



Sanderson Molick 

Lakatos's Methodology and its Application to Logic Research Programs: A Comparative Study

Abstract. Lakatos's (1978) methodology of research programs is renowned for describing the evolution of scientific theories through a combination of heuristic mechanisms that guide their development. Various attempts to apply Lakatos's framework to research programs in logic and mathematics are found in (Priest, 1989; Aberdein and Read, 2009; Hallett, 1979a). The paper draws on Lakatosian assessments of progressive and degenerating mathematical theories found in (Hallett, 1979a) to advance a heuristic proposal for studying the progress and development of logical theories. The upshot is a characterization of Lakatos's framework as applied to logical theories, along with a defense of a holistic approach that acknowledges the iterative relationship between empirical input and logical refinement during the process of development of progressive logical theories.

Keywords: philosophy of logic; research programs; Imre Lakatos; logical disputes

1. Introduction

Modern philosophy of science is marked by the effort to develop adequate theoretical tools for understanding the process of growth of scientific theories. Distinct authors like Popper (2005), Lakatos (2014), and Kuhn (1962) have provided influential conceptual frameworks for understanding the heuristic mechanisms that drive the development of scientific theories. Among the prominent frameworks proposed, Lakatos's methodology of research programs (MSRP) stands out for its structured approach to understanding the progress and demise of scientific theories. This paper aims to extend Lakatos's conceptual scheme to research programs in logic by addressing the problem of what appropriate desiderata

can characterize progressive/degenerative logic research programs in the absence of empirical data, as found in the physical sciences.

Different facets of the problem of applying Lakatos's ideas to logic were explored by various authors. For example, Lakatos's MSRP scheme is employed by [Priest \(1989\)](#) to argue that the classical program is undergoing a degenerating stage. Furthermore, [Aberdein and Read \(2009\)](#) develop a historical reconstruction of some non-classical research programs in logic to analyze issues such as the adequate notion of logical data and the problem of logical revisability. In contrast to the approach developed by Aberdein & Read, this paper proceeds by comparing logical theories with mathematical theories, focusing on the different desiderata employed in the evaluation of progressive mathematical research programs.

For this, the paper draws on Hallett's study of the progress of mathematical theories (see, e.g., [Hallett, 1979a,b](#)) to develop a comparative analysis between the desiderata used for the appraisal of mathematical theories and their application to logical theories. The upshot is a defense of a holistic perspective that emphasizes the interplay between novel empirical intuition and technical refinement through problem-solving activity as central features of progressive research programs. By examining case studies of intuitionistic and paraconsistent programs, we demonstrate how these cycles foster the development of the heuristic mechanisms that characterize Lakatos's MSRP.

The paper is structured as follows: Section 2 introduces Lakatos's framework and its criteria for testing the progress of scientific theories. Section 3 explores how Lakatos's framework can be applied to the evaluation of mathematical theories. Two schools of thought are compared: the first considers empirical intuitions as the central indicator of progress for mathematical theories, while the second emphasizes problem-solving ability. Section 4 characterizes Lakatos's conceptual scheme as applied to logic. Section 5 presents the heuristic reconstruction proposed in the paper, which suggests that progressive research programs in logic alternate between stages of empirical intuition injection and problem-solving activity. This view is illustrated by the historical development of two central research programs in contemporary logic: intuitionistic logic and paraconsistent logic. Section 6 discusses key features of the holistic approach and the challenges faced by any Lakatosian reconstruction of the progress of logical theories. Finally, the paper concludes with final remarks in Section 7.

2. Lakatos on progressive theories

Imre Lakatos's framework for the Methodology of Research Programmes (MSRP), introduced in (Lakatos, 1978), provides a theoretical basis for understanding the process of growth and theory-change in the sciences. According to Lakatos, a research program is said to be **theoretically progressive** if each theory has greater empirical content than its predecessor. Similarly, it is said to be **empirically progressive** if some of the new predictions are confirmed. A research program is then considered **progressive** if it is both theoretically and empirically progressive. Otherwise, it is called a **degenerating** research program.

According to this view, progressive theories are characterized by their ability to increase content, where the new content must explain new scientific facts. Thus, for a new theory to handle the recalcitrant data of the old theory, it must offer a content-increasing view on the body of evidence ignored or not adequately handled by its predecessor. Degenerating programs, on the other hand, only handle recalcitrant data through linguistic reinterpretations, considered by Lakatos as mere ad hoc moves. In (2014), Lakatos summarizes the desiderata for characterizing a progressive theory as follows:

DEFINITION 1 (Lakatos (2014)). Given two theories T and T' , the theory T is said to be falsified by T' iff

- (i) T' has excess empirical content over T ;
- (ii) T' explains the previous success of T ;
- (iii) Some of the excess content of T' is confirmed.

The application of the above criteria for the historical reconstruction of successful theories in scientific fields such as physics, chemistry, or economics is found in various authors (see Hettema, 1995; Backhouse, 2006) with a good understanding of the heuristic mechanisms that drive the theory choice dynamic behind successful research programs.

For scientific fields in which there is no evident analogue for empirical content, the application of Lakatos's criteria is more problematic. This is the case of logic and mathematics. In these fields, comparative criteria such as excess empirical content or the confirmation of novel predictions are not as clear-cut as in the empirical sciences. As a consequence, the problem of finding adequate criteria for understanding the growth of mathematical theories is debated by authors such as Oliveri (2006) and Hallett (1979a).

In (Hallett, 1979a,b), Hallett introduces an influential analysis of Lakatos's framework as a way of understanding the heuristic progress of mathematical theories and the kinds of desiderata historically regarded as hallmarks of progress for mathematical research programs. According to Hallett, two opposing traditions can be identified in the evaluation of mathematical research programs. In what follows, we introduce both traditions and their criteria for assessing progress in mathematics. Afterwards, we examine Hallett's desideratum on the progress of mathematical theories in the context of logical theories.

3. Progress in mathematics: two divergent roads

As discussed in the previous section, any attempt to employ Lakatos's conceptual framework to non-empirical sciences must first identify an appropriate set of criterion for the progress of logical theories. For this, I start by reviewing two traditions in the context of mathematical theories. These traditions are discussed by Hallett (1979a,b), where he develops an application of Lakatos's MSRP to the evaluation of mathematical theories.

