Logic and Logical Philosophy
Online First Articles (2025)
DOI: 10.12775/LLP.2025.024

Tomasz Bigaj

In Defense of Quantifier Generalism: Holism and Infinitary Resources

Abstract. According to quantifier generalism, all facts about the world can be expressed in a language devoid of proper names, whose only referential expressions are variables bound by quantifiers. This paper considers and repels some of the recently raised objections against this position. The central part of the paper presents a critical analysis of the claim advanced by Ted Sider that quantifier generalism is inevitably holistic and therefore requires unusually strong expressive resources when applied to infinite domains. Using an example of arithmetic, it is shown that there is a simple generalistic description of natural numbers that does not resort to any infinitary conjunctions or quantifiers. Such generalistic accounts also exist in many cases involving continua (such as descriptions of matter distribution in a continuous space). Moreover, these accounts are arguably superior to their individualistic counterparts due to their parsimony. In addition to that, Sider's argument alleging that generalism cannot account for the difference between non-isomorphic models of arithmetic is repelled.

Keywords: quantifier generalism; algebraic generalism; individuals; holism; infinitary quantification; arithmetic; non-standard models

1. Introduction

Many philosophers, especially philosophers of science, accept explicitly or implicitly the broad position of qualitativism (also known as generalism), that is the view that the ultimate level of reality is purely qualitative rather than individualistic.[1] One straightforward way to embrace this position is by adopting what many call "quantifier generalism":

[1] This doctrine, in one form or another, is endorsed or at least given serious consideration in (van Fraassen 1991, pp. 465–473; Stachel 2002; Pooley 2006, Glick 2016).

Received June 6, 2025. *Accepted* November 21, 2025. *Published online* December 1, 2025

all fundamental facts about the world should be expressible in a first-order language whose extra-logical terms include only predicates denoting qualitative properties and relations, while excluding proper names or any similar directly referring expressions.[2] Since the only way to formulate a sentence in such a language is by using quantifiers to bind all free variables in a given formula, it follows that worlds described in such a fashion cannot differ merely individualistically, with respect to which individual object is which. In particular, permuting individuals does not give rise to any fundamental difference between worlds.

Quantifier generalism comes under attack from two opposite directions. On the one hand, those who believe that we should include some individualistic facts in our ultimate description of the world will obviously question the limitations imposed by quantifier generalists on the fundamental language. On the other hand, there are some qualitativists who maintain that quantifier generalists do not go far enough, since they keep individuals in their ontology as a fundamental category of entities separate from properties and relations. Since apparently these remnants of individualism may cause some problems, the radical qualitativists opt for an ontology that eliminates individual objects altogether. The most prominent proponent of this approach is Shamik Dasgupta (2009, 2016), whose particular approach to qualitativism is often referred to as algebraic generalism.[3] As for the individualist critics of quantifier generalism

Also, it constitutes the backbone of the influential position of ontic structuralism (for a comparison of generalism with structuralism see Glick 2020).

[2] I leave open the question whether the relation of numerical identity or distinctness is admissible in a generalist language — some believe that it should be replaced by some qualitative relation, such as an appropriate weakly discerning (i.e., symmetric and irreflexive) relation. Also, I will not attempt here to precisely characterize what qualitative properties and relations are, other than they should not "involve" any specific individuals. Thus being taller than the Eiffel Tower is not a qualitative property, while being taller than 330 meters is. For more in-depth discussions on the distinction between qualitative and non-qualitative properties, see (Cowling 2015; Plate 2021).

[3] This approach is based on a formalization of first-order logic in a language devoid of variables, proposed by W. v. O. Quine (1960, 1976) and developed by others (Svenonius 1960; Kuhn 1983). In the current paper I will not critically analyze Dasgupta's radical position; however I would like to stress that in my opinion algebraic generalism suffers from certain shortcomings that make it less attractive than quantifier generalism. My main issue with this approach is that instead of eliminating reference to individuals, it merely hides it in a clever formalism (this was a point made by Quine himself). For some other criticism of algebraic generalism see (Turner 2011, 2016; Glick 2020). For a response see (Diehl 2018).

and qualitativism in general, there are quite a few, to mention Gordon Belot (1995, 2018), Boris Kment (2012) and Ted Sider (2020).

In this paper I will attempt to repel some specific charges against quantifier generalism coming from the camp of individualism. The central part of this paper is a polemic with a recent critique of qualitativism by Sider in (2020). He uses the observation, made first by Dasgupta, that both quantifier generalism and algebraic generalism are holistic in that it is impossible to describe purely qualitatively a given complex situation using separate and independent sentences. Generalists have to formulate one "global" quantified statement, whereas individualists may use independent individualistic propositions. Sider turns this purported holism into an argument against generalism, aiming to show that generalistic language requires some non-standard expressive resources, such as infinitary quantifiers and infinitary conjunctions, in order to account for facts which can be easily accounted for in an individualistic language.[4] As his examples he uses first-order arithmetic (with its well-known non-categoricity) and a simplified theory of material fields in space. In my response I will first question the universal validity of the claim that generalism is holistic, showing that in many cases (including arithmetic) it is possible to offer a purely qualitative description consisting of separately stated facts. This observation can help repel or at least weaken the main charge of insufficient expressive resources. More specifically, I will show that in all cases considered by Sider, the suggested non-standard expressive resources of the generalist are either not necessary, or are equally required by the individualist.

