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## On the Maximality of Positive Properties and Modal Collapse in Variants of Gödel’s Ontological Proof for the Existence of God

**Abstract.** Gödel’s ontological proof for the existence of God — as an inference from the level of rational, logical structure to the level of being — was strongly influenced by Leibniz’s monadology, which is based on the Ur-Monade — God — conceived as absolutely infinite. Gödel defines God as the maximum of positive properties and he characterizes this maximum (of being) with his postulates implicitly as an ultrafilter structure. Using maximality as criterion we have compared the set of positive properties in Scott’s version of Gödel’s theistic theory with modified variants of it to see whether a maximum is still achieved, or not, and what effect this has on modal collapse. The motivation for these, typically weakened, variants of Gödel’s theistic theory has been to avoid modal collapse and the limited determinism associated with it, and to show that the necessary existence of God can still be proved with less strong axioms. A weakening of the maximality criterion of the ultrafilter structure imposed by Gödel’s (and Scott’s) postulates have thus been applied to the set of positive properties in order to allow for alternative models in which modal collapse does not hold. In these alternative models, however, the maximality criterion for the basic quantity of positive properties is abandoned, so that in them God as a “maximum” is not reached. We argue that this is contrary to Gödel’s intention, and that for his conception of God, maximality and absolute infinity are non-negotiable. What Gödel had in mind, as we argue here, was a holistic — absolutely infinite — view of the world, i.e. with modal collapse, but without limited determinism.

**Keywords:** Gödel’s ontological proof; modal collapse; absolute infinity; set theory; filter and ultrafilter structures

## 1. Introduction: Prerequisites and historical context

Gödel’s ontological proof for the existence of God is based on his study of Leibniz, whose work, especially his monadology, strongly influenced him [35, p. 2]. His aim was to create an overall system of the sciences based on a fundamental meta-theory that lay at the interface of philosophy, mathematics, logic and theology:

Gödel’s program in philosophy is to find an exact theory of metaphysics, presumably in the form of a monadology. [...] Gödel characterized his philosophical outlook in this way: “My theory is a monadology with a central monad (namely, God) [addition by Hao Wang]. It is like the monadology by Leibniz in its general structure.” [35, p. 8]

At a first examination Leibniz’ monadology appears to be atomistic, since everything is supposed to consist of individual basic elements, the monads [26, p. 11]. However, on closer inspection of Leibniz’s monadology, this is no longer the case if we look at the nature of the primordial substance — God — of the created monads [26, p. 37; no. 47], which is the cause and, as the primordial substance and inner principle [26, pp. 11–15, 31, 37; no. 1–11, 38, 47], the fundamental essence of every monad. This means that all monads emerge from the primordial substance (primordial monad) [26, p. 37]. Leibniz defines this explicitly as absolutely infinite [26, p. 33; no. 41] and Gödel adopts this primal monad, and thus absolute infinity, as the foundational principle [35, p. 8]. This means that, regardless of what further structures are assumed within the structure of the world — or in the monads that are generated — the basic principle of being always remains absolutely infinite. This is also reflected in Gödel’s work in set theory where he relates absolute infinity as Ackermann’s principle<sup>1</sup> — the notion of the universe of all sets  $V$  as being undefinable — to all other sets, the structures upon them and the axioms related to them, as an emergence from this first Ackermann’s principle [35, pp. 282–283]. This creates a relationship between discrete and homogeneous structures in the consideration of sets, in which elements of sets can be handled as discrete points, as is common in the applications of today’s mathematics. This relationship between the

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<sup>1</sup> It should be noted that the Ackermann principle, as used in many places by Wang and Gödel, should not be confused with the Ackermann set theory [1], since this principle is only indirectly found in the set construction there (e.g. in the overall construction via classes) and goes beyond this on a meta-level.

consideration of points (elements of sets) as discrete objects and the homogeneous-continuous background (Gödel's Ackermann-principle described above) takes on an existential necessity, because the existence of objects, i.e. the points or sets, results precisely from the relation to their constituent background. Only through this distinction, the emergence from the background (figure-ground separation), existence is formed; for more more details see [27, 28]. Without this figure-ground separation, the object would not have any characteristic, it would not be possible to determine or recognize it, i.e. it would not exist. Or, when formulated in relation to mathematics:

The intuitive continuum is characterized by the fact that it has parts and those parts are again continua. Points having no parts are representatives of what is discontinuous and what is the opposition of the continuum. They can form only what is discontinuous. Points appear in partitions only as limits of continua and have in this conception no independent existence at all. And it is inconceivable in the case of this conception that just those points whose existence depends on the continua should constitute the latter. The parting (dividing) would become the parted (divided). [5, p. 171]

