



María del Rosario Martínez-Ordaz 

## Relating Logics Meet Scientific Understanding: On Clustering and the Methodology of Logical Classification

**Abstract.** In this paper, I discuss the legitimacy of the cluster of logics that are considered to be *relating logics*. I argue that even if the cluster ‘‘Relating logics’’<sup>1</sup> satisfies the basic theoretical criteria for legitimate clusters, logicians and philosophers of logic have failed at providing the corresponding pragmatic backing. In response to this, I propose to provide the pragmatic justification of the cluster by showing a domain of application for which ‘‘Relating logics’’ fits more adequately the evidence than any of their rivals do; such a domain is the phenomenon of *scientific understanding*. Finally, I argue that characterizing scientific understanding as a relating phenomenon provides a more accurate description than those offered by rival views.

**Keywords:** relating logics; clustering; scientific understanding

### 1. Introduction

The question that I address here is how we can explain the legitimacy of the cluster labeled ‘‘Relating logics’’? On the one hand, *clustering* is a methodology used to find patterns and group objects according to such patterns; the resulting groups are called *clusters*. However, not all the patterns that are found through clustering are considered legitimate or truly meaningful, in the sense that they correspond to a legitimate underlying grouping. *Legitimate cluster* should exhibit a combination of formal and pragmatic characteristics; which go from considerations

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<sup>1</sup> In what follows, ‘‘Relating logics’’ will be used to designate the cluster that encompasses relating logics.

about degrees of (dis)similarity up to problem-solving applications of the clusters [cf. 20, p. 56]. I discuss these issues in Section 3.

On the other hand, *relating logics* is a family of logics of relating connectives. The key objects of their attention are relations, and their main goal is to account “for the relation of sentences in terms of various connections: content relationship (analyticity, relevance, etc.), causality, temporal order, preference order, etc.” [29, p. 579] — they do this via the implementation of certain connectives. These logics are built on the assumption that, at least in some cases, the relationship that exists between two or more propositions should not be reducible to the combination of the truth values of the individual components. However, the broad way in which relating logics are characterized has led to the counterintuitive inclusion of logics that may appear highly dissimilar into the same cluster, ‘Relating logics’.

The combination of the above, raises the following question: if many of the members of the cluster ‘Relating logics’ can also be classified and easily understood under more traditional categories (relevant, paraconsistent, conditional, etc), why should we consider that there is anything that is shared by these logics that makes them saliently ‘relating’? This is, how can we explain the legitimacy of the cluster labeled ‘Relating logics’?

While tackling this question, my main thesis can be summarized by the following observations:

- Even if the cluster ‘Relating logics’ satisfies the basic theoretical criteria for legitimate clusters, logicians and philosophers of logic have failed at providing the corresponding pragmatic backing.
- A way to provide the pragmatic justification of the cluster is to show a domain of application for which ‘Relating logics’ fit more adequately the evidence than any of their rivals do; such a domain is the phenomenon of *scientific understanding*.
- The characterization of scientific understanding as a relating phenomenon provides a description of it that is more accurate than the ones offered by rival views.

In order to do this, I proceed as follows. First, in Section 2, I provide a general overview of relating logics. In Section 3, I introduce a way to understand families of logics as clusters and I discuss the basic criteria for assessing their legitimacy. In Section 4, I use paraconsistent logics to illustrate how these criteria are implemented. Section 5 is de-

voted to the use of clustering to challenge the legitimacy of the cluster 'Relating logics', particularly its pragmatic justification. In Section 6 and Section 7, I propose a way to deal with such a lack of justification by pointing out a domain of application for which relating logics can account better than any of their rivals. Here, I explain that the phenomenon of scientific understanding can be such a domain of application. Finally, in Section 8, I draw some conclusions.

## 2. Relations and relating logics

This section aims at introducing the basics of Relating logics. For methodological reasons and given the scope of the main concern of this paper, Relating logics will be approached and characterized broadly. Here, I start with some preliminaries about relations (Section 2.1) and I move to sketch the generalities of Relating logics and the corresponding grouping under the label 'Relating logics'.

### 2.1. Preliminaries: relations

A *relation* between  $A$  and  $B$  is a way in which  $A$  and  $B$  are connected, allowing us to state the fact of some link between what  $A$  and  $B$  takes place. Relations are typically understood as abstract entities that exist independently of the things they relate.

Relations can be of different degrees or adicity, but all relations are many-place. Relations can possess certain properties that define their characteristics; which include reflexivity, symmetry, transitivity, and so on. For example, 2-place or binary relations can be either symmetric, non-symmetric, or asymmetric, among others. Relations can have different presentations depending on their objects and the context in which they exist and are interpreted. For example, there are spatial relations (e.g., 'being above of'), temporal relations (e.g., 'before' and 'after'), causal relations (e.g., 'cause and effect'), and logical relations (e.g., 'entails'), among others.

Relations also have different ontological statuses; this depends both on the type of relationship that they instantiate as well as on the theoretical commitments that one decides to endorse when addressing them. On the one hand, metaphysicians have privileged the ontological role of relations like grounding and emergence over other types of relations like

preference order. In addition, some have taken relations to exist as abstract entities in their own right, while others propose that relations are merely conceptual or linguistic constructs that arise from the properties of the entities they relate to.

## 2.2. Relating logics

*Relating logics* are built on the assumption that logic helps us to evaluate not only sentences via models but also the relationships that hold between those sentences [see 22]. What is important to note in this case is that such relations could be of different types: causal, temporal order, and preference order, among others.

The initial motivation for relating logics can be found in some logics proposed by Epstein [see 14], which were designed to account for *content-relationships* in conditional sentences. Epstein logics, initially, preserved classical negation and conjunction, but modified the conditional by assuming the truth condition of material implication and emphasizing a special relationship between the antecedent and the consequent. Epstein logics were called *relatedness logics* and consisted of two systems: the logic **S**, also known as the Symmetric Relatedness Logic, and the logic **R**, i.e., the Non-symmetric Relatedness Logic, which is a sub-logic of **S** [see 29]. In the 1980s, Epstein deepened the analysis of content-relationship by focusing on the inclusion of sentence content. The resulting logics are called *dependence logics*, and consisted mainly of three systems: the logic **D**, also known as Dependence Logic, the logic **DD**, i.e. dual Dependence Logic, and the logic **Eq**, also known as the Logic of Equality of Contents.

