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Logical Deductive Processes as Time Consuming Events: Instance versus execution of logical arguments

Abstract. The point of this paper is to show that deductive arguments may not be executed in deductive inferences, and reciprocally, deductive inferences, if they existed, may not instantiate deductive arguments. Three reasons for making this distinction between instances and executions of logical arguments are offered. For example, the pragmatic expectation of the consequent may instantiate a Modus Ponens argument form, but it does not execute a logical deduction. The proposed distinction clarifies the current state of the art on factual logically deductive inferences in two opposed directions: first, exemplifying a deductive argument is not enough for an inference to be deductive, and second, the same energy and time-consuming process may both execute and instantiate a deductive argument. Future experiments may show or discard the existence of recursive and semi-recursive processes being deductive inferences, that is, being at the same time instance and execution of a deductive argument form.

Keywords: time consuming logicality; deductive argument; deductive inference; instance; execution; realization; recursion; semi-recursion

1. Introduction

After nearly 3000 years explicitly handling deductive phenomena, the nature of deduction still presents crucial open questions. "Deduction is an update which does not require an input from the outside" (Parikh, 2008) is a statement with different meanings at the crossroad of disciplines studying deductive phenomena. There are currently at least three main distinct uses of "deductive". Adventitious conclusions are called "deductive" in many psychological contexts if they stem from an already given premise irrespective of their epistemic qualification and their logi-

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cal features (Evans et al., 2015; Markovits et al., 2018). “Deductive” are also called speech acts inferring conclusions based on how humans define, understand or take conceptual features appearing in their premises (Williamson, 2006; Boghossian, 2014). Finally, “deductive” is a formal update process whose sole input is its premises. This logical use of “deduction” goes back to Aristotle and allows contrasting conceptions such as (Sher, 2018) and (Leitgeb, 2012, 2025). Logical deduction is a family of recursive or (if it lacks negation) recursively enumerable update processes whose conclusions preserve selected properties or designated values of their inputs irrespective of their categorematic content. Thus, logical deduction is purely formal or invariant with respect to the categorematic content of its updates. In this paper, “deductive” is taken in this third sense as identical to “logically deductive”. Wise choices of pertinent properties or designated values preserved by distinct logical consequence relations (truth, probability, demonstrability, relevant entailment, etc.) may differ from silly or absurd choices, since this general definition of logically deductive update needs not to be constrained to specific axioms, rules, languages or frames. Notice that the given definition covers the usual notion of classical bivalent deduction as a particular case.

The following three statements are mainstream truisms in the current state of the art on deductive phenomena:

(i) Deductive inferences are (a) inferential and (b) deductive.
(ii) The process of making deductive inferences is not itself deductive.
(iii) The logical process of deduction is not factual.

For the sake of self-containment, we briefly justify the truistic character of these statements. (ii) is justified because the psychological and social practices explaining how deductive processes happen are not themselves logically deductive. The most influential factual theories of deductive processes describe their execution in non-deductive terms. Mental model counter example production (Johnson-Laird, 2010; Johnson-Laird et al., 2015), mental logic usage criteria (Braine, 2013; Lopez-Astorga, 2017), heuristic procedures (Cosmides et al., 2010) or social practices (Mercier, 2016) are the best available factual descriptions of deductive processes, but they are not themselves deductive. (iii) is justified because “deductively valid” does not apply to events in space-time, since logically deductive arguments are conceived by a significant part of the literature as abstract relations among propositions (Frege, 2014) or probabilities (Adams, 1975). The anti-psychologist stance about logical deduction is mainstream in the philosophical literature and the metalogical tradition, with significant exceptions (Gabbay & Woods, 2001; Woods, 2016).

