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Counterparts as Near-Equals

Abstract. This paper offers an account of the Ship of Theseus paradox along the lines of the so-called *nonstandard primitivism* about vagueness. This account is inspired by a model of the Ship of Theseus paradox offered by Dinis that considers near-equality, in the context of Nonstandard Analysis, as the proper way to model the 'same as' relation. The output is a class of models which unifies the semantic account of vague gradable adjectives recently proposed by Dinis and Jacinto with that of the 'same as' relation. It does so by taking both paradoxes to arise from confusion between relations of marginal difference between vague degrees and "small" precise relations between the things that have those degrees.

Keywords: Ship of Theseus; nonstandard primitivism; near-equality

1. Introduction

According to the *Ship of Theseus paradox*, there is a ship — Theseus's ship — which is solely made of wooden planks. As wood planks get rotten, they are substituted, over time, by new ones. Eventually, all the original planks get substituted, and the rotten planks are used to build a replica of the initial ship. One then might ask which ship is the initial one? The ship with the new planks, the one with the old planks, or none of them?

The *Ship of Theseus paradox* is an instance of a more general puzzle concerning identity over time. What makes one thing at a time identical to some thing at another time?

Our main aim in this essay is to offer an account of the Ship of Theseus paradox, and related puzzles involving cross-temporal identity, along the lines of the *nonstandard primitivist* account of vagueness and

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the Sorites paradox (Dinis and Jacinto, 2025). As we will show, underlying our proposed account of the Ship of Theseus paradox is a unified model of the semantics of vague gradable adjectives and of the 'same as' relation. Accordingly, the present paper can be seen as a proof of concept of NONSTANDARD PRIMITIVISM (see §5) by showing how it can unify the two phenomena in a single theory.[1]

Our starting point for the nonstandard primitivist account of the Ship of Theseus paradox will be the model of the paradox recently offered in (Dinis, 2023). This is a model based on the notion of *near-equality*, which is a notion definable in nonstandard arithmetic: two numbers are *near-equal* if and only if their difference is standard.[2] Near-equality can be seen as an identity-like notion that encapsules a certain vagueness: two objects are *nearly-equal* if their difference is sufficiently small, i.e. *standard*.

We find Dinis's model to be on the right track. According to it, ships differing by a single plank are near-equal, and near-equality is a transitive relation. Yet, ships differing completely with respect to their planks are not near-equals. Dinis's model encapsulates the idea that differences in planks do accumulate, but only imperceptibly.

Still, we will here raise some difficulties for the model, and show how these can be overcome. The difficulties to be raised are the following: (i) how to understand the "near-equality" relation as a relation between ships, rather than a relation between numbers; (ii) how to reconcile the model's presupposition that ships that differ by a single plank are thereby distinct with seemingly true cross-temporal identifications involving them, and (iii) how to reconcile the use of nonstandard analysis (see Nelson, 1987 and Dinis and van den Berg, 2019) with the fact that, in the *Ship of Theseus paradox*, we are in the presence of a *standard* number of ships.

We will address the first difficulty by taking the near-equality relation between numbers to model the relation of *sameness relative to a sortal* (see fn. 4 for the notion of *sortal*).

[1] We are using small caps to indicate that NONSTANDARD PRIMITIVISM is a philosophical position, thus distinguishing it from the mathematical tools often used in nonstandard analysis.

[2] In the context of nonstandard analysis, usually *near-equality* is used in case their difference is infinitesimal. For the present purposes working with the natural numbers is simpler and sufficient, and we adopt this alternative definition of *near-equality*.

We will address the second difficulty by supplementing Dinis's model with Fara's (2008; 2012) counterpart-based model theory. This will make the model flexible, by allowing for true cross-temporal identifications while being neutral on whether ships differing in their planks are, in fact, identical.

Finally, we will address the third difficulty by adopting a nonstandard primitivist model whereby what we have is a nonstandard number of degrees of relative sameness, rather than a nonstandard number of ships. To achieve this we will enrich the model theory with scales based on the so-called ML theory (Dinis and Jacinto, 2025). This affords an understanding of the Ship of Theseus paradox as arising from a confusion between *marginal* and *large* differences.

In (Dinis and Jacinto, submitted) it has been argued that instances of the *Sorites paradox* also involve a nonstandard number of vague degrees (e.g., degrees of baldness), rather than a nonstandard number of objects having those degrees, and that the Sorites paradox likewise arises from a confusion between *marginal* and *large* differences. Accordingly, the nonstandard primitivist model here offered affords a unified understanding of both paradoxes.

The paper is structured as follows. In §2 we present a class of models, called *near-equality* models, which reflect the model of the Ship of Theseus paradox offered in (Dinis, 2023). In §3 the near-equality models are additionally equipped with temporal operators that allow to reflect Fara's distinction between *identity* and *sameness relative to a sortal*. The resulting models are then called *relative sameness models*. §4 is dedicated to further extend relative sameness models in two different ways: (i) by reinterpreting *being a plank of*; and (ii) by incorporating the idea that a thing is equal to its matter. In §5 we present a final class of models that involve ideas from the *nonstandard primitivism* account on vagueness which allow to drop the unrealistic assumption, present in all previous models, that there exist nonstandardly many ships. These models furthermore offer a unification of the Ship of Theseus and the Sorites paradoxes. §6 concludes.

2. Near-equality models

Dinis considers a binary relation $\simeq$ of *near-equality* and a sequence of nonstandardly many near-equal ships $(s_i)_{i=0,\dots,n}$ such that each s_i de-notes the ship obtained after i planks have been replaced. The *Ship of Theseus paradox* is then represented as follows.

$$\begin{cases} s_0 \simeq s_1 \\ s_1 \simeq s_2 \\ \quad \ldots \\ s_{n-1} \simeq s_n \\ s_0 \not\simeq s_n \end{cases}$$

The key idea is that the symbol $\simeq$ is interpreted as meaning 'near-equal but not necessarily equal', which allows to say, e.g. that ships s_0 and s_1 are different but imperceptibly so.[3]

In order to deal with near-equality, Dinis bases his model on the nonstandard theory ENA^- (Dinis and van den Berg, 2019; Nelson, 1987). This theory extends arithmetic by adding to its language a new predicate st — where the intended reading of st(x) is that x is standard –, and to its axioms new axioms governing this predicate (cf. Figure 1).

The nonstandard natural numbers are infinitely large, as they are an "end-extension" of the natural numbers, in the sense that every nonstandard natural number is greater than any standard natural number. We will call *Nelson natural number* to any standard or nonstandard natural number. And, in the spirit of Edward Nelson, we denote by $\mathbb{N}$ the set of Nelson natural numbers. Formulas involving the predicate st are called *external*. The remaining ones, i.e. the formulas in the language of classical mathematics, are called *internal*.

The first two axioms of ENA^- state that 0 is standard and that if a certain number is standard then its successor is also standard. The third axiom postulates the existence of nonstandard natural numbers. The last axiom is a form of induction, called *external induction*. It states that

[3] There is also a second version based on external induction (see axiom 4 in Figure 1) which goes as follows. Consider the relation $E(x, y)$: 'x is imperceptibly different from but not equal to y' and let s_n represent the ship after n planks have been replaced. The paradox is then represented by (where $\mathbb{N}$ is defined in the main text)

$$\begin{cases} \big(E(s_0, s_1) \wedge \forall^{\mathrm{st}} n \in \mathbb{N}\, (E(s_0, s_n) \to E(s_0, s_{n+1}))\big) \to \forall^{\mathrm{st}} n \in \mathbb{N}\, E(s_0, s_n) \\ \exists \omega \in \mathbb{N}\, (\neg E(s_0, s_\omega)) \end{cases}$$

So, external induction seemingly allows to conclude that if only a standard number of planks are substituted, then one is indeed in the presence of a different ship, but only imperceptibly so. In order for differences to be perceptible one is required to replace nonstandard many planks.

