and $\langle v, \mathcal{R}\rangle \models B$,
- $\langle v, \mathcal{R}\rangle \models A \vee B$ if and only if $\langle v, \mathcal{R}\rangle \models A$ or $\langle v, \mathcal{R}\rangle \models B$,
- $\langle v, \mathcal{R}\rangle \models A \rightarrow^{\mathrm{w}} B$ if and only if $A\ \mathcal{R}\ B$ and either $\langle v, \mathcal{R}\rangle \not\models A$ or $\langle v, \mathcal{R}\rangle \models B$.

Note that the evaluation conditions for the connectives are the classical ones, with the only exception of those for implication, which besides having its usual evaluation conditions, require that antecedent and consequent are related by $\mathcal{R}$.

To obtain suitable models for connexive logics in $\mathbf{M}_{\mathcal{L}}$, Jarmużek and Malinowski [15] impose the following (independent) conditions on $\mathcal{R}$:

(a1) for any $A \in \mathcal{L}$, $A\ \tilde{\mathcal{R}}\ \neg A$,
(a2) for any $A \in \mathcal{L}$, $\neg A\ \tilde{\mathcal{R}}\ A$,
(b1) for any $A, B \in \mathcal{L}$: $(A \rightarrow^{\mathrm{w}} B)\ \mathcal{R}\ \neg(A \rightarrow^{\mathrm{w}} \neg B)$ and either $A\ \tilde{\mathcal{R}}\ B$ or $A\ \tilde{\mathcal{R}}\ \neg B$,
(b2) for any $A, B \in \mathcal{L}$: $(A \rightarrow^{\mathrm{w}} \neg B)\ \mathcal{R}\ \neg(A \rightarrow^{\mathrm{w}} B)$ and either $A\ \tilde{\mathcal{R}}\ B$ or $A\ \tilde{\mathcal{R}}\ \neg B$.[6]

Note that (a1) and (a2) jointly imply that $\tilde{R}$ is symmetric for contradictory formulas.

Taking into account the evaluation conditions for the formulas in $\mathcal{L}$, Jarmużek and Malinowski obtain as a corollary the following result: for any $R \subseteq \mathcal{L} \times \mathcal{L}$ and A, $B \in \mathcal{L}$,

- If $\mathcal{R}$ fulfills (a1), then $R \models \neg(A \rightarrow^{\mathrm{w}} \neg A)$;
- If $\mathcal{R}$ fulfills (a2), then $R \models \neg(\neg A \rightarrow^{\mathrm{w}} A)$;
- If $\mathcal{R}$ fulfills (b1), then $R \models (A \rightarrow^{\mathrm{w}} B) \rightarrow^{\mathrm{w}} \neg(A \rightarrow^{\mathrm{w}} \neg B)$;
- If $\mathcal{R}$ fulfills (b2) then $R \models (A \rightarrow^{\mathrm{w}} \neg B) \rightarrow^{\mathrm{w}} \neg(A \rightarrow^{\mathrm{w}} B)$.

The more usual names for the schemas in the consequents are variant of Aristotle's Thesis, Aristotle's Thesis, Boethius' Thesis and Variant of Boethius' Thesis, respectively.

To obtain the converses of these implications, Jarmużek and Malinowski propose that $\mathcal{R}$ be closed under negation, that is:

(c1) $\mathcal{R}$ is *closed under negation* iff for any $A, B \in \mathcal{L}$: if $A\ \mathcal{R}\ B$, then $\neg A\ \mathcal{R}\ \neg B$

This condition is also independent of (a1), (a2), (b1) and (b2) and, as I have said, it allows the move from the appropriate models for connexive

[6] In [15] the second part of this condition appears in both (b1) and (b2). In [21, p. 21] the authors considered the condition "If $A\ \mathcal{R}\ \neg B$ then $A\ \tilde{\mathcal{R}}\ B$" instead.

logics in $\mathbf{M}_{\mathcal{L}}$ to the validity of the schemas (A1)- (B2) and vice versa. Furthermore, imposing (c1) on $\mathcal{R}$ results in the validity of the schema

$$\neg((A \rightarrow^{\mathrm{w}} B) \wedge \neg B \wedge \neg(\neg A \rightarrow^{\mathrm{w}} \neg B)))$$

which is not valid under the conditions (a1), (a2), (b1), (b2) only. For simplicity, in what follows I will speak only of relating models where all the conditions (a1), (a2), (b1), (b2) and (c1) are satisfied.

Finally, it is worth mentioning, especially for the discussion to come, that Detachment, $A, A \rightarrow^{\mathrm{w}} B \models B$, is valid in this semantics for connexive logics (verification is left to the reader).

3. The nature of $\mathcal{R}$

In Epstein's pioneering works on relatedness logic [see, e.g., 7, 42] the relating relation $\mathcal{R}$ was originally understood as a content-connection between antecedent and consequent in an implication. Later on, Jarmużek and Kaczkowski in [14] generalized Epstein's relatedness logic in two important respects. First, the binary relation between formulas is not necessarily regarded as a content-connection and in general is not subject to any constraint; the properties of the relation depend on the target phenomenon. Second, they allowed other connectives besides the implication to be evaluated by the satisfaction of a certain $\mathcal{R}$, not necessarily the same for all connectives. Possible interpretations of the relation are analytic containment, causation, temporal precedence, among many others.