The conjunction of (ii) and (iii) implies that there are not logically deductive inferential events as described in (i): conjunction of logically deductive and time-consuming events. Deductive inference calls for deductive and inferential elements, but their factual component does not correspond to deductive processes (ii), nor their deductive component corresponds with factual processes (iii). This conclusion shows how messy the current informed state of the deductive question is, since it conflicts with the common assumption that there are deductive inferences: there are psychological or neural properties which under normal conditions are present whenever normatively deductive reasoning is produced.

On the other hand, refuting (i) does not rule out deductive inference altogether, but only in its formulation in (i). One alternative is to consider that deductive inferences do not exemplify but rather represent deductive processes (see (Rudnicki & Łukowski, 2024), for examples, in this direction). Thus, in this regard there are no processes being at the same time logically deductive and inferentially factual, (i) can be avoided and deductive reasoning abilities can have a purely normative role in real cognitive systems.[1] This alternative — to my knowledge — as such has never been fully and explicitly presented, but it is a common unspoken default assumption. Any candidate process with the features (1) being logically deductive, and (2) being a factual event in space-time, poses difficulties like the following. Consider the case of Modus Ponens (MP) as an abstract deductive argument pattern concluding q from $\{p$ implies q, $p\}$. This example satisfies (1), but when it comes to satisfy (2) we typically find belief management procedures to infer q from the beliefs p and p implies q. But these procedures are not logically deductive and therefore do not satisfy (1), even if they preserve knowledge and justification[2] (Schechter & Enoch, 2006). This is not an isolated example. Consider the case of Disjunctive Syllogism (DS) concluding q from $\{p \text{ or } q, \text{not } p\}$. This pattern satisfies (1), but the events in space-time which make up this conclusion do not need to be deductive or even inferential. Moreover, consider the case of human argumentative

[1] The epistemic justification of deductive norms is an open problem not tackled here. The factual reality of deductive processes here discussed is usually taken for granted in that epistemic discussion.

[2] Notice that Harman's and Schechter's arguments on MP as a rule of inference and not of logic are inverted here.

practices which occur in space-time and may eventually satisfy (2), but how could they satisfy (1)? These examples suggest that (1) and (2) can be taken as incompatible and still accept (ii) and (iii).

Both the fregean tradition (Frege, 2014) and the new probabilistic tradition (Harman, 1973, 1984; Harman & Khemlani, 2012; Oaksford & Chater, 2007; Oaksford, 2015), agree in rejecting (i) and accepting some versions of (ii) and (iii). The fregean perspective favors (1) and the new inductive tradition favors (2), but both are skeptical towards the reality of time consuming deductive processes. There is in fact in the psychological and neuroscientific literature sparse evidence for the existence of logically deductive inferences as factually distinct from non-deductive processes (Heit & Rotello, 2010; Singmann & Klauer, 2011; Evans & Over, 2013; Ghasemi et al., 2022). The contrast between abstract deductive schemas and real time-consuming deductive processes (Casalegno & Marconi, 2009; Urbański, 2011) has taken many distinct forms in the history of logic and it is still an unresolved conflict.

The objective of this piece is to clarify this complex state of the art on real logical deduction by means of articulating the contrast between two distinct ways of being deductive. Abstract relations among propositions instantiate deductive processes called deductive arguments but inferences producing deductive arguments do not instantiate deductive processes, they execute them in space-time. Inferences and arguments are deductive, if at all, in distinct ways. A time-consuming inferential process may recursively execute and also instantiate a recursive deductive argument, thus producing and exemplifying a deductive inference. If (semi)recursive processing leaves hints in space-time, logical deduction may indeed be factual under some conditions to be experimentally scrutinized. Whether there are or not deductive inferences is a testable statement about the existence of recursive neural or psychological processes accompanying recursive logical computations. The proposed distinction between deductive arguments and inferences should be applicable to any conception of deduction even if the very nature of deductive processes is a disputed issue in philosophy and psychology.