(1) $\mathrm{st}(0)$

(2) $\forall n \in \mathbb{N}(\mathrm{st}(n) \to \mathrm{st}(n+1))$

(3) $\exists \omega \in \mathbb{N}(\neg \mathrm{st}(\omega))$

(4) $(\Phi(0) \wedge \forall^{\mathrm{st}} n \in \mathbb{N}(\Phi(n) \to \Phi(n+1))) \to \forall^{\mathrm{st}} n \in \mathbb{N}\, \Phi(n)$,

where Φ is an arbitrary formula (internal or external) and $\forall^{\mathrm{st}} n \in \mathbb{N}\, \Phi(n)$ is an abbreviation of $\forall n \in \mathbb{N}(\mathrm{st}(n) \to \Phi(n))$.

Figure 1. Axioms of ENA^{-}

any property Φ is true of all standard natural numbers in case Φ holds of zero, and whenever Φ is true of some standard n, then Φ is also true of its successor $n+1$. Note that the usual form of induction is still valid, but only for internal properties.

In order to clarify Dinis's model and its predictions, we will appeal to the following formal languages.

Definition 2.1. For each Nelson natural number n, $\mathfrak{L}_n$ is the first-order language with identity, individual constants $\ulcorner s_i \urcorner$, for each $i \leqslant n$, and binary predicate $\simeq$. The *individual terms* of $\mathfrak{L}_n$ are the variables and individual constants of $\mathfrak{L}_n$, and the m-ary *predicate terms* of $\mathfrak{L}_n$ are the m-ary variables and predicates of $\mathfrak{L}_n$.

Here, s_i refers to the ship obtained by replacing i planks from the initial ship, and $\simeq$ stands for a "near-equality"-like relation. Then, the following models reflect the account of the *Ship of Theseus* paradox given in (Dinis, 2023). We denote by $\mathcal{P}(X)$ the *power set* of X, i.e. the set of all subsets of X.

Definition 2.2. Let $n \in \mathbb{N}$ be *nonstandard*. A *near-equality model* $\mathfrak{A}_n$ for $\mathfrak{L}_n$ is a pair $\langle D_{\mathfrak{A}_n}, I_{\mathfrak{A}_n} \rangle$ such that:

(i) $D_{\mathfrak{A}_n}$ is the set of Nelson natural numbers smaller than or equal to n;

(ii) $I_{\mathfrak{A}_n}$ is an interpretation function such that:
   (a) $I_{\mathfrak{A}_n}(a) \in D_{\mathfrak{A}_n}$, for every individual constant a. In particular, $I_{\mathfrak{A}_n}(s_i) = i$, for each $i \leqslant n$;
   (b) $I_{\mathfrak{A}_n}(\beta) \in \mathcal{P}(D^m_{\mathfrak{A}_n})$ for every m-ary predicate β. In particular:
      i. $I_{\mathfrak{A}_n}(=) = \{\langle d, d \rangle \mid d \in D_{\mathfrak{A}_n}\}$;
      ii. $I_{\mathfrak{A}_n}(\simeq) = \{\langle x, y \rangle \mid x, y \in D_{\mathfrak{A}_n}$ and $\mathrm{st}(|x-y|)\}$.

Variable-assignments and *satisfaction* are defined as usual. A formula φ is *true in a model* M, and we write $M \vDash \varphi$, if and only if it is satisfied by that model under every variable-assignment. As should be clear, the following can then be easily proved:

PROPOSITION 2.1. *For each near-equality model* $\mathfrak{A}_n$*:*
1. $\mathfrak{A}_n \vDash s_i \simeq s_{i+1}$, *for each* $i < n$.
2. $\mathfrak{A}_n \vDash \neg(s_0 \simeq s_n)$.

For each model $\mathfrak{A}_n$ we have that ship s_i is "near-equal" to ship s_{i+1}, and that the initial ship s_0 is not "near-equal" to s_n, the last ship. One question regarding near-equality models is how should 'near-equality' be interpreted, given that we are speaking about *ships* rather than numbers? What does it mean to say that *ships*, rather than numbers, are near-equals?

Relative sameness models also appear to lack the resources for dealing with cross temporal identifications. We have that ships s_i and s_j are distinct whenever i is distinct from j. But — speaking from the moment that the ship was first built (time 0) –, it appears that it is true that there is some plank which is not now a part of s_0 but which will be a part of it in the future. Yet, this is not captured by the model insofar as for each t other than 0, s_t will be distinct from s_0.

3. Counterparts as Near-Equals

The *Ship of Theseus paradox* makes reference to time. In order to better account for this aspect of the paradox, we'll expand the languages from Definition 2.1 with temporal operators:

DEFINITION 3.1. Language $\mathfrak{L}_n^T$ extends $\mathfrak{L}_n$ with:
- (i) the temporal operators $\mathbb{P}$ ('in the past') and $\mathbb{F}$ ('in the future');
- (ii) the binary predicates $\simeq_{i,j}$ ('is, at i, the same boat as, at j'), for each $i, j \leqslant n$;
- (iii) the binary predicate Plk ('is a plank of').

Our first proposed improvement on near-equality models, as accounts of the *Ship of Theseus paradox*, follows the work of Fara (2008, 2012) in distinguishing *identity* from *sameness relative to a sortal*. Fara has argued that distinct objects may nonetheless be *the same* F, for any given sortal concept F.[4] For instance, according to her, each *ship* is *identical* to the portion of matter it is *currently* made up of. Yet, *in the future*, it will be *the same ship as* a *different* portion of matter. So, on her view, ship s_0 is distinct from ship s_i, for every i other than 0. But portions of matter may be distinct while being the same ship. Accordingly, there is some $i > 0$ such that s_0 and s_i are the *same ship*, even though, s_0 and s_i are not *identical*.

A further thesis of Fara's — which our improvement on near-equality models will also incorporate — is that, in modal contexts, claims made in terms of the identity predicate concern facts about *sameness relative to a sortal* rather than *real identity*. For instance, consider the following sentence:

$$\exists x(\neg \mathrm{Plk}(x, s_0) \wedge \mathbb{F}(\mathrm{Plk}(x, s_0))). \tag{$\dagger$}$$

(i.e. some plank which is not now a part of s_0 will be so in the future).

Fara takes sentence (†) to be true just in case there is a plank x which is not a plank of s_0 and there is a point in the future and a ship s_i at that point which is the *same ship* as s_0 is now and which has, at that point, x as one of its planks. Since s_0 and s_i will be composed of different matter (as they will be composed of different planks), they will, according to Fara, be distinct. So, according to Fara, sentences such as (†) are made true by facts which primarily concern not the objects that those sentences are about (s_0) but instead about objects (s_i) which stand in the *relative sameness* relation to them.

