

Logic and Logical Philosophy Volume 34 (2025), 547–560 DOI: 10.12775/LLP.2025.004

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Is p and $\neg p$ a Contradiction?

L'existence de la contradiction n'est pas essentielle, elle est logicielle, elle n'est toutefois pas purement fictionelle et une bonne terminologie permet de bien l'encrer. Baron de Chambourcy

Abstract. We discuss how to formulate and understand contradiction. After emphasizing the importance of a correct formulation for a notion as important as the notion of contradiction, we present a variety of formulations of the proposition corresponding to "p and $\neg p$ ", which is often considered as expressing contradiction. We then discuss the standard example of contradiction in classical logic and the way Wittgenstein defines contradiction in the Tractatus, not using negation. After that, we point out the variety of connectives which are nowadays called negations and often denoted by the same symbol, underlying that negations obeying neither the law of noncontradiction, nor the excluded middle are considered part of the family. We then recall how contradiction is defined within the theory of oppositions, drawing attention to the fact that this theory is against considering that the pair p and $\neg p$ forms a contradiction if we take into account the whole family of negations, including paraconsistent negations. At the end of the journey, we set up a list of notions involving negation and contradiction and propose a terminology that may be useful to dispel confusion and promote understanding.

Keywords: contradiction; negation; symbolism; paraconsistent logic; Newton da Costa; antilogy; Wittgenstein; universal logic; square of opposition

1. Contradiction: what it is and how to name it

It is common nowadays to say that p and $\neg p$ is a contradiction. What does it mean exactly and is it correct?

From a scientific perspective, just because a majority of people think something is right does not mean it is. Science is not democratic (but it is open to all)¹ and an *Argumentum ad populum* is considered as a logical fallacy. However, the consensus of experts is important. It is difficult to explain how exactly such a consensus works, and we will not go here into the details of this very interesting question of philosophy of science. We can just say that scientific consensus is an *understanding* between experts at a certain time, but that it is continuously evolving.

Let us now remind the famous quote of Blaise Pascal, originally in French [39]: "Je ne dispute jamais du nom, pourvu qu'on m'avertisse du sens qu'on lui donne." The standard English translation is: "I never quarrel about a name, provided I am apprised of the sense in which it is understood". Independently of Pascal's own thought, this English version is better. A literal translation of the last part of the original quote in French would be "the sense which is given to it". Understanding is better than arbitrary assignment. This quote of Pascal can be interpreted as meaning that we cannot stay at the superficial level of the language: a few lines after, Pascal uses the expression "playing with words". It is better to avoid this kind of sophistic behavior, this gaming attitude. As Jean Delaveine likes to say: "Si les mathématiques n'étaient qu'un jeu, je préférerais jouer à la poupée."

The articulation between thought, language and reality is very important. Using the right word for the right thing at the right time is the beginning of wisdom. The "right time" is crucial because language is not a fixed phenomenon, the meaning of the words is changing. Moreover, it would be better to say "using the right sign for the right thing at the right time", because here both "¬" and "contradiction" are involved.

Hartley Slater wrote [42]: "If we called what is now 'red', 'blue', and vice versa, would that show that pillar boxes are blue, and the sea is red? Surely the facts wouldn't change, only the mode of expression of them." This comment is about concrete reality, but we can say the same about

¹ It is like an airplane: everybody is welcome on board but, thanks to God, the pilot is not elected by the passengers.

swapping in mathematics (to call what is now '0', '1', and vice versa) or swapping in religion (to call what is now 'Devil', 'God', and vice versa).

This question "Is p and $\neg p$ a contradiction?" is important because contradiction is a central notion of philosophy, logic and mathematics. We should therefore be aware of how we are dealing with it: how we are naming it, defining it, understanding it.

2. Different ways to formulate p and $\neg p$

In the sentence "It is common nowadays to say that p and $\neg p$ form a contradiction", the word "nowadays" is indeed important because some years ago there was not the symbol " \neg ", nor symbols like " \forall , \exists , =, \rightarrow , \vdash " in logic. Logic was not symbolic. The symbolization of logic has many aspects. Modern logic has been symbolized in different ways by Boole, Venn, Peirce and Frege.

Symbolization can be good but it also can be dangerous. It is the case when the symbol turns to be more important than its meaning, and when the meaning becomes highly ambiguous. This takes place in religion and also in politics, in particular with flags. The most striking example is the Nazi flag, based on a religious symbol.

But it is not because of such an extreme case that we should reject symbolism, we should avoid throwing the baby out with the bath water. One of the most famous symbols, that of the balance for justice, example chosen by Saussure [30] to emphasize the dichotomy between arbitrary signs and symbols, is fairly good. Symbolic pictograms are also very useful for traffic rules [see 31]. And what about symbols in science?