2. Holism and the infinitary quantifiers

We are now going to present the full-scale attack on all forms of qualitativism, including quantifier and algebraic generalism, mounted by Ted Sider in (2020, pp. 98–104).[5] The starting point of his argument is

[4] To be fair, Sider himself admits that his arguments against quantifier generalism can be outweighed by positive arguments in its favor. He writes "Generalism and individualism are both live possibilities for being the fundamental theory of the world, and the considerations that bear on the choice between them are tentative, contentious, and intertwined with many difficult issues" (p. 100).

[5] The arguments presented below are directed against quantifier generalism, but can be easily turned into objections against algebraic generalism since there is a one-to-one correlation between sentences of the former and the latter.

the observation, made by Dasgupta and accepted by Sider, that both quantifier and algebraic generalism imply some form of holism. Here is an example illustrating this claim. Suppose that we have two objects a and b such that a possesses a property F, b possesses a property G and a stands in a relation R to b. The individualist can describe this situation using separate sentences: $F(a)$, $G(b)$ and $R(a, b)$.[6] On the other hand, the generalist who wishes to represent the same facts in her language cannot simply replace the individualistic sentences with corresponding existential formulas $\exists x\ F(x)$, $\exists x\ G(x)$ and $\exists x \exists y\ R(x, y)$, since the joint truth of these sentences cannot guarantee the required connections between objects whose existence is stated in separate quantified formulas. In order to achieve this, the generalist has to write one longer sentence:

$$\exists x \exists y [F(x) \wedge G(y) \wedge R(x, y) \wedge x \neq y]. \tag{1}$$

Let us add that formula (1) does not imply that there are *exactly* two objects in the universe, but only that there are *at least* two objects. This feature is shared by the individualistic presentation of the situation in terms of sentences $F(a)$, $G(b)$ and $R(a, b)$, since the truth of their conjunction does not exclude that there may be objects in the domain other than a and b (see ft. 6). Also, we should make sure to exclude the possibility that a could satisfy G and b could satisfy F, as well as the possibility that the relation R would connect b and a. Thus, the most accurate generalistic description of the situation expressed elliptically in the individualistic statements $F(a)$, $G(b)$ and $R(a, b)$, would be much longer:[7]

[6] If need be, we can add to that a complete set of negative sentences such as $\neg R(b, a)$, $\neg G(a)$ and so forth. Interestingly, using purely singular sentences we cannot express the idea that there are no other objects in the universe apart from a and b. We can add a quantified sentence $\forall x(x = a \vee x = b)$, but curiously enough its 'translation' in the form of the individualistic conjunction $(a = a \vee a = b) \wedge (b = a \vee b = b)$ does not exactly express the same thought, unless we accept the metalanguage thesis that the domain consists of objects a and b only. This shows that facts of possessing properties and relations by individuals cannot completely characterize a given situation (Sider admits that as well, albeit somewhat half-heartedly, see p. 103). In order to do that, we would have to add to the list of individualistic facts the "completeness clause" stating that all positive and negative facts on the list are exhaustive, but such a clause itself could not be presented in the form of $F(a)$ or $R(a, b)$ and the like.

[7] There is no need to add the clause $x \neq y$ here, since it is already implied by $F(x)$ and $\neg F(y)$.

$$\exists x \exists y[F(x) \wedge \neg G(x) \wedge G(y) \wedge \neg F(y) \wedge R(x, y) \wedge \neg R(y, x) \wedge \forall z(z = x \vee z = y)]. \quad (1')$$

Dasgupta and Sider extrapolate this particular example and claim that the generalist is unable to completely describe any situation with the help of the conjunction of separate sentences. Here are some quotes from both authors:

> [...] the generalist thinks that the situation *must be* [italics mine] characterized by stating the single fact all in one breath. Unlike the individualist, she cannot decompose the situation into atomic parts the sum of which give the situation as a whole. That is why I call generalism a holistic metaphysics. (Dasgupta 2009, p. 56)

> A complex system must be described using a single quantified sentence. [...] Both forms of generalism, then, are holistic, in that the whole truth about a system is a single fact, and does not reduce to a collection of multiple "smaller" facts. (Sider 2020, p. 99)