Unlike other $\mathcal{R}$s, the $\mathcal{R}$ in the relating models for connexive logic is not given any determinate intuitive meaning. In fact, in [22] $\mathcal{R}$ is implicitly treated as a sui generis relation, and is simply called 'connexive relation'. Nonetheless, the founding paper of Boolean connexive logics does more in terms of elucidating the nature of $\mathcal{R}$. There, Jarmużek and Malinowski say:

> We can even assume that the fact that in a given model $A\ \mathcal{R}\ B$ we shall interpret as A *is connected to* B, whereas the fact that $A\ \tilde{\mathcal{R}}\ B$ can be interpreted as A *is not connected to* B. [15, p. 433]

To tell the truth, "A and B are connected" does not seem very different from "A and B are related". Something else should be said to characterize this (relating) relation of "connectedness".

Jarmużek and Malinowski seem to have in mind that connectedness is a certain relation between contents:

> There are strong indications that by dint of its application we can accurately and directly express the relations between the propositions that lie in the heart of connexive logic. These relations do not have to follow from a common lingual form, but — as mentioned before — from a content similarity that is not always expressible in a logical form.
>
> [15, p. 430]

But then the question is simply moved away, not answered. What is a content and what are the properties of the relation of content-similarity, at least the alleged content-similarity, the connectedness, involved in connexive logic?

Let me start with the notion of content. It is important because Jarmużek and Malinowski invoked it to refute the idea that the validity of the characteristic connexive schemas is sufficient as an analysis of connexivity:

> Still, even if we adopt propositions (A1), (A2), (B1), (B2) as axioms, no common content is guaranteed. For propositions (A1), (A2), (B1), (B2) are valid in binary matrix $\{1, 0\}$ with distinguished value of 1, with classical material implication and negation defined as: $\sim 1 = \sim 0 = 1$. Similarly, these propositions are true in a binary matrix with classical negation and implication defined as: $x \Rightarrow y = 1$ iff $x = y$.
>
> [15, p. 429]

The latter connective not only validates the characteristic connexive schemas, but it also delivers *strong connexivity* in the sense of Kapsner [17][7], "even though it has no virtues of reasonableness whatsoever".

Two issues have been conflated here. One is the unreasonableness of $x \Rightarrow y = 1$ iff $x = y$ (or of $\sim 1 = \sim 0 = 1$ for that matter), and the other is the lack of "connectedness" or "common content". If the former implies the latter, or the other way around, depends on how content and the connection between contents are defined. (And as I will show below, one cannot use the necessary condition for commonality of content from relevance logics.) To illustrate how commonality of content can be

[7] That is, it satisfies the following conditions:

Kapsner-strength 1: In no model, $A \to \mathsf{N}\, A$ is satisfiable (for any A), and in no model, $\mathsf{N}\, A \to A$ is satisfiable (for any A).

Kapsner-strength 2: In no model, $A \to B$ and $A \to \mathsf{N}\, B$ are simultaneously satisfiable (for any A and B).

achieved in this case, to a great extent independently of the evaluation conditions for complex formulas, consider the following case. Let S be a semantics, a set of interpretations. For simplicity, suppose that an interpretation is a function i from $\mathcal{L}$ to the set of values $\{1, 0\}$. Then:

- Let the *content of a formula* A, $\mathrm{Cont}(A)$, be the set of interpretations under which it is not true, that is, $\mathrm{Cont}(A) = \{i \mid i(A) \neq 1\}$.
- Say that A's and B's contents are *connected* if and only if $\mathrm{Cont}(A) \cap \mathrm{Cont}(B) \neq \varnothing$.
- Say that an implication $A \to B$ is *logically false* (in a logic $\boldsymbol{L}$), denoted $\models_{\boldsymbol{L}} {\sim}(A \to B)$, if and only if it has a logically true negation, and that it has a *logically true negation* if and only if $\mathrm{Cont}(A) \cap \mathrm{Cont}(B) = \varnothing$.

Then it is easy to verify that the heterodox valuations of implications and negations notwithstanding, $A \to {\sim} A$ and its variant are logically false: their components have no connected content under any evaluation. This would be so also with the classical valuations.[8]

Also, if the contents of $A \to B$'s components are connected, the contents of $A \to {\sim} B$'s components are not connected. Suppose that $\mathrm{Cont}(A) \cap \mathrm{Cont}(B) \neq \varnothing$, i.e., there is an interpretation i such that $i(A) = i(B) = 0$. Then, whether using classical or the heterodox negation, $i({\sim} B) = 1$, and this is so for any i that meets the previous condition. But then $\mathrm{Cont}(A) \cap \mathrm{Cont}({\sim} B) = \varnothing$.