According to Hallett, the evaluation of mathematical theories is divided into two opposing camps: the *empiricist* tradition and the *problem-solving* tradition. Each of these traditions defended distinct sets of desiderata for the evaluation of mathematical theories. While the empiricist tradition regarded the injection of empirical intuition as a necessary epistemic requirement for the progress of mathematical theories, the problem-solving tradition considered both fruitfulness and the ability to solve problems as appropriate indicators of progressive mathematical theories.

The authors of the empiricist tradition regarded the development of abstract results as a key symptom of degenerating research programs, and the steps toward greater levels of abstraction were viewed as a form of detachment from the original mathematical intuitions. In contrast, the authors from the problem-solving tradition argued that a theory's ability to solve old problems and open new paths of exploration are more important than relying on empirical intuitions. In sum, while the empiricist tradition regarded abstraction as a distinct sign of stagnation for a mathematical theory, the problem-solving tradition viewed abstraction

as a crucial condition for proving novel results and fruitfully bridging different areas of mathematics.

In what follows, I will introduce how the empiricist and the problem-solving desiderata have been employed as standards for evaluating mathematical theories. Subsequently, Hallett's (1979a) own criterion of progress for mathematical theories is presented along with his defense of problem-solving as a core sign of theoretical fruitfulness.

3.1. The empiricist desideratum

The empiricist view on the progress of mathematical theories is rooted in *mathematical empiricism*, a philosophical position that recognized the *a posteriori* nature of mathematical truths. Although not unified by a unique set of theses, a central assumption of the empiricist tradition in the philosophy of mathematics is that experience, broadly construed, plays a pivotal role in mathematical knowledge. Some proponents of this tradition emphasize the role of empirical experience, while others focus on the dynamic and revisable nature of mathematical theories. According to their view, empirical experience is broadly understood as different applications of mathematics, from inductive reasoning to physical interpretations of mathematical theories.

The roots of this perspective were also explored by Lakatos (1976), where he introduced the notion of *quasi-empiricism* in mathematics. However, rather than endorsing a strictly empiricist view, Lakatos argued that mathematical knowledge is fallible due to the dialectical process of proofs and refutations. He mentions authors like Hermann Weyl, László Kalmár, and John von Neumann as the forerunners of this philosophical tradition, in contrast to the traditionally held aprioristic views on mathematical knowledge.

Influenced by Brouwer's intuitionism, mathematicians like László Kalmár and Hermann Weyl challenged the dominant formalist view of mathematics endorsed by Hilbert's program. For instance, Weyl regarded intuition rather than proof as the primary foundation of mathematical knowledge (see Bell and Korté, 2025). Similarly, Kalmár (1967) argued against viewing mathematics as a pure deductive science, a view also shared by Lakatos. Kalmár defended the role of empirical knowledge as a core component of the progress of mathematical theories. According to him, this view opposed the formalist ideal, which erroneously consid-

ered proof as the only and ultimate source of justification for mathematical practice (see [Kalmár, 1967](#)).

As for the criterion of progress for mathematical theories, [Hallett \(1979a\)](#) argues that we encounter in [von Neumann \(1984\)](#) an explicit exposition of the empiricist ideal, according to which the absence of an empirical source may be indicative of a degenerating stage of a mathematical theory. This is remarked by the author along the following lines:

As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired by ideas coming from ‘reality’ it is beset with very grave dangers. It becomes more and more purely aestheticizing, more and more purely *l’art pour l’art*. ([von Neumann, 1984](#), p. 10)

The passage above highlights a fundamental belief in von Neumann’s criterion for progressive theories in mathematics. Even though von Neumann acknowledges the great advance of abstract areas such as modern algebra or real analysis, their inability to find empirical applications remains symptomatic of “art pour art”, i.e, a theoretical activity completely detached from their primary (empirical) concern.

In ([von Neumann, 1984](#)), von Neumann defends the injection of empirical ideas as an often necessary resource to reverse great periods of stagnation in a mathematical theory. Accordingly, purely formal mathematics was considered unfruitful and in need of empirical intuitions to produce fresh ideas. For Hallett, the rise of Hilbert’s program was a key influence on the development of significant opposition, among the authors of the problem-solving tradition, to the kind of view promoted by von Neumann.

3.2. The problem-solving desideratum

Various authors in the philosophy of science have advocated for problem-solving as the central heuristic mechanism driving the development of scientific theories (see [Laudan, 1977](#); [Kuhn, 1962](#)). For these authors, the shift to a new theory is not only determined by its capacity to elaborate novel predictions but rather by its ability to solve problems not adequately addressed by the old theory.

In mathematics, the capacity to solve old problems has always been considered a distinctive feature of an emergent theory. In ([Lakatos, 1964](#)), Lakatos explores how problem-solving may function as a suitable criterion for the progress of mathematical theories, provided the

proposed solutions are not dependent upon ad-hoc maneuvers. This is because ad-hoc moves allow the formulation of new theories designed to handle specific problems without any power of generalization. Lakatos (1964) provides examples of distinct ad-hoc maneuvers employed in the development of Euler's formula $V - E + F = 2$ for three-dimensional polyhedra. These ad-hoc moves were capable of solving specific cases of the theory without providing any general theory about the objects investigated. Hence, problem-solving alone was not sufficient for the progress of a mathematical theory.

According to Hallett (1979a), an explicit defense of problem-solving as an adequate desideratum for the evaluation of mathematical theories was first presented by Hilbert along the following lines:

The acid test of a new theory is its ability to solve problems which, though known for a long time, the theory was not expressly designed to solve. The maxim 'By their fruits ye shall know them' applies also to theories. (Hilbert, 1926, p. 200)

Hilbert's requirement for the success of a new theory is crucial not only for avoiding mere ad-hoc maneuvers but also for demanding the new theory to shed light over old problems. According to Hilbert, the ability to address problems for which the theory was not originally designed is a strong indication of its fruitfulness. In (Hilbert, 1926), the author mentions how Cantor's theory of transfinite numbers illuminated old problems of real analysis and provided novel insights into other branches of mathematics.

