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Revisiting Jaśkowski's Criterion

Abstract. This paper aims to re-examine the so-called Jaśkowski's criterion. This guiding principle was initially proposed as a methodological instrument for delineating formal systems as either satisfactory or unsatisfactory paraconsistent logics. It is a historical fact that Jaśkowski's criterion has been, and remains, frequently overlooked in the relevant literature. In order to provide a detailed explanation of this fact and the argument for it being a regrettable state, there are three main points elaborated upon. Firstly, the philosophical motivations of Jaśkowski's work. Secondly, an overview of some early attempts of meeting requirements of Jaśkowski's criterion. Thirdly, its relevance to contemporary research in the field of philosophical logic, particularly in relation to recent discussions on the notion of 'philosophical interpretation'. From the above, philosophical conclusions are drawn, and avenues for further research are identified.

Keywords: Jaśkowski's criterion; paraconsistency; philosophical interpretation; formal philosophy

1. Introduction

It should not be controversial to state that a fundamental principle of effective X-ing, irrespective of the specific character of X and the definition of 'effective', is to refrain from engaging in ineffective X-ing.[1] However, this raises a rather obvious question of how one can distinguish

[1] Note that this requirement is fulfilled simply by doing nothing, which is not ideal as far as requirements go. The most evident solution would be to stipulate that, in addition to refraining from engaging in ineffective X-ing, one must partake in activity that could be categorized as X-ing to begin with. The author would like to express his gratitude to the anonymous reviewer for pointing out this mankament of the not-so-witty-after-all opening of the article.

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between these two types of X-ing. The most evident way of tackling this issue would be to engage in methodological investigations. There it is important to be cautious of a delicate balance between generalizations and specifications. It is reasonable to assume that no more than the above fundamental principle could be provided for the case of 'effective X-ing', as it is the most general case. It is therefore necessary for further analysis to specify one's concerns. In the paper it is done with regard to the issue of 'effective philosophizing'. The question of whether there is one correct philosophical methodology, or whether there are many depending on, for example, the domain under investigation, is a highly contentious one.[2] However, it is important to note that the undertaking of any philosophical inquiry is invariably accompanied by the adoption of a methodological standpoint, even if only implicit one. Nonetheless, in certain instances, explicit methodological statements can be found. This paper concerns such a case in the domain of philosophical logic.

In seminal (Jaśkowski, 1948) there are three requirements stated, constituting the so-called Jaśkowski's criterion. The aim of this paper is to undertake a re-examination of this guiding principle. It was initially proposed as a methodological instrument for delineating formal systems as either satisfactory or unsatisfactory paraconsistent logics.[3] However, it is a historical fact that Jaśkowski's criterion was, and still is, frequently overlooked in the relevant literature. In order to provide a detailed explanation of this fact and the argument for it being a regrettable state, there are three main points to be elaborated upon. Prior to sketching them, let us put forward the following passage as a basis for the sentiment that is to be haunting the content of this paper (in the rest of the paper

[2] For an overview of possible methodologies and stances on their character, see (Cappelen et al., 2016; Horvath et al, 2025).

[3] It is important to concede that this is an anachronistic reading, since neither the term 'paraconsistent logic' nor any paraconsistent system could have been known to Jaśkowski at that time. Moreover, the extent to which his pioneering ideas correspond to the conception of paraconsistency in different traditions is not a forgone conclusion (for the recent work on this topic, see (Fazio et al, 2025; Rodrigues et al., 2025); for more information on the so-called Polish tradition see the former and (Nicolás-Francisco, 2023); for the work on traditions other than Polish, see (Tanaka, 2003)). To illustrate this point, Jaśkowski did not concern himself with the role of negation, yet this is the divergent point in the treatment of paraconsistency between the so-called Australian and American plans, see (Restall, 2019). The author would like to express his gratitude to the anonymous reviewer who drew attention to the need for this particular clarification.

referenced simply as 'the Sentiment'). In his *Logic and Philosophical Methodology*, John P. Burgess wrote the following:

> [The formal thing] was largely developed in hopes of making itself philosophically useful. Performance, however, has not lived up to promise, and it is not going too far to say that at times modern [the formal thing] has done more to darken rather than to enlighten our understanding. [...] More was done in the way of developing elaborate technical constructions to prove various conventional systems to be formally consistent, than was done in the way of analyzing and explaining the notions [of the formal thing] and their representation in language in order to show the systems in question intuitively intelligible, or where they were not so, to replace them by novel systems that would be.
> (Burgess, 2016, p. 619)

The context of this quote is intentionally omitted, as a point that this sentiment can be advanced to any venture in the domain of philosophical logic. The reader is invited to plug in their own topic of interest in order to illustrate this point.

With the above in mind, the structure of the paper is as follows: in Section 2, Jaśkowski's criterion is presented. In Section 3, the philosophical motivations of Jaśkowski, stemming from his earlier work, are discussed. The significance of this section is rooted in the fact that they are virtually unknown, and insofar not presented in the literature. In Section 4, an overview is provided of some early attempts to meet requirements of Jaśkowski's criterion. Finally, in Section 5, the contemporary relevance of presented matters is described, mainly for the recent discussion on the notion of 'philosophical interpretation'. Using this framework, a case study of the selected modern approach to meeting Jaśkowski's criterion is analyzed, and diagnosed to be severely inadequate. Section 6 is a concise summary of the key findings, along with avenues for future research.

2. Preliminaries: Jaśkowski's criterion

In (Jaśkowski, 1948) the idea of a formal system capable of handling inconsistencies has been adressed. During this period, the matter in question was of a particularly divisive character, due to the ongoing Polish discussion on the principle of non-contradiction. In brief, one position in the discussion was adopted by Polish analytic philosophers, who claimed that formal systems cannot tolerate inconsistencies. They argued that classical logic is the correct one and, since the law of explosion holds in it, any system proving α and $\neg\alpha$ for some α would be trivial and, consequently, of no use. In contrast, Polish Marxist philosophers advanced the position that logic itself should be eschewed, and that dialectical methods should be utilized in philosophical practice instead. This image of the discussion is both simplified and riddled with flaws[4]; nevertheless, it will suffice for aims of this paper.

It is important to note that Jaśkowski did not participate in this discussion in any official capacity, as he did not align with any of the aforementioned perspectives. He was a logician who did not adhere to classical logic as a correct one, but at the same time strived to address certain Marxist intuitions by employing formal methods. This is precisely what he did in formulating the problem of the logic of inconsistent systems as a task to find a logic which:

(Jaśk1) when applied to the inconsistent systems would not always entail their overfilling;

(Jaśk2) would be rich enough to enable practical inference;

(Jaśk3) would have an intuitive justification.[5]

The above requirements constitute the so-called Jaśkowski's criterion (resp. Jaśkowski's problem). Fulfillment of them is the *sine qua non* of satisfactory (i.e., effective) paraconsistent logic. The first requirement determines whether a given system *is* a system of paraconsistent logic. It is somewhat mechanical and thus the easiest one to meet. The subsequent two conditions determine whether a given system is *satisfactory*.[6] As already noted by Jaśkowski, it is much harder to reach an objective conclusion here, as both depend on many factors, including subjective ones. Nevertheless, the entire criterion must be met if one wants to talk about satisfactory (i.e., effective) paraconsistent logic.