For Gödel, this relationship between foreground and background, or object and background, is represented by the fundamental concept of cause [35, p. 315]; for a more detailed description of this causality relationship we refer to Kovač [24, 25]. Due to this necessary relationship, absolute infinity must also exist internally so that points obtain their existence through this background. Due to the absolute infinity of the primordial monad, the absolute infinity of  $V$  is reflected inwards at every point, which is described above as Ackermann's principle, but which Gödel also described as the reflection principle:

Ackermann's principle, or the reflection principle in a more restricted sense:  $V$  cannot be defined by a structural property not containing  $V$  — no property definable from the elements of  $V$  can determine  $V$ . [35, p. 283]

## 2. Basic assumptions of Gödel's ontological proof

In the light of the above observations, we now examine Gödel's ontological proof and its basic assumptions in some more detail. Instead of

working with Gödel’s original text [18], we consider Scott’s version [30], as shown in Figure 1.<sup>2</sup> The modifications do not change the essential content or structure of the argument and preserve the original course of the proof; see also [21, pp. 368–369] and [34].

Gödel defines God as the being that possesses all positive properties. This proof for the existence of God stays in the tradition of ontological proofs of God, such as those of Anselm of Canterbury, Descartes or Leibniz, in which conclusions are drawn from the logical conceptual level to the level of being, i.e. the ontological level. God is understood as the maximum, beyond which nothing greater can be conceived. With Gödel, this maximum is only represented by the “being that possesses all positive properties.” [35, pp. 114–115]

This means that the usual attributes such as “omniscient” and “allmighty” can also be represented by *all positive properties* [35, p. 118]. What exactly positive properties are, however, must be deduced elsewhere:

Gödel uses the notion of a positive property as primitive. He says that positive means positive in the moral aesthetic sense, independent of the accidental structure of the world, and that it may also mean pure attribution — that is, the disjunctive normal form in terms of elementary propositions (or properties) contains a member without negation — as opposed to privation (or containing privation).

[20, p. 404; cited after: 35, pp. 113–114]

Regarding the basic requirements, we need to consider first that Gödel defines causality as a fundamental ontological concept, i.e. the connection of every object to its causal background, which also applies to space, time and possibilities and is important because the proof for the existence of God is an ontological one, i.e. it deduces from the conceptual level to the level of being [35, p. 315]. It does not matter whether objects are actual or only possible and in which time they exist, because

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<sup>2</sup> In Gödel’s original version there is a definition of essence that contains an error corrected by Scott. The uncorrected version of the definition of essence leads to a contradiction, as already mentioned by Adams and demonstrated in [10, p. 97]. Adams [2, p. 392] already hints at such an inconsistency; he states: “if there is a property that is necessarily false of everything, then it must be an essence of x” and he notes that undesirable conclusions can be drawn from this. Although he does not use the word inconsistency directly, he apparently recognized the problem, which was independently discovered by the theorem prover Leo-II, who reported a derivation of falsity from Gödel’s premises in experiments.

**Definition 1** A *God-like* being possesses all positive properties:

$$G(x) \equiv \forall\varphi[P(\varphi) \supset \varphi(x)]$$

**Definition 2** An *essence* of an individual is a property possessed by it and necessarily implying any of its properties:<sup>a</sup>

$$\varphi \text{ ess. } x \equiv \varphi(x) \wedge \forall\psi(\psi(x) \supset \Box\forall y(\varphi(y) \supset \psi(y)))$$

**Definition 3** *Necessary existence* of an individual is the necessary exemplification of all its essences:

$$NE(x) \equiv \forall\varphi[\varphi \text{ ess. } x \supset \Box\exists y\varphi(y)]$$

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**Axiom 1** Either a property or its negation is positive, but not both:

$$\forall\varphi[P(\neg\varphi) \equiv \neg P(\varphi)]$$

**Axiom 2** A property necessarily implied by a positive property is positive:

$$\forall\varphi\forall\psi[(P(\varphi) \wedge \Box\forall x[\varphi(x) \supset \psi(x)]) \supset P(\psi)]$$

**Axiom 3** The property of being God-like is positive:<sup>b</sup>

$$P(G)$$

**Axiom 4** Positive properties are necessarily positive:

$$\forall\varphi[P(\varphi) \supset \Box P(\varphi)]$$

**Axiom 5** Necessary existence is a positive property:

$$P(NE)$$

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**Theorem 1** Positive properties are possibly exemplified:

$$\forall\varphi[P(\varphi) \supset \Diamond\exists x\varphi(x)]$$

**Corollary C** Possibly, God exists:

$$\Diamond\exists xG(x)$$

**Theorem 2** Being God-like is an essence of any God-like being:

$$\forall x[G(x) \supset G \text{ ess. } x]$$

**Theorem 3** Necessarily, God exists:

$$\Box\exists xG(x)$$

<sup>a</sup> The addition of the fragment " $\varphi(x) \wedge$ " prevents the inconsistency [cf. 11].