However, taken at face value these pronouncements are false. It is possible to describe the above-presented situation in a purely generalist language using a conjunction of separate sentences. Here is one way to do it:

$$\exists x F(x), \quad (2)$$

$$\forall x[F(x) \to \neg G(x)], \quad (3)$$

$$\forall x[F(x) \to \exists y(R(x, y) \wedge G(y))]. \quad (4)$$

It is easy to prove that (1) is logically entailed by the conjunction of (2)–(4). From (2) and (4) it follows that there is an object possessing F that stands in relation R to an object possessing G. And condition (3) ensures that F-objects and G-objects are distinct (see Appendix for a more formal proof). On the other hand, if (1) is true, then (2)–(4) are true, provided that the individualistic description of the situation in question is complete, i.e., there are no further atomic facts regarding objects a and b, and no further individual objects.[8] All in all, it may be claimed that the situation can be decomposed into separate parts whose sum gives the situation as a whole.[9]

---

[8] As we have already pointed out in footnote 6, this assumption has to be made by the individualist too, if he wants to insist that his description in terms of individualistic sentences is exhaustive.

[9] I have encountered one peculiar objection against my non-holistic and non-

Dasgupta and Sider would probably respond that facts described in (2)–(4) are not atomic. I am not exactly sure what their meaning of "atomic" is — if they assume that atomic facts are facts regarding the possession of properties by individuals and relations holding among individuals, then obviously the generalist cannot be an atomist in this sense. But this does not make her a holist either. A slightly better objection to the above example is that sentences (2)–(4) have been selected in a rather haphazard fashion, with no discernible pattern that could be applied to all possible situations. In other words, I have not presented a principled method of how to decompose any situation into separate generalistic facts. Perhaps such a principled method can be invented, perhaps it can't. But at the very least, the thesis of holism formulated by Dasgupta and Sider should be made conditional: the generalist cannot decompose a given situation into separate facts *satisfying certain conditions*. It is Dasgupta and Sider's responsibility to spell out precisely what conditions they have in mind.

Sider uses the purported holistic character of generalism to wheel out his main argument. He writes:

> Since both quantifier and algebraic generalism require the whole fundamental truth about the largest system, the system consisting of the entire universe, to be describable by a single sentence in a fundamental language, this requires the fundamental language to have strong expressive resources — infinitary quantification and conjunction, say — if the universe is infinite. (Sider 2020, p. 99)

Let us see how this argument works in the simplest of all cases involving an infinite universe, that is the set of natural numbers ordered by the binary relation S of being a successor. The individualist can describe this universe using an infinite sequence of "positive" individualistic sentences:

individualistic description given in formulas (2)–(4) that I would like to comment on. The critic argues that these formulas by themselves cannot secure that in each of them we speak about the same objects, in contrast to the holistic variant (1) or the individualistic descriptions using proper names a and b. Now, this objection is baffling to me. In order to interpret any formal language, we have to select a particular domain of quantification. Since formulas (2)–(4) belong to the same language, the presupposed domain must be the same in virtue of our stipulation. Observe that this assumption must be made with respect to the individualistic language as well, otherwise we could not be sure that each occurrence of the letter "a" in formulas $F(a)$ and $R(a, b)$ refers to one and the same individual. Thus, the objectors, if they wanted to be consistent, would have to question even the non-holistic and individualistic description of the considered situation.

$$S(1,0);\ S(2,1);\ S(3,2);\ \dots$$

plus an array of negative sentences:

$$\begin{array}{l}\neg S(0,0);\ \neg S(0,1);\ \neg S(0,2);\ \dots\\ \neg S(1,1);\ \neg S(1,2);\ \neg S(1,3);\ \dots\\ \neg S(2,0);\ \neg S(2,2);\ \neg S(2,3);\ \dots\end{array}$$

and so on.[10]

Now, Sider claims that the generalist can only express all these facts in one enormous existential sentence arrived at by first replacing every numeral n with a variable x_n, then combining all the separate formulas into one infinite conjunction and finally binding the infinity of variables with an infinite number of quantifiers. The result would look something like that:

$$\exists x_0 \exists x_1 \exists x_2 \dots\dots [S(x_1,x_0) \wedge S(x_2,x_1) \wedge \dots \wedge \neg S(x_0,x_0) \wedge \\ \neg S(x_0,x_1) \wedge \dots \wedge \neg S(x_1,x_1) \wedge \neg S(x_1,x_2) \wedge \dots]$$

Indeed, the formula seems to be rather monstrous. However, as an aside, one may wonder why the ellipses "…" within the range of the quantifiers are so much worse than the ellipses used in the array of individualistic sentences written earlier. In order to completely describe the set of all natural numbers, the individualist has to resort to an infinite number of individualistic sentences and an infinite number of constants. These infinite resources correspond exactly to infinitary conjunctions and infinitary quantifiers of the generalist, so the individualist does not seem to be in a better position than his opponent.