It must be acknowledged that the presence of the Jaśkowski's criterion within the history of paraconsistency in general, and contemporary research in particular, is faint; or at the very least, its philosophical

[4] For more information on the discussion *per se*, see (Woźniak, 2021). For more information on the flaws in its standard account, see (Krawczyk, 2023).

[5] References to the English text are based on (Jaśkowski, 1999), which is a translation of his original paper that was written in Polish.

[6] For an alternative framing, namely one that focuses on the distinction between *pure* and *applied* logics, see (Rodrigues et al., 2025).

significance is so. Instead, the framework of the so-called da Costa's criterion (see daCosta, 1974) is primarily used, which posit the following four requirements for a satisfactory system of paraconsistent logic:

(daC1) in these systems the principle of non-contradiction, in the form $\neg(\alpha \wedge \neg\alpha)$, should not be a valid schema;

(daC2) from two contradictory formulae, α and $\neg\alpha$, it would not in general be possible to deduce an arbitrary formula β;

(daC3) it should be simple to extend these calculi to corresponding predicate calculi (with or without equality);

(daC4) they should contain the most part of the schemata and rules of the classical propositional calculus which do not interfere with the first condition.

Some of the above requirements may be regarded as analogous to ones posited by Jaśkowski, i.e. (daC2) with (Jaśk1), and (daC4) with (Jaśk2). However, there are notable discrepancies. Firstly, da Costa's concern is with the principle of non-contradiction, a concept that Jaśkowski does not deem to be of particular importance in the context of the task at hand. Secondly, the most staggering difference is that da Costa's primary goal in terms of 'satisfaction' is to provide a simple way for extending propositional systems into first-order ones, without any mention of an intuitive justification for such formal system. This gave rise to the following observation:

> [O]n the one hand, da Costa's preoccupation with the answer to [the requirement (daC3)] gives one of the main reasons why he is often suggested by some authors to be the 'real founder of paraconsistent logic' [. . .], despite Jaśkowski's prior construction. On the other hand, perhaps the absence of [(Jaśk3)] from da Costa's set of requisites gives an excuse for the difficulty one may encounter in establishing intuitive interpretations for the calculi [proposed by da Costa].
>
> (Marcos, 2005, p. 54)

In light of the Sentiment, it is evident that these priorities may be misguided. Furthermore, an explicit argument is provided in preference of the latter criterion, namely:

> It is hard to believe in fact that any proposer of a paraconsistent logic would willingly acknowledge or aim his own logic not to satisfy any of [Jaśkowski's] clauses. This way, the problem of Jaśkowski reveals itself as nothing but the most general problem of paraconsistency. [. . .] The

> problem of da Costa, as we shall call the problem of defining paraconsistent logics respecting clauses [(daC1)-(daC4)], looks much more determinate. (Marcos, 2005, p. 54)

Thus, the idea is that problems posed by da Costa's criterion are significantly more trackable and determinate (not to say, formal),[7] while requirements posed by Jaśkowski's criterion are more general and therefore less concrete (not to say, philosophical).

In the subsequent section, the philosophical motivations of Jaśkowski are presented, providing a starting point for countering the above point of view. The contention is advanced that the lack of both trackability and determinability (not to say, formalness) of Jaśkowski's proposal is not its vice, but its virtue.

3. Philosophical motivations of Jaśkowski

Starting in the middle of the seventeenth line of (Jaśkowski, 1999), the following sentence is to be found: "In a paper by the present author [Jaśkowski, 1947] the reader can find certain introductory explanations concerned with the issue here under discussion". Said paper is "Zagadnienia logiczne a matematyka" (*eng.* "Logical problems and mathematics"), published in 1947 in *Myśl Współczesna*. The case of this paper is a peculiar one, as it provides the philosophical underpinnings for the work undertaken in (Jaśkowski, 1948), yet is mostly absent in the literature. To illustrate this particular point: it has neither been translated into English (which is a minor issue), nor is it substantially referenced in the existing literature on Jaśkowski's scientific work, that is (Kotas and Pieczkowski, 1967; Dubikajtis, 1975; Indrzejczak, 2018). This is a major issue. In these papers (Jaśkowski, 1947) is referenced as just an example of the paper where "the author presents his opinion on actual problems", is referenced in the list of publications but not in the main text, and not referenced neither in bibliography nor main text, respectively.

[7] It is interesting to note that da Costa's opinion was that "[e]vidently, the last two conditions are vague" (daCosta, 1974, p. 498). That being said, it does not imply that they are inherently vague and could not be refined and made more precise by means of formalization; a process that arguably indeed have occurred. For instance, one could take (daC4) to be expressing the idea of the so-called classical recapture. For more details, see (Tajer, 2020).

It is not possible to refer back to this paper in the aforementioned ones. Nevertheless, there remains the possibility of emphasizing its significance and, optimally, translating it into English to provide access for a broader audience. In the following sections, quotations from (Jaśkowski, 1947) are provided for the purpose of elucidating the philosophical motivations of Jaśkowski. The present author has translated each one, with the original text indicated in the footnote. The section is structured into three parts. Firstly, the views of Jaśkowski on the nature of logic are presented. Secondly, his opinions on the project of Łukasiewicz are described. Finally, the foreshadowing of (Jaśkowski, 1948) is elucidated.

3.1. Jaśkowski on the status of logic

Discussions surrounding the status of logic are nothing new, but recently there has been a resurgence of interest in this topic (see, e.g., da Costa and Arenhart, 2018; Sher, 2020; Peregrin and Svoboda, 2022). The two primary issues under consideration are those of logic's (alleged) normativity and its (alleged) uniqueness. It is evident that Jaśkowski had to have some opinions on these matters, as he undertook the project of providing a formal system that would differ from classical logic. He starts with the following passage:

> The work of a mathematician consists in conducting subtle reasoning; efficient logic is a tool for him, and its advantages and flaws are reflected in the results of his work. Consequently, mathematicians, akin to other workers, engage in reflection regarding the tools they utilize particularly in instances of malfunction. [...] The catalyst for such reflections are the difficulties that sometimes unexpectedly arise. This raises logical issues for mathematicians — these questions are of a more general nature than those encountered in traditional mathematical problems, and thus hold relevance not only for mathematicians but also for researchers in the field of philosophy.[8] (Jaśkowski, 1947, p. 58)

[8] In Polish: Cała praca matematyka polega na przeprowadzaniu subtelnych rozumowań, sprawnie działająca logika jest dla niego narzędziem, którego zalety i usterki odbijają się na wynikach pracy. Toteż matematyk, jak każdy pracownik, zastanawia się nad używanym narzędziem, zwłaszcza wówczas gdy ono zawodzi. [...] Bezpośrednim bodźcem do takich badań są dopiero trudności, które nieoczekiwanie wyłaniają się co pewien czas. Nasuwają się wówczas matematykom zagadnienia logiczne — pytania, które posiadają charakter ogólniejszy niż właściwe zadania matematyczne i budzą zainteresowanie już nie tylko matematyków, lecz również badaczy filozofii.

Note that logic is presented here primarily as a tool. Evidently, it may also be investigated, albeit only in the occurrence of an extraordinary situation. These 'unanticipated difficulties' constituting extraordinary situation are, in fact, antinomies and other phenomena that are predicated on inconsistencies. It follows that the investigation of logic is a task that is equally pertinent to philosophers, in stark contrast to traditional mathematical problems.