In Hallett's view, the problem-solving criterion is a more suitable measure of progress for mathematical theories since many of them have experienced great stages of development without relying on empirical intuition. These are theories that have undergone intense stages of abstract development and proved results with fruitful consequences for other branches of mathematics.

According to the problem-solving tradition, progressive mathematical theories trigger novel stages of development by offering fresh insights into other branches of mathematics or by aiding in the resolution of long-standing problems. Thus, even if mathematical principles originate from empirical intuition, periods of abstract developments do not necessarily equate to stagnation within the research program.

Despite advocating for the problem-solving desideratum, Hallett acknowledges shortcomings in Hilbert's criterion, such as the fact that a

theory may supply only a partial solution to a problem or, differently, it may solve a problem that emerged only after its formulation. As a result, Hallett proposes a modified version of Hilbert's requirement. He suggests interpreting it as a minimal requirement for progressive theories and introduces a modified version as follows:

DEFINITION 2 (Hallett's criterion). A new mathematical theory T_2 constitutes progress with respect to its predecessor T_1 iff T_2 is used in at least one problem P which T_1 did not solve, provided that P is not of T_2 's own making and that the statement solving P was not used in the construction of T_2 .

To avoid theories that arise from ad-hoc maneuvers, Hallett's formulation imposes a new condition by which the new theory cannot employ the statement that solves the problem P in its very construction. Thus, akin to Hilbert and Lakatos, Hallett aims to exclude cases where the proposed solution for a mathematical problem is tailored only to specific instances rather than constructing a general solution.

In the subsequent section, I will examine how the empiricist and the problem-solving criterion serve as guides for measuring the progress of logical theories. However, unlike Hallett's perspective on the progress of mathematical theories, I argue that neither empirical intuition nor problem-solving alone suffice as desiderata for assessing the progress of logical theories. Instead, the resulting position advocates for a holistic perspective, according to which progressive logical theories alternate between stages of empirical intuition and technical refinement through problem-solving activity.

4. Logical theories through lakatosian lenses

In this section, I detail the methodology used to apply Lakatos's MSRP to research programs in logic. This involves adapting Lakatos's conceptual scheme to the unique features of logical theories. Different facets of the problem of applying Lakatos's ideas to logic were explored by other authors. For example, Lakatos's MSRP scheme is employed by [Priest \(1989\)](#) to argue that the classical program is undergoing a degenerating stage. Furthermore, [Aberdein and Read \(2009\)](#) developed a historical reconstruction of some non-classical research programs in logic to analyze issues such as the problem of revisability in logic, and which heuristic framework best represents logical disagreements.

This paper focuses on one particular facet of the problem of applying Lakatos's ideas to logic that has not been explored by the aforementioned authors: determining the suitable criteria for characterizing progressive logical theories. Unlike Priest, the purpose of the heuristic model constructed in the paper is not to resolve logical disputes, but rather to enable the historical reconstruction of progressive programs. By establishing criteria for progressiveness and degeneracy in logical research programs, the paper offers a systematic approach to modeling the development of logical theories. Before introducing the holistic view proposed in this paper, I first provide a characterization of Lakatos's conceptual scheme as applied to logical theories.

4.1. Characterization

Lakatos's methodology of research programs is defined by four central components¹: 1) the hard-core, usually defined as the central tenets of the theory, which considered irrefutable hypotheses; 2) the positive heuristics: the set of concepts, predictions, and tools responsible for driving the development of the theory; 3) the protective belt: the set of auxiliary constraints responsible for absorbing anomalous data; and 4) the negative heuristics: the set of constraints that works in conjunction with the protective belt to protect the hardcore. The components of a research program, while applied to logic, may be characterized as follows:

1. **Hard core:** The hard core consists of foundational principles for a class of logics. The central inferences that characterize the intended notion of validity along with core philosophical assumptions. Elements of the hard core are the axioms of formal systems and other core principles such as bivalence, constructivity, and so on.

2. **Positive heuristics:** The positive heuristic is responsible for driving the development and expansion of the program. It involves the creation of new logical systems, proof techniques, and suitable syntactical and semantical constraints that align with the tenets established by the hard core.

3. **Negative heuristics:** The negative heuristic includes criteria for rejecting or revising existing theories or systems developed according to the tenets of the hard core. For instance, if a logical system is found to be inconsistent or unable to capture certain semantical intuition, it may be rejected or revised.

¹ The reader may check ([Chalmers, 2013](#)) for an introductory presentation.

4. **Protective belt:** The protective belt consists of auxiliary hypotheses, supporting arguments, and applications that defend the core of the program. Different strategies may be applied by the protective belt in response to new challenges without abandoning the core principles. For example, modifications to inference rules, the formulation of translation schema, or semantical alterations aimed at enhancing the expressive/inferential power of a theory may be implemented in response to theoretical objections or the discovery of anomalous data.²

Overall, these elements shape a research program in logic by establishing its foundational principles, guiding its explorations, setting criteria for evaluation and revision, as well as providing theoretical tools to defend the hard core against challenges. Due to the dynamics of scientific discovery, research programs develop through a cycle of revision and expansion. While some elements of the hard core may drive expansion through the positive heuristic, new elements can be incorporated to the hard core through the discovery of novel results. For example, the intuitionist program was initiated by a set of philosophical theses proposed by Brouwer. However, it was only after the development of the positive heuristics through the BHK-semantic clauses that new requirements related to constructivity were incorporated into the hard core of the program.

5. Progress in logic: a holistic view

This section introduces a heuristic model that integrates both key facets considered hallmarks of progress for mathematical theories: the injection of empirical intuition and the development through problem-solving pursuit. According to the view defended in this paper, progressive logical theories alternate between iterative cycles of empirical intuition injection and refinement through problem-solving activity.

I begin by characterizing what most appropriately qualifies as *empirical intuition injection* in the context of logical theories. While the empiricist tradition has often emphasized desiderata such as the physical applicability of mathematical theories, logical theories admit a broader range of applications, from philosophical frameworks to mathematical

² A more detailed analysis of how these strategies are employed by research programs in logic can be found in (Aberdein and Read, 2009; Haack, 1978). A similar characterization of these elements is also found in (Priest, 1989).

formalisms. Consequently, the empirical intuition—understood here as *practical application*—that informs the development of logical theories may arise from various domains where logical practice is involved³. Unlike mathematical theories, where problem-solving often addresses external applications, problem-solving within logical theories typically targets issues internal to the theory's development. Common examples include devising an appropriate semantic interpretation for a given logical system, formulating suitable axiomatizations, constructing proof-theoretical tools, and solving similar demands.