Epstein contributions to this topic are vast, but, at the same time, most of them are of narrow interest: the relations that hide behind conditional sentences. For Epstein, a conditional sentence is true *iff*: the antecedent is true or the consequence is false, as well as if both parts are related because of their content [cf. 14, 15]. This relation is often of one of these two types: *overlap* and *inclusion*. The former means that

$A$  is content-related to  $B$  *iff* the content of  $A$  and the content of  $B$  have something in common. [29, p. 6]

The latter, the inclusion of content, means that

$A$  is content-related to  $B$  iff the content of  $B$  is included in

- (B1) the content of  $A$
- (B2) the content of  $A$  is included in the content of  $B$
- (B3) the content of  $A$  and the content of  $B$  are the same. [29, p. 6]

One of the main features of Epstein’s work is the conceptual openness that surrounds the notion of relatedness. Because his main intention was to be able to capture as many relationships as possible, the logical constraints of this notion are quite fuzzy. This is, while Epstein often considers that the minimal property that relatedness has to fulfill is reflexivity — all propositions should relate at least to themselves, he is quite open to the idea of, depending on the object of study, requiring other properties such as symmetry or transitivity.

From a methodological perspective, there are three main aspects of Epstein’s understanding of *relatedness*: conceptually is a primitive notion, and it indicates a type of relation that holds between objects denoted by propositional variables. In addition, in Epstein’s view, the implementation of logic in the analysis of the relatedness relations that hold between sets of formulae facilitates a more specific and fine-grained understanding of the relatedness of sentences.

While the work of Epstein is considered to be a touchstone for the philosophical and formal development of relating logics, the label of “relating logics” and the current understanding of it is due to the work of Jarmužek [see 22], and Jarmužek and Kaczkowski [see 23]. In this respect, their most important contribution to the literature consists of a broader approach to relating logic and relating semantics.

According to them, relating logics are logics of *relating connectives*; this emphasizes the idea that different (non-monadic) logical operators can be relating connectives — this is, relatedness is not exclusive to conditional sentences. Intentional interpretations of both conjunction and implication are the clearest examples of this. For instance, in those cases in which conjunction is used to express any type of order (temporal, preference, etc), it becomes evident that the link that holds between the conjuncts runs deeper than just the truth condition of the classical conjunction.

The following works in the direction of broadening the scope of the research on relatedness have also involved the incorporation of multi-relating models, many-valued (or subtly graded) evaluations of the relationships, as well as the enrichment of these logics with possible world semantics, among others. Formally speaking, these ideas have headed in very interesting directions; however, there are still important philosophical questions concerning the generalities of the grounds of relating logics.

To date, given the formal and philosophical complexity of relating logics there are significant philosophical issues associated with them. Some of these issues include questions concerning the nature of relations in general and whether relating logics can help to improve our current understanding of specific relationships — such as connexivity [see 24]). Another subset of philosophical issues concerns the expressivity of these logics and whether they actually can succeed at portraying the salient features of different types of relations. Finally, another

set of philosophical challenges that these logics face concerns their applicability and domain-specificity; in particular, whether there are inherent limitations to their applicability in different domains and whether there is a preferred intended domain that all relating logics should attempt to account for.

The plan for the rest of the paper consists in tackling a methodological challenge that emerges from both philosophical assumptions about the understanding of relations that is assumed in relating logics and from the (narrow) scope of applicability of these logics. In order to do so, the next section is devoted to addressing the methodological criteria followed when grouping families of logics in philosophical logic.

### 3. Families of logics as clusters

This section aims at explaining a way to understand and assess the legitimacy of families of logics. In order to do so, I use some of the basic elements of cluster analysis and I propose to characterize families of logics as clusters. I start with some preliminaries about clusters. Then, I proceed to discuss what it would mean to characterize families of logics as clusters. Finally, I provide some insight on how to determine their legitimacy.

#### 3.1. The basics about clustering

*Clusters* are groups of objects that share a set of attributes or characteristics.<sup>2</sup> *Clustering* is the methodology that is used to identify such groups. Broadly speaking, clustering is used to find patterns, however, not all the patterns that are found (and the corresponding clusters) are considered *truly meaningful*—in the sense that they correspond to a legitimate underlying grouping.<sup>3</sup>

While there are still many open discussions concerning the necessary and sufficient conditions that a cluster should satisfy for being legitimate, there is a common agreement on some of the desired theoretical characteristics, including the following:

- The dissimilarities within-cluster are small.
- The dissimilarities between-cluster are large.
- Members of a cluster should be well represented by its centroid.

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<sup>2</sup> It is important to notice that, while in the last decades clustering has evolved to become a statistical method, it is, at least partially, rooted in the philosophical practice of clustering into categories, types, and (natural) kinds [cf. 20, pp. 53–55].

<sup>3</sup> In the literature about cluster analysis, *legitimate clusters* are often also called *true clusters*. However, this latter label has a realist connotation, closely linked to natural kinds, that is not relevant for the purposes of the paper, and thus it will be avoided.

- Clusters should be characterizable using a small number of variables.
- Features should be approximately independent within clusters. [cf. 20, p. 56]
- The validity of a cluster depends on its capability of best fitting natural partitions without any a priori class information. [cf. 49]

In addition to these characteristics, there is a saliently pragmatic criterion that should not be overlooked when assessing the legitimacy of a cluster. This criterion is dual:

- On the one hand, it is expected that there is a given well-determined application for having chosen a particular partition — this is, the chosen partition should have a problem-solving relevance.
- On the other hand, it is desired that the partition corresponds to an independent domain (independent from the one that underlies the explanation of such partition and its relevance).

The idea behind this pragmatic component is that the clusters that are identified can aid the clarification and transparent comparison of methods as well as the explanation and understanding of the domain that the partition corresponds to.

Finally, it is important to note that clustering aims to build bridges between abstract patterns and the most salient epistemic goals of different scientific disciplines.

### 3.2. Families of logics as *legitimate* clusters

Throughout the history of (philosophical) logic, logicians and philosophers have classified different logical frameworks into *families* considering their most salient characteristics — including principles, theorems, and aims, among others. The justification behind this type of grouping is manifold.

First, it emerges from the intuitive recognition of patterns and common features that are shared among different and independent logical systems. This is, logics that have been grouped together have an important level of family resemblance either syntactically, semantically, structurally, or philosophically. However, these similarities cannot be superfluous and should be significantly nested within the core of each of the logics that belong to a family. This reinforces the intuitive legitimacy of the pattern that is shared.

Second, methodologically speaking, the classification of logics into families eases the scrutiny, explanation, and understanding of the features of each of the members of these groups. It also allows analogical reasoning to play an important role in some of these explorations. Furthermore, this type of classification aids the identification of distinct methodological approaches to reasoning and formalization that are shared (or could/should be shared) among logics that have other types of characteristics in common.