Upon the investigation of logic as a subject, certain metalogical results are obtained, including consistency proofs and measures of proof-theoretic strength. The following is a passage on the status of such results according to Jaśkowski:

> [...] the [metalogic][9] is not normative in nature. It does not issue direct commands, such as 'You must accept these axioms and not others' or 'You must conduct proofs in this way and not another'. Rather, it states objectively that a particular method of proof allows one to obtain theorems that cannot be obtained by other methods. By presenting the properties of various possible formalized systems and their respective advantages and disadvantages, the [metalogic] leaves the choice of system to those who wish to use it.[10] (Jaśkowski, 1947, p. 62)

It would be challenging to find a more unambiguous perspective on this matter. Jaśkowski's position is that objectivity of metalogical results does not entail their normativity. The focus on the selected metalogical features of a given formal system is contingent upon the stated goals of its prospective users. It is not possible to determine whether the given choice is 'right' or 'wrong' in the goalless vacuum. In order to accentuate this point even further, consider the following:

> The relationship of this research to practical life is the same as that of all mathematical work. No mathematical calculation will tell us: "A

[9] It is important to note that in the original text, Jaśkowski uses the term 'methodology' (of deductive sciences). This term was historically used for metalogical investigations and results. In view of the fact that the present paper is concerned with methodology and metalogic in their contemporary meanings, suitable modifications have also been made to the quotations.

[10] In Polish: [...] [metalogika] nie posiada charakteru normatywnego, nie wydaje wprost poleceń: 'Należy przyjąć takie a nie inne aksjomaty, należy tak a nie inaczej prowadzić dowody'. Stwierdza tylko obiektywnie, że np. taka metoda dowodzenia pozwala otrzymać twierdzenia, których nie daje inna. Przedstawiając własności różnych możliwych systemów sformalizowanych, ich zalety i wady, pozostawia decyzję wyboru tym, którzy odpowiedni system chcą stosować.

> bridge should be built according to this design and no other", as it can only lead us to the conclusion: "A bridge built according to this design meets the requirements for strength, price, and so on". The decision on how to build the bridge is a separate matter. It may be guided by other considerations, even such as habit or a desire to break with convention.[11] (Jaśkowski, 1947, p. 62)

It is important to note the permissiveness of Jaśkowski's position. In logic, if one wants to do something differently, one can even provide a justification along the lines of wanting to break with convention. It is evident that this constitutes a rather weak justification, yet it remains a justification in its own right (cf. Jacquette, 1994). It is still possible to evaluate the outcomes of a project; however, it is not possible to do so in advance. Consequently, it is not possible to prevent an individual from pursuing some alternative approach, as metalogical results are not providing any normative constrains, they do not dictate any orders.

3.2. Jaśkowski on Łukasiewicz's project

In consideration of the fact that the primary focus of Jaśkowski's logical endeavours pertained to non-classical logics, it is relevant to examine his perspective on an earlier project undertaken in Poland by Jan Łukasiewicz. The following point is made:

> [$Ł_3$] can be interpreted in a similar way to two-valued logic, but using a greater number of values. To this end, we need to establish the values that compound expressions should receive: 'if *p*, then *q*', depending on the values of *p* and *q*. Once this has been established, we can create a matrix (interpretation table) with selected values. This provides us with a correct definition of a new formalized system that we can examine [metalogically] — regardless of whether we assign any intuitive meaning to the values.[12] (Jaśkowski, 1947, p. 66)

[11] In Polish: Stosunek tych badań do praktyki życiowej jest taki sam, jak wszelkich prac matematycznych. Rachunek nie daje nakazu: "Należy zbudować most według tego a nie innego projektu", może tylko doprowadzić do wniosku: "Most zbudowany według tego projektu czyni zadość wymaganiom co do wytrzymałości, ceny itd." Decyzja, jak most zbudować — to już sprawa dalsza, przy jej powzięciu można kierować się innymi względami, nawet przyzwyczajeniem lub chęcią zerwania z szablonem.

[12] "[$Ł_3$] można interpretować podobnie jak logikę dwuwartościową, lecz przy użyciu większej ilości wartości. Należy w tym celu ustalić, jakie wartości mają otrzymywać wyrażenia złożone: 'jeżeli *p*, to *q*' itd. zależnie od wartości zdań *p* i *q*. Z chwilą, gdy

Note that Jaśkowski is outlining the general mechanism required to establish a new formal system. In propositional cases, it is sufficient to establish a method of evaluating more complex logical expressions based on the value of less complex ones. This is known as compositionality. Upon completion, the formal system can be subjected to metalogical analysis, which will yield specific results. It is important to note that this formal work can be conducted even in the absence of any intuitive understanding of either new truth values or the mechanism by which compound expressions acquire their values.

This is the precise philosophical problem that has been identified in the literature concerning Łukasiewicz's project, namely the interpretation of truth-values as probabilities, and the peculiar behaviour of connectives, particularly implication. Jaśkowski is pointing exactly that saying:

> Łukasiewicz's systems have not yet found application in mathematics. The very principle of accepting probabilities as logical values raises doubts here. [...] Therefore, there is no basis for assigning the value 1 to the conditional: 'if p, then q' in every case where both p and q have the value 1/2, as Łukasiewicz does. Although, as we can see, the implementation raises certain reservations, Łukasiewicz's idea points the way to further research in this direction.[13]
>
> (Jaśkowski, 1947, p. 66)

It should not be surprising then that in the context of his own research, Jaśkowski asserts that intuitive justification is among the most crucial elements of the entire endeavour. The focus of his studies is not simply the provision of metalogical results for a newly established system, but rather the provision of a formal tool for a philosophically important undertaking. In (Jaśkowski, 1948) he is echoing his earlier position noting that:

to ustaliliśmy, mówimy, że mamy macierz (tabelę interpretacyjną) o danych wartościach. Otrzymujemy tym samym poprawną definicję nowego systemu sformalizowanego, który możemy badać metodologicznie — nawet bez względu na to, czy wartościom przypisujemy jakieś znaczenie intuicyjnie zrozumiałe, czy nie.

[13] In Polish: Systemy Łukasiewicza nie znalazły dotychczas zastosowania w matematyce. Wątpliwości budzi tutaj sama zasada przyjęcia prawdopodobieństw za wartości logiczne. [...] A więc nie ma podstawy do nadania wartości 1 okresowi: 'jeżeli p, to q' w każdym przypadku, w którym zarówno p jak i q mają wartość 1/2, jak to czyni Łukasiewicz. Chociaż realizacja budzi, jak widzimy, pewne zastrzeżenia, to jednak pomysł Łukasiewicza wskazuje drogę do dalszych poszukiwań w tym kierunku.

> As is known, even sets of those inscriptions which have no intuitive meaning at all can be turned into a formalized deductive system.
>
> (Jaśkowski, 1999, p. 43)

In the ensuing portion of the text, he proposes a formal system that addresses the requirements outlined by Jaśkowski's criterion. Notably, in order to eliminate any concerns or difficulties that could arise on a similar basis to that of Łukasiewicz's project, Jaśkowski takes great care (the details will be covered in Section 4.4) to provide an intuitive meaning to expressions of the proposed system.

3.3. Jaśkowski's foreshadowing

Before proceeding to an examination of the proposed solutions to Jaśkowski's problem, including his own, it is first important to consider the foreshadowing that can be found in (Jaśkowski, 1947). In a recent paper by Urchs (2025), titled "The beginnings of Toruń logic", the existence of Jaśkowski's paper (1947) is acknowledged precisely for this reason. The author notes that at the end of the paper, Jaśkowski states that it would be highly interesting to have a non-explosive methodological underpinning for reasoning in the foundations of mathematics. Furthermore, given the context of publication "[i]t is hard to imagine that Jaśkowski just lightly formulated his remark in a very serious journal, without having a well thought-out concept up his sleeve" (Urchs, 2025, p. 16). Urchs' opinion is that the matter presented in both (Jaśkowski, 1947) and (Jaśkowski, 1948) was already developed during the war and has since been reconstructed.