2. Instance vs execution: argument vs inference

In the cloud of inferential practices there are all sorts of heuristic procedures which cognitive systems use to maximize informational content or preserve important semantical properties such as truth, probability, demonstrability or entailment. Eventually, also other epistemically irrelevant properties or even malign or absurd ones. Which of these practices are arguments is difficult to define (Goodman, 2018). However, a full definition is not required to identify basic unconventional features that arguments have, and inferences do not. These features are based on a distinction between instances (type/token relationships) and executions (realization relationships), a distinction so basic that it is neutral with respect to all competing conceptions of deduction. In this section we focus on three features discerning instances from executions: (i) instances are exemplified as tokens of types, while executions are not, (ii) executions are factive, while types and tokens may not be factive, (iii) instances share the features they exemplify or tokenize, but executions do not always have the features they instantiate.

Arguments are instantiable or allow instances, examples or tokens. As opposed to fragments, parts or other sorts of samples, instances of x exemplify x completely or wholly. An instance of x is a case or exemplar of x, where x is not a part or a feature of x but x itself (Mertz, 2016). Types, tropes, things, persons, propositions, decisions, images, joga exercises are instantiable. Irreal, false, fictive, imaginary things or processes are also instantiable. Deductive arguments allow instances in the following sense: one and the same argument is present in its full integrity in distinct formats or modalities (for example linguistic, visual or agentive) and in distinct supports (biological, psychological, social) and contexts (for example occasions in time and space). We have just seen MP and DS are examples of argument or argument patterns. Notice that since arguments allow for instances, arguments are in fact argument patterns.

Deductive arguments are in the philosophical tradition conspicuously instantiable as abstract objects in the platonic and fregean sense or as inscriptions in the nominalist tradition (Rossberg & Cohnitz, 2009). Inscriptions exemplify formal arguments as propositions instantiate them irrespective of their differences in the understanding of instance–allowance. Following Harman (1973, 1984, 2002) we define arguments or implications as relations among propositions and let nominalistic notation aside together with the conceptual and technical difficulties (regarding finitism, for example) it comes with. Please notice how (unlike Harman) no logical constraint is placed on which relations are implicative. The only requirement is the usual recursive definition of deduction as a recursive process (Boolos, Burgess & Jeffrey, 2002; Epstein & Carnielli, 2018).

Harman (1973, 1984, 2002) distinguishes arguments (as relations between propositions) from inferences as time consuming events in spacetime. This intuitive basic distinction is later developed in Harman's influential normative distinction between logic and reasoning. There is however a basic equivocation here between instance and execution of logical arguments. Instances imply, first, that something is instantiable (a type, for example), and second instances are specified at something carrying the instance (a token, for example). Neither instantiables nor instances need to consume time, or energy, or somehow happen. Types or instantiables are less specific than their tokens or instantiations, but they can be just as real or irreal. Some instantiables are typically abstract, for example propositions, while others are considered concrete individuals, for example material bodies. On the other hand, instances can also be abstract tokens. The dimension INSTANCE discriminates types (instantiables) from tokens (instances) in terms of their specificity, not in terms of their reality. In this regard, a "concrete" instance is still an instance and not a realization. For example, an instance of a DS argument may be specified this evening at this raven's brain (Mody & Carey, 2017) and still no inferential time-consuming process has been executed.

Not everything allows instances. Events, proferences, autographic works of art and inferences do not allow instances. They certainly allow weaker samples: partial incomplete, imperfect repetitions, when for example a proference or a Saturday morning is said to be repeated. Relations weaker than identity may relate cases of 'the same' utterance or 'the same' smile. These are not examples of instances as defined above. If x and y share some crucial functional properties, they may be considered relaxed or imperfect samples of the same event, but never genuine instances. Putting my trousers this Sunday morning is certainly located or situated in time (a summer 2024 morning, for example). But time location should be distinguished from time consumption. Many real instances of arguments may be located in time, but only inferences are time-consuming.