<sup>b</sup> Instead of postulating  $P(G)$  one can alternatively use the following, more general, third-order axiom expressing that the (possibly infinite) conjunction  $X$  of properties taken from a (possibly infinite) set of positive properties  $Z$  is positive:  $\forall Z.(Pos(Z) \longrightarrow \forall X.(X \sqcap Z \longrightarrow PX))$ . While Gödel hinted at such a postulate in his scriptum with his footnote stating “\* and for any number of summands”, from which  $P(G)$  follows, Scott simplified the argument by directly using  $P(G)$  instead; for more on this see [7, Section 4.4], where a formalised version of such a third-order axiom has been studied.

Figure 1. The definitions, axioms and theorems in Scott’s variant of Gödel’s ontological argument; cited after: 10.

Gödel assumes that there is no uniform independent time or space as one common background, but that all times and all spatial forms as well as all possibilities exist simultaneously in parallel, or rather *in each other* actually [23, 36, 37]. Thus, in order to obtain the *maximum* of positive properties for the proof for the existence of God, the maximum of being must be worked out. In Gödel’s notes, the following can be

found with regard to the positive properties, which fits in with his fundamental ontological concept of causality. Apart from the description as “moral/aesthetic”, „*positive property*” is described as follows:

It is possible to interpret the positive as perfective; that is, ‚purely good‘, that is, such as implies no negation of ‚purely good.‘<sup>x</sup> The chief axiom runs then (essentially): A property is a perfective if and only if it implies no negation of a perfective. All that is needed besides this is the axioms:<sup>+</sup> The necessity of a perfective is a perfective,<sup>Φ</sup> and being is a perfective.\* [...]

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<sup>x</sup> Cannot be replaced by ‚good‘.

<sup>+</sup> It need *not* be assumed that always either  $\varphi$  or  $\neg\varphi$  is positive.

<sup>Φ</sup> Or if  $M\varphi$  is a perfective, then  $\varphi$  is too.

\* Or: there is a perfective. From this it follows that being is a perfective, since it implies the possibility of this perfective. [20, p. 435]

Wang comments on this:

His general line of thought is familiar from the history of philosophy. Descartes, for example, spoke of perfections instead of positive properties, but the crucial steps of his argument in the Fifth Meditation are similar to Gödel’s: (a) God is the subject of all perfections, by definition and in accordance with our clear and distinct idea; and (b) existence is perfection. [35, p. 116]

Hence, *existence* is a *perfective* and, on the basis of Gödel’s fundamental ontology of causality, it is the most fundamental positive property that every object has *a priori* and to which other possible positive properties can then be added afterwards. To clarify the terms used, we can understand *being* as a connection between *existence* and *essence* and describe Gödel’s fundamental ontological causal relationship of an object to its background (which then constitutes the object in its entire being) also as an existential relationship to the defining background. Merely by this existential relationship of every possible object, we can cover reality in its entirety (in a set theoretic sense understood as cardinality), because this basic ‚set‘ of the space-time (or world/universe) is not discrete, but is kept maximally open — as a non-discrete continuum.<sup>3</sup> However, Gödel’s general fundamental (= necessary) positive property of existence (and ‘NE’ necessary existence as the interplay of existence and essence) is to

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<sup>3</sup> Hence, this ‚set‘ can not be a ‚set‘ in the classical mathematical sense anymore and it must be examined to what extent the concept of ‚class‘ is sufficient.

be distinguished here from the existence of God and the modal collapse, i.e. the ‘one’ world. God’s existence is to be proven on the basis of the ontological axioms and then afterwards the modal collapse [23, pp. 327–328] follows.<sup>4</sup> By using the existential relationship as a fundament, an ultrafilter is used over the basic quantity (resp. universal continuum) of the existence property of all entities, which implies the existence of all possibilities and all time forms. In the following we will use the term ‘basic quantity’ for this universal homogeneous continuum of existential relations. Ultrafilters are those filters that cannot be further refined [For a detailed definition, description and function of filters and ultrafilters see: 13] (in their original definition, they work on sets, but they can also be applied analogously to classes). However, the basic quantity of positive properties remains homogeneously continuous, since all possibilities as well as temporal and spatial forms remain included and no restriction to a discrete set is introduced. Rather, the ultrafilter of positive properties remains unbounded within the open basic quantity  $V$ , the universal class of all sets, which Gödel described as undefinable or not conclusively structurable. This characterizes the absolute maximum of positive properties, namely absolute infinity. Due to this unboundedness and the absence of any refinement of the ultrafilter, the basic quantity  $V$  and the collection of positive properties coincide at the level of absolute infinity and therefore admit no distinction in absolute size. This observation is made here at the conceptual level, since Gödel’s axioms do not enforce a maximally infinite model structure, but they enable it. One could, thus, argue that suitable further axioms of infinity could or should be added to Gödel’s theistic theory. There are no restrictions and the filter set thus remains as maximally open as the basic quantity. The ultrafilter only has to filter out the most basic *perfective* of being. However, there is a second challenge with regard to the set of all positive properties (in addition to obtaining the maximum), namely to ensure that all positive properties are compatible with each other. For this purpose, Gödel, according to Wang, additionally describes the properties as simple “in the sense that they are not combinations of other properties” and that “we can envisage at least all Boolean combinations of them by conjunction, disjunction,