Our guiding ideas in what follows are that Dinis's "near-equality"-like relation is the *relative sameness* relation, and that the truth-conditions of sentences in temporal contexts are to be spelled out in terms of *relative sameness* in the way that Fara has suggested. In Definition 3.2 we offer "relative sameness" models of the Ship of Theseus paradox which incorporate these ideas. But before doing so, it will be helpful to say a bit more about some of the formal properties of *relative sameness*.

Fara treats *relative sameness* as a relation between pairs of the form $\langle x, t\rangle$, where x is an object and t is a time: it is that relation that obtains between $\langle x, t\rangle$ and $\langle y, t'\rangle$ if and only if, given how x is at t, it is the same sortal F as y is at t'. Accordingly, each binary predicate $\simeq_{t,t'}$ of $\mathfrak{L}_n^T$

[4] Where a concept is a *sortal concept* just in case it gives criteria of identity and distinctness for the things falling under it — and so tells us how to count things of that sort –, as well as criteria of identity over time for each thing of that sort.

stands for the relation in which x and y stand if and only if x, as x is at t, is the same ship as y, as y is at t'.

Furthermore, Fara takes *relative sameness* to be a *rolemate* relation, i.e., a *weak equivalence* relation between pairs which is *functional on its first argument*. Recall that (i) a weak equivalence relation is one which is *symmetric*, *transitive*, and *weakly reflexive* (i.e., any pair that bears the relation to some pair, also bears it to itself); and (ii) a relation between pairs is *functional on its first argument* if and only if, if $\langle x, t\rangle$ stands in the relation with both $\langle y, t'\rangle$ and $\langle z, t'\rangle$, then $y = z$.

Relative sameness models are then defined as follows:

DEFINITION 3.2. Let $n \in \mathbb{N}$ be *nonstandard*. A *relative sameness model* $\mathfrak{M}_n$ for $\mathcal{L}_n^T$ is a quintuple $\langle T_{\mathfrak{M}_n}, A_{\mathfrak{M}_n}, D_{\mathfrak{M}_n}, I_{\mathfrak{M}_n}, S_{\mathfrak{M}_n}\rangle$ such that:

(i) $T_{\mathfrak{M}_n}$ is the set of natural numbers smaller than or equal to n;

(ii) $A_{\mathfrak{M}_n}$ is a non-empty set;

(iii) $D_{\mathfrak{M}_n} \in \mathcal{P}(A_{\mathfrak{M}_n}^m)^{T_{\mathfrak{M}_n}}$ is a function from $T_{\mathfrak{M}_n}$ to $\mathcal{P}(A_{\mathfrak{M}_n})$ such that $\bigcup_{t\in T_{\mathfrak{M}_n}} D_{\mathfrak{M}_n}(t) = A_{\mathfrak{M}_n}$;

(iv) $S_{\mathfrak{M}_n}$ is a relation on $T_{\mathfrak{M}_n} \times A_{\mathfrak{M}_n}$;

(v) $I_{\mathfrak{M}_n}$ is an interpretation function such that:

  (a) $I_{\mathfrak{M}_n}(a) \in \bigcup_{t\in T_{\mathfrak{M}_n}} D_{\mathfrak{M}_n}(t)$, for every individual constant a. In particular, $I_{\mathfrak{M}_n}(s_t) \in D_{\mathfrak{M}_n}(t)$ for each $t \in T_{\mathfrak{M}_n}$;

  (b) $I_{\mathfrak{M}_n}(\beta) \in \mathcal{P}(A_{\mathfrak{M}_n}^m)^{T_{\mathfrak{M}_n}}$ for every m-ary predicate β. In particular, for every $t \in T_{\mathfrak{M}_n}$:

    i. $I_{\mathfrak{M}_n}(=)(t) = \{\langle d, d\rangle \mid d \in A_{\mathfrak{M}_n}\}$;

    ii. $I_{\mathfrak{M}_n}(\simeq_{i,j})(t) = \{\langle c, d\rangle \mid S_{\mathfrak{M}_n}(\langle c, i\rangle, \langle d, j\rangle)\}$;

(vi) $S_{\mathfrak{M}_n}$ is a rolemate relation such that, for all $t, t' \in T_{\mathfrak{M}_n}$:

  ($\star$) $S_{\mathfrak{M}_n}(\langle I_{\mathfrak{M}_n}(s_t), t\rangle, \langle I_{\mathfrak{M}_n}(s_{t'}), t'\rangle)$ if and only if $|t' - t|$ is standard.

Relative sameness models are models for quantified temporal logic in which times — the elements of $T_{\mathfrak{M}_n}$ — are represented by the natural numbers smaller than or equal to a nonstandard n. They are variable-domain models since to different times are assigned possibly different domains. The set $A_{\mathfrak{M}_n}$ is the set of objects existing at some time or other, and $D_{\mathfrak{M}_n}$ assigns to each time a subset $D_{\mathfrak{M}_n}(t)$ of $A_{\mathfrak{M}_n}$ which represents the set of objects existing at time t.

We require each ship s_t to exist at time t (i.e., the interpretation function $I_{\mathfrak{M}_n}$ assigns to s_t an element in $D_{\mathfrak{M}_n}(t)$).

The relation $S_{\mathfrak{M}_n}$ represents the relative-sameness relation, and so is required to be a rolemate relation. The special clause ($\star$) aims to capture Dinis's idea that relative sameness between ships is to be modeled via near-equality. Since each ship will correspond to a particular time, and times will be represented by Nelson natural numbers, Dinis's proposal can be complied with by taking ships to be the same just in case the times at which they exist are near-equal.

Given a model $\mathfrak{M}_n$, a rolemate function can be defined which maps each object, as it is at a time t, to its rolemate at time t':

Definition 3.3. An element y of $A_{\mathfrak{M}_n}$ is the *rolemate* at t' of x as it is at t, written $y = \lceil x_t \rfloor_{t'}$, if $S_{\mathfrak{M}_n}(\langle x, t\rangle, \langle y, t'\rangle)$. If there is no z such that $S(\langle x, t\rangle, \langle z, t'\rangle)$, then we take y to be the whole $A_{\mathfrak{M}_n}$.

The purpose of the second part of the definition is to ensure that $\lceil x_t \rfloor_{t'}$ is always defined. The choice of $A_{\mathfrak{M}_n}$ is arbitrary. It works insofar as $A_{\mathfrak{M}_n} \neq z$, for all $z \in A_{\mathfrak{M}_n}$.

The main novelty of the model theory based on relative sameness models, in comparison with the standard model theory for quantified temporal logic (Fara, 2008), concerns the definition of *satisfaction*. Now, it is common to define satisfaction, and denotation functions, in terms of variable-assignments. But, and by contrast with Fara's (2008) original approach, it turns out to be easier to work directly with denotation functions instead of with variable-assignments (in particular, this simplifies notation substantially). Denotation functions are then defined as follows:

Definition 3.4. A *denotation function* is a function σ mapping each individual term into $A_{\mathfrak{M}_n} \cup \{A_{\mathfrak{M}_n}\}$ and each m-ary predicate term into $\mathcal{P}(A^m_{\mathfrak{M}_n})^{T_{\mathfrak{M}_n}}$.