With these considerations in mind, I propose a generalized form of Hallett's problem-solving criterion. However, unlike Hallett, the purpose is not to apply it as the single relevant criterion for the progress of a logical theory. As I shall demonstrate, fruitful problem-solving must be accompanied by novel semantical intuition.⁴

DEFINITION 3 (Generalized Hallett's criterion). A new logical theory T_2 constitutes progress with respect to its predecessor T_1 if T_2 is used in at least one problem P which T_1 did not solve, and if T_2 introduces novel semantic intuition regarding the relevant logical vocabulary.

Although problem-solving may be considered an important component of the progress of a logical theory, Hallett's criterion is not sufficient for testing logical theories. Non-classical theories often incorporate the problem P (which they intend to solve) directly into their formulation. For example, most non-classical logics impose semantic/syntactic restrictions over the system to invalidate the principle in dispute with classical logic. This could be interpreted as an ad-hoc move by the classical advocate. Given the generality of classical logic, non-classical theories are bound to rediscover patterns of inference overlooked by classical logic.⁵

³ From this perspective, our reasoning practices provide empirically-informed material for the foundation of our logical theories. This view aligns with a practice-based account of logical concepts. See (Martin, 2023) for an empirically-informed account of logical practice or (Tahko, 2017) for a broader characterization of empirical intuition. In (Arenhart and Molick, 2025), the authors present an empirically-informed account of logical practice based on the interplay between *pure* and *applied* logic.

⁴ Given that semantical intuitions are obtained through logical practice, I shall use the terms 'semantical intuition' and 'empirical intuition' interchangeably.

⁵ Hallett also discusses mathematical theories that are too general, like ZFC, and so makes the novel theories bound to grow on the backbone of these established theories.

Thus, their only possible path of development is by producing novel intuitions regarding patterns of inference neglected by classical logic. These inferences shall become characteristic principles for the new notion of validity that aims to compete with classical logic.

The generality of classical logic positions non-classical theories as sub-theories of classical theories, rendering them unable to produce novel facts. As a result, if Hallett's condition is accepted as an adequate criterion for the progress of logical theories, it will cut too deep, ruling out a great share of the non-classical program. Only contra-classical theories, which validate arguments not accepted by classical logic, may be considered good candidates for progressive logical theories since they are not based on the failure of classical principles.

Our generalized form of Hallett's criterion seems to be more aligned with logical practice. A new logical theory constitutes progress as long as it provides novel intuition over its predecessor. These fresh intuitions should form the foundation for novel concepts over the relevant logical vocabulary. For example, novel intuitions about contradictory theories paved the way for non-classical conceptions of negation. Similarly, new insights into the relationship between premises and conclusions helped establish the grounds for the relevantist conception of validity. Whether the novel intuitions ought to be regarded as ad-hoc moves is left for debate by the logical community and the partisans of the program. Most non-classical theories fit our generalized Hallett's criterion since they are born out of a problem not adequately handled by classical theory and employ a novel interpretation of the relevant logical vocabulary. For example, in the case of relevant logics, the relevant vocabulary is interpreted according to constraints of relevance, and therefore gives rise to operation of intensional conjunction, negation, and similar operations.

Overall, Hallett's criterion shows that problem-solving alone is not sufficient for testing the progress of logical theories. Stages of problem-solving must be preceded by the injection of novel semantical intuitions. In the development of logical theories, these semantical intuitions, which establish the ground for the hard core, drive the research agenda to be pursued by the positive heuristic. In modern logic, this is evidenced by the birth of non-classical concepts, arisen from semantical intuitions neglected (or judged to be ill-formed) by classical theories.⁶ Non-classical

⁶ This view aligns with the tenets of logical anti-exceptionalism (Hjortland, 2017), according to which logical concepts may result from data discovered through vari-

theories depart from classical principles to address specific limitations or to accommodate alternative philosophical positions.

According to our heuristic model, while empirical intuition plays a crucial role in the initial development of a logical theory, a subsequent problem-solving stage is necessary to develop the necessary technical machinery capable of semantically representing the empirical source. This technical machinery can later find new applications in different domains, thereby broadening the scope of the theory. For example, this was the case with modal logic. Only after Kripke's possible world semantics were introduced did new results and applications emerge, contrasting with the previous Carnapian semantic formulation. Kripke's semantics not only triggered a novel stage of problem-solving activity but also provided new intuitions regarding old metaphysical problems. Thus, the injection of new semantic intuition paved the way for a productive stage of technical development.

To summarize: according to the view defended here, the progress of logical theories should be understood through a holistic perspective that considers the injection of empirical ideas and the technical development via problem-solving pursuit. While empirical intuition lays the groundwork for exploring different avenues in our reasoning practices, it is through problem-solving efforts that these intuitions are harmonized in the design of new logical systems.

In the following, I explore two case studies in the history of logic, namely, the development of intuitionistic and paraconsistent logic. I will argue that the evolution of these research programs involved an interplay between the injection of empirical ideas and problem-solving pursuit.⁷

5.1. Case study: the intuitionist program

The intuitionist program stands as an influential rival to the classical program. Initiated during the *foundational crisis*, the modern develop-

ous sources: from mathematical practice to philosophical ideas, and so on. Thus, non-classical theories are motivated by rediscovering chunks of data ignored or not adequately handled by classical theories.

⁷ The purpose of the case studies is to illustrate the role of the heuristic mechanisms of empirical intuition injection and problem-solving in the development of a research program. It is also important to highlight that the reconstruction to be presented concerns only the propositional part of those programs. Even though both programs had a high reach by fomenting novel research into mathematics, the question of how to apply the same heuristical reconstruction is out of scope of this paper.

ment of the non-classical program was largely influenced by the intuitionist program. The chief dispute between intuitionism and classical logic is centered on the notion of constructive proofs. A rational reconstruction of the intuitionist research program can be characterized by the following three stages: (i) an initial problem driven by novel philosophical intuitions over the nature of mathematical truths and its methods of proof; (ii) its positive and negative heuristics developed through problem-solving activity; and (iii) the injection of fresh intuitions over the notion of constructivity.