Third, the classification of logics into families highlights the fact that certain families of logics have been developed and studied specifically to address particular applications or domains. These applications can be very abstract—going from our theoretical understanding of necessity and possibility, up to our intuitions regarding the general behavior of conditionals and relations, or can be as concrete as trying to provide a logical representation for specific scientific theories or particular social and epistemic practices.

Fourth, the classification of logics into families also has a pedagogical role. It provides a systematic way to introduce different logics as well as formal and philosophical features that are present in them. This allows students to see and understand logic as a structured field of inquiry.

It is important to note that the classification of logics into families is neither rigid nor definitive. On the one hand, there are significant overlaps and variations between families, because depending on the features that are considered salient a logic can meet the conditions necessary to fit into more than one family. On the other hand, these classifications are most of the time pragmatically driven and therefore, tend to change depending on the goals that are considered. This is, most of the time, there is a specific methodological/epistemic/philosophical goal that is sought to be met through the implementation of a particular classification.

All this considered, the common usage of “families” in philosophical logic can be seen as a form of clustering. In particular, families of logics emerge from the initial identification of a few key similarities that are deemed significant within a specific research program. However, as it occurs in cluster analysis, not all potential groupings are (equally) legitimate. With this in mind, I propose to take the following as basic characteristics for legitimate clusters in philosophical logic:

1. **Salient features:** A cluster is characterized using a small number of concrete variables, which can be semantic, syntactic, philosophical, etc.<sup>4</sup>
2. **Selective filtering:** When combined, the selected salient features should work as a selective filter. This is, they should allow for the selection and grouping of items that meet certain criteria while simultaneously filtering out items that do not meet those criteria.
3. **Centroid:** The centroid of a cluster consists of the list of the selected salient features.
4. **Necessity and sufficiency:** The centroid should encompass the necessary and sufficient conditions that a logic must meet within a given framework to qualify as a member of the cluster.

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<sup>4</sup> It is important to note that these features are considered noteworthy within a specific research program, theoretical framework, or given goal.

5. **Centroid's relation:** Logics within a cluster should be represented by its centroid.
6. **Degrees of (dis)similarities:** While the dissimilarities within-cluster should be small (or overlookable), the dissimilarities between-clusters should be large.
7. **(Independent) Partition:** The legitimacy of a cluster of logics depends on its capability of best fitting an independent or natural partition. This provides justification for the claim that the cluster is motivated by an independent domain/phenomenon.
8. **Applicability:** A legitimate cluster should have a well-determined application — preferably with problem-solving purposes.

Summing up, families of logics are clusters grouped having in mind a minimal set of well-defined characteristics that are shared by all of the members of the group. These characteristics not only unite the logics within the family under a common label but also serve to distinguish them from other types of logics. Legitimate clusters of logics should satisfy the criteria (1)–(8).

From the outset, I want to be clear about the dialectic. None of these criteria, individually or combined, suffice for justifying any metaphysical claims about the utter legitimacy of families or sets of logics. Even if a cluster is legitimate, this only concerns its methodological legitimacy; this is, the cluster is a legitimate grouping of objects of study that is more useful than its rivals in a particular context for a problem-solving enterprise.

#### 4. An example: $\lrcorner$ Paraconsistent logics $\lrcorner$

Here, I briefly illustrate the use and evaluation of clusters in the philosophy of logic using paraconsistent logics as an example.

##### 4.1. The cluster $\lrcorner$ Paraconsistent logics $\lrcorner$

Take, for instance, the case of the family of the so-called paraconsistent logics. “A logic is paraconsistent *iff* its logical consequence relation ( $\models$ , either semantic or proof-theoretic) is not explosive” [41].

**Salient features:** The features used to characterize the members of the cluster labeled  $\lrcorner$ Paraconsistent logics $\lrcorner$  are:

- (i<sub>P</sub>) a logic is paraconsistent if it does not validate the Principle of Explosion,
- (ii<sub>P</sub>) there is a further justification for doing so.<sup>5</sup>

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<sup>5</sup> It is important to emphasize that this characterization is set by following some pragmatic criteria and that it does not aim at being joint carving about the nature of paraconsistency.

**The centroid:** is the combination of  $(i_P)$  and  $(ii_P)$ .

## 4.2. Is it a cluster?

In order to assess whether the broad group of paraconsistent logics can be recognized as a cluster, let's explain the roles that  $(i_P)$  and  $(ii_P)$  play in determining the centroid of the cluster.

**Selective filtering:** First,  $(i_P)$  and  $(ii_P)$  are components of a minimal characterization of paraconsistent logics, which when combined become a selective filter.

On the one hand,  $(i_P)$  works as a filter in the sense that it distinguishes a specific group of logics from a broader sample that includes explosive logics (going from classical logic and its extensions, up to some 'non-classical' logics, like intuitionist logic). On the other hand,  $(i_P)$  is inclusive enough to allow for a variety of logics to be covered by the same label, even if they satisfy this condition in very different manners and for very diverse reasons. This is reflected in the fact that the cluster 'Paraconsistent logics' includes different families of logics — for example Discussive Logic [see 26], Non-Adjunctive logics [see 42], Adaptive logics [see 1, 2], Logics of Formal Inconsistency (see [6], [36]), and Relevant logics (see [17], [44], [43]), among others.

Furthermore,  $(ii_P)$  points out the fact that paraconsistent logics are individually motivated to be paraconsistent and that there is an explanation for this motivation within their individual frameworks — either justified on philosophical reasons, a domain of application, etc. While these explanations might seem very different from each other, they all (at least partially) tackle the same object:  $(i_P)$ .

**Necessity and sufficiency:** Second,  $(i_P)$  and  $(ii_P)$  constitute a minimal list of necessary and sufficient conditions that a logic must meet within a given framework to qualify as paraconsistent. And more importantly, even though the differences between families of paraconsistent logics are very clear (and sometimes, even dramatic), all of the members of these families satisfy both. In this sense,  $(i_P)$  and  $(ii_P)$  constitute the centroid of the cluster and all members of the corresponding cluster are represented by it.

**Degrees of (dis)similarities:** Following the criteria given above for clusters of logics it is worth noting that the dissimilarities within-cluster are small or overlookable in different senses. Philosophically speaking, for instance, even if there is no uniformity in the number of truth values that ground all of these logics, it is clear that there is a common motivation for a weak acceptance of contradictions. This is, while across the different paraconsistent frameworks, the truth value of contradictions might vary, there is a clear shared commitment

to treating contradictions as not irremediably threatening.<sup>6</sup> In addition, the dissimilarities between-clusters are large at their cores if we compare the cluster  $\lceil$ Paraconsistent logics $\rceil$  with the cluster that contains Classical Logic and its extensions.