Executions are factive processes, while instances need not be factive. This is a relevant distinction, since instantiating x does not consume time or energy in the real world, while executing x does in fact consume time and energy. Correspondingly, inferences are factive processes while arguments can be factual processes or not. We know, repeat, understand and learn Gauss's arguments but not his inferential processes.

Let us intuitively illustrate the core ideas of this instance/execution distinction with an example and compare it with how a computer program works. A program's byte-code is an abstract set of instructions that can be executed multiple times under different conditions and environments. This byte-code can be considered an instance of the algorithm: it preserves the logical structure and functionality, regardless of when or where it is used. By contrast, the run-time trace of the program corresponds to the specific, physical record of a concrete execution — which instructions were triggered, in what order, how much time and energy were consumed, and what resources were involved. This execution leaves measurable traces in time and space and depends on contingent factors, such as system load or hardware configuration. This analogy clarifies that instantiating a deductive argument is not necessarily the same as executing it, and an execution may occur without instantiating a canonical deductive pattern.

Executions of programs are not instantiable, as opposed to algorithms realized in such executions. Notice that programs have properties which their executions lack. A program can be expensive or impossible, it can be repeated or corrected, it is multiple realizable in several conditions, but the actions executing it may not be expensive or impossible, they are not repeatable or correctible, they are not multiple realizable. The execution of P does not in general have P — it produces it but may not have it. For the reasons sketched in this section, arguments and in particular deductive arguments as relations among propositions differ from deductive inferences as factive processes. The distinction does not presuppose any specific notion of deduction beyond the instantiable nature of deductive arguments.

3. How would a candidate cognitive process execute (not just instantiate) a deductive argument

A logical argument is executed by computations and/or inferences which can also instantiate it. The execution or realization of a logical argument is a recursively enumerable process which consumes time and energy and may leave a measurable physical or functional trait. These processes are definable in recursive enumerable (RE) languages, but the processes themselves are physical events and not linguistic entities or sets of numbers. This definition does not preclude the existence or not of such executions, which is an open empirical matter. An inferential process both instantiating and executing a logical argument is a deductive inference. According to Kripke's version of Church Thesis, semicomputable or recursively enumerable sets are those defined in the first order language RE (Pour-El & Kripke, 1967; Boolos, Burgess & Jeffrey, 2002). There is no need to assume this Thesis here, since only ulterior research can determine how recursive enumerable processes constraint which deductive features are realizable or executable. The point is that an instance of a logical argument is recognized by the logical structure it shares with the argument pattern it instantiates, that is, its logical structure and logical complexity. In contrast, the inferential execution of a deductive argument is not a priori verified or refuted. There are in general many ways of executing a process and as we have seen executions of x do not need to instantiate x's features.

Assuming that logical arguments are a priori not executable or realizable is tempting only if no distinction is made between instances and executions of logical arguments. The following four indicators point at refutable and measurable eventual evidence for time and energy consuming executions of logical arguments.

First indicator. Recursive and Semi-recursive processes produce empirical signatures which non-recursive processes do not produce (Martins, 2012; Fischmeister et al., 2017). Since deductive inferences execute deductive arguments, they must be recursively enumerable (in the conceptual frame here defended) and hence eventually become physically or functionally discriminable. In fact, cognitive neuroscientific research on recursive processes in movement (Martins et al., 2019), in music (Vuust et al., 2022), in grammar (Friederici et al., 2011), in numerical calculation (Mikhailov, 2024) and in logically valid inferences (Salto et al., 2021) shows specific traits of recursive cerebral activity. At this point, the neuroscientific evidence is not fully consistent, and it is limited both from quantitative and qualitative perspectives. For example, the neuroelectric processing of complex images is modulated by order, repetition and identity of stimuli, but not by negation, suggesting a recursive enumerable but not recursive process (Merino et al., 2019).