Bearing upon the topological notion of connectedness, saying that the contents of A and B are connected if $\mathrm{Cont}(A) \cap \mathrm{Cont}(B) \neq \varnothing$ seems on the right track. Note, however, that this would induce properties for $\mathcal{R}$ beyond (a1), (a2), (b1), (b2). In particular, $\mathcal{R}$ would then be reflexive ($A \mathrel{\mathcal{R}} A$ for any A) and symmetric (if $A \mathrel{\mathcal{R}} B$ then $B \mathrel{\mathcal{R}} A$, for all A, B).[9]

In the next section, I will examine in more detail these further properties of $\mathcal{R}$. In the remainder of this section, I investigate whether 'connectedness' is just another name for *relevance*. Relevance is thought of

[8] The above notion of content can be found in [28] and can be traced back to certain ideas of Wittgenstein and Popper. The notion of content-connection is also familiar, but it is more commonly called 'content-sharing'. The only definition that might have no ancestor is the last one. Nonetheless, I do not mean to endorse the definitions just presented as superior to other related notions, but merely as aids to make some formal observations.

[9] Recall that *dis*connectedness was already assumed to be symmetric for contradictory formulas in virtue of (a1) and (a2).

as a reflexive and symmetric relation, like connectedness.[10] Actually, Routley [37, p. 393] said that

> the connexion requirement is commonly imposed as a requirement of meaning or content connexion between antecedent and consequent of valid implications. This requirement coincides with the broad requirement of relevance: for if antecedent and consequent enjoy a meaning connexion then they are relevant in meaning to one another, and if they are relevant in meaning to one another then they have through the relevance relation a connexion in meaning. Thus the general classes of connexive and relevant logics are one and the same.

Nonetheless, connectedness — or whatever name one uses for the $\mathcal{R}$ for connexive logic — is different from relevance. A hitherto unnoticed fact is that the relating approach allows to see it more clearly why a logic that is both relevant and connexive is contradictory. Let $\mathrm{Var}(A)$ be the set of propositional variables in A. A relatively weak notion of relevance, perhaps a necessary condition for it, is *variable-sharing*: A and B are relevant (to each other) iff they share at least one propositional variable; in symbols, iff $\mathrm{Var}(A) \cap Var(B) \neq \varnothing$. Now, suppose that we impose the following condition on $\mathcal{R}$:

(r1) for any $A, B \in \mathcal{L}$ such that $\mathrm{Var}(A) \cap \mathrm{Var}(B) \neq \varnothing$, we have $A\ \mathcal{R}\ B$.

Suppose furthermore that $\mathcal{R}$ is both (a1) and (r1), i.e., both connexive and relevant. Consider $A \to {\sim}A$. By (r1), $A\ \mathcal{R}\ {\sim}A$. But by (a1), $A\ \tilde{\mathcal{R}}\ {\sim}A$, but semantic contradictions of this kind are too much for most relevantists and connexivists.[11] (As a corollary, the content of A, $\mathrm{Cont}(A)$, is not the same as $\mathrm{Var}(A)$.)

This is but an example of the versatility of the relating approach. But it is also a signal that one must be extremely careful in determining what are the properties of the $\mathcal{R}$ for connexive logics.

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[10] See for example [35]. Nevertheless, Dunn [6] gives a quite different story, greatly influenced by the notation he uses, according to which relevance is reflexive but also transitive and non-symmetric.

[11] Since his [24], McCall claimed that connexive logics are related to relevance logic, but that the requisites for content-connection are stricter in the former. This seems to be confirmed in the proof above. See also [10] for an attempt to give more substance to McCall's claim.

4. Compatibility

As I have recalled in the introduction, some authors think that the relation involved in the understanding of connexivity is *(in)compatibility*. Pizzi [32] isolated the following properties of compatibility — or *cotenability*, as he prefers to call it — here suitable rewritten in terms of a relating relation $\mathcal{R}$:

- *Reflexivity*: for any A, we have $A \mathcal{R} A$.
- *Subalternation*: if $A > B$ is true, then $A \mathcal{R} B$.
- *Non-triviality*: $A \mathcal{R} B$ does not entail that $A > B$ is true.
- *Reflexivity of implication*: for any A, we have $A \tilde{\mathcal{R}} \mathsf{N} A$.
- *Modal compatibility*: for any A, we have $\Diamond A \mathcal{R} A$.

In a language with $\Diamond$, it will depend on the evaluation conditions of $\Diamond A$ whether Modal compatibility will be satisfied or not. Since I am working in a demodalized language, I leave Modal compatibility out of consideration. As for the other conditions, one can easily verify that $A \rightarrow^{\mathrm{w}} B$ and the $\mathcal{R}$ in the relating semantics for Boolean connexive logics satisfy all of them, with the only exception of Reflexivity: Subalternation and Non-triviality follow from the evaluation conditions for $A \rightarrow^{\mathrm{w}} B$, and the reflexivity of implication is just the condition (a1) for $\mathcal{R}$.

Not only Pizzi, but also Nelson before him, thought that the connexive implication is defined entirely in terms of compatibility. However, in the case of the relating semantics, the relation in the relating semantics and its restrictions are not alone for obtaining a connexive implication, but the truth condition is also needed.[12]

In the previous section, we found that the relating relation $\mathcal{R}$ between antecedent and consequent in an implication that holds good should enjoy Reflexivity and Symmetry, for all A and B, if $A \mathcal{R} B$ then $B \mathcal{R} A$, at least if we build it upon the (topological) notion of connectedness. Nonetheless, some logicians, especially from the relevance tradition, think that a relation that satisfies Reflexivity cannot be understood as compatibility. Here is Restall's argument against the reflexivity of compatibility[13]:

[12] We thank a reviewer for insisting that this must be said explicitly.