But these misgivings are inessential, since it is untrue that the generalist cannot provide us with separate finitary sentences whose infinite conjunction completely describes the set of natural numbers. A list of such generalistic sentences can be easily constructed as follows:

$$\exists x_0 \forall y\, \neg S(x_0,y)$$
$$\exists x_1 \exists x_0 [\forall y\, \neg S(x_0,y) \wedge S(x_1,x_0) \wedge \neg S(x_1,x_1)];$$

[10] These facts can be conveniently presented in the form of an infinite matrix containing 0s and 1s, where 0 in the n-th column and the k-th row indicates that the number $n-1$ is not a successor of $k-1$, and 1 indicates that it is. Obviously, all ones will be placed in a straight line shifted from the diagonal by one number to the right.

$$\exists x_2 \exists x_1 \exists x_0 [\forall y\, \neg S(x_0, y) \wedge S(x_1, x_0) \wedge S(x_2, x_1) \wedge \neg S(x_1, x_1) \wedge \neg S(x_1, x_2) \wedge \neg S(x_2, x_2)];$$
$$\vdots$$

The pattern is clearly visible. The first sentence states the existence of zero (an element that is not a successor of any element). Subsequent sentences in the sequence describe completely all facts regarding initial segments of natural numbers $\{0, 1\}$, $\{0, 1, 2\}$ and so on. None of the sentences on the list require any unusual resources in the form of infinitary quantifiers or infinitary conjunctions. And yet an infinite conjunction of these statements provides us with a complete description of the set of natural numbers (of course the completeness is limited by the non-categoricity of first-order arithmetic, of which more later). Sider's claim about the need of infinitary resources for the generalist in the case of natural numbers is demonstrably false.

One may complain that the above list is peculiar in that every sentence on the list logically implies all the preceding sentences (hence the sentences are not mutually independent, as may be required from the "atomic" ones). Consequently, we end up with a set of "nested" facts rather than independent ones. Due to that peculiarity, we can discard any initial sequence of sentences, and the remaining set will do the job just fine. However, we cannot discard all sentences save one. To do that, we would have to resort to non-standard expressive resources, as described earlier.

But the generalist can do even better than that. She can provide direct translations of every single individualistic sentence on the list given earlier. Here is how this can be done in the case of any arbitrary individualistic sentence $\Phi(n, m)$ stating either that n is a successor of m, or that it is not. Let us assume that $n \geqslant m$ (the other case can be done analogously). The generalist translation of $\Phi(n, m)$ will be as follows:

$$\exists x_n \exists x_m \exists x_0 \ldots \exists x_{n-1} [\forall y\, \neg S(x_0, y) \wedge S(x_1, x_0) \wedge \cdots \wedge S(x_m, x_{m-1}) \wedge \cdots \wedge S(x_n, x_{n-1}) \wedge \Phi(x_m, x_n)].$$

Given the assumption that every object has a unique successor, the above formula is true precisely if and only if $\Phi(n, m)$. Clearly, the obtained formula is finite (the ellipses used within it are finitary — they only indicate a finite number of iterations). Thus, we can claim that for the generalist the entire situation can be decomposed into separate generalistic facts plus perhaps one universal fact regarding the uniqueness of successor.[11]

The question can be asked whether this strategy is universally applicable in cases beyond the arithmetic of natural numbers. I do not have a general answer, but my impression is that it depends on the details of the particular theory whose generalist and non-holistic translation we attempt to find. A necessary condition for the existence of such a translation is, as it seems, that the objects in the domain of the theory be qualitatively distinguishable from one another (for each object there should be a qualitative formula true of this object and only of it). A simple example of a situation in which this condition is not satisfied is provided by the set of integers ordered by the "greater than" relation, without any privileged point designated as 0. In such a case a sample individualistic sentence "$5 > -3$" cannot be expressed generalistically, simply because there is no qualitative formula that could replace numerals "5" and "–3". However, once we expand our language, for instance by introducing the addition operator $+$, a generalistic translation becomes available. We simply define zero with the help of the qualitative formula $x + x = x$, and then follow a procedure analogous to the one described earlier in the case of natural numbers. Each integer number is qualitatively identifiable by appropriate iterations of the successor operation starting with zero (keeping in mind that with respect to negative numbers we have to go "backwards", that is we have to stipulate that it is zero that is an iterated successor of a given negative number).

With respect to broader theories, such as the theory of real numbers, things become complicated due to the fact that we do not have at our disposal an uncountable number of formulas. It remains to be seen whether this hindrance affects only generalism or perhaps individualism as well (see the next section for a more in-depth discussion). In any case, my primary goal was to argue that the universal claim of holism is invalid, hence one good counterexample is sufficient. The case of natural numbers shows that the generalist can give a non-holistic reformulation of an individualistic theory, without resorting to any suspicious expressive resources. However, the case against generalism does not stop here. There is more trouble on the horizon.

[11] We can artificially hide this fact by introducing a successor function $S(x)$ rather than the binary relation of being a successor. But note that the individualist can also be accused of artificially hiding this very fact by applying proper names of which it is presupposed that each picks out exactly one and distinct object.