In view of the above, the following quotations are provided from the concluding section of (Jaśkowski, 1947). In this section, Jaśkowski returns to the discussion of the relationship between metalogical results and the goals of individuals who use formal systems. He opens with the following:

> So far, I have deliberately avoided considering the attitudes of non-mathematicians towards the issues discussed, as this would take us beyond the intended scope of the article. However, since philosophy aims to synthesize all scientific knowledge, to create a worldview, it must also consider the contribution of mathematics to this very process.[14]
>
> (Jaśkowski, 1947, pp. 68–69)

[14] In Polish: Celowo pomijałem dotychczas stosunek nie-matematyków do omawianych zagadnień, rozważania dotyczące tego stosunku wykroczyłyby poza zamierzony

Subsequently, Jaśkowski issues a warning that when comparing the views of philosophers and mathematicians, caution should be exercised, since they are accustomed to employing the same notions in often very divergent manners. Nevertheless, the most significant element of this section is the following question: 'What conclusions does philosophy draw from recent research on the foundations of mathematics?'. Jaśkowski's response is as follows:

> Many results of [metalogic] have a destructive, negative rather than positive significance for the matter of synthesis. They show the difficulties faced by those who set themselves the goal of synthesizing sciences, of creating a scientific view of the world. In 1931, Gödel proved his famous theorem [. . .]. It is, in a sense, proof of the non-existence of the "philosopher's stone" — a formal, consistent synthesis of all information — even within a single discipline, that is mathematics.[15]
>
> (Jaśkowski, 1947, pp. 69–70)

Following this, Jaśkowski offers a commentary on one of the most notable methods employed in addressing various paradoxes within the domain of logic, namely the implementation of some kind of syntactic restriction. Two of the most notable illustrations of this approach are typed set theory in the foundations of mathematics and Tarskian typed approach in formal theories of truth. According to Jaśkowski's preferences:

> Meanwhile, the language used to formulate the synthesis must be rich; we would not want to impose barriers that would effectively prevent us from uttering certain comprehensible sentences.[16]
>
> (Jaśkowski, 1947, p. 70)

It is a significant methodological standpoint which was formalized by the set of axioms of ZF within the framework of set theory, and, much later,

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temat artykułu. Tym niemniej, skoro zadaniem filozofii jest naukowa synteza wszystkich wiadomości naukowych, stworzenie poglądu na świat, to musi ona interesować się również tym, co do tej syntezy wnosi matematyka.

[15] In Polish: Wiele wyników [metalogiki] posiada dla zagadnień syntezy znaczenie raczej destrukcyjne, negatywne, niż pozytywne. Wykazują one, jakie trudności stoją przed tym, kto stawia sobie za cel syntezę nauk, naukowy pogląd na świat. W roku 1931 Gödel udowodnił słynne twierdzenie [. . .]. Jest to niejako dowód nieistnienia 'kamienia filozoficznego' — formalnej niesprzecznej syntezy wszystkich wiadomości — już w obrębie jednej dyscypliny, w matematyce.

[16] In Polish: Tymczasem język służący do formułowania syntezy musi być bogaty, nie chcielibyśmy stawiać mu zapór, które faktycznie uniemożliwiłyby wypowiedzenie pewnych zrozumiałych zdań.

it was formalized in celebrated (Kripke, 1975) within the framework of formal theories of truth. In short, one does not want to impose any artificial barriers on expressions of the language used in a given theory. In the context of the task of synthesizing scientific knowledge, such language could be, for example, some semi-formalized part of natural language. Then, since a high level of expressiveness is required to connect various parts of scientific knowledge, any restrictions could be detrimental. It is evident that this predicament gives rise to numerous questions, and Jaśkowski provides the following ones:[17]

> To what extent is synthesis possible without contradictions, without antinomies? Should one avoid inconsistencies at all costs, even if it means renouncing the complete synthesis of knowledge? [...] Or should one not fear inconsistencies and rely on a logic that takes their sting out and does not allow conclusions to be drawn from what is obviously wrong? Or maybe it will ultimately come down to applying different logics to different theories?[18] (Jaśkowski, 1947, p. 70)

These questions are of considerable significance and are the subject of much contention. Jaśkowski concludes with stating that "[l]et us hope that further logical research will shed new light on these issues."[19]

4. Overview of the proposed solutions

In this section a concise overview of some of the proposed solutions to Jaśkowski's problem is provided. In order to ensure optimal brevity, the ensuing blueprint is employed for the purpose of summarizing the selected papers. Firstly, the manner in which Jaśkowski's problem is presented. Secondly, the motivations for addressing it. Lastly, the main result(s) that have been declared. In the final subsection, Jaśkowski's own solution is presented for comparision with the overviewed ones.

[17] This is also the passage quoted in (Urchs, 2025).

[18] In Polish: W jakim zakresie możliwa jest synteza bez sprzeczności, bez antynomii? Czy unikać sprzeczności za wszelką cenę, choćby wypadło zrezygnować z pełnej syntezy wiedzy? [...] Czy też nie obawiać się sprzeczności a przyjąć taką logikę która stępi ich ostrze i nie pozwoli na wyprowadzenie z nich jawnego fałszu? Czy może w różnych teoriach wypadnie stosować odmienne logiki?

[19] In Polish: Miejmy nadzieję, że dalsze badania logiczne rzucą nowe światło na te zagadnienia.

4.1. "Sur un probleme de Jaśkowski" of da Costa & D'Ottaviano

The paper by da Costa and D'Ottaviano (1970) is the first to address Jaśkowski's problem following his own work. It is noteworthy that the former author is N. C. A. da Costa, who is regarded by some as the 'true founder of paraconsistency' and is responsible for the formulation of an alternative set of requirements for satisfactory paraconsistent logic. It is also worthy of note that the work has been written in French, thus still failing to broaden international appeal.

The authors set out Jaśkowski's problem as a task "of constructing a propositional calculus with the following properties:

(1) an inconsistent system based on such calculus is not always trivial;
(2) calculus must be sufficiently rich to enable most common reasoning;
(3) the calculus must have an intuitive meaning."[20]

Thus, the presentation of the problem is identical to that in (Jaśkowski, 1948), with the exception of the mention of the system being propositional.

Subsequently, the authors indicated their motivation stating that "[a]mong other results, we present here a new solution to this problem".[21]. This new solution to Jaśkowski's problem (and, hence, the main result of the paper) is a three-valued calculus with a conditional corresponding to the implication proposed in (Jaśkowski, 1948), hence fulfilling (to a certain degree) the second requirement. Moreover, as neither of $(\alpha \wedge \neg\alpha) \to \beta$, $\neg\alpha \to (\alpha \to \beta)$ is a valid scheme, this calculus is not always trivialized in a wake of inconsistencies, hence fulfilling the first requirement. Last but not least, as the authors argue, the calculus in question is easily providable with an intuitive interpretation, namely:

> in the preliminary phase of the elaboration of a theory (mathematical, physical, etc.) contradictions may appear which, in the definitive formulation, are eliminated; 0, 1, and 1/2 are the truth values, where 0 represents 'false', 1 represents 'true', and 1/2 represents the provisional value of a proposition A, such that A and $\neg A$ are theses of the theory under consideration in its provisional formulation; In the final form of

[20] In the original: de la construciton d'un propositionel jouissant des propériétes suivantes: (1) un système inconsistant, fondé sur un tel calcul, n'est pas toujours trivial; (2) la calcul doit être suffisamment riche pour rendre possibles la plupart des raisonnements usuels; (3) le calcul doit avoir un sesne intuitif.