### 4.3. Is it *legitimate*?

Now, what has been said here so far, only allows us to understand how to understand a broad group of logics as the cluster  $\lceil$ Paraconsistent logics $\rceil$ . Yet, this is not enough for determining their legitimacy as such. To do so, one should also give a motivation for the naturalness of the partition to which the cluster corresponds as well as provide a well-determined application for such logics.

For the case of  $\lceil$ Paraconsistent logics $\rceil$  the motivation is multiple but convergent. Some paraconsistent logics are driven by the counter-intuitiveness of Principle of Explosion when addressing human reasoning — claiming that if human rationality is closed under a logical consequence relation, this relation cannot be explosive because human agents often work with and from inconsistent information (Cf. [40]). Some paraconsistent logics are motivated by appealing to the lack of relevance that underlies explosion. And some others are motivated by the detection of inconsistent non-trivial theories in different scientific disciplines (Cf. [37]). Other motivations are more mathematically oriented and others are driven by the recognition of patterns present in information-transmitting human practices, among other independent phenomena.<sup>7</sup>

**(Independent) Partition.** What is interesting about all these different motivations for paraconsistent logics is that all of them reflect, broadly speaking, the same inferential phenomenon: the nonsatisfaction of the Principle of Explosion in inconsistent contexts. The independence of each of these motivations with respect to the others feeds the idea of the legitimacy of such inferential phenomenon, and when doing so, it also strengthens the possibility that the cluster  $\lceil$ Paraconsistent logics $\rceil$  is capturing an independent partition.

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<sup>6</sup> This common ground, as well as the emerging philosophical differences, have been described by Beall and Restall (see [3]) under the label of *grades of paraconsistent involvement*. The first grade is the weakest which only consists of the rejection of explosion as a valid inference, and it becomes a key point when determining the basic characteristics of all elements from the cluster, ( $i_P$ ). The second grade is the acknowledgment of interesting contradictory non-trivial theories, this is part of the possible ways in which logicians justify ( $ii_P$ ). The third is the commitment towards the possibility of *some* inconsistent but not-trivial theories to be true, which is also part of a strengthened version of ( $ii_P$ ). Finally, the fourth grade consists in adopting the truth of contradictions.

<sup>7</sup> For a comprehensive analysis of different interpretations of the Principle of Explosion and their role motivating different forms of paraconsistency see [16].

**Applicability.** The assessment of well-determined applications of these logics comes from the implementation of paraconsistent logics for the modeling and explanation of different phenomena (paradoxes, scientific theories, scientific reasoning, argumentation, etc.) combined with the fact that some of these logics are intentionally extended to account for phenomena that other paraconsistent logics were initially motivated by.

I take this section to have shown the way in which clustering can be used to scrutinize, explain and evaluate the legitimacy of groupings of logics in philosophical logic. In the next section, I apply the same methodology to the case of relating logics.

## 5. Challenging the legitimacy of ‘‘Relating logics’’

This section is devoted to the importance of addressing (and challenging) the legitimacy of the cluster ‘‘Relating logics’’. I start with discussing whether ‘‘Relating logics’’ can be seen as a legitimate cluster. I proceed to present a plan to satisfactorily account for the legitimacy of the cluster by pointing out an independent partition that justifies the validity of the cluster ‘‘Relating logics’’.

### 5.1. The basics of ‘‘Relating logics’’ as a cluster

**Salient features:** the features used to characterize ‘‘Relating logics’’ are:

- (i<sub>R</sub>) A logic is relating if it is designed to capture (and reason about) different relations (causal, temporal order, and preference order, among others).
- (ii<sub>R</sub>) A logic is relating if, methodologically speaking, it was built under the assumption that, a relating connective is that for which

the logical value of a given complex proposition is the result of two things:

- the logical values of the main components of this complex proposition; supplemented with
- a valuation of the relation between these components. [25]

**Centroid:** consists of the combination of (i<sub>R</sub>) and (ii<sub>R</sub>).

**Selective filtering:** When combined, (i<sub>R</sub>) and (ii<sub>R</sub>) work as a selective filter grouping all logics whose main focus lies on relations of different types, and filtering out all logics that even though are concerned about relations do not incorporate connectives whose truth value is not determined solely by the truth values of the individual components.

**Centroid’s relation:** The cluster encompasses a large variety of logics, going from Epstein logics, which main focus lies on conditionals and their content,

up to more contemporary ones, that deal with other types of relations, like preference or temporal order. For this reason, the cluster includes logics that are alternatively/more traditionally classified as conditional, relevant, paraconsistent, connexive, and temporal logics, among others. All of these logics are well-represented by the cluster's centroid.

## 5.2. The challenges of 'Relating logics' as a cluster

**Degrees of (dis)similarities:** While the other steps of explaining the grouping of 'Relating logics' were straightforward, things get a little bit tricky when having to assess both the dissimilarities within-cluster and the dissimilarities between-clusters.

First of all, because of  $(i_R)$ , 'Relating logics' includes logics that are motivated by very different types of relations. In particular, our philosophical intuitions about the relations usually addressed with Relating logics are such that all these 'relations' are significantly different from one another — even to a point in which any joint-carving analysis of them would suggest that they shouldn't be modeled with the same formal resource.

For instance, in metaphysics, causality, grounding, and emergence are distinct relational concepts that play essential but different roles in understanding the fundamental nature of reality — as a matter of fact, the objects of each of these relations are very different. *Causality*, for instance, refers to the relationship between cause and effect. It entails that a cause brings about a specific effect, implying a direct or indirect connection between events or states of affairs. In contrast, *grounding* concerns the relationship between entities or facts where one is considered more fundamental or ontologically prior to another. Grounding establishes a dependency relation, where the grounded entity or fact relies on the grounding entity or fact for its existence or intelligibility [cf. 4]. *Emergence*, however, refers to the phenomenon where novel properties, entities, or patterns arise from the interactions or arrangements of simpler components or systems. Emergent properties are often seen as non-reducible and irreducible to lower-level descriptions, indicating that they possess unique causal powers and ontological significance [cf. 39]). All this considered, causality involves the temporal connection between cause and effect, grounding emphasizes the ontological dependence between entities or facts, and emergence highlights the generation of novel properties or entities from simpler components. While the first doesn't require any further metaphysical commitments and it can and has been approached via anti-realist views, the two others are extremely metaphysically-laden. Furthermore, the nature of these relations contrasts significantly with those of preference and temporal order, which tend to be much more accidental and even extremely subjective.