3. Non-standard models of arithmetic and Democritean physics

Using further the example of arithmetic, Sider puts forward another argument against generalism. As is well known, first-order arithmetic possesses non-standard models, that is models which are not isomorphic to the set of natural numbers. Sider claims, without providing much detail, that the generalist is incapable of discerning non-isomorphic models of arithmetic, while the individualist can do that by relying on an infinite list of individualistic facts.[12] In what follows I will argue, pace Sider, that both the generalist and individualist are in the exact same position regarding the recognition of differences between non-standard models of arithmetic.

Let us spell out more precisely the mathematical fact of the non-categoricity of arithmetic.[13] Let $\mathfrak{N} = \langle N, 0, S, +, \cdot \rangle$ be the standard model of natural numbers characterized by a distinguished element 0 and three functions: the one-argument successor function and the two-argument addition and multiplication functions. Structure $\mathfrak{N}$ can be easily expanded by distinguishing all numbers following 0 and giving them unique names, abbreviating iterations of the successor function, as follows: $1 = S(0)$, $2 = SS(0)$, and so on. Now, let us consider the set Σ of all sentences true in $\mathfrak{N}$. We can prove that there is a structure $\mathfrak{M} = \langle M, 0, S, +, \cdot \rangle$ which is non-isomorphic to $\mathfrak{N}$ (even though the cardinality of N equals the cardinality of M) and in which all sentences Σ are true. Proof of this fact is as follows: first we expand the language by adding a new constant a, and then we create a new set of sentences $\Gamma = \Sigma \cup \{a \neq 0, a \neq 1, a \neq 2, \ldots\}$. It is straightforward to notice that

[12] At least this is my best interpretation of his somewhat cryptic remarks on that subject. He definitely subscribes to the first part of the above statement: “Exactly the same name-free first-order sentences are true in this [i.e. non-standard] world and in a world in which the numbers are “normal”, and thus the worlds share the same fundamental facts given either form of generalism” (Sider 2020, p. 100). Then Sider invokes the principle of the supervenience of all facts on the fundamental facts in order to claim that there are certain fundamental facts which cannot be recognized by the generalist (these are the facts that differentiate between the standard and non-standard models). Presumably those facts must be recognized by the individualist, otherwise the argument would work against him as well. But as I will show in the main text, the generalist and individualist are in exactly the same position with respect to this problem; namely they both cannot express the difference between the non-isomorphic models in the language of arithmetic, but can do it in an extended language.

[13] I am following here (Boolos & Jeffrey 1980, p. 193).

every finite subset Γ' of Γ has a model consisting of the set of natural numbers with a denoting a number that is greater than the maximal number n such that formula $a \neq n$ is included in Γ' (there is only a finite number of such formulas in Γ', so obviously n exists). But the compactness theorem ensures that a set whose every finite subset has a model also has a model. In this model, which we can call $\mathfrak{M}$, there is an element which is distinct from every natural number, and hence $\mathfrak{M}$ cannot be isomorphic to $\mathfrak{N}$. And yet all sentences true in $\mathfrak{N}$ are also true in $\mathfrak{M}$ by construction.

The key point here is that models $\mathfrak{M}$ and $\mathfrak{N}$ are indistinguishable in the language of arithmetic, whether or not this language contains names for all natural numbers. Thus, both the individualist and generalist have to admit that no arithmetically-expressible fact can differentiate the two models. However, it may be replied that the individualist can discern the models in the extended language which includes the new constant a. The individualist can point out that the infinite collection of individualistic facts expressed in sentences $a \neq 0$, $a \neq 1$, ... obtain in model $\mathfrak{M}$ and not in $\mathfrak{N}$, thereby accounting for the ontological difference between the two structures. But the use of individualistic formulas is inessential for that purpose: we could equally well present the difference between the standard and non-standard models by expanding the language of arithmetic in a purely generalist fashion. Here is one way to do that. Instead of the name a, we introduce into our language a unary primitive predicate P, and we add to the set Σ the following sentences:

$$\exists x \forall y [P(y) \leftrightarrow y = x],\ \exists x[P(x) \land x \neq 0],$$
$$\exists x[P(x) \land x \neq S(0)],\ \exists x[P(x) \land x \neq SS(0)], \text{ and so on.}$$

(Of course we can also eliminate the symbol 0 in favor of the generalistic formula for the property of not being a successor of anything.) The same argument as before shows that the extended set of sentences has a non-standard model. And obviously this model is distinguishable from the standard one in that the infinite collection of generalist facts stated in the above-introduced formulas obtain in one but not the other.