[21] In the original: Entre autres résultats, nous présentons ici une nouvelle solution de ce problème.

> the theory, the value 1/2 will reduce, at least in principle, to 0 or 1.[22]
> (da Costa and D'Ottaviano, 1970, p. 1351)

Hence, fulfilling the third requirement.

In the second part of the paper, the authors proceed to bring the investigation under the framework of da Costa's criterion by extending the proposed calculus to first-order logic (i.e., fulfilling (daC3)). As is argued, by these means "[we] can construct inconsistent and apparently non-trivial systems of set theory"[23], which is a task reserved for future work. It is important to note that the sole extending of the calculus to first-order logic can be argued to help fulfill (Jaśk2) to a greater degree. In addition, the possible construction of set theory on the basis of proposed calculus has potential to provide new ways of dealing with paradoxes in the foundation of mathematics, which is the subject that was of major interest in (Jaśkowski, 1947).

In general, the work conducted in this paper can be considered to be of significant value from the perspective of Jaśkowski's methodology and goals.

4.2. "On some modal logical systems defined in connexion with Jaśkowski's problem" of Kotas and da Costa

The second address of Jaśkowski's problem is in the paper by Kotas and da Costa (1977). On this occasion, da Costa is collaborating with Jerzy Kotas, a former student of Jaśkowski and a mathematician, a fact which is deemed to be relevant.

The authors present Jaśkowski's problem as:

> the problem of finding logical systems which could be employed as underlying logics of deductive systems not devoid of inconsistency. This is a very important problem, not only from the philosophical but also from the practical and the mathematical points of view.
> (Kotas and da Costa, 1977, p. 57)

[22] In the original: dans la phase préliminaire de l'élaboration d'une théorie (mathématique, physique, etc.), peuvent apparatire des contradictions qui, dans la formulation définitive, sont éliminées; 0, 1 et 1/2 sont les valeurs de vérité, où 0 représente le 'faux', 1 le 'vrai' et 1/2 la valeur provisoire d'une proposition A, telle que A et ¬A sont des thèses de la théorie en considération dans sa formulation provisoire; dans la forme définitive de la théorie, la valeur 1/2 se réduira, au moins en principe, à 0 ou à 1.

[23] In the original: nous pouvons construire des systèmes da théorie des ensembles inconsistants et apparemment non triviaux (da Costa and D'Ottaviano, 1970, p. 1353)

Note that in this instance, the three requirements constituting Jaśkowski's criterion are not mentioned, and instead the first one is solely highlighted. It is acknowledged that this could be interpreted as a convenient abbreviation for the commonly recognized issue. However, this would be an overly generous explanation. Prior to arguing this on the basis of paper's motivations and primary results, it is important to recognize that the authors emphasis that Jaśkowski's problem is very important "not only from the philosophical [...] point of view" can be read as implying that the investigation of this problem has been predominantly focused on from that standpoint. This is certainly not the case. Additionally, this does not concur with Jaśkowski's remark that the problem he is addressing is not a conventional mathematical one, but rather one that demonstrates both mathematical and philosophical intricacy. Subsequently, the authors allude to 'the practical and the mathematical point of view', and provide a summary of Jaśkowski's own solution, stating that:

> In the mentioned paper [i.e. (Jaśkowski, 1948)] Jaśkowski presented one solution of the problem at the level of the propositional calculus. He defined by an interpretation in Lewis' $\mathsf{S5}$ propositional calculus a new logical system $\mathsf{D2}$ which he called discussive (or discoursive) propositional calculus. (Kotas and da Costa, 1977, p. 57)

Once again, note that exclusively formal character of Jaśkowski's own solution is being underscored. On top of this, the authors draw attention to certain observations relating Jaśkowski's solution to different modal systems studied in the literature, pointing out that one may identify D_2 with $\Diamond$-S_5, where the latter is the set of all formulas that become theses of S_5 when preceded by the possibility sign '$\Diamond$'.

The above observation prompts them to adopt the standard formal strategy of doing logic, namely that of generalizations. Thus, given any modal system S, $\Diamond^k$-S denotes the set of all formulae which become theses of S when preceded k times by '$\Diamond$'. The authors' objective (and, consequently, the main result) is:

> to survey certain modal systems obtained by some authors in the course of investigations of [$\Diamond^k$-counterparts] of well-known modal logics , and to present some new interesting properties of their [$\Diamond^k$-counterparts]. (Kotas and da Costa, 1977, p. 57)

It can thus be concluded that the work carried out is purely formal and that a connection with Jaśkowski's problem is, at best, barely there. In light of the Sentiment, Kotas and da Costa (1977) cannot be considered to be of significant value from the perspective of Jaśkowski's methodology and goals.

4.3. "On the problem of Jaśkowski and the logic of Łukasiewicz" of da Costa and Kotas

A year later da Costa and Kotas (1978) was published, once more addressing Jaśkowski's problem. On this occasion, the investigation does not relate to modal logics, but instead to many-valued systems of Łukasiewicz.

The authors begin by providing an overview of the reasons that one could be interested in a formal systems capable of handling inconsistencies, and then continue by presenting Jaśkowski's problem as the task to:

> [...] construct logics satisfying the following conditions: 1) when they are employed as underlying logics of inconsistent propositional systems, inconsistency does not necessarily imply triviality; 2) they are rich enough to make possible most common inferences.
>
> (da Costa and Kotas, 1978, p. 128)

Strikingly, something is missing there. The authors opted to present the problem in a requirement-like manner, yet they omitted the third one. It is acknowledged once more that this could be interpreted as a convenient abbreviation. It could be argued that (Jaśk3) is implied by (Jaśk2), as most common inferences could not be made possible without an intuitive justification for the system.[24]. Nevertheless, this appears to be a flawed argument, and thus an overly generous explanation. Firstly, recall that (Jaśk2) may be regarded as analogous to (daC4), i.e. requirement stating that the proposed system 'should contain the most part of the schemata and rules of the classical propositional calculus which do not interfere with the first condition'. But this merely indicates that certain inferences pertaining to this-and-that syntactical shapes are to hold in the system. Secondly, the investigation of the meaning of these

[24] For such a view, see (Carnielli and Rodrigues, 2021). Moreover, it is worth acknowledging the fact that the importance of Jaśkowski's criterion is noted by the authors of this paper, as they confess that "[a]ctually, we see [(Jaśk2)] and [(Jaśk3)] as expressing a central question for the philosophy of paraconsistency: how to explain, in a plausible and intuitive manner, the nature of contradictions tolerated by paraconsistent logics?" (Carnielli and Rodrigues, 2021, p. 577)

inferences in the new system, especially the involved connectives, is a different matter. Despite the fact that this is frequently overlooked in both practice and literature alike, it is surely an indispensable component within the domain of paraconsistent logics. One of the most significant contemporary questions in this domain pertains to the notion of paraconsistent negations, and whether they can be considered negations at all. This is a matter of considerable interest from both a formal and a philosophical perspective. As such, it is evident that a system may be in close relation to classical logic inferences-wise, thereby fulfilling both (Jaśk2) and (daC4), yet lacking any intuitive justification. Consequently, the omission of (Jaśk3) is a major philosophical blunder.