Second indicator. The psychometric literature does not offer any experimental construct able to measure deductivity, let alone logical deductivity. In this regard, there is no direct tool to determine if a neural, psychological or social process executes a deduction. This absence is probably only provisional, since psychological and neural deductive measures have been proposed (Álvarez-Merino et al., 2020) and indirect tools are available to confirm if a cognitive process executes a deductive argument. For example, experimental paradigms in cognitive neuroscience allow to measure the relative effect of altering the logical complexity and/or the relational complexity of inferential tasks. This methodology has associated magnetic resonance (Monti and Osherson, 2012), electrical (Salto et al., 2021) and ondulatory (Toro et al., 2023) traits with deductive processes.

Third indicator. Distinguishing between instances and executions of logical arguments contributes to the understanding of multiple modalities in logical argumentation. Consider visual, verbal or agentive instances of the same logical argument. Consider also organisms with distinct evolutive history which may instantiate the same logical argument. By definition, all these instances share the same logical structure, but their execution may respond to distinct algorithms and procedures. It is therefore not to be expected that there is a unique physical or functional execution of logical arguments, and the execution (unlike instantiation) is contingent[3] and historical.

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[3] This note presents a specific paradigm designed to empirically test whether the proposed indicators of deductive execution can be validated or refuted through neuroimaging (EEG, MEG, or time-resolved fMRI). The paradigm is focused on simple MP inferences with material conditional in purely positive contexts (without explicit negation and without any modal operators). Pragmatic implicatures and biconditional interpretations of the conditional operator are neutralized either through participant training or sample selection. In this way, a material reading of the conditional is assured. Since the neuroscientific literature lacks direct tools for measuring neural recursivity, the experiment contrasts the neuroelectric effects of logical complexity (number of occurrences of the material conditional operator) and relational complexity (number of non-logical variables). The first paradigm isolates logical complexity by increasing the number of embedded conditionals while keeping the semantic content constant. In contrast, the second paradigm increases relational or semantic complexity—that is, the number of non-logical variables—while maintaining a constant logical structure in the form of a single Modus Ponens inference.

If an observed neural signal—such as variations in response latency, ERP components (e.g., P3b, N400, Late Frontal Negativity), or β-band connectivity clusters—reflects the execution of logical structure and it is not modulated by content processing, then it signals a candidate-1 deductive execution. See the Table 1 annexed as supplementary material. Potentials and connectivity clusters are based on prior published research. A candidate-1 deductive execution which is invariant across distinct inferential formats (linguistic, visual, agentive) is a candidate-2 deductive execution. To qualify as candidate-2, a collection of signals must meet the criteria for candidate-1 across all inferential formats. Candidate-2 deductive executions (if they

There is most probably not enough space-time to execute all logical deductions. Moreover, we certainly do not have to assume that most or even a great part of the logical inferential abilities is deductively systematized. But still, some logical deductions may be in fact be both executed and instantiated. Even if at the end they are not, introducing the notion of deductive execution is needed to verify or refute the existence of deductive inferences as time consuming events.

4. Instances of deductive arguments which are not realized in deductive inferences

If you want an example of a deductive inference, even in scientific and philosophical contexts, it is usual to take an instance of a deductive argument, normally instantiated in psychological, neural or social terms. For example, if x is a real instance of MP or DS, since both are deductive by definition, it is assumed that x is realized in a deductive inference. This assumption is common, but wrong if the argument in this section is sound. Notice that the general abstract distinction between instances and executions, that is, between arguments and inferences, is clearly insufficient and useless in practice if it does not affect specific real cases of deductive instances of arguments.

To proof that the execution of a deductive argument does not need to be a deductive inference, we show instances of deductive arguments (MP and DS) which are not executed by deductive inferences.