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<sup>4</sup> Indeed, the positiveness of an ‘existence’ property such as  $\lambda x.\exists y. y = x$ , even for the cases when actualist quantifiers lacking existential import and/or world dependent Leibniz equality are used, is a consequence of the Axioms A1 and A2 from Figure 1; this has been confirmed in experiments using Isabelle/HOL (not shown here; the respective experimentation files can be provided upon request).

and negation” [35, p. 117]. God therefore not only possesses all simple positive properties, but also all their Boolean combinations, which are positive in the sense mentioned above. Through the conjunction of all positive properties we receive that all simple positive attributes are compatible if and only if this also applies to all positive properties, since this conjunction also includes that of the simple positive attributes. This is equivalent to a disjunction, which includes the conjunction of all simple positive properties as a member, which was described by Gödel as the “disjunctive normal form containing some member without negation” [20, p. 404]. Gödel’s reduction to the simple positive properties, in conjunction with the restriction to the *necessarily* positive properties, solves the task of proving the compatibility of the properties. Being simple ensures that they are mutually independent. Being necessarily simple ensures that they are not mutually exclusive, even if this limits the choice of possible properties [35, p. 118]. With *existence* as the most fundamental positive property, we have a candidate that fulfills all conditions, because every existence is different and not further reducible due to the respective reference to the background.<sup>5</sup> We can therefore also refer to existence as an inauthentic property because it is *before* every other property. Gödel summarizes this as: “The fundamental philosophical concept is cause.” and “The affirmation of being is the cause of the world.” [20, p. 433]. Thus, God is connected with the identification of the primordial ground of the world and the affirmation (confirmation) of being (and thus of existence). Gödel explicitly connects the primordial ground of the world with the proof of the existence of an *a priori* proof for the existence of God: “according to the Principle of Sufficient Reason the world must have a cause. This must be necessary in itself (otherwise it would require a further cause). Proof of the existence of an a priori proof of the existence of God (the proof it contains fails to be one).” [20, p. 431] And from the above statements concerning the affirmation of being it can be concluded: “Implicit in [the] assertions [...] is the idea that the affirmation of being is a positive value, or perhaps the only ultimate (positive) value.” [35, p. 120]

Gödel additionally proves, like Leibniz (only the proof is different), but for example in contrast to Anselm of Canterbury, that God’s ex-

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<sup>5</sup> *Background* is to be understood as background and foreground, because the existential relationship is also given by the fact that one thing is the background for another.

istence is possible, and does not take it as a presupposition. Possible existence is identified with the consistency of the system of all positive attributes: “In this argument, God’s possible existence is identified with the compatibility of the system of all positive properties, which is identified with the consistency of the system of their corresponding propositions.” [35, p. 115] With Axiom 2 of the original proof [18, p. 403] (in Scott’s version Axiom 1, see above) Gödel distinguishes the possible properties into positive and negative ones, whereby the latter are negations of the former [20, p. 432]. Thus, a property is positive if and only if its disjunctive normal form contains a member without negation, i.e. it is either a simple positive property or a conjunction of simple positive properties [20, p. 437; cited from: 35, p. 117]. In general, he bases his proof and the distinction between positive and negative properties strongly on Leibniz [35, p. 87]. The last important point for our purposes is that Gödel’s axioms are so strong that they cause a modal collapse, which consists in the fact that, within Gödel’s theory, one can prove a thesis according to which the modality  $\Box$  becomes inessential (that is, every formula of the form  $A \leftrightarrow \Box A$  is provable) [31]; see also [10, 23]. There is a view to assume that this aspect was consciously intended by Gödel and that it is even part of his entire philosophical intuition, which can be interpreted from various statements by Gödel (for a detailed elaboration of this view, see: [23, p. 327]). In addition, for example, in a correspondence with Hao Wang [35, pp. 105–108], he discusses his view that God may have created the world in a sequence of different (sub-)worlds, but that it exists as a whole in a rational way (p. 105). In [19, p. 435] he also writes:

That the necessity of a positive property is positive is the essential presupposition for the ontological proof. If  $\varphi(x) \supset N\varphi(x)$  is assumed [as following from the essence of  $x$ ], then it is easily provable that for every compatible system of properties there is a thing, but that is the *inferior way* [*der schlechte Weg*]. Rather  $\varphi(x) \supset N\varphi(x)$  should follow first from the existence of God. [19, p. 435]

Hence, the existence of God is intended to be proven from the logical axioms first and only then the modal collapse should follow [23, p. 328]. The latter should therefore not itself be a presupposition of the existence of God.

### 3. Comparison of the original proof with modified proofs: modal collapse and the maximum of positive properties

The fact that a modal collapse followed from Gödel's proof has been criticized by many as an undesirable, negative side effect of his postulates, since it can be interpreted as a restriction of free will [for an overview see: 32, pp. 132–140; 218–222]. Therefore, attempts have been made to modify Gödel's postulates and proof in such a way that the consistency and the necessary existence of a divine being is preserved, but the modal collapse is avoided. These attempts comprise, for example, the variants of Anderson [3, 4], Fitting [17] and Benzmüller [8, 9].

We will in the remainder compare some of the proposed changes, and their implied consequences, with the original proof of Gödel in the variant of Scott. However, first, some terms need to be clarified. One relevant modification concerns the notion of positive properties: Is this notion intended to range over extensions or intensions of properties? While Scott [30] and Anderson [3] work with intensions of positive properties in their proofs, Fitting [17], for example, chooses extensions of positive properties in his modification. Hence, we briefly need to clarify the difference. The *extension* of a property  $\varphi$  concerns the scope of the property within a particular possible world, i.e. the set of entities in that world that exemplify that property. The *intension*  $\varphi'$  of a property concerns its conceptualized meaning, i.e. the descriptive, world independent meaning of it, which is not only directly connected with its exemplifying entities, but indirectly, namely modulo its interpretation in a concrete possible world, whereby the extension of  $\varphi'$  within this world is then determined. One may thus view (and formalize) intensions as world-parameterized terms that may change their extension from world to world. Benzmüller and Fuenmayor [6] have compared various modifications of Gödel's proof, that differ among others, with regard to an intensional or extensional take on Gödel's notion of positive properties, and checked these modifications for consistency, correctness and further implications. We also briefly explain, how the following terms are used in the remainder: ‚rigid‘ means that world-independent terms have identical evaluations in all possible worlds, i.e. they are independent of the conditions in the respective world and are thus considered fixed. Note that any extensions of a property  $\varphi$  can be turned into a rigid conceptualization  $\varphi'$  of it by simple parameterization over worlds, thereby transporting their fixed meaning from a given world to all other worlds.

Since the distinction between extensional and intensional positive properties in Gödel's proof affects the modal filter and ultrafilter structures considered, we briefly present the description of the associated modal ultrafilter notions as introduced in the study of Benz Müller and Fuenmayor [6, pp. 131–133, 143–144]: A  $\gamma$ -ultrafilter is defined on the power set of concepts, i.e. on property intensions. A  $\delta$ -ultrafilter, in contrast, is defined on the power set of individuals, i.e. on property extensions. In addition, in Benz Müller and Fuenmayor's analysis, sets  $P$  of intensional positive properties are contrasted with sets  $P'$  of positive properties, which contain all properties  $\varphi$  whose rigid conceptualizations of their extensions (in a given world) lie in  $P$  [6, p. 138]. Using the terms as introduced, we can summarize the characteristics of the different proof variants studied as follows:

**Characteristics of Scott's (and Gödel's) proof [cf. 6, p. 137]:**

- The notion of positive properties  $P$  in the definition of God ( $G$ ) ranges over intensions of properties, i.e. the properties quantified over are of the type  $\gamma$ .
- The equivalence in Axiom 1 can be split into A1a and A1b<sup>6</sup>:  
 (A1a)  $\forall X.P(\rightarrow X) \rightarrow \neg(PX)$ , where  $\rightarrow$  is a set/predicate negation  
 (A1b)  $\forall X.\neg(PX) \rightarrow P(\rightarrow X)$
- Modal collapse follows from the premises.
- The set  $P$  of positive properties is a  $\gamma$ -ultrafilter.
- Existence as the most fundamental positive property corresponds to a rigid intensional property (also rigid intensional extension), since there is only one concept of existence (causality/connection between object and background), but this is covered in all worlds, possibilities, times.
- All other intensional positive properties, in addition to existence, are added afterwards.
- The sets  $P$  and  $P'$  are the same.