In the following, *denotation function variants* play the same role as variable-assignment variants:

Definition 3.5. For each denotation function σ, each $o \in A_{\mathfrak{M}_n}$ and each individual term α, a *denotation function variant* $\sigma[o/\alpha]$ is a function such that, for each term τ:

$$\sigma[o/\alpha](\tau) = \begin{cases} \sigma(\tau) & \text{if } \tau \neq \alpha \\ o & \text{if } \tau = \alpha \end{cases}$$

The denotation functions that agree with the model's interpretation function have a special role insofar as they will allow us to define truth at a time:

Definition 3.6. A denotation function σ is a *denotation function of model* $\mathfrak{M}_n$ if and only if it coincides with the interpretation function of $\mathfrak{M}_n$ with respect to all individual constants and predicates.

The crux of Fara's proposal lies in having formulae occurring in the scope of temporal operators be satisfied relative to *rolemate* denotation functions. Note that, by contrast with Definition 3.3, in Definition 3.7 are defined rolemates of *denotation functions*, and not simply rolemates of *objects*.[5]

Definition 3.7. For each denotation function σ, and $t, t' \in T_{\mathfrak{M}_n}$, its *rolemate* $\lceil \sigma_{t,\varphi} \rfloor_{t'}$ at t', as σ is at t (with respect to formula φ), is a function such that, for each term τ:

$$\lceil \sigma_{t,\varphi} \rfloor_{t'}(\tau) = \begin{cases} \lceil \sigma(\tau)_t \rfloor_{t'} & \text{if } \tau \text{ is a constant occurring free in } \varphi \\ \sigma(\tau) & \text{otherwise.} \end{cases}$$

Satisfaction is then defined as follows:

Definition 3.8. The *satisfaction* of a formula φ in a model $\mathfrak{M}_n$, at time t and relative to a denotation function σ, denoted $\mathfrak{M}_n \vDash^{\sigma}_{t} \varphi$, is defined according to the following clauses

1. $\mathfrak{M}_n \vDash^{\sigma}_{t} \beta(a^1, \ldots, a^m)$ iff $\langle \sigma(a^1), \ldots, \sigma(a^m) \rangle \in \sigma(\beta)(t)$;
2. $\mathfrak{M}_n \vDash^{\sigma}_{t} \neg\varphi$ iff $\mathfrak{M}_n \nvDash^{\sigma}_{t} \varphi$;
3. $\mathfrak{M}_n \vDash^{\sigma}_{t} \varphi \wedge \psi$ iff $\mathfrak{M}_n \vDash^{\sigma}_{t} \varphi$ and $\mathfrak{M}_n \vDash^{\sigma}_{t} \psi$;
4. $\mathfrak{M}_n \vDash^{\sigma}_{t} \exists v\, \varphi$ iff there is some $y \in D_{\mathfrak{M}_n}(t)$ s.t. $\mathfrak{M}_n \vDash^{\sigma[y/x]}_{t} \varphi$;
5. $\mathfrak{M}_n \vDash^{\sigma}_{t} \mathbb{F}\varphi$ iff $\exists\, t' > t : \mathfrak{M}_n \vDash^{\lceil \sigma_{t,\varphi} \rfloor_{t'}}_{t'} \varphi$;
6. $\mathfrak{M}_n \vDash^{\sigma}_{t} \mathbb{P}\varphi$ iff $\exists\, t' < t : \mathfrak{M}_n \vDash^{\lceil \sigma_{t,\varphi} \rfloor_{t'}}_{t'} \varphi$.

The only clauses which are not usual with respect to standard quantified temporal logic are clauses 5 and 6. These require that satisfaction of a formula $\#\varphi$ at time t relative to a denotation function σ, for $\#$ a temporal operator, be determined in terms of whether φ is satisfied at possibly other times t' relative to σ's *rolemate* $\lceil \sigma_{t,\varphi} \rfloor_{t'}$, rather than in terms of σ itself. So, $\varphi(\alpha)$ is required to be satisfied, at t', by $\sigma(\alpha)$'s *rolemate*, $\lceil \sigma(\alpha)_t \rfloor_{t'}$, rather than by $\sigma(\alpha)$ itself.

Finally, *truth at a time* and *truth* are defined as follows:

[5] Fara's treatment of *quantification* also resorts to the rolemate relation. But since our main points can be made without this extra complication, we leave it out. Details can be found in (Fara, 2008, §D, Definition $\mathbf{f}^{\mathbf{x}}_{\mathbf{c}}$).

Definition 3.9. A formula φ is *true* at $t \in T_{\mathfrak{M}_n}$ and model $\mathfrak{M}_n$, $\mathfrak{M}_n \vDash_t \varphi$, if and only if $\mathfrak{M}_n \vDash_t^\sigma \varphi$ for every denotation function σ of the model; φ is *true* at $\mathfrak{M}_n$, $\mathfrak{M}_n \vDash \varphi$, if and only if $\mathfrak{M}_n \vDash_t \varphi$ for every $t \in T_{\mathfrak{M}_n}$.

Relative sameness models align with Dinis's (2023) insight that ships differing by exactly one plank are "nearly equal" (i.e., they are the *same ship*), whereas the initial ship and the ship resulting from replacing all of the initial ship's planks are not (i.e., they are *different ships*):

Proposition 3.1. *For each relative sameness model* $\mathfrak{M}_n$*:*
1. $\mathfrak{M}_n \vDash s_i \simeq_{i,i+1} s_{i+1}$, *for each* $i < n$.
2. $\mathfrak{M}_n \vDash \neg(s_0 \simeq_{0,n} s_n)$.

Proof. Let σ be an arbitrary denotation function of $\mathfrak{M}_n$. By Def. 3.6 we have that $\sigma(s_0) = I_{\mathfrak{M}_n}(s_0)$, $\sigma(s_i) = I_{\mathfrak{M}_n}(s_i)$, $\sigma(s_{i+1}) = I_{\mathfrak{M}_n}(s_{i+1})$, $\sigma(\simeq_{0,n}) = I_{\mathfrak{M}_n}(\simeq_{0,n})$ and $\sigma(\simeq_{i,i+1}) = I_{\mathfrak{M}_n}(\simeq_{i,i+1})$.

So, by clause ($\star$) of Def. 3.2, it is not the case that $S_{\mathfrak{M}_n}(\langle\sigma(s_0), 0\rangle, \langle\sigma(s_n), n\rangle)$ since n is nonstandard and so $|n - 0|$ is nonstandard. We also have that $S_{\mathfrak{M}_n}(\langle\sigma(s_i), i\rangle, \langle\sigma(s_{i+1}), i+1\rangle)$, as $|i + 1 - i| = 1$ is standard. Let t in $T_{\mathfrak{M}_n}$ be arbitrary. Then, $\langle\sigma(s_0), \sigma(s_n)\rangle \notin \sigma(\simeq_{0,n})(t)$ and $\langle\sigma(s_i), \sigma(s_{i+1})\rangle \in \sigma(\simeq_{i,i+1})(t)$, by clause (b)ii of Def. 3.2.

Hence, $\mathfrak{M}_n \vDash_t^\sigma \neg(s_0 \simeq_{0,n} s_n)$, and $\mathfrak{M}_n \vDash_t^\sigma s_i \simeq_{i,i+1} s_{i+1}$, by Def. 3.8. So, $\mathfrak{M}_n \vDash \neg(s_0 \simeq_{0,n} s_n)$, and $\mathfrak{M}_n \vDash s_i \simeq_{i,i+1} s_{i+1}$ as σ and t were arbitrary, by Def. 3.9. $\dashv$

Yet, so far, relative sameness models are neutral with respect to the truth of (†). For they leave open the interpretation of the 'Plk' predicate. We turn to this in the next section.