As agreed by historians such as [Troelstra and van Dalen \(2014\)](#), [van Atten \(2023\)](#), the initial stage of the intuitionist program is marked by Brouwer's philosophical concerns over the relation between mathematical proofs and the limits of our mental constructions. Previously, other mathematicians had shown similar concerns about constructive features of proofs under the influence of Hilbert's program. This is acknowledged by [Brouwer \(1952\)](#) himself by mentioning mathematicians like Poincaré, Borel and Lebesgue as the forerunners of the intuitionist school.⁸ However, it is Brouwer's philosophical theses that will establish the unrefutable part of the intuitionist program in its first stage of development.

While the classical program did not acknowledge the non-constructive character of reduction proofs, Brouwer was the first to explicitly investigate the role of non-constructive proof techniques in classical mathematics and their connection to the unrestricted validity of the excluded middle. Many of the philosophical tenets that will shape the intuitionistic program are already presented in Brouwer's PhD Thesis ([Brouwer, 1907](#)). According to ([van Atten, 2023](#)), these core tenets of the intuitionist program may be summarized along the following theses: (i) pure mathematics consists in the act of making specific kinds of mental constructions; (ii) the foundations of mental constructions are intuitions on the flow of time; (iii) the foundations of mathematics must be built solely on constructions allowed by intuition; (iv) logical reasoning mirrors mathematical activity; and (v) valid inferences must preserve constructibility. Finally, Brouwer's intuitionism introduced an explicit anti-realist view of mathematical objects, establishing the program's foundational commitments.

⁸ It is important to distinguish the history of intuitionistic logic from the broader history of intuitionistic mathematics. In this paper, we focus only on the former, particularly in its modern development through the influence of Brouwer.

As explained by [Heyting \(1966\)](#) and [Brouwer \(1952\)](#), the unrefutable part of the intuitionistic program is determined by the view according to which the program of investigating mental mathematical constructions requires the rejection of the principle of excluded middle, as classical mathematics allows the construction of objects for which no mental intuition is available. Consequently, from the intuitionist perspective, mathematics is characterized as “the study of certain functions of the mind” ([Heyting, 1966](#), p. 10). In his PhD thesis, Brouwer analyzed various mathematical proofs that relied on the law of excluded middle, including standard proofs of the Intermediate Value Theorem and the Weierstrass Approximation Theorem. As stated by him:

From the intuitionist standpoint the dogma of the universal validity of the principle of the excluded third in mathematics can only be considered as a phenomenon of history of civilization, of the same order as the former belief in the rationality of π or in the rotation of the firmament about the earth. ([Brouwer, 1952](#), p. 142)

As the philosophical tenets of the hard core of the program were solidified under the influence of Brouwer's views on classical mathematics, the positive heuristics of the program unfolded through the introduction of novel concepts central to the intuitionist program. Among the most significant are the interpretation of the negation of a proposition A as “assuming A leads to contradiction” the understanding of mathematical assertions as “I have constructed A in my mind”, and the view of mathematical existence as synonymous with “to be constructed” ([Heyting, 1966](#), p. 3). By this stage, the intuitionist program had already developed alternative interpretations of logical vocabulary, allowing it to depart from the principle of excluded middle and other central inferences of classical mathematics. The rejection of non-constructive proofs and the development of constructive proofs for fundamental mathematical results were established as central epistemic requirements for the program's positive heuristic.

Subsequently, the program entered its problem-solving stage by developing formal requirements capable of refining Brouwer's philosophical intuitions. In particular, Heyting's work in establishing methodological principles for intuitionistic logic's semantics played a crucial role in the program's development. As discussed by [Sundholm \(1983\)](#) and [Troelstra \(1990\)](#), the early history of intuitionistic logic is marked by Heyting's attempts to systematically interpret intuitionistic logic in terms of

constructive proofs rather than truth-values. As is well-known, a key development was the interpretation of intuitionistic negation as *refutability* rather than complementary falsity, a concept introduced by Heyting that became central to intuitionistic semantics.⁹

Throughout the problem-solving stage of the program, distinct researchers such as Kolmogorov, Glivenko, among others, contributed to the development of a suitable interpretation of intuitionistic logic and its subsystems, thus clarifying key concepts for the hard core of the program like strong negation, disjunction property, impredicativity, proof transformations, and so on.

Other key results played a central role for the negative heuristic of the program, such as the proof of Glivenko's double negation translation, and the development of intuitionistic arithmetic through Gödel's and Gentzen's work. All these results played a key role in replying to objections aimed at the expressive power of intuitionistic logic in comparison to classical logic. While Glivenko's theorem established that classical and intuitionistic logic are equiconsistent, Gödel sought to develop a provability intuitionistic calculus over classical concepts. One may summarize the key results responsible for the progress of the intuitionist program as follows¹⁰:

- Development of the BHK-interpretation of logical constants;
- Proof of Glivenko's theorem;
- Development of intuitionistic arithmetic and provability calculus through Gödel's and Kolmogorov's work;
- Development of the proof-theory in intuitionistic logic by Gentzen;
- Kripke's possible-world semantics for intuitionistic logic.

As described by the heuristic model proposed in this paper, the development of the intuitionist program occurred through the injection of new semantical intuitions acquired from mathematical practice and explored by Brouwer. These intuitions were subsequently formalized during the problem-solving stage of the program. Today, the intuitionist program continues to produce novel results in distinct areas of mathematics. For example, the Curry-Howard correspondence revealed a deep connection

⁹ For a detailed account of the genesis of the BHK interpretation, see (Troelstra, 1990). Other developments for the semantic played a key role in this stage, such as Beth's semantics.

¹⁰ The list is not intended to be exhaustive of all central results for the development of the program but rather to illustrate the development of key aspects.

between intuitionistic logic and type theory, thus establishing the foundation for the development of intuitionistic type theory and demonstrating the practical applicability of intuitionistic principles in computer science.