**Characteristics of Anderson's variant [cf. 6, pp. 139–141]:**

- Scott's requirements A1a and A1b are weakened. A1b is dropped and only A1a remains: "If a property is positive, then its negation is

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<sup>6</sup> Scott's Axiom 1 is an equivalence statement. Breaking this down into two directions is done to better clarify the differences in regard to Anderson's axioms, which omit one of these two directions. In addition, since we are following the work of Benz Müller and Fuenmayor [6] in this presentation, we also retain their notation.

not positive”. This means that the exclusive-or between positiveness and non-positiveness of properties is abandoned.

- This modification has the further effect that the necessary existence of God would no longer follow. Therefore, further modifications were introduced by Anderson concerning the concepts of God-likeness (we call this now  $G^A$ ) and essence ( $\mathcal{E}^A$ ):
  - $G^A$ : an individual  $x$  is God-like  $G^A$  if, and only if, all and only the necessary/essential properties of  $x$  are positive, i.e.  $G^A x \equiv \forall Y (PY \leftrightarrow \Box(Yx))$ .
  - $\mathcal{E}^A$ : a property  $Y$  is an essence  $\mathcal{E}^A$  of an individual  $x$  if, and only if, all necessary/essential properties of  $x$  are implied by  $Y$  and, conversely, all properties implied by  $Y$  are necessary/essential properties of  $x$ .
- Modal collapse is avoided in Anderson’s variant; the simplest countermodel to modal collapse consists of two worlds and one divine entity.
- The set of positive properties  $P$  in Anderson’s variant is not a  $\gamma$ -ultrafilter. However, the set  $P'$  of all properties  $\varphi$  whose rigid conceptualizations  $\varphi'$  lie in  $P$  still is a  $\gamma$ -ultrafilter. This means that the sets  $P$  and  $P'$  are no longer equal.

**Characteristics of Fitting’s variant [cf. 6, pp. 139–141]:**

- The notion of positive properties  $P$  in the definition of God ( $G$ ) ranges over extensions of properties, i.e. these properties are of type  $\delta$  and not of type  $\gamma$  anymore (intensional properties) as in Scott’s and Anderson’s proofs.
- According to this understanding, positive properties are apriori fixed from world to world in Fitting’s variant, whereby they are not in Scott’s and Anderson’s variants. Scott (and Gödel) define  $Gx$  as  $\forall Y_{\gamma}.PY \longrightarrow Yx$ , whereas Fitting changes this to  $\forall Y_{\delta}.PY \longrightarrow (|Yx|)$ , where  $|\cdot|$  is rigidifying the application term  $Yx$  for all possible worlds. (The concept of essence is changed in a similar way).
- Fitting’s set  $P$  of positive properties represents a  $\delta$ -ultrafilter.
- Modal collapse is avoided.

Overall, in Anderson’s and Fitting’s modifications, the necessary existence of a God-like being is preserved and modal collapse is avoided. With the above distinction between the set of (intensional) positive properties  $P$  and the set  $P'$  of positive properties whose rigid conceptualiza-

tions lie in  $P$ , the following then applies to the three variants: in Scott's proof,  $P$  and  $P'$  coincide and are  $\gamma$ -ultrafilters; in Anderson's variant,  $P$  and  $P'$  do not coincide and only  $P'$  but not  $P$  is a  $\gamma$ -ultrafilter; in Fitting's variant, the set  $P$  (which ranges over property extensions and thus corresponds conceptually to the set  $P'$  in Scott's and Anderson's variants) constitutes a  $\delta$ -ultrafilter. The concept of positive properties in Gödel's original ontological argument corresponds to the mathematical concept of a (principal) modal ultrafilter on intensional properties [6, p. 144]. "Filters, especially ultrafilters, measure the size of subsets of a set; a set is large in this sense if it lies in the filter" [12, p. 3].

In Anderson's variant there may now be properties in  $P'$  which are not in  $P$ . Hence, the modifications of Anderson not only result in the modal collapse being avoided, but they also cause a change in the set of positive properties  $P$ , which now may become smaller. In Anderson's model, the basic quantity of (intensional) positive properties is no longer a  $\gamma$ -ultrafilter. By removing the equivalence in Axiom A1, which prevents a modal collapse, the set in the filter is reduced, as only a portion of the intended positive properties are filtered out. This means that the *maximum* of the basic quantity may no longer be mapped in the filter, as Gödel should have intended.