4. Extending the model

4.1. More planks in the future

A first way of extending relative sameness models thus concerns the interpretation of the binary predicate Plk (standing for the binary relation *is a plank of*). Our interpretation will reflect the fact that a ship's planks are progressively replaced, one at a time.

We start by introducing the following notational conventions:

$$I_{\mathfrak{M}_n}(\mathrm{Plk}(s_t)) := \{x : \langle x, I_{\mathfrak{M}_n}(s_t)\rangle \in I_{\mathfrak{M}_n}(\mathrm{Plk})(t)\}$$

That is, $I_{\mathfrak{M}_n}(\text{Plk}(s_t))$ is the set of planks of s_t at time t.

$$I_{\mathfrak{M}_n}(\text{Plk}(s_t \cap s_{t'})) := I_{\mathfrak{M}_n}(\text{Plk}(s_t)) \cap I_{\mathfrak{M}_n}(\text{Plk}(s_{t'}))$$

That is, $I_{\mathfrak{M}_n}(\text{Plk}(s_t \cap s_{t'}))$ is the set of planks in common between s_t at time t and $s_{t'}$ at time t'.

$$I_{\mathfrak{M}_n}(\text{Plk}(s_t \cup s_{t'})) := I_{\mathfrak{M}_n}(\text{Plk}(s_t)) \cup I_{\mathfrak{M}_n}(\text{Plk}(s_{t'}))$$

That is, $I_{\mathfrak{M}_n}(\text{Plk}(s_t \cup s_{t'}))$ is the set of planks in either s_t at time t or $s_{t'}$ at time t'.

Relative sameness models can then be extended by requiring that they satisfy the following requirements, for every $t < t' \in T_{\mathfrak{M}_n}$ (we will write $A \approx B$ for the existence of a bijection between sets A and B):

(A) $I_{\mathfrak{M}_n,t}(\text{Plk}(s_t)) \approx I_{\mathfrak{M}_n,t'}(\text{Plk}(s_{t'}))$;
(i.e., all ships have the same number of planks);

(B) $I_{\mathfrak{M}_n}(\text{Plk}(s_0)) \cap I_{\mathfrak{M}_n}(\text{Plk}(s_n)) = \emptyset$;
(i.e., s_0 and s_n have no planks in common);

(C) $I_{\mathfrak{M}_n}(\text{Plk}(s_t)) = I_{\mathfrak{M}_n}(\text{Plk}(s_t \cap s_0)) \cup I_{\mathfrak{M}_n}(\text{Plk}(s_t \cap s_n))$;
(i.e., every ship's planks come from s_0 or from s_n);

(D) $I_{\mathfrak{M}_n}(\text{Plk}(s_{t'} \cap s_0)) \subset I_{\mathfrak{M}_n}(\text{Plk}(s_t \cap s_0))$;
(i.e., as time goes by, ships lose planks from s_0);

(E) $\exists! x \in I_{\mathfrak{M}_n}(\text{Plk}(s_t \cap s_0))$ and $x \notin I_{\mathfrak{M}_n}(\text{Plk}(s_{t+1} \cap s_0))$;
(i.e., as a unit of time passes, ships lose exactly one plank from s_0);

(F) $I_{\mathfrak{M}_n}(\text{Plk}(s_t \cap s_n)) \subset I_{\mathfrak{M}_n}(\text{Plk}(s_{t'} \cap s_n))$;
(as time goes by, ships gain planks from s_n);

(G) $\exists! x \in I_{\mathfrak{M}_n}(\text{Plk}(s_{t+1} \cap s_n))$ and $x \notin I_{\mathfrak{M}_n}(\text{Plk}(s_t \cap s_n))$;
(i.e., as a unit of time passes, ships gain exactly one plank from s_n);

(H) $\bigcup_{t \in T_{\mathfrak{M}_n}} I_{\mathfrak{M}_n}(\text{Plk}(s_t)) \subseteq \bigcap_{t \in T_{\mathfrak{M}_n}} D_{\mathfrak{M}_n}(t)$;
(i.e., each ship's plank exists at every time).

We can now show that relative sameness models yield the correct predictions with respect to the truth of sentence (†) at time 0:

Example 4.1. We will prove that $\mathfrak{M}_n \vDash_0 \exists x(\neg\text{Plk}(x, s_0) \wedge \mathbb{F}(\text{Plk}(x, s_0)))$. Let $\mathfrak{M}_n$ be a relative sameness model such that $I_{\mathfrak{M}_n}(s_i) \neq I_{\mathfrak{M}_n}(s_j)$ whenever $i \neq j$, and σ an arbitrary denotation function of $\mathfrak{M}_n$. We have that $\exists! d \in I_{\mathfrak{M}_n}(\text{Plk}(s_1))$ such that $d \notin I_{\mathfrak{M}_n}(\text{Plk}(s_0))$, by Df. 3.2, clause (iv) and (G).

So, $\langle d, I_{\mathfrak{M}_n}(s_0)\rangle \notin I_{\mathfrak{M}_n}(\mathrm{Plk})(0)$ and $\langle d, I_{\mathfrak{M}_n}(s_1)\rangle \in I_{\mathfrak{M}_n}(\mathrm{Plk})(1)$. Moreover, $\sigma[x/d](x) = d$, $\sigma[d/x](s_0) = I_{\mathfrak{M}_n}(s_0)$, and $\sigma[d/x](\mathrm{Plk}) = I_{\mathfrak{M}_n}(\mathrm{Plk})$, by Dfs. 3.5 and 3.6. So,

$$\langle \sigma[x/d](x), \sigma[x/d](s_0)\rangle \notin \sigma[x/d](\mathrm{Plk})(0).$$

Hence, by Df. 3.8 it holds that $\mathfrak{M}_n \vDash_0^{\sigma[d/x]} \neg\mathrm{Plk}(x, s_0)$.

Let A stand for $\mathrm{Plk}(x, s_0)$. Then, we have that

$$\lceil \sigma[d/x]_{0,A} \rfloor_1(x) = \sigma[d/x](x) = d,$$
$$\lceil \sigma[d/x]_{0,A} \rfloor_1(\mathrm{Plk}) = \sigma[d/x](\mathrm{Plk}) = I_{\mathfrak{M}_n}(\mathrm{Plk}),$$

by Df. 3.3, and:[6]

$$\begin{aligned}
\lceil \sigma[d/x]_{0,A} \rfloor_1(s_0) &= \lceil \sigma[d/x](s_0)_0 \rfloor_1 && \text{(by Df. 3.7)}\\
&= \lceil I_{\mathfrak{M}_n}(s_0)_0 \rfloor_1 && \text{(by Dfs. 3.5 \& 3.6)}\\
&= I_{\mathfrak{M}_n}(s_1) && \text{(by Df. 3.3 \& Df. 3.2}(\star))
\end{aligned}$$

Therefore,

$$\langle \lceil \sigma[d/x]_{0,A} \rfloor_1(x), \lceil \sigma[d/x]_{0,A} \rfloor_1(s_0)\rangle \in \lceil \sigma[d/x]_{0,A} \rfloor_1(\mathrm{Plk})(1).$$