It is possible to deduce the reason for the aforementioned blunder, and consequently the motivations of the paper under discussion, based on the preceding subsection. Nevertheless, the blueprint should be applied in any case. The primary observation that forms the basis for the authors' work is predicated on remarks pertaining to natural language communication and the manner in which it might be formalized using D_2 and $\mathsf{Ł}_3$:

> In every-day life the process of assertion of sentences is very complex. We may suppose that we consider any sentence as true when our conviction about its truth is strong enough or, in other words, when a 'logical value' sufficiently large corresponds to the sentence. Clearly, the assertion of a sentence as probable is made analogously. If we restrict ourselves to D_2 or $\mathsf{Ł}_3$, then we have to consider as probable all sentences which have (in our conviction, of course) a 'logical value' greater than 0, and moreover we are constrained to assume that 'to be probable' and 'to be true' have practically the same meaning. This attitude, although convenient and full of interesting consequences, is only a crude approximation to the actual procedure.
>
> (da Costa and Kotas, 1978, p. 128)

Therefore, when truth-values of sentences are considered subjectively, that is to say, as convictions about their 'real' status, the use of two or three values to model the process of assertion is not fine-grained enough. However, it should be noted that one of the primary issues with Łukasiewicz's project, as also identified by Jaśkowski, pertains to the conceptualization of probabilities as truth-values. In the above observation, subjective probabilities are taken for granted without any further elaboration. The problematic relationship between convictions about the truth value of a sentence and its 'real' truth value is also left entirely unaddressed.

The main result, in accordance with the stated aim, is then:

> [...] to show that if we take as bases the finite or infinite logics of Łukasiewicz, then it is possible to define a class of logical calculi, some of which are solutions of Jaśkowski's problem.
>
> (da Costa and Kotas, 1978, p. 129)

Once more, it is a compromised version of Jaśkowski's problem, arguably lacking its most crucial element. Furthermore, in a manner akin to (Kotas and da Costa, 1977), the work adopts a purely formal approach, as its philosophical underpinnings are rather weak in their own right and do remain unelaborated in the subsequent sections of the paper in question. The results obtained and the open questions stated are both of purely formal character. In light of the Sentiment, da Costa and Kotas (1978) cannot be considered to be of significant value from the perspective of Jaśkowski's methodology and goals.

4.4. Jaśkowski's own solution

Jaśkowski proposed his own solution in the same paper he formulated the problem, that is in (Jaśkowski, 1948). Interestingly enough, its status is disputed in the literature. For example, da Costa and D'Ottaviano (1970) stating that its a solution in its own right, while Ciuciura (2003) claims that "Jaśkowski, as we shall see, ignores to some extent the need to justify that the D_2 system satisfies conditions [(Jaśk2)] and [(Jaśk3)]".[25]

In the interest of utilizing the blueprint, note that the manner in which Jaśkowski's problem is presented was provided in Section 2, while the motivations for addressing it were provided in Section 3. Hence, what is left is to present an overview of Jaśkowski's main result, that is his system of discussive logic D_2.

As the name itself suggests, this system is proposed as a means of formalizing some aspects of natural language communication. Jaśkowski starts with the following observation:

> The same [i.e., lack of uniform opinion] happens if the theses advanced by several participants in a discourse are combined into a single system

[25] In Polish: "Jaśkowski, jak zobaczymy, ignoruje w pewnej mierze potrzebę uzasadnienia, że system D_2 czyni zadość warunkom [(Jaśk2)] i [(Jaśk3)]" (Ciuciura, 2003, p. 289).

> [...]. Let such a system which cannot be said to include theses that express opinions in agreement with one another, be termed *discussive system*. To bring out the nature of the theses of such a system it would be proper to precede each thesis by the reservation: "in accordance with the opinion of one of the participants in the discussion" [...]. Hence the joining of a thesis to a discussive system has a different intuitive meaning than has assertion in an ordinary system.
>
> (Jaśkowski, 1999, p. 43)

In order to make a formal sense of such reserved assertion[26], Jaśkowski proposed to simply preceed any asserted sentence with the possibility sign '$\Diamond$'. The idea is that if one of the participants of the discussion is asserting something, then they are adding it to the space of possibilities shared by discussion's participants.[27] Subsequently, Jaśkowski demonstrates that the above intuitions cannot be formalized in a standard system of two-valued modal logic, for the reason that $\Diamond(\phi \to \psi) \to (\Diamond\phi \to \Diamond\psi)$ is not its thesis. Intuitively, if one participant of the discussion states that 'if ϕ, then ψ' (i.e., 'it is possible, that if ϕ, then ψ'), and some participant of it states that ϕ (i.e., 'it is possible, that ϕ'), then it does not intuitively follow that ψ (i.e., 'it is possible, that ψ'). Consequently, Jaśkowski's primary task is to identify a propositional function that, when applied to discussive theses, plays an analogous role to implication in ordinary systems. The connective dubbed 'discussive implication' defined in the following manner is provided as one of the solutions:

DEFINITION 4.1 (Discussive implication). Let φ, ψ be any formulae of the given language. *Discussive implication* is defined as $\phi \to_{\mathrm{d}} \psi := \Diamond\phi \to \psi$.

In the modal terms, the above formula is stating that 'if it is possible, that ϕ, then ψ', while in the discussive terms it may be read as 'if someone states that ϕ, then ψ'. The above problem is defused then, as $\Diamond(\Diamond\phi \to \psi) \to (\Diamond\phi \to \Diamond\psi)$ is a thesis of two-valued modal system. Once again, intuitively, if one participant of the discussion states (in the accepted sense) that 'if ϕ, then ψ' (i.e., 'if it is possible that ϕ, then ψ'), and some participant of it states that ϕ (i.e., 'it is possible, that

[26] For a contemporary overview of various approaches to assertion, see (Goldberg, 2020).

[27] Note the strong similarities to the later work on assertion that was carried out in (Stalnaker, 1978, 2002). The obvious difference is that the latter could make use of the full framework of possible worlds semantics.

ϕ'), then it does indeed intuitively follow that ψ (i.e, 'it is possible, that ψ). Therefore, such defined discussive implication does formally what one would like it to do philosophically. Jaśkowski goes on to define D_2 as a two-valued modal system with deductive implication, in which the theses are derived from those of the standard modal calculus by preceding them with the possibility symbol '$\Diamond$'. In the remainder of the paper, Jaśkowski provides a formal analysis of the proposed system and offers a philosophical commentary on these results.

It can thus be concluded that the work carried out is not purely formal, as Jaśkowski takes great care to provide an intuitive meaning to expressions of the proposed system, hence fulfilling (Jaśk3). Consequently, it is to be considered of significant value from the perspective of Jaśkowski's methodology and goals. This fact should not come as a surprise

5. Contemporary relevance

This section presents the relevance of Jaśkowski's criterion to contemporary research. The focus is on philosophical logic, particularly recent discussions on the notion of 'philosophical interpretation'. A case study of a selected modern approach to Jaśkowski's problem is then presented. Using the methodological framework revisited in this paper, the aforementioned approach is identified as being severely inadequate.