5.2. Case study: the paraconsistent program

Another central non-classical program in modern logic is the paraconsistent program. Paraconsistent logics are known to accommodate contradictions without leading to triviality. The philosophical intuitions that motivated the development of paraconsistent systems cast doubt on the truth of a central principle of classical logic, namely the principle of explosion. Similarly to the intuitionist program, a rational reconstruction of the paraconsistent program can be characterized by: (i) an initial problem driven by novel philosophical intuitions over the role of contradictions in mathematical practice; (ii) its positive and negative heuristic developed through problem-solving activity; and (iii) the injection of fresh intuitions regarding the concept of contradiction and its role in formal systems.

The pre-history of the paraconsistent program is demarcated by [Priest \(2007\)](#) as the attacks directed by various authors to the validity of the Law of Non-Contradiction. Philosophers including Wittgenstein, Meinong, and Hegel are cited as early proponents of arguments against the law. In addition, historians like [Gomes and D'Ottaviano \(2017\)](#) trace the roots of the program even further back to sources in Ancient Greek and Medieval thought.

The initial stage of development of the paraconsistent program is marked by the injection of novel philosophical intuition motivated by different arguments, ranging from the discovery of different kinds of paradoxes (like the semantic and set-theoretic paradoxes) to considerations over the adequate relation between premises and conclusion of a given argument. As discussed by different authors (see [da Costa et al., 2007](#); [Priest et al., 2022](#); [Gomes and D'Ottaviano, 2017](#)), authors like Popper, Vasiliev, and Lukasiewicz, provided distinct motivations for the development of logical theories capable of accommodating contradictory theses. However, it is through the works of S. Jaskowski and N. da Costa that the problem-solving stage of the paraconsistent program is initiated, with efforts focused on developing adequate logical systems capable of handling contradictory theories without leading to trivialization.

The rejection of the principle of explosion (also called *ex contradictione quodlibet*) constitute the core tenet of the paraconsistent program. According to the principle of explosion, validated by classical logic, any formula is derivable from a contradiction ($A, \neg A \vdash B$). Thus, a central methodological requirement of the positive heuristic of the program is to develop theoretical tools capable of defusing the “explosive” character of classical negation. The different approaches to fulfilling this requirement have given rise to diverse schools of paraconsistency, such as the inconsistency-adaptive logics school, which developed systems based on failures of monotonicity, the Polish school of paraconsistency, which developed paraconsistent systems based on modal logic, and the Brazilian school, which developed systems based on consistency operators.

Similarly to the intuitionist program, the negative heuristics of the paraconsistent program demanded theoretical tools to counterbalance the lost of strength caused by the failure of explosion. This gave rise to theoretical innovations such as different kinds of recapture results, novel formulations of naive set theory, as well as additional developments in paraconsistent truth-theories. The following results can be highlighted as landmarks in the development of the paraconsistent program.

- Vasiliev’s imaginary semantics;
- Jaskowski’s formulation of the logic $D2$;
- da Costa’s first-order axiomatization;
- Asenjo’s three-valued calculus;
- Priest’s defense of dialetheism.

The same dynamic of development is found in other research programs central to contemporary logic, such as non-monotonic logics, many-valued logics and, more recently, substructural logics. All these programs are motivated by a rediscovery of patterns of inference neglected by classical logic. Hence they explore philosophical intuitions that challenge the justification of some classical concepts.

Even though the development of the paraconsistent program was intertwined with considerations motivated by logicians working in the relevant logic tradition, the landmarks mentioned here for the historical development of paraconsistent logics sufficiently demonstrate how the interplay between the injection of empirical intuition and problem-solving pursuit was crucial for establishing the paraconsistent research program as a central contribution within the non-classical program.

While the initial problem P considered by the paraconsistent program was classical logic ineffective way of handling contradictions by conflating inconsistency and triviality, the intuitionistic program focused on classical logic's inability to produce constructive proofs. Both research programs examined in this section illustrated cases in which novel semantic intuition over the logical constants were developed in response to specific requirements for invalidating classical principles. Whereas the intuitionistic program rejected the law of excluded middle, the paraconsistent program rejected the principle of explosion. As the history of these programs demonstrates, such requirements led to the development of novel formal techniques during their problem-solving stages.

6. Lakatosian reconstructions in logic and some limitations

The examples explored in the previous section serve to illustrate how the iterative process of refining logical theories in response to novel empirical intuition is crucial for their progress. This involves modifying auxiliary hypotheses through the syntactical and semantical apparatus to match the theoretical requirements of the hard core.

As defended by some authors from the empiricist tradition, mathematical concepts originate from empirical intuitions that need to be abstracted to obtain an accurate formal description (Kalmár, 2011; von Neumann, 1984). According to this view, the development of logical theories occurs in conformity with the empiricist desideratum, as the development of non-classical concepts originates from intuitions contrary to those held by classical concepts. Logical concepts emerge from empirical intuitions discovered through various domains of our experience of the world. However, contrary to what is defended by the authors in the empiricist tradition, periods of abstract development do not represent the stagnation of a logical theory.

Examples of this kind show that neither empirical intuitions nor problem-solving activity alone are sufficient to describe the growth of logical theories. Logical theories may not need the injection of novel intuition until the period of problem-solving activity has solved some of their central problems. In the case of modal logics, after the establishment of central proof-theoretical and model-theoretical results for modal systems, fresh intuitions over Kripke's semantics allowed new applications of modal systems in a wide range of scientific fields (see van Benthem et al., 2010; Blackburn et al., 2001).

According to the holistic view developed in this paper, degenerating research programs are characterized by a set of features that preclude the program from progressing through the iterative cycles of empirical intuition injection and theoretical refinement through problem-solving. These central features include a lack of novel intuitions over the logical vocabulary, the inability to solve the problems posed by the hard core, over-reliance on ad-hoc maneuvers, and the inability to find new applications across different areas of research. Research programs so characterized are gradually abandoned by the scientific community due to their unfruitfulness.

Following Lakatos, Graham Priest also suggests in (Priest, 1989) that degenerating programs are characterized by their inability to produce novel predictions as well as novel intuitions or formal results. They are bound to constantly produce auxiliary moves by the protective belt as a way to absorb anomalies. Hence, Priest's view offers a characterization of degenerative programs in line with the one defended here: degenerating programs neither produce novel semantic applications nor engender new formal results.