Hence, by Df. 3.8 it holds that $\mathfrak{M}_n \vDash_1^{\lceil \sigma[d/x]_{0,A} \rfloor_1} \mathrm{Plk}(x, s_0)$. So, by Df. 3.8, we have that

$$\mathfrak{M}_n \vDash_0^{\sigma[d/x]} \mathbb{F}(\mathrm{Plk}(x, s_0))$$

So,

$$\mathfrak{M}_n \vDash_0^{\sigma[d/x]} \neg\mathrm{Plk}(x, s_0) \wedge \mathbb{F}(\mathrm{Plk}(x, s_0))$$

Hence,

$$\mathfrak{M}_n \vDash_0^{\sigma} \exists x(\neg\mathrm{Plk}(x, s_0) \wedge \mathbb{F}(\mathrm{Plk}(x, s_0))).$$

Since σ was an arbitrary denotation of the model, we finally obtain the desired result, by Df. 3.9 it holds that $\mathfrak{M}_n \vDash_0 \exists x(\neg\mathrm{Plk}(x, s_0) \wedge \mathbb{F}(\mathrm{Plk}(x, s_0)))$. ⊣

The previous example shows that relative sameness models are indeed capable of accommodating the truth of sentences such as (†) while preserving the idea that, in the Ship of Theseus paradox, the ships existing at different times are not really *identical*, even if they are *the same ship* (and so "nearly-equal").

[6] That is, since s_0 is an individual term occurring free in A, the rolemate at 1 of variable-assignment $\sigma[d/x]$ as it is at 0, with respect to A, assigns to s_0 the rolemate at 1 of $\sigma[d/x](s_0)$ as it is at 0. Since $\sigma[d/x](s_0) = \sigma(s_0)$ is just $I_{\mathfrak{M}_n}(s_0)$, the rolemate at 1 of variable-assignment $\sigma[d/x]$ as it is at 0 assigns to s_0 the rolemate at 1 of $I_{\mathfrak{M}_n}(s_0)$ as it is at 0, which is nothing but $I_{\mathfrak{M}_n}(s_1)$.

4.2. Things are identical to their matter

Now, besides advocating that identity and relative sameness are different relations, Fara also advocates that each thing is *identical* to the matter that it is made up of. That is, according to Fara, ship s_0 is identical to its matter — and so is not identical to any of the ships s_t, for any time t greater than 0 (as none of these are constituted by exactly the same matter that constitutes s_0). It is just that the truth of a sentence, at a time, whose main operator is a temporal operator, and in which a given term occurs, depends not on what is true of the referent of that term at (possibly) other times, but rather on what is true of the referent's rolemate at those other times.

Relative sameness models can be extended to incorporate the view that a thing is identical to its matter as follows. We first extend our language with *matter* constants, i.e., individual constants s_t^*, for every Nelson natural number $t \leqslant n$ (we will sometimes refer to the language's original individual constants as *ship* constants). Each term s_t^* is intended to stand for the *matter* that s_t is made up of at time t.

We then extend relative sameness models to reflect this idea by requiring that they satisfy the following condition, for every time $t \in T_{\mathfrak{M}_n}$:

$$I_{\mathfrak{M}_n}(s_t^*) = I_{\mathfrak{M}_n}(\mathrm{Plk}(s_t))$$

That is, we equate the matter of each ship with the planks it is made up of, and require it to exist throughout all times.

Now that we have an interpretation for the matter that each ship is made up of, the next step is to require our models to identify ships with their matter. This is done by further extending relative sameness models by requiring that they satisfy the following condition, for every time $t \in T_{\mathfrak{M}_n}$:

$$I_{\mathfrak{M}_n}(s_t^*) = I_{\mathfrak{M}_n}(s_t).$$

While we have identified ships with their matter, the truth of temporal statements formulated in terms of *matter constants* should be sensitive not to what objects stand in the *same ship as* relation but rather to what objects stand in the *same matter as* relation. One way to do so would be to extend once more relative sameness models, this time with a rolemate relation representing the *same matter as* relation. But since, according to Fara, a piece of matter is the *same matter as* another piece of matter just in case it is *identical to it*, it is easier to impose the condition that the rolemate of a denotation function σ assign to a "matter term" whatever σ originally assigned to that term. That is:

$$\lceil \sigma_{t,\varphi} \rfloor_{t'}(\tau) = \begin{cases} \lceil \sigma(\tau)_t \rfloor_{t'} & \text{if } \tau \text{ is a ship constant} \\ \sigma(\tau) & \text{otherwise} \end{cases}$$

This extended model would then render the following sentence true:

$$s_0 = s_0^* \wedge \mathbb{F}(\neg(s_0 = s_0^*) \wedge s_0 = s_1^*)$$

(i.e. Ship s_0 is identical to its matter, though in the future it will not be identical to that matter, but will instead be identical to s_1's matter.)

Example 4.2. We will prove that $\mathfrak{M}_n \vDash_0 s_0 = s_0^* \wedge \mathbb{F}(\neg(s_0 = s_0^*) \wedge s_0 = s_1^*)$. Let $\mathfrak{M}_n$ be any (extended) relative sameness model and σ an arbitrary denotation function of $\mathfrak{M}_n$. We have that $I_{\mathfrak{M}_n}(s_0) = I_{\mathfrak{M}_n}(s_0^*)$. So, $\sigma(s_0) = \sigma(s_0^*)$, by Df. 3.6. So, $\langle \sigma(s_0), \sigma(s_0^*) \rangle \in I_{\mathfrak{M}_n}(=)(0)$, by Dfs. 3.2 & 3.8. So, $\langle \sigma(s_0), \sigma(s_0^*) \rangle \in \sigma(=)(0)$, again by Df. 3.6. Therefore, by Df. 3.8, $\mathfrak{M}_n \vDash_0^\sigma s_0 = s_0^*$.

Now, let B be the formula $\neg(s_0 = s_0^*) \wedge s_0 = s_1^*$. We have that $\lceil I_{\mathfrak{M}_n}(s_0)_0 \rfloor_1 = I_{\mathfrak{M}_n}(s_1)$. So, by Df. 3.6:

$$\lceil \sigma(s_0)_0 \rfloor_1 = \lceil I_{\mathfrak{M}_n}(s_0)_0 \rfloor_1 = I_{\mathfrak{M}_n}(s_1) = I_{\mathfrak{M}_n}(s_1^*) = \sigma(s_1^*).$$

Moreover, $\lceil \sigma(s_0)_0 \rfloor_1 = \lceil \sigma_{0,B} \rfloor_1(s_0)$ and $\sigma(s_1^*) = \lceil \sigma_{0,B} \rfloor_1(s_1^*)$, by Df. 3.7. So, $\lceil \sigma_{0,B} \rfloor_1(s_0) = \lceil \sigma_{0,B} \rfloor_1(s_1^*)$. Hence,

$$\langle \lceil \sigma_{0,B} \rfloor_1(s_0), \lceil \sigma_{0,B} \rfloor_1(s_1^*) \rangle \in \sigma(=)(1).$$

So, by Df. 3.8, $\mathfrak{M}_n \vDash_1^{\lceil \sigma_{0,B} \rfloor_1} s_0 = s_1^*$.