5.1. The notion of 'philosophical interpretation'

As stated in the final paragraph of Section 2, the contention advanced in this paper is that the lack of trackability and determinability of Jaśkowski's criterion is its virtue. As noted in the literature, the problem of fulfilling its requirements has a distinctively philosophical, rather than formal, character. The notion of 'philosophical interpretation' is introduced below with the intention to convey the same meaning as 'intuitive justification' of (Jaśk3). Two points must be noted here. Firstly, there might be some differences between 'interpretation' and 'justification' on the epistemological ground, but investigating this would go beyond the scope of this paper. Secondly, the 'philosophical' in 'philosophical interpretation' does not necessarily invoke any philosophical character *per se*; rather, it indicates a distinct philosophical attitude towards the need to provide an interpretation. That being said, it is important to emphasize that the matter of philosophical interpretation is a philosophical issue rather than a mathematical one. This is not only because it is a *philosophical* interpretation, but also because of the following fact:

> [t]here is no hope of giving a fully, formally rigorous mathematical proof of the coincide between some rigorously defined notion and some intuitive notion, since all the notions involved in a fully, formally rigorous mathematical proof must be rigorously-defined and not intuitive ones. (Burgess, 2016, p. 617)

Nevertheless, if a logical system is to have any significance beyond that of a mere mathematical structure, then it appears that it must be possible to endow it with some sort of interpretation.[28]

Important work on elucidating the murky notion of philosophical inspiration has recently been presented in (Tajer and Fiore, 2022). Discussing this notion, the authors provide an example of two formally dual but philosophically distinct logical systems, concluding that the difference lies in their philosophical backgrounds, that is:

Definition 5.1 (Philosophical background.[29]). Let $\langle \mathcal{L}, \models \rangle_{\mathsf{D},\mathsf{I}}$ be the result of applying the pure logic $\langle \mathcal{L}, \models \rangle$ to the domain D under the metatheoretical interpretation I. To give a *philosophical background* for $\langle \mathcal{L}, \models \rangle_{\mathsf{D},\mathsf{I}}$ is to give a philosophical account of the notions involved in, or relevantly related to, the application of $\langle \mathcal{L}, \models \rangle$ to D under I.

In this sense, a ‘philosophical background’ is a philosophical interpretation of the logical calculus in question. This definition is vague, as explaining the murky notion of ‘philosophical background’ requires the use of the equally hazy notion of ‘philosophical account’. However, it suffices to illuminate the following intuition: providing a philosophical

[28] As remarked in (Antunes and Szmuc, 2022), “and this hold true even in those cases in which only a certain technical application is envisaged”.

[29] For more details, especially on other quasi-formal notions involved, see (Tajer and Fiore, 2022), pp. 212–217. Note that the term ‘background’ should suggest that it is something stable against which something else acts. However, the above definition allows for the absence of *the* philosophical background, and for the existence of multiple valid ones. Thus, such a background is ‘backgrounding’ only when initially fixed, there is no such thing as a single background that would be in some sense correct or absolute. This could be considered an issue, but this possibility will not be entertained here given the content of Section 3.1. Nonetheless, the author would like to express his gratitude to the anonymous reviewer for signaling this potential issue.

interpretation of a formal system involves telling a (hopefully convincing) story about the meaning of the formal machinery in question. Consequently, relying on a presentation of model-theoretic semantics for a given system misses the point of a philosophical interpretation. If this were not the case, one could simply replace 'philosophical account' with 'model-theoretic semantics'. That is something that is intentionally not done. This standpoint is similarly upheld in the recent defense of the notion of philosophical interpretation:

> So, the notion of philosophical interpretation is clearly distinguished from formal interpretation. Indeed, this latter notion has to do with model theoretical apparatus that interprets the language of **L** and defines the model-theoretical consequence relation $\models_{\mathbf{L}}$. On the other hand, as we argued before, philosophical interpretations are informal.
> (Barrio et al, 2025, p. 209)

Notice that philosophical interpretations being essentially informal must also be neither straightforwardly trackable nor mechanically determinable. But these are exactly the points made against using Jaśkowski's criterion as a methodological tool in dealing with research on paraconsistent logics. Furthermore, note that the focus on providing just formal interpretations fuels the Sentiment. It occurs to us that paying more attention to Jaśkowski's third criterion, and stating it in terms of 'would have a philosophical interpretation', could be a remedy for this problem. Let us explore this idea through a case study of the selected modern approach to Jaśkowski's problem.

5.2. Case study

Here, a case study of the selected modern tackling of Jaśkowski's problem is analyzed, namely one provided in the paper by Béziau (2006), titled "The paraconsistent logic Z. A possible solution to Jaśkowski's problem".[30] The following analysis is structured around the blueprint from Section 4, with the commentary provided along the way.

[30] One could argue that the case study is flawed from the very beginning, as the picked paper is not a heavyhitter in this field of research. However, this diagnosis would not be correct; if there are any such papers, Béziau's is certainly one of them. Firstly, it is a rare one — the author is not aware of any others in the XXI century — focusing on Jaśkowski's criterion. Secondly, it is a fruitful one as it has inspired research on Z-like formal systems, for example (Omori and Waragai, 2008; Osorio et al, 2014; Mruczek-Nasieniewska and Nasieniewski, 2018). It is worth noting that none of these are concerned with Jaśkowski's criterion, but rather with the formal

The author presents Jaśkowski's problem as follows:

> Jaśkowski in his 1948's paper [i.e., (Jaśkowski, 1948)] already clearly stated the problem of combining these three features, and this has been called Jaśkowski's problem [cf., e.g., (Kotas and da Costa, 1977)]. It is not misleading to say that Jaśkowski's problem is the basic problem of paraconsistent logic. (Béziau, 2006, p. 100)

It would be beneficial to provide an elaboration on the 'three features' in question, however, as will be demonstrated hereafter, it is not clear what some of these features are, nor is it clear why there is a supposition that they have been observed already by Jaśkowski. In order to identify the first of the said features, the opening paragraph of the investigated paper is to be considered:

> It seems that nowadays the main open problem in the field of paraconsistent logic is still: Does paraconsistent logic really exist? This means: can we find a good paraconsistent logic? Of course one logic may be good for Mr. Black and bad for Mr. White. In other words, there is no formal definition of what is a good logic. Anyway all the known systems presented thus far bear serious defects. At least there is no paraconsistent logic which is recognized unanimously as a good paraconsistent logic. (Béziau, 2006, p. 99)

Two elements of the above merit attention. Firstly, it is as problematic as it is deficient to paraphrase a question on the existence of paraconsistent logic as one regarding the existence of a satisfactory one. It should be recalled that there is a marked distinction between (Jaśk1) and (Jaśk2)-(Jaśk3). The former is about there being a paraconsistent logic, whereas the latter are about this logic being a satisfactory one. In the course of examining Jaśkowski's own solution, it became evident that significant emphasis was placed on the fulfillment of the last two requirements. The rationale for this pertained to the inherent philosophical concerns that were of Jaśkowski's interest. It is possible to devise a system of paraconsistent logic; however, this does not entail the system will be satisfactory from the point of view of some particular goal. Secondly, as outlined in the section on Jaśkowski's motivations, it is evident that "one logic may be good for Mr. Black and bad for Mr. White." However, this is not due to their subjective opinions, but rather due to the possibility

properties of systems inspired by Béziau (2006). This fact only adds further punch to the judgment of this section. The author would like to express his gratitude to the anonymous reviewer for pointing out this possible worry.

that they may pursue divergent goals. The notion of 'good logic' in a vacuum is a fallacious one. Furthermore, given that the status of a logical system is contingent on its philosophical interpretation, which is informal, it is unsurprising that there is no formal definition of what constitutes a good logic. There cannot be. Consequently, if Béziau is trying to state that one of the research objectives should be to seek out some objectively good paraconsistent logic and, moreover, is suggesting that it was Jaśkowski's idea, then — in the light of Section 3 — he is simply mistaken and incorrect, respectively.