[13] Here is Restall *verbatim*:

> Given our interpretation of states, we do not want C to be reflexive, since some states are inconsistent. We would expect there to be states x such that xCx fails, for if xCx, then if $x \models \sim A$ we cannot have $x \models A$, as you would expect, since x is 'self-compatible', or as we will say from now on, x is *consistent*. [36, p. 61]

1. If x is an inconsistent state, there is an A such that $x \models \mathsf{N}\, A$ and $x \models A$.
2. $x \models \mathsf{N}\, A$ iff for every y such that xCy, $y \not\models A$.
3. Suppose that xCx.
4. Suppose that x is inconsistent.
5. There is an A such that $x \models \mathsf{N}\, A$ and $x \models A$
6. If $x \models \mathsf{N}\, A$ then $x \not\models A$.
7. $x \models \mathsf{N}\, A$.
8. $x \models A$.
9. $x \not\models A$.
10. $x \models A$ and $x \not\models A$.

1 is half of the definition of ‘inconsistent state’; 2 is the evaluation condition for negation in terms of compatibility between states and 3 is the assumption that compatibility is reflexive. 5 follows from 1 and 4; 6 follows from 2 and 3; 7 and 8 follow from 5. Finally, 9 follows from 6 and 7, and then 10 follows from 8 and 9, which is a semantic contradiction and, as I have said, that is not good even for many paraconsistentists.

The alleged moral of this argument is that compatibility is not reflexive, that is, that premise 3 above is untrue. But for others, the moral to be drawn is different. On this argument, De and Omori [5, p. 20] say:

> We would not say of an information state that supports A and not-A that it is self-incompatible, at least if we are going by our intuitive understanding of the notion. The fact that a state supports a contradiction gives us no reason to think that it is self-incompatible in the intuitive sense. [...] And if worlds, taken under any sense of the word [including “information states”], are not self-incompatible, then the paraconsistentist has strong reasons for rejecting compatibility semantics [i.e., negation’s evaluation condition in terms of compatibility between states].

Thus, they prefer the American Plan for evaluating negation just in terms of truth values, with no appeal to states and special relations between them.

De and Omori do not have connexive logic in sight[14], but their reaction might be shared by a connexivist, especially by one sympathetic

A similar argument against the reflexivity of compatibility is advanced in [1], and then Restall and Berto join forces in [2].

[14] Not consciously, at least, but it is worth mentioning that Omori is a noticeable supporter of connexive logics.

to the Incompatibility Approach. In fact, De and Omori's rejoinder echoes Everett Nelson's take on impossibility and inconsistency (I have changed Nelson's 'consistency' to 'compatibility' to make the similarity more visible):

> Prof. C. I. Lewis has defined "p and q are [compatible]" as "it is possible that p and q both be true". [...] For example, since "$2 + 2 \neq 4$" is impossible, and consequently cannot possibly be true, it follows, according to the above definition, that "$2+2 \neq 4$" is [incompatible] with any proposition whatsoever, regardless of its meaning. This I think is false. There is nothing in the meaning of "$2 + 2 \neq 4$" which is, or whose consequences are, [incompatible] with the meaning of every proposition. In general terms, p may be impossible but nevertheless [compatible] with some proposition q. Now Prof. Lewis's notion of [compatibility] is too narrow, for it excludes cases of propositions that are [compatible]; e.g., "$2 + 2 \neq 4$" and "$2 + 2 \neq 4$"; "$2 + 2 \neq 4$" and "Napoleon defeated Wellington". And its negative "[incompatibility]" is too broad, for it holds certain propositions to be [incompatible] which are not; e.g., the illustrations in the last sentence.

Here Nelson expresses his dissatisfaction with a theory that bases incompatibility between propositions on their joint impossibility (and especially when the joint impossibility is due only to one of the propositions), just as De and Omori rejected basing negation on incompatibility between states. To be more explicit: De and Omori reject the analysis of negation based on incompatibility between states because it leads to say that inconsistent states are not self-compatible, which is counterintuitive for them. Nelson, on the other hand, rejected the analysis of incompatibility based on the notion of impossibility, because impossibilities would be incompatible with every proposition, including themselves, which is not the case according to Nelson. Thus, they think that even contradictions (in the De and Omori case) and impossibilities more generally (in Nelson's case) are self-compatible, that is, that compatibility is reflexive.

So much for Reflexivity. What about Symmetry for $\mathcal{R}$? We have seen above that it makes sense for connectedness, especially connection of contents, understood as the non-empty *intersection* of contents. I will say a bit more about this issue in the next section. So far, I will close this section pointing out that Symmetry has been far less controversial than Reflexivity.

5. Implications

Another problem with current relating semantics for Boolean connexive logics is that no general properties of $\mathcal{R}$ to evaluate the truth of arbitrary implications are given. The conditions on $\mathcal{R}$ are just the minimal ones to validate the connexive schemas and Detachment. (And having as a consequence the invalidity of the symmetry of implication.) This is worth pointing out especially coming from logicians, like Jarmużek and Malinowski, interested in the reasonableness of certain connectives. Is meeting those requirements enough to claim that the binary connective is an implication?