It may be objected at this point that the generalist-friendly description of a non-standard model for arithmetic is a mere formal trick, since we cannot exclude the possibility that the primitive predicate P is just an individual name in disguise, of the "Pegasizes" type. It is standard to address such misgivings by resorting to the ontological-level distinction between pure (qualitative) and impure (non-qualitative) properties, and argue that in the case at hand the intended denotation for the above-introduced predicate P is a pure property. However, I doubt that a sharp distinction between pure and impure properties can be drawn within mathematics. Is the relation of being a successor pure or impure? If interpreted extensionally, as the set of ordered pairs $\{\langle 1,0\rangle, \langle 2,1\rangle, \ldots\}$, it may very well be claimed to be impure. This seems to collapse any useful distinction between "genuine" and "Pegasizes-type" predicates. The essence of Sider's claim is that the difference between the standard and non-standard models of arithmetic cannot be expressed in a language which does not use any individual constants, only predicates and quantifiers. This claim is provably (and perhaps trivially) false, regardless of the scruples one may have concerning the "hidden nature" of the used predicate.[14]

Sider's arguments against generalism are not limited to arithmetic. He admits that due to the simplicity of natural numbers, it is relatively easy to express facts about them in a name-free language (p. 101). Indeed, the linear and discrete character of natural numbers is definitely helpful in fulfilling this task. However, things may not be so simple when we consider a three-dimensional continuum of objects such as points in space. As a next challenge for the generalist, Sider discusses a Democritean physics, in which every point in space can be either occupied by homogeneous matter or not. Thus, every point can possess one of two exclusive properties: "on" (occupied) or "off" (not occupied). Even though such a model of the physical universe is still extremely simple, Sider argues that the generalist is unable to properly express it without infinitary resources. The number of distinct functions from an uncount-

---

[14] Yet another way to highlight the ontological (modal) difference between individual constants (proper names) and predicates is by pointing out that the former function as rigid designators, i.e. they are supposed to refer in any possible world to the same individual. (Predicates can also function as rigid designators if they are treated as names for natural kinds. I will ignore this caveat and treat all predicates non-rigidly, as picking out in any possible world objects that possess appropriate properties.) Due to this difference it may be argued that propositions involving individual constants are not invariant under permutations of elements of models, while qualitative propositions containing only predicates are. However, it is extremely difficult if not outright impossible to apply the concepts of rigid designators, permutations and identity across possible worlds to mathematical theories and models. It hardly makes sense to seriously consider a possible scenario in which the numbers five and ten swap their places in the structure of natural numbers.

ably infinite set to the two-element set of values {on, off} exceeds continuum. And yet in standard generalistic language with no extra resources the set of all sentences is only countably infinite, and hence there are only continuum-many subsets of such sentences. As Sider concludes, "a quantifier generalist seems to need an infinitary language allowing infinitary quantification and conjunction, in order to state a ramsey sentence with continuum-many existential quantifiers and (at least) continuum-many conjuncts" (p. 101).

There are several corrections and clarifications to be made here. First off, Sider's estimation of the number of distinct Democritean worlds is made from the perspective of an individualist, not a generalist. Clearly, not every distinct on-off function on the set of spatial points corresponds to a distinct world for a generalist, so the resulting number of distinct worlds is grossly overestimated. In order to properly count them, we have to know what additional structure the total set of points possesses. In the simplest scenario there is no additional structure – points are just unstructured, intrinsically indistinguishable entities. In such a case the only qualitative differences between worlds would be with respect to the cardinality of the set of occupied ("on") points and its complement. Thus, there would be exactly one world with no "on" points, exactly one world with one such point, and so on. And surely the difference between worlds with different finite numbers of "on" points is expressible in the generalist language. In the case of infinite sets we need stronger expressive resources in the form of either second-order logic or full-blown set theory, but this is not something unique to and caused by generalism.

When we add some additional structures, such as topological or metric ones, the number of distinct worlds does increase. Now we can identify only those worlds that can be transformed into one another by the symmetries of the assumed structure. For instance, in the case of Euclidean geometry, all worlds that can be transformed into each other by rotations or translations are considered identical by the generalist (formally we take the quotient of the original set of worlds with respect to the equivalence relation of being connected by a symmetry). But it may be replied that the remaining number of distinct worlds is still too big for them to be discerned in a generalist language. Obviously, the number of distinctly shaped areas in a continuous space exceeds the limit of infinite countability. Does this mean that the individualist ultimately wins?

Not so fast. My second commentary to Sider's Democritean example is that his argument is based on the premise which we have already refuted, namely holism and its associated claim that the only way to produce a generalistic description of a given situation is via ramseifying its individualistic representation. But this is patently not true. There are plenty of cases in which the generalist can describe a given Democritean world in a simple sentence (or sentences) without resorting to any infinitary tools. Consider a situation in which there is a sphere of radius r which contains all and only "on" points. Such a universe is easily describable with the help of a straightforward formula:

$$\exists x(C(x) \wedge \forall y[d(x, y) \leqslant r \rightarrow On(y)]),$$

where $C(x)$ is assumed to be satisfied by exactly one element ($\exists x \forall y[C(y) \leftrightarrow x = y]$), and d is a distance function. On the other hand, even in this simplistic case the individualist is obliged to continue using a language with an uncountable number of individual constants, since he has to distinguish among all qualitatively identical worlds that differ only with respect to any Euclidean transformation (translation or rotation). In this case (and in any analogous case involving regular shapes describable in simple mathematical formulas) the generalist has an obvious advantage over her rival with respect to parsimony.