The fact that all these differences are at the core of the characterization of each of these relations, combined with the role they play in motivating and constraining each of the logics designed to model them, suggests that the differences within-cluster are larger than desirable. While in the case of  $\lceil$ Paraconsistent logics $\rceil$  one could still argue that the understanding of what a contradiction is as well as the interpretation of the Principle of Explosion could vary from logic to logic, it was clear that they both were uniformly objects of concern of every member of the cluster. However, in this case, logics that are motivated by causal connections might not have assumed a take on preference order.

In addition, there are logics that fully satisfy  $(i_R)$  that are not part of the cluster  $\lceil$ Relating logics $\rceil$ , because they don't do so by following  $(ii_R)$ . This makes that the dissimilarities between-clusters aren't as crisp and large as desired.

The underlying intuition is that  $\lceil$ Relating logics $\rceil$  lack the same level of efficient clustering found in other logics, such as  $\lceil$ Paraconsistent logics $\rceil$ . This disparity arises because, unlike labels such as paraconsistent logic or relevant logic, relating logics encompass a broader range of elements in a less uniform manner — one of the components of the centroid refers to an independent phenomenon and the other refers to a methodological criterion to follow when building a logic. While the paraconsistent grouping encompasses anything that fails to validate explosion, the relating grouping incorporates elements primarily focused on relations, regardless of their shared similarities or lack thereof, and a way to model them.

This is strengthened by the expectation of an object of clustering to fulfill certain sufficient and necessary conditions to be classified under specific groupings or categories. For instance, if a given logic, L1, does not satisfy the requirement of taking Explosion as a valid inference, it falls into the cluster of  $\lceil$ Paraconsistent logics $\rceil$ . Similarly, suppose a logic, L2, describes inferential behavior of, or has a domain of discourse centered around, relations. In that case, it becomes meaningful to refer to it as a member of  $\lceil$ Relating logics $\rceil$  in contrast with other logics.

Yet, the problem here might be that separated  $(i_R)$  and  $(ii_R)$  lead to different not fully convergent partitions. Following  $(i_R)$ , relating logics are defined as those that investigate relationships as objects of logical behavior, these includes many of the most traditional logics that were concerned with conditionals, without necessarily making any changes on the way to interpret the values of those conditionals. However,  $(ii_R)$  leads us to see relating logics as those whose interpretation of connectives is methodologically relating, such as the Belnap-Dunn logics, even if these logics are not fully motivated by one specific (and philosophically characterized) type of relation.

In addition, one should not mistake how the cluster  $\lceil$ Relating logics $\rceil$  is formed: the logics it encompasses are not grouped based on their mathemat-

ical similarities concerning model theory and proof theory. While it is true that many of these logics share formal structure, what justifies their classification within this cluster is not their structural resemblance at this level, but rather their alignment with the two core conditions ( $i_R$ ) and ( $ii_R$ ).<sup>8</sup> That is, they are unified by their commitment to capturing and reasoning about relations, as well as by their methodological approach to defining connectives in a way that integrates relational evaluations alongside the logical values of their components. Thus, treating the cluster as one grounded merely in technical formal similarities would overlook its defining philosophical and methodological motivations.

The above raises the question of if many of the alleged relating logics can be also classified and easily understood under more traditional and not fully convergent categories, why should we consider that there is anything that is shared by these logics that makes them saliently ‘relating’? This is, how can we explain the legitimacy of the label ‘Relating logics’?

### 5.3. The legitimacy of ‘Relating logics’ as a cluster

The lack of clarity about the extent of the similarities shared by the members of the cluster ‘Relating logics’ gives rise to at least two alternative explanations concerning the legitimacy of the cluster itself.

On the one hand, one might interpret the difficulty in grouping ‘Relating logics’ as a signal that the clustering criteria are being misapplied. Rather than viewing ( $i_R$ ) and ( $ii_R$ ) as descriptive features of a well-defined family of logical theories, one could instead treat them as methodological guidelines—indicating a strategy for developing logics in response to a particular type of motivation, namely, the need to formally capture diverse relational phenomena. On this reading, the cluster does not so much delineate a homogeneous class of logics as it foregrounds a shared approach to logical construction. Such a shift in perspective could lead to a more appropriate understanding of the cluster, one that emphasizes methodological unity and responsiveness to relational complexity over formal uniformity.<sup>9</sup> While I consider this interpretative shift both interesting and promising—especially as a way to unify seemingly heterogeneous systems—I will not pursue it further here. My main aim here is to do justice to the explicit motivations that have been articulated in the literature when introducing and developing different relating logics.

On the other hand, one might see the grouping difficulty as a call for showing the cluster’s capability of best fitting an independent partition as well as

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<sup>8</sup> Thanks to Alejandro Estrada-Girón for the pointers.

<sup>9</sup> I’m grateful to Moisés Macías-Bustos for proposing this interpretation. The idea of reading ( $i_R$ ) and ( $ii_R$ ) as methodological rather than taxonomical was first brought into focus through our conversations.

having a well-determined application. The problem with ‘‘Relating logics’’, however, is that while they do point out independent domains that can motivate their legitimacy, these domains individually correspond to only part of the general phenomenon that underlies the partition—either they focus on causality, analyticity, preference order, etc. This is, individually, relating logics have tackled different relations, but not yet provided a domain that justifies the grouping of the broad diversity of these relations.

While one could accept this as a methodological starting point, saying that there is a partition that comes from our philosophical understanding of relations in general, the domains of applications that have been suggested for relating logics are also partial in the same sense; either these logics have been used to model and explain causal relations, or temporal orders, etc. but never to model an integrative relational phenomenon. To do so is the objective of the next section.

## 6. ‘‘Relating logics’’ meets scientific understanding (I)

In this section, I characterize scientific understanding as a relational phenomenon and I explain the role that it plays when providing an independent partition to justify the legitimacy of the cluster ‘‘Relating logics’’. In order to do so, the section is divided into two parts. First, I introduce the methodological grounds of this research. Second, I introduce scientific understanding as a relational phenomenon.

### 6.1. Methodological grounds

First, the *legitimacy* of a given cluster depends on the combination of both the cluster being motivated by an independent partition as well as being applied in/for a well-determined problem.

Second, a *relational phenomenon* involves relationships or interactions between entities or individuals, and it emphasizes the interconnectedness and interdependence of elements within the phenomenon, system, or domain. In the study of relational phenomena, it is of crucial importance to focus on the scrutiny of the relations themselves rather than solely examining the individual elements or components involved.