To give more content to the reasonableness talk, I will focus on the validities usually expected from an implication worth the name. I endorse the following working principle:

- If in a logic there are implicative logical truths, asking $A > A$ to be among them is not so unreasonable.

I am not claiming that $A > A$ must be a logical truth in any logic with an implication. There are logics, such as $\mathbf{K}_3$ or **FDE**, with connectives recognizable as implications, for example, through their evaluation conditions, yet with no implicative logical truths, because this is prevented by the kind of admissible interpretations. I am merely saying that it is not unreasonable to expect the validity of $A > A$ in a logic with logical truths.

But not even this obliges one to necessarily ask for the validity of $A > A$ in all logics with implicative logical truths. When implication is understood in certain ways, there might be implicative logical truths without $A > A$ being among them. For example, in Goddard and Sylvan's works on reason-giving implications and non-ponible reasoning [see, e.g., 13] $A \ni B$, "A is a reason for B", is analyzed as "A (relevantly) implies B and A is prior enhancing information to B". This delivers Aristotle's Thesis, for A is not a reason against itself, but not $A \ni A$ because, in general, a proposition is not a reason for itself. In [11], a connexive logic based on discussive ideas where $A > A$ fails can be found.

There are no indications that implication in Boolean connexive logics is being understood as in Goddard and Sylvan's logics or as in Estrada-González, Nasieniewski and Nicolás-Francisco's, though. Then, in the presence of other implicative principles, such as Boethius' Theses, it is not so unreasonable to ask for the validity of $A \rightarrow^{\mathrm{w}} A$ as well. But if one insists on the validity of $A \rightarrow^{\mathrm{w}} A$, the relating relation $\mathcal{R}$ must be reflexive.

Let me briefly discuss an argument by Sylvan in [39] to the effect that understanding connexivity through compatibility is incompatible with accepting the logical validity of $A > A$, at least to the eyes of Chrysippus, the Megarians and other supporters of the third position mentioned by Sextus ('$\oplus$' stands for a generic disjunction):

1. $(A \otimes \mathsf{N}\, A) > A$ Simplification
2. $A > (A \oplus \mathsf{N}\, A)$ Addition
3. $(A \otimes \mathsf{N}\, A) > (A \oplus \mathsf{N}\, A)$ Transitivity 1, 2
4. $(A \otimes \mathsf{N}\, A) > \mathsf{N}(A \otimes \mathsf{N}\, A)$ Substitution 3, De Morgan
5. $(A \otimes \mathsf{N}\, A) \circ \mathsf{N}(A \otimes \mathsf{N}\, A)$ (IA2), 4
6. $\mathsf{N}((A \otimes \mathsf{N}\, A) > (A \otimes \mathsf{N}\, A))$ (IA0), 5

The last step is the negation of an instance of $A > A$, strongly suggesting the invalidity of $A > A$.

There are at least two comments to make regarding this argument. First, note that the contradiction between $(A \otimes \mathsf{N}\, A) > (A \otimes \mathsf{N}\, A)$ and $\mathsf{N}((A \otimes \mathsf{N}\, A) > (A \otimes \mathsf{N}\, A))$ is not the only contradiction that might look damaging to a connexivist: Step 4 already contradicts Aristotle's Thesis.[15] For me, Sylvan's argument is just another example of how easily connexive principles lead to contradiction; that a connexive implication leads to a contradiction in the presence of Simplification is no wonder. Perhaps the novelty here is that contradictions reach $A > A$.

That is related to my second comment. The step from '$(A > A)$ is acceptable (or valid)' to '$\mathsf{N}(A > A)$ is unacceptable (or invalid)' is invalid. It might be valid for the historic logicians that Sylvan has in mind, but in general it is not. **M3V** is a connexive logic where both $\models_{\mathbf{M3V}} (A \rightarrow A)$ and $\models_{\mathbf{M3V}} {\sim}(A \rightarrow A)$ are valid and, in that sense, both would be acceptable, at least to the eyes of an **M3V** supporter.[16] One can stare incredulously to **M3V**, doubt that its implication is reasonable at all and say that it has no bearing at all on Sylvan's argument. I disagree on all counts. It seems unsurprising that contradictions around a "core" principle of implications (like $A > A$) arise when a good account (the Incompatibil-

[15] In fact, [26] presented the argument from 1 to 4 as a *reductio* of both Simplification and Addition.

[16] **M3V** was introduced in [24] and "popularized" in [29]; it has been further studied in [8, 40].

ity Approach) of a good implication (a connexive one) is combined with certain principles like Simplification, known for producing contradictions for connexive implications. Does this contradiction make the arrow in **M3V** a non-implication, so to speak? It hardly does that. The implication in that logic validates the usual expected argument schemas, for example, Identity, Detachment, and at least its truth condition[17] seems implication-ish, even if convoluted, and it invalidates argument schemas that could make it another connectivesee, for example, $A \models_{\mathbf{M3V}} A \rightarrow B$ or $A \rightarrow B \models_{\mathbf{M3V}} A$. That it validates contradictions makes it contradictory, not a non-implication.