In Fitting's model the number of positive properties is similarly reduced. Transferred to our concept of existence as the most fundamental positive property, this would also apply to this concept, since it would then only be counted if something exists in all worlds and no longer variably in individual, or even only possible, worlds. As a result, in some of the models satisfying Fitting's premises the maximum of the original basic quantity can no longer be mapped here either.

While the premises of the modified ontological proofs are still consistent and the necessary existence of a divine being is still deducible from them, the nature of this divine being now changes in some of the models satisfying the premises. The reduction of the set of positive properties that is filtered out in these models results in a ,smaller' God that is no longer the maximum ,than which nothing greater can be conceived'.

### **The classical version of Gödel's argument with Christian's Axiom A1'**

In Benz Müller's [9] simplification of the argument the number of axioms is reduced as depicted in Figure 2. Axioms A4 and A5, which address necessity conditions of properties, are dropped and Axioms 1 and 2 are modified. In particular, Axiom 1' is used, which had already

**Definition 1** A *God-like* being possesses all positive properties:

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**Axiom 1'** Self-difference is not a positive property:  $\neg P(\lambda x.x \neq x)$

**Axiom 2'** A property implied by a positive property is positive:

$$\forall \varphi \forall \psi [(P(\varphi) \wedge \forall x [\varphi(x) \supset \psi(x)]) \supset P(\psi)]$$

**Axiom 3** The property of being God-like is positive:<sup>a</sup>  $P(G)$

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<sup>a</sup> Instead of postulating  $P(G)$  one may again use the more general axiom  $\forall Z.(Pos(Z) \rightarrow \forall X.(X \cap Z \rightarrow PX))$  (stating that the conjunction of any collection of positive properties is positive) and infer  $P(G)$  from it.

Figure 2. Simplified set of axioms as studied by Benzmüller [9].

been mentioned by Christian [14, p. 9] and by Czermak [15, p. 5], and which was later modified by Świątorzecka and Łyczak [33]. Generally, albeit discovered independently, Figure 2 can be seen as a combination of the approaches by Christian [14, p. 9] and Essler [16, pp. 309–319], who first presented a non-modal version of Gödel’s ontological proof. As demonstrated by Benzmüller [9], the possible and necessary existence of a God-like entity possessing all positive properties is still provable from the depicted reduced set of axioms using essentially the same argumentation steps as in Scott’s or Gödel’s proofs. However, analogous to our observations as discussed above, the elimination of these axioms leads to a generalization where additional models of the premises are now enabled in some of which the maximality criterion of an ultrafilter is no longer satisfied. And as above, these additional models are apparently in conflict with what Gödel, in the tradition of Leibniz, originally had in mind when trying to model his concept of a supreme entity, God. While the additional axioms are thus not needed for proving existence (of a generalized structure), they are required, however, to adequately characterize Gödel’s notion of God. Gödel introduced the restriction to the necessary positive properties so that different positive properties are not mutually exclusive and are compatible with each other [35, p. 118]. In addition, the change in A2 means that not only the properties that are necessarily implied by positive properties are positive, but also those that are implied without necessity become positive. However, from the standpoint of a metaphysical interpretation not captured by the logical formalisation above: if existence is taken to be the fundamental positive property and the distinction between necessarily positive and

merely positive properties is eliminated — as in the modification of A2 introduced above — then depending on this standpoint, every other property would also be positive. In that case, there could be no genuinely negative properties, since possessing any property presupposes existence. Gödel's distinction between the necessary and the possible makes no further distinction between the necessary and the sufficient. We therefore read necessary here as a direct cause and thus as sufficient. In the case of existence, a property can still be negative, because its negativeness does not necessarily follow from existence, but possibly from something else. Being negative also requires existence, but only indirectly. The direct causality of being negative can follow from something else.

The problem of modal collapse must now be explained in more detail, as this was the primary motivation for the modifications. The reason for the criticism of modal collapse is that it is concluded to cause a determinism that excludes free will. However, determinism only excludes free will if it is itself limited, i.e. if it assumes a limited and thus predetermined basic quantity (of circumstances or possibilities). However, if absolute infinity is assumed, then determinism also remains open due to the openness of the basic quantity. In other words, since the maximum is conceived through absolute infinity due to the actual existence of all times and possibilities, determinism and free will are not mutually exclusive (since they are already existent as possibilities). This view of Gödel also becomes clear from a conversation between Gödel and Rudy Rucker [see in detail: 29, p. 168]:

Gödel seemed to believe that not only is the future already there, but worse, that it is, in principle, possible to predict completely the actions of some given person. [...] ‘There is no contradiction between free will and knowing in advance precisely what one will do. If one knows oneself completely then this is the situation. One does not deliberately do the opposite of what one wants.’