According to Priest, this is the current stage of the classical program. Not only can the classical program not produce novel facts and predictions, but it also fails to generate important new formal results. During the 20th century, the program reached its peak in development by providing all the central metatheoretic results for contemporary logic. However, this progressive stage has finished. Its future is now limited to defensive moves by the negative heuristics while novel logics are discovered by continually challenging its former predictions. The author mentions how most philosophical projects based on classical logic end up failing, such as logicism, Wittgenstein's atomism and Carnap's constructionism. As remarked by the author "[...] classical logic has ceased to be an important source of insights into logic." (p. 136).

This view was later opposed by Williamson (2018). According to the author, the success of the classical program is explained by its extensive application in mathematics. Classical concepts have been rigorously tested in contemporary logic and have consistently achieved a high degree of success. Moreover, all instances of recalcitrant data explored by non-classical theories are adequately handled by the protective belt of classical theory. As for the mathematical practice, classical concepts remain widely used in the metatheory of most non-classical theories.

This sort of dispute demonstrates how the heuristic mechanisms employed by logic research programs might struggle with ambiguity when deciding between a progressive or a degenerative research program. [Aberdein and Read \(2009\)](#) argue that the same body of logical evidence can be used to motivate conservative reforms within an existing research program, as well as to justify a deviating reform in a novel research program. The Liar Paradox is cited by the authors as an example of how the same set of data can be used to motivate conservative revisions within classical logic as well as deviations toward non-classical truth-theories. Thus, in accordance with Lakatos's conceptual framework, logical evidence can be used by the negative heuristic in a conservative program and, concurrently, by the positive heuristic in a deviating program. As a result, logic research programs may face the challenge of deciding whether the available evidence represents a novel prediction by a rival theory or an anomaly to be managed by the protective belt.

Impasses of this kind are not new to applications of Lakatos's MSRP framework. [Lakatos \(1970\)](#) himself discusses how conflicting biases may infiltrate the rational reconstruction of a rival research program through his distinction between internal and external reconstructions. This seems to be the case in ([Priest, 1989](#)), where Priest defends the paraconsistent dialetheist program as the adequate surrogate for the classical program.

While the heuristic model proposed in this paper proved useful for comprehending the development of progressive and degenerating research programs in modern logic, it remains an open path for future exploration to investigate how it can be enhanced to handle the ambiguity problems mentioned above and thus shed new light on logical disputes.

7. Final remarks

The paper applied Lakatos's methodology for appraising the progress of scientific research programs to the field of logic. A comparative study is conducted to examine how the desiderata used for evaluating mathematical theories can be applied to logical theories. Two contrasting traditions and their associated desiderata were analyzed in relation to logical theories. Due to the shortcomings of each desideratum when applied to logical theories, the paper proposes a dual perspective where progressive logical theories alternate between the injection of empirical intuition and formal refinement through problem-solving activity.

The development of two logical research programs is examined to illustrate the interplay between semantic intuition and problem-solving activity during the emergence and consolidation of a logic research program. Finally, the paper highlights how heuristic mechanisms in logic programs may face ambiguity when distinguishing between degenerating and progressive theories. This ambiguity poses a significant challenge to any Lakatosian assessment of logical theories. Additional challenges for lakatosian reconstructions, such as an adequate notion of logical data and excessive empirical content, or the distinction between internal/external reconstructions in logical debates, are left for future exploration.

Acknowledgments. The authors is thankful to an anonymous referee for suggestions regarding earlier versions of this paper.

References

- Aberdein, A. and S. Read. (2009). The philosophy of alternative logics. In L. Haaparanta (ed.), *The Development of Modern Logic*. Oxford Scholarship Online, chapter 14, pages 613–723. Oxford University Press. DOI: [10.1093/acprof:oso/9780195137316.003.0041](https://doi.org/10.1093/acprof:oso/9780195137316.003.0041)
- Arenhart, J. R. and S. Molick. (2025). Anti-exceptionalism about logic and the a priori/a posteriori divide. *Revista Perspectiva Filosófica*, 52: 22–52. DOI: [10.51359/2357-9986.2025.263122](https://doi.org/10.51359/2357-9986.2025.263122)
- Backhouse, R. E. (2006). *Explorations in Economic Methodology: From Lakatos to Empirical Philosophy of Science*. Routledge. DOI: [10.4324/9780203029978](https://doi.org/10.4324/9780203029978)
- Bell, J. L. and H. Korté. (2025). Hermann Weyl. In E. N. Zalta, and U. Nodelman (eds.), *The Stanford Encyclopedia of Philosophy*. Stanford University, Fall 2025 edition. <https://plato.stanford.edu/archives/spr2022/entries/logic-paraconsistent/>
- Blackburn, P., M. De Rijke, and Y. Venema, (2001). *Modal logic*, volume 53. Cambridge University Press. DOI: [10.1017/CB09781107050884](https://doi.org/10.1017/CB09781107050884)
- Brouwer, L. E. J. (1907). On the foundations of mathematics. *Collected works*, 1: 11–101. DOI: [10.1016/C2013-0-11893-4](https://doi.org/10.1016/C2013-0-11893-4)
- Brouwer, L. E. J. (1952). Historical background, principles and methods of intuitionism. *South African Journal of Science*, 49(3–4): 139.
- Chalmers, A. (2013). *What is this thing called science?* Hackett Publishing.