4.1. The case of non-deductive anticipation

Consider the following sets of propositions:

(a) $\{\ulcorner$If you are my grandmother, you are older than me$\urcorner$, $\ulcorner$You are my grandmother$\urcorner\}$, hence $\ulcorner$you are older than me$\urcorner$
(b) $\{\ulcorner$If you are my grand mother, it is Sunday$\urcorner$, $\ulcorner$You are my grand mother$\urcorner\}$, hence $\ulcorner$it is Sunday$\urcorner$
(c) $\{\ulcorner$If you are my grandmother, you are my grandmother$\urcorner$, $\ulcorner$You are my grand mother$\urcorner\}$, hence $\ulcorner$you are my grandmother$\urcorner$

existed) may happen to be invariant across distinct types of cognitive systems, but then they probably must be functionally defined.

The cases (a), (b), (c) are deductive arguments, all real instances of the same MP pattern. All of them are deductive in the sense that their conclusions (the propositions after "hence") are logically (and probabilistically) deduced from their premises (the propositions within "{}". The use of "hence" and "{}" is just a kind of politeness for the reader to identify more easily conclusions and premises. They are not stricto senso components of the argument. We have also deleted the repeated proposition ⌈You are my grandmother⌉ for politeness and convention, since the hyperset {⌈You are my grandmother⌉, ⌈You are my grandmother⌉} is standardly identical to {⌈You are my grandmother⌉}.

So, the task is to proof that (a), (b), (c) can be executed in a non-deductive inference. In the presence of conditionals, the human brain anticipates both the antecedent and (with less intensity) the consequent of the conditional (Bonnefond & Van der Henst, 2009; Bonnefond et al., 2012; Barthel et al., 2024). For example, the conditional "if you are my grandmother, you are older than me" anticipates the antecedent "you are my grandmother" and the consequent "you are older than me". Moreover, it occurs with other kinds of sentential operators (disjunction or conjunction). Anticipation, like salivation, may not be conceived as an inferential process at all, but still it is exclusively based on the premises and their features. This anticipation leaves a measurable electrical hint in the cortical activity, as shown by the lesser amplitude of the electrical potential N200 evoked by antecedents and consequents in contrast with arbitrary unexpected stimuli (Szewczyk & Shriefers, 2013). There is ample neurocognitive evidence of these anticipations, offered by the lesser amplitude in the cerebral electrical reaction to antecedents and consequents already present in conditionals. New or unexpected sentences and terms produce greater electrical amplitude, both at early attentional stages and at processing level (Goel, 2019). Anticipation is a non-deductive process, since it occurs in presence of a conditional both with and without the presence of a minor additional premise. Therefore, a non-deductive execution may instantiate a deductive MP argument in the cases (a), (b), (c).

A narrative exposition of the argument may help. Consider the following factual series of steps of proposition:

(1) ⌈If you are my grandmother, you are older than me⌉

Expressed by an utterance of "If you are my grandmother, you are older than me", produces the cortical anticipation of its antecedent

(2) ⌈you are my grandmother⌉

The premise (1) also anticipates its consequent:

(3) ⌈you are older than me⌉

This inferential process is clearly non deductive, but it is an instance of an MP argument.

Another factual series also begins with (1) and (2). By error or chance (2) becomes asserted:

(3*) ⌈you are my grandmother⌉
(4*) ⌈you are older than me⌉ is deduced from (1), (3*)

Which is also non-deductive.

The anticipation as a cortical process reflects expectancies, regularities or habits more than reasoning. But not only words or terms are anticipated, but propositional contents of conditionals, as evidenced by the electrical differences between "if" and "only if" (Barthel et al., 2024). Thus, an instance of an MP argument appears in an undeductive sequence of contents. This example is not as extraordinary as it may seem. Examples of "Modus Ponens without deduction" (Barcelo, ms) are also found in logically valid contexts in which epistemic properties (knowledge, justification) are not preserved.

4.2. The case of non-deductive directional inference.

The directionality of the conditional (if p, q) changes if it is presented in the inverse format (q, if p). Changing the directionality of the conditional does not alter its propositional content, since it is the same conditional with the same antecedent and consequent. The arguments:

- (q if p), p, hence q (if p, q), p, hence q

have only superficial differences that are not reflected in the truth values of direct and inverse conditionals. They are in fact from the point of view of bivalent truth identical and deductive. There are deductive systems (like Ticket entailment) sensible to directionality in which both arguments are in fact logically unequivalent (Mendez et al., 2012).