One could insist that, like the ancient third positioners, Boolean connexive logics are after *consistent* connexive logics, so the validity of any contradictory pairs, such as $\mathsf{N}((A \otimes \mathsf{N}\, A) > \mathsf{N}(A \otimes \mathsf{N}\, A))$ and $(A \otimes \mathsf{N}\, A) > \mathsf{N}(A \otimes \mathsf{N}\, A)$, or $(A > A)$ and $\mathsf{N}(A > A)$, is unacceptable. But contradictions are not obtained in Boolean connexive logics. As we have seen, implicative logical truths are sparse in them. Indeed, none of steps 1 to 4 are valid in Boolean connexive logics.[18]

The question now is whether some of the steps 1 to 4 in Sylvan's argument should be valid alongside $A > A$. For some authors, e.g., [45], the failure of Simplification is "highly unintuitive", to the point that they prefer accepting negation-inconsistency instead of losing Simplification. As I have said, negation-inconsistency is typically not an option for supporters of Boolean connexive logics. Moreover, given that Boethius' Thesis and Detachment are valid in Boolean connexive logics, Simplification cannot hold on pain of contradiction, as was already observed by Routley [38, p. 82]: from $(A \otimes \mathsf{N}\, A) > A$ and $((A \otimes \mathsf{N}\, A) > A) > \mathsf{N}((A \otimes \mathsf{N}\, A) > \mathsf{N}\, A))$, an instance of Boethius' theses, one obtains $\mathsf{N}((A \otimes \mathsf{N}\, A) > \mathsf{N}\, A)$ by Detachment, which contradicts $(A \otimes \mathsf{N}\, A) > \mathsf{N}\, A$.

But are there arguments for the invalidity of Simplification that come directly from the very nature of $\mathcal{R}$, and not indirectly from the fact that it entails contradictions? Said otherwise, what are the reasons for preferring $(A \otimes B)\ \tilde{\mathcal{R}}\ A$ and $(A \otimes B)\ \tilde{\mathcal{R}}\ B$ rather than $(A \otimes B)\ \mathcal{R}\ A$

[17] In **M3V**, the evaluation conditions for implication are the following: (i) $(A \rightarrow B)$ is true if and only if A is not true, or B is not false, or both A is false and B is true; (ii) $(A \rightarrow B)$ is false if and only if A is true, or A is false and either B is true or B is false.

[18] Simply put $(A \otimes \mathsf{N}\, A)\ \tilde{\mathcal{R}}\ A$ and $A\ \tilde{\mathcal{R}}\ (A \oplus \mathsf{N}\, A)$. Whether those countermodels make sense at all will be discussed below.

and $(A \otimes B)\ \mathcal{R}\ B$; indeed, for positing the former in order to avoid contradictions? The answer is basically Nelson's in [30]. Consider $A \otimes B$ and A. A is related to (is compatible with) A, but that is not sufficient to claim that A is related to $A \otimes B$, because B may be $\mathsf{N}\, A$ or any other proposition incompatible with A, which would prevent the relation between A and the whole of $A \otimes B$. This does not mean that any instance of Simplification is invalid, from which one could get $\mathsf{N}((A \otimes B) > A)$: $(A \otimes A) > A$ is valid, because the consequent is related to $A \otimes A$ in being related to each conjunct.

Someone might argue that all instances of Simplification fail because the antecedent does not imply the consequent. In all cases of Simplification there is a part of the antecedent that is either redundant (if B is A) or irrelevant for obtaining the consequent. Let me call 'the strong effective use approach' such explanation of the failure of Simplification. Nelson himself suggested this approach to implication.[19] However, the strong effective use approach is different from the compatibility approach, as the latter need not distinguish occurrences of formulas: both As in $A \otimes A$, not only one of them, are compatible with A.[20]

How could one recover Symmetry from considerations about implication? Pizzi [32] proved that compatibility is symmetric if and only if the connexive implication (defined in terms of compatibility) contraposes, i.e., if $A > B$ if and only if $\mathsf{N}\, B > \mathsf{N}\, A$. The proof cannot be copied verbatim in a relating semantics, though, as Pizzi makes a considerable number of crucial assumptions, in particular, that all forms of contraposition (in particular, that $A > B \models \mathsf{N}\, B > \mathsf{N}\, A$ and $\mathsf{N}\, B > \mathsf{N}\, A \models A > B$, on the hand, and $A > \mathsf{N}\, B \models B > \mathsf{N}\, A$, on the other) are equivalent. A proof of the symmetry of $\mathcal{R}$ through the contraposible nature of $A \to^{\mathrm{w}} B$ would crucially require that $\mathcal{R}$ satisfies the following condition:

(c2) $\mathsf{N}\,\mathsf{N}\, A\ \mathcal{R}\ B$ if and only if $A\ \mathcal{R}\ \mathsf{N}\,\mathsf{N}\, B$.