Furthermore, we have that:

$$I_{\mathfrak{M}_n}(s_1) = I_{\mathfrak{M}_n}(s_1^*) = I_{\mathfrak{M}_n}(\mathrm{Plk}(s_1)) \neq I_{\mathfrak{M}_n}(\mathrm{Plk}(s_0)) = I_{\mathfrak{M}_n}(s_0^*).$$

But, by Dfs. 3.2 & 3.6, $\lceil \sigma(s_0^*)_0 \rfloor_1 = \sigma(s_0^*) = I_{\mathfrak{M}_n}(s_0^*)$. Then, since $\lceil \sigma(s_0)_0 \rfloor_1 = I_{\mathfrak{M}_n}(s_1)$, we have that $\lceil \sigma(s_0)_0 \rfloor_1 \neq \lceil \sigma(s_0^*)_0 \rfloor_1$. So, by Df. 3.7, $\lceil \sigma_{0,B} \rfloor_1(s_0) \neq \lceil \sigma_{0,B} \rfloor_1(s_0^*)$. Therefore, by Df. 3.8,

$$\mathfrak{M}_n \vDash_1^{\lceil \sigma_{0,B} \rfloor_1} \neg(s_0 = s_0^*) \text{ and } \mathfrak{M}_n \vDash_1^{\lceil \sigma_{0,B} \rfloor_1} \neg(s_0 = s_0^*) \wedge s_0 = s_1^*.$$

So,

$$\mathfrak{M}_n \vDash_0^\sigma \mathbb{F}(\neg(s_0 = s_0^*) \wedge s_0 = s_1^*)$$
$$\mathfrak{M}_n \vDash_0^\sigma s_0 = s_0^* \wedge \mathbb{F}(\neg(s_0 = s_0^*) \wedge s_0 = s_1^*).$$

And since σ was an arbitrary denotation function of $\mathfrak{M}_n$, it can finally be appreciated that $\mathfrak{M}_n \vDash_0 s_0 = s_0^* \wedge \mathbb{F}(\neg(s_0 = s_0^*) \wedge s_0 = s_1^*)$. ⊣

Notwithstanding, there is one aspect of relative sameness models which still appears quite counterintuitive. The crucial result, Proposition 3.1, whereby "adjacent" ships are the same, though the first and the last are not, presupposes that there is a nonstandard number of ships, and that a nonstandard number of planks has been removed from the original ship. But this assumption is surely unrealistic. For it requires the existence of nonstandardly many ships differing by at least one plank. Yet, the Ship of Theseus paradox is just as suasive if there are only standardly many ships differing by at least one plank. Presumably, what this shows is that the way in which *near-equality* underlies *relative sameness* isn't quite the one which is presupposed by relative sameness models.

For this reason in the next section we present a model which arguably better accounts for the relationship between relative sameness and near-equality, while preserving the merits of relative sameness models. To anticipate, it will be the difference between the *degrees of relative sameness* of, respectively, the initial ship and the ship resulting from substitution of all the initial ship's planks, which will be infinitely large, rather than the number of ships or planks involved.

5. A Nonstandard primitivist model

Our final model of the Ship of Theseus paradox will involve ideas from so-called NONSTANDARD PRIMITIVISM (Dinis and Jacinto, submitted, 2025), which consists of the following view on vagueness:

1. There are quantities, referred to as *vague quantities*, whose degrees are vague in that the having of such a degree by an entity is neither reducible to nor supervenient on the having of a precise degree by that entity;[7]
2. There exist two binary relations, the *marginally smaller than* and the *largely smaller than* relations, whose common field consists exactly in the vague degrees;
3. Neither the obtaining of the *marginally smaller than* relation nor of the *largely smaller than* relation (between vague degrees) is reducible

[7] For more on vague quantities, see (Dinis and Jacinto, submitted), where, following the lines of Fara's (2000) interest-relativism about vagueness, we offer one way of understanding vague quantities as reflections of the interests of agents.

to or supervenes on the obtaining of precise relations between precise degrees;

4. The ML theory is true of the *marginally smaller than* and *largely smaller than* relations.

This is not the place to reintroduce the ML theory in full detail (we refer the reader to (Dinis and Jacinto, 2025) and also to (Dinis and Jacinto, submitted) for a simplification). Still, its core idea — and to make a long story short — is that vague degrees have a structure mirroring that of nonstandard arithmetic and nonstandard analysis, and marginal difference is well-represented by *near-equality*.

NONSTANDARD PRIMIVISM and the ML theory have been used to offer a diagnosis of versions of the Sorites paradox involving vague gradable adjectives (i.e., adjectives such as as 'tall', 'rich', 'bald', etc.). The main idea is that there is a confusion between differing marginally (as well as between differing largely) and differing by a fixed precise amount. It is this confusion that gives rise to the Sorites paradox.

For instance, consider a soritical series for baldness whose first and last elements are, respectively, Michael Jordan and Cristiano Ronaldo, and such that each element in the series has exactly one hair more than its predecessor (whenever it has one). It is tempting to think that adjacent members in this series — who differ by the fixed precise amount of a single hair — differ *marginally* with respect to how bald they are. Yet, since marginal difference is *transitive* (just as near-equality is), if adjacent members differed marginally rather than largely, then Jordan and Ronaldo would differ marginally with respect to how bald they are — and so the former would be bald if and only if the other were. But, Jordan and Ronaldo differ largely rather than marginally: Jordan is bald whereas Ronaldo is not.

So, adjacent members in a soritical series do not (always) differ marginally. Some differ largely. But this is not because they differ by more than some fixed precise amount. After all, adjacent members in our soritical series for baldness differ by nothing but a single hair, though some will differ largely rather than marginally.[8] Marginal difference is neither supervenient on nor reducible to any fixed precise difference.

[8] Furthermore, if one thinks that differences in baldness track some precise dimension other than number of hairs (e.g., hair density), we know that it will be possible to construct soritical series for this other dimension.

But this need not mean that it is senseless to think that baldness comes in degrees. Rather, what needs to be acknowledged is that baldness degrees are not to be equated with hair number, hair density, or whatnot. They are irreducible to degrees of precise quantities. Furthermore, according to NONSTANDARD PRIMITIVISM, the relations of *marginal* and *large* difference hold primarily between degrees of baldness, and only derivatively between people having those degrees.

Now, we've said that, because Michael Jordan and Cristiano Ronaldo differ largely rather than marginally with respect to how bald they are, not all adjacent members in our soritical series for baldness differ marginally (owing to the transitivity of marginal difference). But note that this is so only insofar as Jordan and Ronaldo differ by a *standard* number of hairs. If they differed by a *nonstandard* number of hairs, then the fact that they differ largely would no longer entitle us to conclude that some adjacent members in the soritical series differ largely. But, of course, nonstandard primitivists will take the thought that Jordan and Ronaldo differ by a *non*standard number of hairs to arise owing to nothing but a confusion between precise quantities and vague ones.

The nonstandard primitivist solution to the Sorites paradox paves the way to a significant improvement upon our models of the Ship of Theseus paradox. The idea will be that underlying those models is a confusion between a precise relation — difference by a standard number of planks — and a vague relation — relative sameness. And just as with the nonstandard primitivist solution to the Sorites paradox, we will assume that there are degrees of relative sameness — which themselves can stand in relations of marginal and large difference. When they differ marginally, the objects that have those degrees are the same ship. Otherwise, they're not. Near-equality still gets into the picture, but now as a model of the marginal difference relation.