- da Costa, N. C., D. Krause, and O. Bueno. (2007). Paraconsistent logics and paraconsistency. In *Philosophy of Logic*, pages 791–911. Elsevier. DOI: [10.1016/B978-044451541-4/50023-3](https://doi.org/10.1016/B978-044451541-4/50023-3)
- Gomes, E. L., and I. M. L. D'Ottaviano, (2017). *Para além das Colunas de Hércules, uma história da paraconsistência: de Heráclito a Newton da Costa*. Editora da Unicamp.
- Haack, S. (1978). *Philosophy of Logics*. Cambridge University Press. DOI: [10.1017/CB09780511812866](https://doi.org/10.1017/CB09780511812866)
- Hallett, M. (1979a). Towards a theory of mathematical research programmes (i). *The British Journal for the Philosophy of Science*, 30(1): 1–25. DOI: [10.1093/bjps/30.1.1](https://doi.org/10.1093/bjps/30.1.1)
- Hallett, M. (1979b). Towards a theory of mathematical research programmes (ii). *The British Journal for the Philosophy of Science*, 30(2): 135–159. DOI: [10.1093/bjps/30.2.135](https://doi.org/10.1093/bjps/30.2.135)
- Hettema, H. (1995). Bohr's theory of the atom 1913–1923: A case study in the progress of scientific research programmes. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 26(3): 307–323. DOI: [10.1016/1355-2198\(95\)00018-6](https://doi.org/10.1016/1355-2198(95)00018-6)
- Heyting, A. (1966). *Intuitionism: an introduction*, volume 41. Elsevier. DOI: [10.1016/S0049-237X\(08\)71461-7](https://doi.org/10.1016/S0049-237X(08)71461-7)
- Hilbert, D. (1926). On the infinite. *Mathematische Annalen*, 95: 161–190. DOI: [10.1007/BF01206605](https://doi.org/10.1007/BF01206605)
- Hjortland, O. T. (2017). Anti-exceptionalism about logic. *Philosophical Studies*, 174: 631–658. DOI: [10.1007/s11098-016-0701-8](https://doi.org/10.1007/s11098-016-0701-8)
- Kalmár, L. (1967). Foundations of mathematics—whither now? In *Studies in Logic and the Foundations of Mathematics*, volume 47, pages 187–207. Elsevier.
- Kalmár, L. (2011). *The Development of Mathematical Rigor from Intuition to Axiomatic Method*. Springer. DOI: [10.1007/978-3-7091-0177-3_12](https://doi.org/10.1007/978-3-7091-0177-3_12)
- Kuhn, T. S. (1962). The structure of scientific revolutions. *The University of Chicago Press*, 2: 90.
- Lakatos, I. (1964). Proofs and refutations. *The British Journal for the Philosophy of Science*, 14(56): 296–342. DOI: [10.1093/bjps/XIV.56.296](https://doi.org/10.1093/bjps/XIV.56.296)
- Lakatos, I. (1970). History of science and its rational reconstructions. In *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, volume 1970, pages 91–136. Cambridge University Press. DOI: [10.1086/psaprocbienmeetp.1970.495757](https://doi.org/10.1086/psaprocbienmeetp.1970.495757)

- Lakatos, I. (1976). A renaissance of empiricism in the recent philosophy of mathematics. *The British Journal for the Philosophy of Science*, 27(3): 201–223. DOI: [10.1093/bjps/27.3.201](https://doi.org/10.1093/bjps/27.3.201)
- Lakatos, I. (1978). *The Methodology of Scientific Research Programmes: Ed by John Worrall and Gregory Currie*. Cambridge University Press. DOI: [10.1017/CB09780511621123](https://doi.org/10.1017/CB09780511621123)
- Lakatos, I. (2014). Falsification and the methodology of scientific research programmes. In *Philosophy, Science, and History*, pages 89–94. Routledge.
- Laudan, L. (1977). *Progress and its Problems: Towards a Theory of Scientific Growth*, volume 282. University of California Press.
- Martin, B. (2023). The philosophy of logical practice. *Examining Philosophy Itself*, pages 151–174. DOI: [10.1002/9781394160747.ch8](https://doi.org/10.1002/9781394160747.ch8)
- Oliveri, G. (2006). Mathematics as a quasi-empirical science. *Foundations of Science*, 11: 41–79. DOI: [10.1007/s10699-004-5912-3](https://doi.org/10.1007/s10699-004-5912-3)
- Popper, K. (2005). *The Logic of Scientific Discovery*. Routledge.
- Priest, G. (1989). Classical logic aufgehoben. *Paraconsistent Logic: Essays on the Inconsistent*. Munich: Philosophia Verlag, pages 131–150. DOI: [10.2307/j.ctv2x8v8c7.7](https://doi.org/10.2307/j.ctv2x8v8c7.7)
- Priest, G. (2007). Paraconsistency and dialetheism. In *Handbook of the History of Logic*, volume 8, pages 129–204. Elsevier. DOI: [10.1016/S1874-5857\(07\)80006-9](https://doi.org/10.1016/S1874-5857(07)80006-9)
- Priest, G., K. Tanaka, and Z. Weber. (2022). Paraconsistent Logic. In E.N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*. Stanford University, Spring 2022 edition. <https://plato.stanford.edu/archives/spr2022/entries/logic-paraconsistent/>
- Sundholm, G. (1983). Constructions, proofs and the meaning of logical constants. *Journal of Philosophical Logic*, 12: 151–172. DOI: [10.1007/BF00247187](https://doi.org/10.1007/BF00247187)
- Tahko, T.E. (2017). Empirically-informed modal rationalism. In F. Bob, and F. Leon (eds.), *Modal Epistemology after Rationalism*, Studies in Epistemology, Logic, Methodology, and Philosophy of Science, chapter 3, pages 29–45. Springer. DOI: [10.1007/978-3-319-44309-6_3](https://doi.org/10.1007/978-3-319-44309-6_3)
- Troelstra, A.S. (1990). On the early history of intuitionistic logic. In *Mathematical Logic*, pages 3–17. Springer. DOI: [10.1007/978-1-4613-0609-2_1](https://doi.org/10.1007/978-1-4613-0609-2_1)
- Troelstra, A.S., and D. van Dalen. (2014). *Constructivism in Mathematics*, Vol. 2. Elsevier.

- van Atten, M. (2023). The Development of Intuitionistic Logic. In E. N. Zalta and U. Nodelman (eds.), *The Stanford Encyclopedia of Philosophy*. Stanford University, Fall 2023 edition. <https://plato.stanford.edu/archives/fall2023/entries/intuitionistic-logic-development/>
- van Benthem, J. (2010). *Modal Logic for Open Minds*. No. 199 of series CSLI Lecture Notes Number 199. Stanford.
- von Neumann, J. (1984). The mathematician. *Heywood [Hey47]*.
- Williamson, T. (2018). Alternative logics and applied mathematics. *Philosophical Issues*, 28(1): 399–424. DOI: [10.1111/phis.12131](https://doi.org/10.1111/phis.12131)

SANDERSON MOLICK
Division of Humanities
Federal Institute of Pará, Brazil
smolicks@gmail.com
<https://orcid.org/0000-0003-2918-1055>