Relational phenomena involve interdependence between the entities or elements involved. The relations between these entities shape and influence their behavior, properties, and outcomes. In addition, relations in relational phenomena often give rise to emergent properties that cannot be reduced solely to the characteristics of individual elements. Furthermore, relational phenomena tend to involve complex causal webs and feedback loops. The relations

between entities can create intricate causal chains, where the effects of actions and events propagate through the network of relations, leading to indirect or delayed consequences. All this considered, methodologically speaking, the focus of relations is crucial for the scrutiny of relational phenomena as it can shed light on the ways in which certain elements of the phenomenon affect others, and in the long run, the way in which the whole system works. In sum, ignoring the relational aspect of relational phenomena leads to an incomplete understanding of the phenomenon.

Third, while every relational phenomenon might be of use when motivating the analysis of certain types of relations; the identification of a domain that is considered *saliently relating* and that can be of importance for the application of relating logics is a more intricate task. In Section 5.1, I have claimed that the relatedness of a given logic comes from both:

- (i<sub>R</sub>) being designed to capture (and reason about) different relations (causal, temporal order, and preference order, among others).
- (ii<sub>R</sub>) being built under the assumption that, a relating connective is that for which “the logical value of a given complex proposition is the result of two things: the logical values of the main components of this complex proposition; supplemented with a valuation of the relation between these components” [25, p. 563].

From this, it seems sensible to say that, when looking for a domain of application for relating logics, this domain should host scenarios in which the following conditions are satisfied:

- (i<sub>R</sub><sup>\*</sup>) A domain is *saliently relating* if it portrays the interaction of different relations (causal, temporal order, and preference order, among others).
- (ii<sub>R</sub><sup>\*</sup>) A domain is *saliently relating* if the relations that hold in that domain are constrained by connectives that evaluate complex propositions as resulting from the logical values of the main components in addition of a valuation of the relation between these components.

Fourth, applicability is an important criterion when addressing the philosophical robustness of a given logic. In particular, the methodological and epistemological value of applicability has been explained and justified through (methodological) *abductivist* frameworks.

*Abductivism* is the view according to which, similarly to scientists, logicians are justified to choose a logical theory over another whenever having evidence in favor of it fitting more adequately the evidence, but also when ranking it higher than its rivals with respect to virtues such as strength, simplicity, and unifying power [cf. 50, 21, 48]. If pursuing such a line of research, one must either show that either (a) relating logics can explain or justify certain theorems that we consider important to preserve in a better way than their rivals; or, one must show that (b) relating logics can found a domain of application for

which they fit more adequately the evidence than any of their rivals do — this is, a domain that is saliently relating.

In what follows, I focus on a case of (b). In the rest of this section, I argue that the cluster ‘‘Relating logics’’ can find a domain of application in the phenomenon of scientific understanding, in particular, that scientific understanding is both a relational phenomenon and that it constitutes a domain that is saliently relating. This, I take, constitutes evidence in favor of their philosophical and methodological value as well as of the legitimacy of the cluster ‘‘Relating logics’’.

## 6.2. Scientific understanding as a relational phenomenon

*Scientific understanding* (henceforth, *understanding*) has been traditionally considered to ‘‘consist of knowledge about relations of dependence. When one understands something, one can make all kinds of correct inferences about it’’ [51, p. 100]. Understanding plays an indisputably essential role in the success of scientific endeavors. It enables scientists to construct comprehensive representations of specific aspects of the world and facilitates the integration of seemingly disconnected theoretical frameworks. Advancing research in the sciences hinges upon the attainment of understanding.

Understanding is both a psychological and epistemic phenomenon that emerges when an epistemic agent *grasps* bits of independent domains through the use of theories, models, and explanations, among others. Understanding consists in combining disconnected pieces of scientific knowledge into a more cohesive picture of the world. When a scientist understands a phenomenon  $X$ , she can identify the phenomenon in different contexts, meaning that she can recognize different exemplars of the phenomenon [cf. 12]; she can address the conditions that were needed for bringing about that particular phenomenon, and she can relate it to other phenomena described through her best scientific descriptions and explanations of the world.

There are two major agreements about understanding in science: the first is that understanding is a relational phenomenon, and the second is that understanding possesses a standing state when compared to other epistemic products such as knowledge and explanation.<sup>10</sup> The combination of these two facts has made understanding into a multilayered epistemic construct that relates bits of scientific knowledge in order to build broader and more robust images of the world. However, the remaining features of understanding are still subjects of deep philosophical debates. The most important of them being *the status of the content of understanding* and *the relata of understanding*.

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<sup>10</sup> Evidence of this is that both knowledge and explanation seem to be elements of understanding but understanding is not included in any of those.

The problem of determining which type of elements can figure into the content of understanding can also be characterized as the chase for the necessary conditions that something has to satisfy to be included in what is genuinely understood. The three most common alternatives are:

- that we understand only true propositions, this is called *factivism*,
- that we understand true propositions but sometimes, we *use* some non-true propositions as stepping stones while pursuing understanding; this view is called *quasi-factivism* [cf. 34],
- that some non-true propositions can be included in the content of what is being understood if and only if they play a crucial role in the facilitation of understanding; this is called *non-factivism* [cf. 11, 12].

It is important to stress that one of the main goals of the debate about the content of understanding is to shed light on the mechanisms that underlie the reliability and robustness of understanding.

In addition to this debate, there is also the discussion about which are the entities that are connected through understanding. Some epistemologists have claimed that understanding is a phenomenon of relating bits of explanatory knowledge. This view assumes that understanding comes only after having obtained explanatory knowledge; this type of understanding has received the name of *explanatory understanding* [cf. 30, 18, 19, 38, 46, 47, 27, 45, 32, 33]. Some others have argued that explanatory knowledge is only one type of entity from many that can be related through understanding. According to this view, understanding can come from establishing relations between knowledge about facts, procedures, orders, structures, etc.; and all of them are crucial (and sometimes sufficient) for the achievement of understanding [cf. 11, 10, 12, 7, 8, 28, 9, 35].

In light of the lingering uncertainties surrounding both the truth of the content of understanding and the nature of its relata, it becomes evident that our sole certainty lies in the indispensability of (different types of) relationships for understanding. The questions discussed here prompt us to acknowledge that it is the relations themselves that hold the key to the analysis, description, and explanation of understanding. It is through the establishment of connections and associations between knowledge and beliefs (about theories, concepts, data, phenomena, etc.) that understanding emerges.

Because of this, it seems sensible to claim that the most salient feature of understanding is relations—note that this doesn't mean that relations alone suffice for understanding. This means that the prominent aspects of understanding are connections and the ways in which epistemic bits are arranged so that they give rise to a more comprehensive image of the domain that is being understood. And methodologically speaking, an adequate analysis of understanding is grounded on a previous comprehension of 'relations' as a global

subject that encompasses causal connections, dependencies, influences, as well as other forms of relations.