Later on we will endeavor to show that this model delivers intuitively true predictions about the Ship of Theseus paradox. Notwithstanding, the model is based on the idea that there are such things as *degrees of relative sameness*. Yet, degrees of relative sameness do not seem to be the same as degrees of, e.g., baldness. Can sense be made of degrees of relative sameness?

We think so. In our view, degrees of relative sameness concern how close a representative of a sortal F is to being a fixed F. For instance, in the case of the sortal *ship*, the degrees concern how close a *ship* is to being a fixed ship at a fixed time, which, for the sake of argument, we consider to be the initial Ship of Theseus. This fixed ship acts as a prototype for *being the Ship of Theseus* in the same way that in the psychological theory of prototypes whether an object counts as a ship depends on how close it is to a prototypical ship (Rosch and Mervis, 1975).[9] Accordingly, this view would seem to fit with some psychological evidence to the effect that we categorize in terms of prototypes. The idea is then that we categorize not only sorts of things according to prototypes, but also individual instances of those sorts. That is, we categorize in terms of prototypes not only what things fall under properties typically had by many things, but also what things fall under properties commonly thought to be had by no more than one thing, such as the property of being the Ship of Theseus.

The primitivist model will be obtained by adding a sortal scale to the relative sameness model, that is, an assignment of Nelson natural numbers to the different ships. One important difference between the scales used to measure relative sameness and those involving vague degrees is that degrees of relative sameness will be ordered so that they have a *minimum* — the degree had by the *prototype*.

Primitivist models are akin to relative sameness models. The domain function assigning a set of individuals to each time will be defined as in relative sameness models. By contrast with relative sameness models, times will represented only by standard Nelson natural numbers. Primitivist models also possess a scale assigning to each individual-time pair a degree, which will be represented by a Nelson natural number. Moreover, the relative sameness relation will now be modeled by the relation in which two individual-time pairs stand whenever their degrees of relative sameness differ by some standard natural number.

Definition 5.1. Let $n \geqslant 2$ be a (possibly standard) Nelson natural number. A *primitivist model* $\mathfrak{A}_n$ is a tuple $\langle T_{\mathfrak{A}_n}, A_{\mathfrak{A}n}, D_{\mathfrak{A}_n}, I_{\mathfrak{A}_n}, S_{\mathfrak{A}_n}, h_{\mathfrak{A}_n}\rangle$ which is a model for $\mathcal{L}n_T$ such that:

(i) $T_{\mathfrak{A}_n} := \{i : 0 \leqslant i \leqslant n\}$;

(ii) The sets $A_{\mathfrak{A}_n}$, $D_{\mathfrak{A}_n}$ and $I_{\mathfrak{A}_n}$ are defined as in Definition 3.2;

(iii) The mapping $h_{\mathfrak{A}_n}$:$\{\langle x, t\rangle \mid t \in T_{\mathfrak{A}_n} \text{ and } x \in D_{\mathfrak{A}_n}(t)\} \to \mathbb{N}$ is a "sortal scale", i.e. is such that:

[9] In this example, it seems to make sense that the prototype is the initial ship. But of course, what ship is considered to be the prototype might be a contextual matter, psychological matter, or to be explained in some other way. We leave these issues for future work.

(a) $h_{\mathfrak{A}_n}(\langle s_0, 0\rangle) = 0$;
(b) For every $t < k$, if $h_{\mathfrak{A}_n}(\langle s_t, t\rangle) = i$, then $h_{\mathfrak{A}_n}(\langle s_{t+1}, t+1\rangle) = j$, for some j such that $j - i = l$, for some standard natural number l;
(c) $h_{\mathfrak{A}_n}(\langle s_n, n\rangle) = k$, for some nonstandard k;

(iv) $S_{\mathfrak{A}_n}$ is defined in terms of $h_{\mathfrak{A}_n}$ as follows:

$$S_{\mathfrak{A}_n}(\langle x, t\rangle, \langle y, t'\rangle) \text{ iff } \mathrm{st}(|h_{\mathfrak{A}_n}(\langle x, t\rangle) - h_{\mathfrak{A}_n}(\langle y, t'\rangle)|).$$

The sortal scale $h_{\mathfrak{A}_n}$ assigns to the original ship the minimal degree of relative sameness. The degree assigned to a ship at time $t+1$ will be determined by the degree assigned to the ship present at time t, so that the difference between the latter and the former's degrees is a standard natural number. Furthermore, the degree of the ship present at the last time is a nonstandard number. This represents the idea that the last of the ships is a *different ship* from the initial one.

The model-theoretic definitions of variable-assignments, satisfaction, etc. are defined as in the case for relative sameness models. Now, how well does the primitivist model fare with respect to the criticisms presented to the previous models? As previously mentioned, it possesses all the virtues of relative sameness models. Yet, it does not presuppose that the *Ship of Theseus paradox* requires that there be a nonstandard number of ships or planks involved. As mentioned, nonstandardness gets into the picture via the structure of degrees of relative sameness, which are assigned by a sortal scale. It is these degrees that differ by a nonstandard quantity.

As previously mentioned, there's also the question of how the replica ship and the initial ship relate. Are they the same ship? Our models are neutral on that question. It will all depend on what degree is assigned to the replica ship. Still, we may wonder, what degree *should* be assigned to it? What is the *intended* model?

In (Dinis and Jacinto, submitted) it is argued that the degrees of vague quantities which are posited by NONSTANDARD PRIMITIVISM encode both information about the world and information about the agent's interests. For instance, in the case of *baldness*, degrees of this quantity encode both information concerning number of hairs and information about the agent's interests in distinguishing people on such grounds. Perhaps something similar can be said about degrees of relative sameness. How close the replica ship is to the original Ship of Theseus would then be a function not only of what planks they share but also of the interests of the agent in distinguishing the ships (with respect to ship related matters — e.g., their history). For some purposes, ships' differing history might be of significance, and so agents will distinguish them on such grounds. Otherwise, being composed of the same planks is enough to identify them.

If this is right, then different models are needed to reflect the interests of different agents. That is, what is the "intended model" depends on the conversation which is taking place and the interests of those involved. Each model will vary with respect to its scale — which is, according to this picture, as it should be.

6. Conclusion

In this paper we have provided several classes of models that make it possible to reason about the *Ship of Theseus paradox*. The models that we have introduced rely on nonstandard arithmetic and on the notion of near-equality and are divided into two main categories: relative sameness models and nonstandard primitivist models. Relative sameness models are models for quantified temporal logic in which times are represented by the natural numbers smaller than or equal to some nonstandard natural number. The way that *near-equality* enters the picture is by supposing the existence of a nonstandard number of ships between the original ship and the one resulting from replacing all planks in the original one. This rather unrealistic assumption is overcome in nonstandard primitivist models by resorting instead to degrees of relative sameness, assigned by a sortal scale, differing by a nonstandard quantity.

Overall, we get a unified model of Sorites paradoxes involving vague gradable adjectives and of the Ship of Theseus and related paradoxes involving cross temporal identifications. Underlying paradoxes of both kinds is a confusion between relations of marginal difference between vague degrees and relations of difference by small but precise degree between objects.

It is because we confuse the two relations that we are led to a contradiction. Nonstandard primitivist models enable us to rigorously distinguish the precise relations between objects from the relation of marginal difference obtaining between their degrees, whilst maintaining the main insights of Dinis (2023).

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