The second of three features is identified in the following paragraph:

> The central problem of paraconsistent logic is to find a negation which is a *paraconsistent* negation in the sense that [$\alpha, \neg\alpha \nvdash \beta$], and which at the same time is a paraconsistent *negation* in the sense that it has enough strong properties to be called a negation.
>
> (Béziau, 2006, p. 99)

This represents a distinctly contemporary interpretation of the paraconsistency problem. A substantial body of literature exists on the subject of the definition of explosiveness (a feature considered by many to be of fundamental importance to paraconsistent logics). The investigation of negation-like connectives is a pivotal element in this field. Furthermore, Béziau's observations highlight the importance of not only the formal features of such a connective, but also the need to justify initial categorization of it as a negation. For instance, if one were to define negation as a contradictions-forming operator, then paraconsistent negation could never be considered a negation in this sense. The issue is further complicated by the existence of definitions of explosiveness that do not employ negation (see Basu and Roy, 2022). That being said, it is important to note that this is a significant area of research (in which the author of this paper also takes great interest), however, the assertion that it is 'the central problem of paraconsistent logic' is arguably an exaggeration.

Finally, in order to elaborate further on the remark concerning the issue of negation, there is the following third feature to which Béziau pays attention:

> Furthermore, such kind of paraconsistent negation, if it has not to be an artificial construct, a mere abstract object of a formal meaningless game, must have an intuitive background. (Béziau, 2006, p. 99)

It is rather straightforward. However, it is important to acknowledge the notion of an 'intuitive background' that is purportedly involved. Previously invoked were notions of 'intuitive justification' and 'philosophical background'. Now there is also some kind of a mingle between those two. It is reasonable to assume that the idea to which Béziau is alluding is that of a 'philosophical interpretation'. Thus, what is remarked is that (Jaśk3) must be fulfilled in regard to any deviations from classic logical connectives.

This is the point in the paper at which the remark regarding Jaśkowski articulating the problem of integrating these three features is made. Furthermore, the springboard behind Béziau's paper is the conviction that "Jaśkowski himself presented a system, called discussive logic, but it does not seem to be a solution to his problem" (Béziau, 2006, p. 100). Accordingly, the primary motivation (and, consequently, the main result) of his paper is to:

> [...] present a paraconsistent logic Z which seems to be a possible solution to Jaśkowski's problem: this logic is paraconsistent, has a very intuitive semantics, is axiomatizable and its negation is quite strong, in particular the replacement theorem holds for it.
>
> (Béziau, 2006, p. 100)

In consideration of the scope of the current paper, it is of no interest to address issues pertaining to whether the given system is explosive, its axiomatization, or formal features of its negation. What there is a place for is to explore the notion of 'a very intuitive semantics'. This specific formulation, in light of the preceding subsection's content, should already evoke a sense of concern.

Béziau starts the section appropriately named 'Intuitive explanation' with the following passage:

> Semantically speaking, the basic idea of paraconsistent negation is that a proposition and its negation can both be true. In modern logic, even from the viewpoint of classical logic, true does not mean necessary true in the real world, but true in some possible worlds, or some models. Note therefore that the idea of paraconsistent negation is not so strange and does not commit one to believe in true contradictions. The point is that for paraconsistent logic we have to consider paraconsistent worlds (or structures, or models), i.e. worlds in which a proposition and its negation can both be true. (Béziau, 2006, p. 100)

The point is that the framework of possible worlds semantics, a well-known idea in the tradition of analytic philosophy, can be appled to elucidate the notion of paraconsistent negation. This is achieved by introducing slight modifications (i.e., dualization) to the standard construction. It is proposed that, given the efficacy of possible world semantics in various logical systems, such as intuitionistic logic, this strategy would surely yield an intuitive framework for the paraconsistent system. Let us even take all that for granted. Nevertheless, something bizarre ensues next. In the subsequent, more formal section named 'Mathematical definition', Béziau states the following:

> We consider a standard set of zero-order formulas For_Z built with three binary connectives $\wedge, \vee, \rightarrow$, and one unary connective $\neg$. To simplify the definition we consider here bivaluations, i.e., functions from For_Z to $\{0, 1\}$, rather than possible worlds. Therefore a cosmos [i.e., set of possible worlds] is here a set of bivaluations. (Béziau, 2006, p. 101)

Notice the sudden shift in the narration. In order to address the requirement outlined by (Jaśk3), Béziau presented a conventional narrative revolving around possible worlds and all the related concepts. However, when engaging with formal elements of his investigation, a transition to bivaluations occurred (for the sake of formal simplicity). While it is not inherently problematic to employ bivaluations, it is important to recognize that the intuitiveness of justification does not simply transition from one framing to another. In order to provide a philosophical intepretation for the formal machinery used in his paper, Béziau must tell a different story about what it amounts to. The point being made here is not that this is impossible or otherwise difficult, but rather that it is not performed, and moreover, that this whole, methodologically important, issue appears to be entirely invisible to the author.

Last but not least, the main result of the paper under discussion, as acknowledged by Béziau himself, is:

> a completeness proof for this axiomatic system, inspired by, but different from, the standard proofs of completeness in modal logic. (Béziau, 2006, p. 111)

Therefore, it can be said that the results obtained are of purely formal character, as the philosophical work is carried out in a careless manner at best, and not at all at worst. In light of the Sentiment, Béziau (2006) cannot be considered to be of significant value from the perspective of Jaśkowski's methodology and goals.

6. Summary

The primary motivation behind this paper was that reintroducing Jaśkowski's criterion to contemporary research could offer a novel perspective on some of the current issues in philosophical logic and facilitate their resolution. Particularly noteworthy is the link between the most non-formal of Jaśkowski's requirements, that is (Jaśk3), and ongoing discussion on the notion of 'philosophical interpretation'. This subject matter is of immense significance, as it gives proper attention to the Sentiment.

Finding the way out of the Sentimental situation is the key to effective philosophizing in the domain of paraconsistent logic in particular, and in the domain of philosophical logic in general. It is hoped that the present paper suffices in demonstrating that Jaśkowski's criterion (in some contemporary form[31]) can greatly help in this manoeuvring.

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[31] As stated in Footnote 3, neither the term 'paraconsistent logic' nor any paraconsistent system could have been known to Jaśkowski in the 1940s Furthermore, a great deal of work, both formal and conceptual, has been carried out since his pioneering work. Examples include the importance of the notion of negation, the strategy of classical recapture, and different ways of specifying the notion of intuitive justification. Consequently, it would be reasonable to reformulate Jaśkowski's criterion along more contemporary lines, as this would render it more applicable to the ongoing discussions and current problems. It is possible to argue that this is anachronistic; consequently, it could be more appropriate not to attribute a modernized criterion to Jaśkowski, but rather to consider it as a novum that draws heavily on the spirit of the original one. For the last time, the author would like to express his gratitude to the anonymous reviewer for highlighting need for this particular elaboration.

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