---

[19] Although not as strong as described here. Nelson [30, p. 447] says:

> Naturally, in view of the fact that a conjunction must function as a unity, it cannot be asserted that the conjunction of p and q entails p, for q may be totally irrelevant to and independent of p, in which case p and q do not entail p, but it is only p that entails p. I can see no reason for saying that p and q entail p, when p alone does and q is irrelevant, and hence does not function as a premise in the entailing.

Note that he does not count redundancy as a reason to block Simplification.

[20] Incidentally, providing a sound and complete semantics for Nelson's logic **NL** [for a perspicuous presentation, see 23] is an important open problem. See [34] for an interesting yet currently incomplete attempt to solve it using relating semantics.

Are these reasonable assumptions to make for a connexive implication? This is difficult to determine. Contraposition is involved in certain derivations of contradictions[21], but once Simplification has been given up, there are no obstacles to accepting Contraposition. On the other hand, the additional constraint on $\mathcal{R}$ to obtain a contraposable implication fits well with the Boolean nature of negation. This is recognized by Jarmużek and Malinowski in [16, p. 229], where they consider (c2) but as a constraint for a relating relation between indexes of evaluation that validates the dualities between $\Box A$ and $\Diamond A$:

> Although formulas: A and $\neg\neg A$ are different from the syntactic point of view, let us note that we operate with the classical, Boolean negation $\neg A$. So it looks reasonable to add as the next constraint:
>
> $$A \,\mathcal{R}_w\, B \Longleftrightarrow \neg\neg A \,\mathcal{R}_w\, B \Longleftrightarrow A \,\mathcal{R}_w\, \neg\neg B. \tag{$\neg\neg$}$$
>
> It says that double (classical!) negation has no influence on being connected. Condition ($\neg\neg$) imposed on the models could be the next enhancement of our modal Boolean connexive logics, of course.

What they say concerns the expected properties of negation, not indexes, so this property could equally be assumed in the non-modal case.

6. Boolean connexive logics and the Incompatibility Approach

At the very beginning of this paper, I claimed that the relating semantics for Boolean connexive logics can be understood as a version of the

[21] For example, Aristotle's earliest one in *Prior Analytics* 57^b3:

1.	$(A \otimes \mathsf{N}\, A) > A$	Simplification
2.	$(A \otimes \mathsf{N}\, A) > \mathsf{N}\, A$	Simplification
3.	$\mathsf{N}\, A > \mathsf{N}(A \otimes \mathsf{N}\, A)$	1, Contraposition
4.	$(A \otimes \mathsf{N}\, A) > \mathsf{N}(A \otimes \mathsf{N}\, A)$	2, 3, Transitivity of implication

where 4 contradicts Aristotle's Thesis, or Alberic of Paris more general one:

1.	$A > B$	Hypothesis
2.	$\mathsf{N}\, B > \mathsf{N}\, A$	1, Contraposition
3.	$(A \otimes \mathsf{N}\, B) > A$	Simplification
4.	$\mathsf{N}\, A > \mathsf{N}(A \otimes \mathsf{N}\, B)$	3, Contraposition
5.	$(A \otimes \mathsf{N}\, B) > \mathsf{N}\, B$	Simplification
6.	$(A \otimes \mathsf{N}\, B) > \mathsf{N}\, A$	2, 5, Transitivity of implication
7.	$(A \otimes \mathsf{N}\, B) > \mathsf{N}(A \otimes \mathsf{N}\, B)$	4, 6, Transitivity of implication

where 7 contradicts again Aristotle's Thesis. On the rejection of Simplification as a solution to the latter, see [9].

Incompatibility Approach and, moreover, that such a semantics can be naturally enriched so as to make it indistinguishable from the Incompatibility Approach.

Let me first guarantee that the relating relation $\mathcal{R}$ in the semantics for Boolean connexive logic plays the role of compatibility, at least in the sense of Pizzi. Recall those properties of compatibility:

- *Reflexivity*: for any A, we have $A\,\mathcal{R}\,A$.
- *Subalternation*: if $A > B$ is true, then $A\,\mathcal{R}\,B$.
- *Non-triviality*: $A\,\mathcal{R}\,B$ does not entail that $A > B$ is true.
- *Reflexivity of implication*: for any A, we have $A\,\tilde{\mathcal{R}}\,\mathsf{N}\,A$.

We found that, if something like the notion of connectedness is behind the notion of connexive connection, $\mathcal{R}$ must be reflexive. Subalternation is satisfied in virtue of the evaluation condition of $\rightarrow^{\mathrm{w}}$. Also, the evaluation condition of $\rightarrow^{\mathrm{w}}$ ensures that Non-triviality is the case.

In the previous section, I discussed whether $A > A$ can be reasonably required from any $>$. I argued that it can be so required if the logic has implicative truths. By the evaluation conditions of $\rightarrow^{\mathrm{w}}$, the validity of $A \rightarrow^{\mathrm{w}} A$ entails, for all A, v and $\mathcal{R}$: $A\,\mathcal{R}\,A$ and either $\langle v, \mathcal{R}\rangle \not\models A$ or $\langle v, \mathcal{R}\rangle \models A$. But then, by (a1) or any of the conditions (b), $A\,\tilde{\mathcal{R}}\,\mathsf{N}\,A$. Therefore, $\mathcal{R}$ has the Pizzian properties of compatibility. In such a case let me say that $\mathcal{R}$ is a compatibility relation, or a Pizzian relation.