However superficial their differences as arguments, they show significant inferential differences discovered decades ago by cognitive science (Oberauer & Wilhelm, 2000). The inferential process concluding q from (q if p), p is not the same as the one concluding q from (if p, q), p, because the reaction time in the inverted inference is significantly higher than in the direct one, only direct MP and not inverse MP is automatic (Reverberi et al., 2012) and directionality produces significant electrical differences in the premise integration phase at about 300ms after the minor premise presentation. Importantly, pragmatic presuppositions carried by direct conditionals are massively lost in inversed versions (Espino & Ramírez, 2018). Since expectancies management is not a deductive issue, this is an example of a non-deductive inference instantiating MP.

4.3. The case of distinct modalities and formats

The individuation conditions for arguments are clearly different from those of inferences as shown by the fact that one and the same argument form allows distinct modalities and formats, while one and the same inferential time-consuming process allows limited variation in physical or functional features. Consider DS as the deductive argument concluding q from (p or q), not p. This pattern appears in linguistic (Newstead & Griggs, 2013), visual (Cesana-Arlotti et al., 2018) and agentive (Bermúdez, 2006) formats, where grown human subjects and toddlers (Mody & Carey, 2017; Gautam et al., 2021) realize disjunctive, or exclusion inferences whose conclusions are non-arbitrary sentences, terms, images or agencies. Consider a sequence of such premises and conclusion instantiating classical negation and inclusive disjunction. That is, a sequence of sentences, terms, images or agencies instantiating DS. This sequence has DS logical structure, but the inferences producing it are crucially different in linguistic, visual or agentive formats. For example, the cortical spatio-temporal dynamics of visual DS is distinct from linguistic DS. Since distinct formats or modalities generate distinct inferences for one and the same DS argument, this is a prima facie example of deductive argument which is not known to be realized in a congruent deductive inference corpus.

Since the Greek antiquity until early research, an important number of experiments has evidenced disjunctive and exclusion inferences produced by birds (doves, parrots, ravens) and mammals (monkeys, dogs, sea-wolfs). One recent example among many is (Jones & Call, 2024). All these organisms have a distinct evolutive history and distinct functional and physical properties mobilized in their inferences. It is not established in the literature if the sequences of objects, images and agencies realized or executed by these animals instantiate inclusive or exclusive disjunction, and a standard Boolean negation. However, these sequences are not logically arbitrary or trivial, and therefore they instantiate some version of DS, even if in these cases any kind of symbolic understanding of DS is excluded.

5. Deductive inferences which do not instantiate deductive arguments

The usual criterion available to consider any time-consuming event a deductive inference is that it instantiates a deductive argument. We have just argued that this usual procedure is wrong and confounds inference and argument (instantiation and production). Knowing that an inference is deductive requires finding factual features eventually producing the argument in space-time. Those properties may not exist, of course, or may not be known. Notice that there is a clue as to which factual events may constitute deductive inferences: the recursive nature of deductive processes may leave a physical mark of deductivity.

Consider an inductive process such as pattern recognition as it happens in learning machines. It is a probabilistic learning process instantiable in computers, organisms and institutions, for example as it is described in Harman & Khemlani (2012). Alternatively, consider as a second example the invalid inferences made by a player of an inductive memory game such as SET (Falco, 2019). These two examples do not instantiate a deductive argument in the sense that both are inductive processes. However, in the inductive process itself, microdeductions are needed to correct wrong hypothesis (Modus Tollens) or to continue with a successful process (Modus Ponens). Both inferences are isolated deductive steps, but the machine or game does not instantiate any deductive argument. An experimental result on microdeductions is shown in (Salto et al., 2021), where logically invalid inferences seem to require cortical deductive processes at the millisecond scale.