My final point is a general one (I have already hinted at it earlier). Even if Sider was right that the only way to produce a generalistic description of any situation involving an infinite number of elements is by using infinitary conjunctions and quantifiers, still the situation of the individualist does not seem to be better in that respect. He definitely has to use an infinite (uncountable) number of constants together with infinitary conjunctions.[15] On the other hand, the generalist does not need constants, but instead introduces infinitary quantifiers. Why is it so much worse? To me this looks like a reasonable trade-off. Sider responds to this challenge by resorting to the well-known distinction

[15] An objection can be made here that infinitary conjunctions are not needed: all we need is an infinite list of atomic sentences, each of which is finite. To my mind, this is just a word play. When I select an infinite list of sentences and stipulate that all of them must be true in a given domain, this is essentially the same as accepting one infinite conjunction. At least the truth conditions for both seem to be the same. Of course, the issue of acceptance is a delicate one — one can refuse to accept a particular sentence not because it is false, but for other reasons, such as for instance that the form of the sentence violates some more or less arbitrary restrictions. What I am saying here is only that I do not see an obvious reason why we should refuse to admit an infinitary conjunction while accepting an infinite list of atomic sentences.

between the parsimony regarding the number of posited individuals and regarding the number of distinct kinds of individuals. The individualist merely increases the number of individual entities (names), while the generalist expands the number of kinds, says Sider. But I do not see how infinitary conjunctions and quantifiers relate to the infinite numbers of kinds. This is not explained by Sider — he merely states "if the infinitary logical concepts should be classified with "kinds" for these purposes, then the argument stands" (p. 102). But this is a big "if", and no further justification is forthcoming.[16]

To sum up: Sider's arguments fall short of proving that in each case involving infinite universes, the generalist requires some unusually strong expressive resources which are avoidable under individualism. In some cases such resources may indeed be necessary, but the individualist is arguably in a similar position. On the other hand, in some other situations the individualist must use infinitary descriptions while the generalist can resort to finite, simple formulas. Also, the necessity of an individualistic treatment of the case of non-standard models of arithmetic is questionable. In the absence of further arguments, I submit that generalism survives the onslaught.

4. Conclusion

Quantifier generalism has a lot going for it. It possesses a familiar, well-developed logical structure, with a perspicuous ontology and semantics. It dovetails well with scientific practice, since most if not all scientific theories and laws can be easily reconstructed within its logical framework. One crucial advantage of quantifier generalism, which I discuss at length elsewhere (in a manuscript being prepared for publication), is that it dispenses with the mysterious directly referring terms and their attendant primitive individuals (bare, property-less substrata) whose identities lie beyond our epistemic ken. In contrast to individualism, it does not admit an infinite number of distinct but indistinguishable situations obtained by permuting individuals, and thus is more parsimonious in this regard. In spite of what its critics imply, quantifier generalism is neither unavoidably holistic, nor does it require some unusually strong expressive

[16] Instead of explaining why infinitary quantifiers should imply any multiplication of kinds, Sider moves on to argue for the parsimony with respect to kinds by invoking laws. This is of course reasonable, but does not address the most controversial issue.

resources across the board which are absent from individualism. All in all, it is a serious candidate for a fundamental metaphysical theory of the world.

Acknowledgments. I am grateful to the anonymous referee for his/her suggestions how to improve my manuscript. The work on this article was supported by grant no. 2021/43/B/HS1/01399 from the National Science Center, Poland.

References

Belot, G. (1995), "New work for counterpart theorists: Determinism", *The British Journal for the Philosophy of Science*, 46: 185–195.

Belot, G. (2018), "Fifty million Elvis fans can't be wrong", *Noûs*, 52: 946–981.

Boolos, G. S, and R. C. Jeffrey (1980), *Computability and Logic*, Second edition, Cambridge: Cambridge University Press.

Cowling, S. (2015), "Non-qualitative properties", *Erkenntnis*, 80: 275–301.

Dasgupta, S. (2009), "Individuals: An essay in revisionary metaphysics", *Philosophical Studies*, 145: 35–67.

Dasgupta, S. (2016), "Can we do without fundamental individuals? Yes", pages 7–23 in E. Barnes (ed.), *Current Controversies in Metaphysics*, New York: Routledge.

Diehl, C. (2018), "A language for ontological nihilism", *Ergo*, 5(37): 971–996.

Glick, D. (2016), "Minimal structural essentialism. Why physics doesn't care which is which", pages 207–225 in A. Guay and T. Pradeau (eds.), *Individuals Across the Sciences*, Oxford: Oxford University Press.

Glick, D. (2020), "Generalism and the metaphysics of ontic structural realism", *The British Journal for the Philosophy of Science*, 71: 751–772.