In this context, he concluded with regard to logic: “[...] In the same way, time-travel is possible, but no person will ever manage to kill his past self. The *a priori* is greatly neglected. Logic is very powerful.” [Gödel according to: 29, p. 168] If we consider these statements with respect to Gödel's world view mentioned above, the world is not seen as limited, from which a limited determinism would follow. World and mathematics belong together and the supreme principle is Ackermann's principle —  $V$  is unattainable — or in the other formulation: the foundation is the

primordial monad of absolute infinity, from which follows an unlimited determinism that also contains free will.

Furthermore, we see that Gödel's ultrafilter, in contrast to the filters used in the modified proofs, uses an open concept of set. His used set of positive properties is left open, it is not conceived as a point continuum, since the concept of existence functions arbitrarily for all possible objects in the homogeneous continuum. The boundaries of objects and properties are considered only as contours within the homogeneous continuum, generated by our intuition, and not as actual boundaries in the continuum. This means that clear boundaries of objects and properties remain a projection in the continuum and the continuum remains as a medium for this projection in the filter. This is not the case in the modified models, because the restrictions in the filter replace the medium with the projections, creating fixed objects and properties and thus a continuum of points. Even if we cannot assume that Gödel's handling of this homogeneous set was explicitly about introducing a new concept of set, he nevertheless used this concept effectively.

An interesting open question is whether Gödel's proof can eventually be transferred to intuitionistic logic, whereby a core challenge is to prove the possible existence of God while avoiding the use of an *reductio ad absurdum* argument [22]. One may argue that the use of the open concept of set is not influenced by the logic used, since the concept of set, that is used for the basic quantity on which the filter is supposed to function, precedes any logic. However, further work is required to settle this question.<sup>7</sup>

In addition to avoiding modal collapse, some of the proof modifications were also motivated by the desire to show that a 'God-like entity' can be proved with fewer assumptions. By considering less restrictive conditions, the space of possible models expands to the set of generalized models, some of which now violate Gödel's apparent intention regarding maximality, as discussed earlier. While the existence of such an entity

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<sup>7</sup> From the intuitionistic or proof-theoretical perspective, it could also be objected here that there is no pre-existence of concepts (thus also not of sets) since from this perspective all concepts should only appear afterwards from the formal construction. In this perspective, only formalism exists at the beginning. However, the very idea of the formal-logical ontological proof for the existence of God rests, as we see it now, precisely in the connection between ontology and formal logic, which have a previously defined concept of God, and thus also a concept of world or reality as their object of investigation, which makes these previous concepts necessary.

remains provable, the entire theistic theory changes in a way that can thus be criticized.

A possible synthesis between the motivation of the modified proofs and Gödel's intention of modal collapse is perhaps the one already noted above: assuming modal collapse for a finite world provides a limiting determinism for which the modified proofs have shown that it can be avoided and that God also exists in two worlds. This means that, on the one hand, the modified proofs provide a counter-argument for a limited world, but on the other hand, this assumption of a limited world is not compatible with God as a maximum anyway, since every limitation also represents a deficiency in the maximum, or in God. Perhaps the error lies only in the assumption that the world is limited. Those who want to prevent modal collapse, because they assume that the one world intended by Gödel is ‚determining too small‘ create models that produce multiple worlds by modifying the filters on the basic quantity of positive properties. Precisely because of this — in the opposite —, however, a smaller determinate world is created, because the new filters cannot map the full continuous power of the basic quantity. Thus, if we aim to prove God as the absolute maximum, such limitations must be removed, and maximality and absolute infinity must be both possible and required (in all models considered). This in turn gives us *one* — absolutely infinite — world, i.e. with modal collapse, but without limited determinism, because it is open and unlimited. In other words, we can also break this down into two possible cases:

1. One limited world is assumed, which does not correspond to the maximum: then adding another limited world to this may be regarded as a larger situation, and may avoid modal collapse, but — with respect to the absolute maximum — is just as limited as a single limited world.
2. If a world is absolutely infinite: then a model containing two (limited) worlds is necessarily smaller, since absolute infinity admits only one world as the absolute maximum that contains everything. Consequently, the restrictions arising from proof modifications within the basic quantity of positive properties establish God only with respect to a non-maximal subset. Such a ‚God‘ cannot be the true God in Gödel's sense, who is possible only in the single world of absolute infinity.

From this we can conclude that the maximum of positive properties inevitably results in a modal collapse, as required by Gödel — for an absolutely infinite world.

To adequately capture this intended notion of an absolutely infinite world in formal logic remains a challenge for further work, in particular concerning the tension between a set-theoretic perspective, a type-theoretic perspective as adopted in existing formalisations and the meta-physical interpretation of both.

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