6. Alignment of instance/execution in Marr's levels for logical deduction

D. Marr's influential distinction between the computational, algorithmic, and implementational levels of explanation in information processing is a cornerstone in the study of cognitive processes (Peebles & Cooper, 2015;

Colombo & Knauff, 2020). Applying this tripartite distinction to logical deductive processes is generally fruitful—for example, in the earlier differentiation between arguments (computational level) and inferences (algorithmic or representational level). However, certain aspects of deductive phenomena require a refinement of these explanatory levels, particularly through the introduction of the instance/execution distinction.

One first concern is the ambivalent role of the computational level as normative and as factive stance in the case of logical deduction. Even if arguments based on classical bivalent logical consequence and arguments based on constructive or relevant consequence may belong to the same explanatory schema, the explanation at the metalevel is obviously different and alien to Marr's levels.

But the main concern with logical deduction in distinct explanatory levels is the fact that deductive processes below the computational level may be explained in non-logical deductive terms. For example, consider a probabilistic approach to the algorithmic or representational level, in which extreme probability values 0, 1 behave consistently with classical logic at the computational level. The literature has developed at least two families of probabilistic representational or algorithmic level explanations of classically valid logical arguments: one is based on Adam's probabilistic semantics (Adams, 1975) and another one in Bayesian semantics (Oaksford & Chater, 2007). As we saw before, there are also pragmatic approaches to conditional rules such as MP (Bonnefond et al., 2012) which are based on expectations and not on deductions. The empirical evaluation of such proposals is an open issue in which we do not need to enter (Gazzo et al., 2023). The crucial point is that Marr's levels alone offer no tools to discern among logically deductive and non-logically deductive processes. This is when the instance/execution divide is useful to lay out the concept of deductive executions of deductive inferences and eventually verify or refute the reality of such deductive processes. For this reason, at the implementational or physical Marr level we can also at least conceptually define time-consuming processes which execute logically deductive processes at all Marr levels.

Let us state this situation in more general terms. Consider a normative property P which is naturally defined at the computational level. A cognitive process may not instantiate P even if it does explain it at some Marr level. Most cognitive process do not execute P at any Marr level. In fact, even if a process neither instantiates nor executes P, it could explain it, for example if it explains representations of P. But still, P

might be instantiated and executed under special conditions at any of Marr's levels. Deductivity could be such a property P.

7. Spatio-temporal constraints and perspectives on deductive inference

We have seen in previous sections that instantiating a deductive argument–form is not enough for an inference to be considered deductive. Therefore, instantiating a truth preserving argument (like MP or DS) does not make an inference logically deductive. We have also seen that deductive processes may literally happen in space-time if their execution and their instantiation coincide. Recursive or semi-recursive processes in the nervous system may instantiate and execute recursive or semi-recursive logical processes. Whether these processes exist or not in nature is empirically testable or refutable, and not a categorical confusion between abstract and time-consuming objects.

The execution of logical arguments faces a priori limitations whose consequences are to be further studied. Deductive arguments are systematic in a sense in which their execution is not (Stenning & Van Lambalgen, 2012). Closure properties usually found in deductive arguments are probably lost in deductive inferences. Moreover, compositionality (Baggio, 2021) and propositionality (Leitgeb, 2012) of deductive inferences is certainly constrained by nature and evolution in ways deductive arguments are not. The limits and reach of such constraints are not known at this moment, but they probably pose a limit to deductivity as a time and energy consuming reality.

8. Conclusion

Instances of deductive arguments do not need to be executed in deductive inferences and conversely executions of deductive inferences may not instantiate deductive arguments. The corps of logically deductive inferences as factive events in space-time is still to be experimentally proven or refuted: it is not a category mistake to conceive executions of logical arguments which may also instantiate them. Their recursive nature could offer a clue for factually proving in the future the existence or inexistence of deductive inferences as time consuming processes.

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