Now, what could be the (relating) evaluation condition for a binary compatibility connective, $A \circ B$? One could say that A and B are compatible if and only if they are both true, or one is not related to the negation of the other, that is[22]

- $\langle v, \mathcal{R}\rangle \models A \circ B$ if and only if either $A\,\tilde{\mathcal{R}}\,\mathsf{N}\,B$ or both $\langle v, \mathcal{R}\rangle \models A$ and $\langle v, \mathcal{R}\rangle \models B$.

Recall the Incompatibility Approach:

(IA0) $A > B =_{\mathrm{def.}} \mathsf{N}(A \circ \mathsf{N}\,B)$
(IA1) $\models_{\boldsymbol{L}} A \circ A$
(IA2) $A > B \models_{\boldsymbol{L}} A \circ B$

[22] The condition below makes quite evident that compatibility is in general different from intensional conjunction, a relevantist fusion-like connective with the following evaluation condition:

- $\langle v, \mathcal{R}\rangle \models A \,\&\, B$ if and only if $A\,\mathcal{R}\,B$, $\langle v, \mathcal{R}\rangle \models A$ and $\langle v, \mathcal{R}\rangle \models B$.

In a Boolean connexive logic $\boldsymbol{L}$ with both $\circ$ and & one would have $A \,\&\, B \models_{\boldsymbol{L}} A \circ B$ but not vice versa. This fact is not sufficiently stressed in, for example, [34].

(IA3) ©$(\dots A \dots) \dashv_{\boldsymbol{L}}\models$ ©$(\dots \mathsf{N}\,\mathsf{N}\,A \dots)$
(IA4) If $\mathit{\Gamma}, A \models_{\boldsymbol{L}} B$ then $\mathit{\Gamma} \models_{\boldsymbol{L}} A > B$

It is easy to verify that, in a Boolean connexive logic $\boldsymbol{L}$, $\models_{\boldsymbol{L}} A \circ A$, by (a1). Thus, one has (IA1). If $\mathcal{R}$ is Pizzian, and I have just shown that it should be so in a Boolean connexive logic, $A \circ B$ iff $\mathsf{N}(A > \mathsf{N}\,B)$, and hence $A > B$ iff $\mathsf{N}(A \circ \mathsf{N}\,B)$, satisfying thus (IA0).

Now suppose that $\langle v, \mathcal{R}\rangle \models A \rightarrow^{\mathrm{w}} B$. Therefore, either $\langle v, \mathcal{R}\rangle \not\models A$ and $A\,\mathcal{R}\,B$, or $\langle v, \mathcal{R}\rangle \models B$ and $A\,\mathcal{R}\,B$. In either case, $A\,\mathcal{R}\,B$. Therefore, by the (b) conditions, $A\,\tilde{\mathcal{R}}\,\mathsf{N}\,B$, which is enough for $\langle v, \mathcal{R}\rangle \models A \circ B$. Therefore, one has (IA2). (IA3) comes almost for free by (c2), whereas the verification of (IA4) is routine and left to the reader.

Summarizing: With some extra plausible conditions for $\mathcal{R}$, obtained by analyzing the notions of connectedness and implication, it can be shown that $\mathcal{R}$ is a compatibility relation, at least in the sense of Pizzi, and that one can obtain the desiderata of the Incompatibility Approach.

7. Final words

In this paper, I started from the observation that, in its current state, relating semantics to Boolean connexive logics does not give an account of what connexivity is, but merely provides machinery to validate the usual connexive schemas, no more, no less. Nonetheless, I showed, using both the informal paraphrases of the relating logicians regarding the $\mathcal{R}$ in Boolean connexive logics and some considerations about the nature of implication, that the relation in the relating semantics for Boolean connexive logics is a compatibility relation, at least in the sense of Pizzi [32]. Moreover, I showed that the relating semantics can be naturally enriched so as to make it virtually indistinguishable from the Incompatibility Approach. According to that account of connexivity, an implication holds good if and only if the antecedent is incompatible with a negation of the consequent. In the process, I showed that connexivity and minimal relevance (understood as variable sharing) are contradictory, and discussed some common relevantist arguments against the reflexivity of compatibility.

8. Epilogue

Alas, the times of research and writing are not the times of publication. After completing the first full draft of this paper, I was fortunate enough to meet Mateusz Klonowski. He shared the general concern about how to understand the $\mathcal{R}$ in the relating semantics for Boolean connexive logics. His deep knowledge of relating logic and Epstein's set-assignment semantics led him to suggest that, in the case of connexive logic, the relation $\mathcal{R}$ represents either a form of content-inclusion or a form of content-equality, although this needed to be proved. Klonowski did it in [19], thus showing that relating semantics for Boolean connexive logics can be developed into a full, good account of connexivity based on some forms of content-sharing. Impressive as [19] is, mainly due to Klonowski's insights, I still prefer the understanding of connexivity through (in)compatibility, and thus reading $\mathcal{R}$ in terms of compatibility. If compatibility can be formally connected to content inclusion or content equality as defined in [19], it remains to be shown.

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