All this considered, understanding should be seen as a relational phenomenon par excellence.

**(Independent) Partition.** In addition and with respect to the legitimacy of the cluster  $\lceil$ Relating logics $\rceil$ , the phenomenon of scientific understanding in itself can be taken as an instance of an independent partition that justifies the grouping of the whole cluster. While the majority of the members of  $\lceil$ Relating logics $\rceil$  are concerned with the behavior of specific relations, the whole cluster aims at addressing a large variety of them in an integrative manner, which is exactly what grounds the notion of “understanding”. The fact that the characterization of understanding has been built independently from that of relating logics feeds the possibility that the cluster  $\lceil$ Relating logics $\rceil$  is capturing an independent partition.

## 7. $\lceil$ Relating logics $\rceil$ meets scientific understanding (II)

This section aims at characterizing scientific understanding as a relating phenomenon and domain for future application of relating logics. In order to do so, the section is divided into two parts. First, I explain the relating nature of scientific understanding. Second, I illustrate this with an example and address the value of this domain for the legitimacy of the cluster  $\lceil$ Relating logics $\rceil$ .

### 7.1. Scientific understanding as a relating domain

So far, I have explained the conditions under which understanding should be seen as a relational phenomenon; yet, the question of whether understanding fulfills conditions (i $^*_R$ ) and (ii $^*_R$ ) remains to be answered. In order to tackle this question, I characterize understanding as the *building of epistemic networks* and I explain how this characterization allows us to recognize its relating nature.

First of all, understanding is not a matter of just combining pieces of information but it consists of the capability of determining the different relations that might hold between such pieces of data (which are often also about relations) and the connections that might exist, as well as those that should not exist, between different fragments of information. Thus, understanding is about determining privileged arrangements of epistemic bits in such a way that they provide the most cohesive and comprehensive representation of a given domain.<sup>11</sup>

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<sup>11</sup> The salient features of the networks of understanding, compared to other epistemic networks, are the type of products that one can get and the comprehensiveness

The interpretation of understanding as the building of epistemic networks is formed on the basis of three elements:

- **Nodes** are epistemic bits. Nodes encompass beliefs and knowledge of different types (including explanatory, factual, procedural, etc.). While the large majority of them are true, some of them can be non-true.

The content of the nodes is composed of information of different types some of which is about relations (causal, spatial, temporal, etc.), some other bits of information are solely about facts, and others are about theoretical principles, and so on. For instance, in the following image, the first node, (EP) designates a theoretical assumption, the second node, (M,E)(Sat) designates factual information, while the last one, designates information about the spacial relation held by the Moon, the Earth and the Sun when all are aligned.

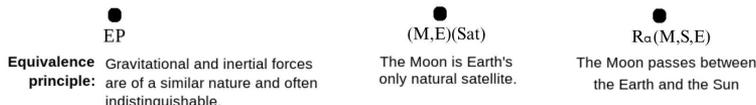


Figure 1. Nodes of networks of understanding

It is worth noting that epistemic agents have different doxastic attitudes toward the information that is contained in the nodes, some of the bits of data that are included in the content of understanding are taken to be true and some others, while epistemically useful, might be known to be false as they result from idealizations or fictions [cf. 12].

- **Links** are the relations that underlie the privileged arrangements of epistemic bits (nodes). The purpose of these links is dual: first, they determine the relationships that hold between what the nodes contain (whether they are causal, temporal, or entailment relations, among others). Second, the links also have the purpose of expressing the ways in which two or more nodes can interact, and particularly, under which conditions, nodes with non-true information, can behave safely (inferentially speaking) while still giving rise to the most adequate grasp of the domain that is seeking to be understood. In particular, for the case in which one part of the *relata* is known to be non-true, it is extremely important that the relationships that link this bit of data with the rest of the network can account for its relevance within the network and not only for its truth value.

In the image below, two different types of links are represented: one that is causal and that connects information about two facts, one being the effect of the other, and another that illustrates an entailment relation that holds between theoretical assumptions within the same theoretical framework.

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of such outputs. In this sense, the outcomes of understanding surpass the ones of knowledge.

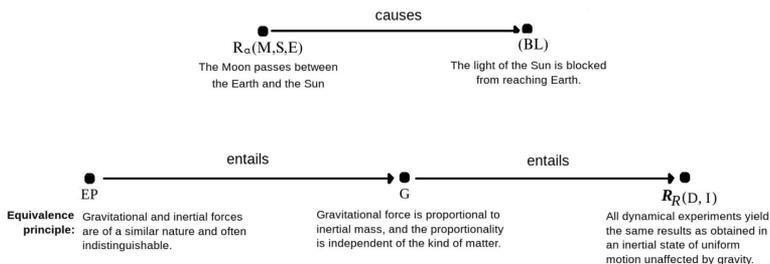


Figure 2. Links of networks of understanding

It is important to say that that the relations that hold between nodes have different purposes. Some of them are truth-preserving, mostly when the content of the connected nodes is regarded as true; but, some others aim at *preserving reliability* of the information, especially when the content of the nodes is not true.<sup>12</sup> And more importantly, the large majority of the relations that are held within a network aim at *reinforcing* the content of a given node or set of nodes in such a way that the resulting picture is more robust than the one that would have emerged from the mere conjunction of the bits of the information being put together.<sup>13</sup>

**The general constraints of the entire network.** Are of an inferential type, and they allow determining the consequences of a network that can and should be legitimately inferred when all relevant information has been linked in an appropriate manner. This makes the constraints resemble a consequence relation.

The combination of these three elements gives rise to a view for which understanding emerges from establishing relations that hold between epistemic bits. The evaluation of the links that hold within a network is determined partially by the truth values of the information that is contained in the nodes, as well as by an additional valuation of the type or relation that is held between specific nodes. The combination of this reinforces the idea that understanding consists of determining privileged arrangements — and distinguishing them from other possible, less epistemically fruitful, arrangements within the same information.

<sup>12</sup> For a more detailed account of preservation of reliability, see [5].

<sup>13</sup> Two nodes in a network *reinforce* each other if either “T provides a “rationale” for (a part of) T1” [31, p. 54]; or if, at least, one supports the basic assumptions of the other, or explains mechanisms of the second node, or clarifies the concepts of the first node [cf. 13].

## 7.2. Relating understanding

Here, I aim at providing an example of the relatedness of understanding, this in order to show the fruitfulness of understanding as a domain for future application of relating logics.