Kment, B. (2012), "Haecceitism, chance and counterfactuals", *Philosophical Review*, 121(4): 573–609.

Kuhn, S. T. (1983), "An axiomatization of predicate functor logic", *Notre Dame Journal of Formal Logic*, 24(2): 233–241.

Plate, J. (2022), "Qualitative properties and relations", *Philosophical Studies*, 179(4): 1297–1322. DOI: 10.1007/s11098-021-01708-y

Pooley, O. (2006), “Points, particles, and structural realism”, pages 83–120 in D. Rickles, S. French and J. Saatsi (eds.), *The Structural Foundations of Quantum Gravity*, Oxford: Oxford University Press.

Quine, W. v. O. (1960), “Variables explained away”, *Proceedings of the American Philosophical Society*, 104(3): 343–347.

Quine, W. v. O. (1976), “Algebraic logic and predicate functors”, pages 283–307 in *The Ways of Paradox and Other Essays*, Second Edition, Cambridge Mass.: Harvard University Press.

Sider, T. (2020), *The Tools of Metaphysics and the Metaphysics of Science*, Oxford: Clarendon Press.

Stachel, J. (2002), “ ‘The relations between things’ versus ‘the things between relations’: The Deeper meaning of the hole argument”, pages 231–266 in D. Malament (ed.), *Reading Natural Philosophy. Essays in the History and Philosophy of Science and Mathematics*, Chicago: Open Court.

Svenonius, L. (1960), *Some Problems in Logical Model Theory*, Library of Theoria, Lund.

Turner, J. (2011), “Ontological nihilism”, pages 3–52 in K. Bennett and D. W. Zimmerman (eds.), *Oxford Studies in Metaphysics*, Vol. 6, Oxford: Oxford University Press.

Turner, J. (2016), “Can we do without fundamental individuals? No”, pages 24-34 in E. Barnes (ed.), *Current Controversies in Metaphysics*, New York: Routledge.

van Fraassen, B. (1991), *Quantum Mechanics: An Empiricist View*, Oxford: Clarendon Press.

Appendix

Below I present a semi-formal proof of the equivalence between formula (1) and the conjunction of (2)–(4).

Proof of the direction from (2)*,* (3)*,* (4) *to* (1)*.* Due to assumption (2), we can select an object satisfying F and give it an arbitrary name a: $F(a)$. By the rule of Universal Instantiation (UI), we get from (3):

$$F(a) \to \neg G(a),$$

and, by Modus Ponens (MP), we get $\neg G(a)$. Now, from (4) using (UI) we get:

$$F(a) \rightarrow \exists y(R(a,y) \wedge G(y)),$$

and by MP, we obtain: $\exists y(R(a,y) \wedge G(y))$. This formula again enables us to select an object b satisfying: $R(a,b) \wedge G(b)$. So, we obtain $a \neq b$ since $\neg G(a)$ and $G(b)$. Therefore, we obtain:

$$F(a) \wedge G(b) \wedge R(a,b) \wedge a \neq b.$$

So, applying Existential Generalization (EG), we obtain (1).

Proof of the direction from (1) *to* (2), (3), (4). As explained in the main text, the proof will go through under the additional assumption that the domain consists of exactly two objects a and b, and that the individualistic description provided by Dasgupta and Sider is complete. Thus, we can simply replace (1) with the complete generalistic premise (1′).

From (1′) we infer that there are objects a and b satisfying the entire formula under quantifiers:

$$F(a) \wedge \neg G(a) \wedge G(b) \wedge \neg F(b) \wedge R(a,b) \wedge \neg R(b,a) \wedge \forall z(z = a \vee z = b).$$

So, firstly, we obtain $\exists x\ F(x)$, i.e. (2). Secondly, also $F(a) \wedge \neg G(a) \wedge \neg F(b)$. Hence, by ordinary rules of propositional logic, we get:

$$(F(a) \rightarrow \neg G(a)) \wedge (F(b) \rightarrow \neg G(b))$$

So, since $\forall z(z = a \vee z = b)$, we can generalize the above and obtain (3).

Finally, we also obtain:

$$F(a) \wedge \neg G(a) \wedge G(b) \wedge \neg F(b) \wedge R(a,b).$$

Hence, by propositional logic, we get:

$$[F(a) \rightarrow (R(a,b) \wedge G(b))] \wedge [F(b) \rightarrow (R(b,b) \wedge G(b))].$$

We can now apply Existential Generalization to the consequents of both implications:

$$[F(a) \rightarrow \exists y(R(a,y) \wedge G(y))] \wedge [F(b) \rightarrow \exists y(R(b,y) \wedge G(y))],$$

and since $\forall z(z = a \vee z = b)$, we replace the conjunction of two formulas containing names a and b with a universally quantified statement (4).

TOMASZ BIGAJ
Faculty of Philosophy
University of Warsaw, Poland
t.f.bigaj@uw.edu.pl
https://orcid.org/0000-0002-8121-9789