Consider the following case: a student, S, who has recently gained understanding of the phenomenon of solar eclipses through Einstein's relativity theory. According to the approach proposed above, this would mean that Susan has built a network that combines her theoretical knowledge about relativity theory with her knowledge regarding the motion of planets, asteroids, and other space objects. Because a genuine understanding of a phenomenon is expected to be generalized and instantiated through, at least, the most salient exemplars [cf. 12]; a consequence of Susan's understanding of solar eclipses should be the understanding of how solar eclipses can affect and be perceived from Earth.

At first glance, it could seem that understanding the phenomenon is just a matter of explaining that a solar eclipse is a phenomenon where a celestial object like a moon moves in a direct path between the Earth and the Sun, resulting in the partial or complete obstruction of sunlight. However, a deeper examination reveals that this explanation is rooted in fundamental assumptions derived from various theoretical frameworks. Grasping the concept of a solar eclipse entails establishing connections between these underlying assumptions.

The content of some of the relevant epistemic bits that are crucial for the understanding of solar eclipses includes the following theoretical assumptions:

- R(GR): The theory of General Relativity is reliable — it accurately describes the behavior of gravity and the motion of bodies in the presence of massive objects and gravitational fields.
- (Mo, Go): Massive objects generate gravitational fields.
- (Mo, Go)(Co): Massive objects cause curvature in the spacetime around them as a consequence of their gravitational field.
- (L): Light travels along geodesics — the shortest possible paths in curved spacetime.

The conjunction of these nodes, when supplemented with the information about the Sun, the Earth, and the Moon should suffice for the building of a network of understanding around the phenomenon of solar eclipses perceived from Earth. Empirical assumptions that might result relevant for this matter include:

- (M, E)(Sat): The Moon is the natural satellite of Earth.
- $R_a(M, S, E)$ : The Moon, the Sun, and the Earth are aligned.

The combination of these bits of information should suffice for the understanding of the phenomenon of solar eclipses as perceived from Earth, which should have as its consequence the conjunction of the following information:

- $R_a(M, S, E)(B)$ : During a solar eclipse, the Moon passes between the Sun and the Earth, blocking the direct light from the Sun.
- $(S, O)(CC)$ : The gravitational field of the Sun causes curvature in the space-time around it.
- $(CC, L)(BL)$ : The curvature of spacetime near the Sun bends the path of light rays coming from it.

The resulting network of understanding could be represented as follows.

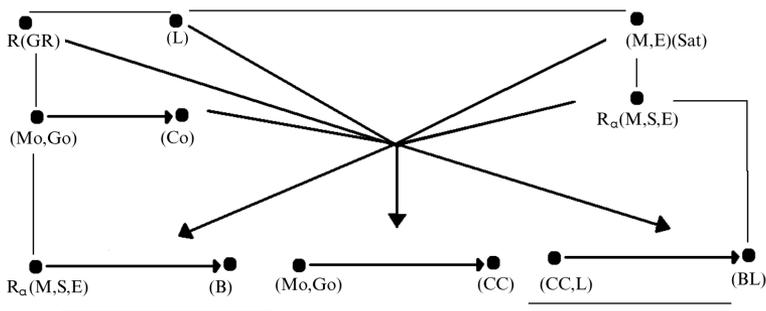


Figure 3. Network of understanding: a solar eclipse. Horizontal arrows indicate causal connections and vertical and tilted arrows indicate entailment-like relations

So far, what intuitively seems to connect all the nodes from the theoretical and empirical assumptions is a simple conjunction; the key element of their combination is putting them together. So, the relations that hold between nodes, unless said otherwise, are quite straightforward conjunctions—for instance, no node holds a causal relation with any other node. However, and despite this intuitive reading, the relations between some nodes are deeper: they reinforce the role that the nodes play for the general goal of understanding the phenomenon.

More importantly, through the network presented in Figure 3, the importance of recognizing and evaluating complex propositions is shown, having in mind more than just the truth values of the components of the propositions. For instance, take the case of node (L), the information contained in this node results from idealization—it assumes that light travels along geodesics, which are the shortest paths in curved spacetime. However, it does not account for factors such as the scattering and refraction of light caused by the Earth's atmosphere or other intervening media. This causes that (L) cannot be taken as true and if the relation that holds between (L) and the rest of the theoretical bits were a classical conjunction then, at least, that segment of the network would be false. But what we can see from the way in which (L) relates with

other nodes is that it actually eases the connections between them (particularly between (G) and  $(R_a(M, S, E))$ ) and makes easy to infer the explanations for the particular case of solar eclipses as seen from Earth.

**Applicability.** All the above considered, it is sensible to say that the characterization of understanding as the building of epistemic networks allows highlighting two main properties of understanding: first, it consists of the set up and interaction of different relations (causal, temporal order, and preference order, among others) that hold between epistemic bits,  $(i^*_R)$ . Second, the evaluations of the relations that are held within a network of understanding are constrained by the evaluation of both the logical values of the data contained in the nodes as well as by an additional valuation given the type of relationship that exists between those nodes,  $(ii^*_R)$ .

While more could be said here, I hope it is clear that a network approach to understanding highlights its relating nature. I tentatively conclude that this should suffice to consider understanding both an independent domain that can motivate the exploration of relating logics as a legitimate grouping as well as a worth of pursuit domain of application for the members of the cluster  $\lceil$ Relating logics $\rceil$ .

## 8. Final remarks

Here I challenged the legitimacy of the cluster  $\lceil$ Relating logics $\rceil$  as it has been characterized in recent literature. I argued that even if the cluster  $\lceil$ Relating logics $\rceil$  satisfies the basic theoretical criteria for legitimate clusters, logicians and philosophers of logic have failed at providing the corresponding pragmatic backing.

In addition I defended that in order to offer a pragmatic justification of the cluster one should show a domain of application for which  $\lceil$ Relating logics $\rceil$  fit more adequately the evidence than any of their rivals do; such a domain is the phenomenon of *scientific understanding*.

Finally, I characterized scientific understanding as both a *relational phenomenon*—and argued that this suffices for legitimately motivating the independent partition behind relating logics, and as a *relating domain* which provides a novel field for the application of relating logics.

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MARÍA DEL ROSARIO MARTÍNEZ-ORDAZ  
Institute for Philosophical Research  
National Autonomous University of Mexico  
Universidad Nacional Autónoma de México  
Circuito, Mario de La Cueva s/n, C.U.  
Coyoacán, 04510 Ciudad de México, CDMX, Mexico  
[martinezordazm@gmail.com](mailto:martinezordazm@gmail.com)  
<https://orcid.org/0000-0003-2118-3515>