A symbol can help to fix an idea but we have to keep in mind that the meaning of a symbol is neither precise, nor completely determined. This is the case even in mathematics, for example with the symbol "1". It is not because we are writing "1" that we exactly know what we are talking about, this symbol is used to denote many different things [see 16].

Instead of writing "p and $\neg p$ ", we could have written other things, as described by Table 1.

The first line (1) is the expression we have chosen. One may prefer to put some quotes (2). This is not our choice because we want to go beyond the surface of the language. Someone who thinks that sentences are tastier than propositions would prefer the expression of line (3). A conceptual girl may prefer to use the letter "x" (4), and this for two rea-

1	$p \text{ and } \neg p$
2	" p and $\neg p$ "
3	$s \text{ and } \neg s$
4	$x \text{ and } \neg x$
5	$p \text{ and } \neg p$
6	$p \wedge \neg p$
7	p and not p
8	$\{p, \neg p\}$
9	the pair p and not p

Table 1.

sons: on the one hand to promote a higher level of abstraction (including facts, information, data), on the other hand to emphasize that this is a variable. A straight guy may prefer not to use italics (5), but the use of italics is a common practice in mathematics for variables (and constants). One may want to get more formal, using a symbol for the conjunction (6) or a set-theoretical notation (8), or less formal, not using a symbol for negation (7), or using a set-theoretical framework but without heavy symbolism (9), following Paul Halmos (the author of *Naïve set theory* [35]) motto: "The best notation is no notation" [36].

What we want to focus on is the *Bedeutung* and the expression "p and $\neg p$ " (or alternatively "the pair p and $\neg p$ ") provides a good access to this *Bedeutung* through the *Sinn* it is conveying [cf. 33]. And when we consider a formula like $p \land \neg p$ a good reason not to put quotes is because we want to talk about its meaning, similarly as when we write $0 \times 4 = 0$.

It is important to emphasize that the use of the symbol "¬" is here ambiguous because it can be interpreted in different ways. Modern logic has taught us the relativization of signs in mathematics, even of a constant like "4", and we have to also keep this in mind for logic itself.

3. Contradiction, negation and antilogy

Independently of all the ways to formulate "p and $\neg p$ ", when we say that p and $\neg p$ is a contradiction, the notion of contradiction here presented depends on the notion of negation, or to speak in a lighter way, involves negation, by contrast to Wittgenstein's approach in the Tractatus.

If we have a clear and precise definition of negation, then no problem. For example, we can define negation as follows:

 $\neg p$ is false if and only if p is true.

This is symbolically expressed by the famous truth table:

p	$\neg p$
0	1
1	0

This is what is called "classical negation". According to this classical definition, a contradiction is always false. This is a particular case of an *antilogy*, the opposite of the notion of *tautology*, a proposition which is always true, a *logical truth*.

But does it make sense to call any antilogy, a contradiction? This would be the same as calling any star, a sun, or any neutral element a one, or, to stay at the logical level, to call any tautology, an excluded middle. The excluded middle, p or $\neg p$, is always true but this is a particular case of tautology. In classical logic, there are many other cases of tautologies, and also there are many other cases of antilogies, other than contradiction, for example $\neg(p \to p)$. In fact, in classical logic, any negation of a tautology is an antilogy. And vice versa, tautology and antilogy are twin antagonist sisters.

The guy who is guilty of having identified contradiction with antilogy is Ludwig Wittgenstein. In the *Tractatus Logico-Philosophicus* [43, 4.46.], he introduced and defined the notion of tautology, using the word "tautology" and at the same time defines the notion of antilogy using the word "contradiction". ("Kontradiktion" in German²). The name "tautology" is used in 4.46 for a proposition which is always true and the name "contradiction" for something which is always false. This terminology unfortunately lacks symmetry both by the expressions which are used and their semiotic aspects. Better terminologies are the pairs: tautology/antilogy, logical truth/logical falsehood.

Wittgenstein defines *contradiction* (antilogy) in the same way that he defines *tautology*, independently of any specific connective or formula. Wittgenstein's definition of contradiction (antilogy) is negationless. This goes towards the general *theory of bivaluations* developed by Newton da Costa [28] and *universal logic* developed by the present author [4], as

² Here Wittgenstein does not use the most common word for contradiction in German which is "Widerspruch", even in traditional logic, where the principle of non-contradiction is called "Das Prinzip vom Widerspruch".

a general theory of logic systems. In this perspective, one defines for example the notion of maximal non-trivial theory as a theory which is not trivial and which has no non strict non-trivial extension, a definition which is more general than the one of maximal consistent theory depending on negation.

4. The variety of negations

There is nowadays a consensus among logicians to consider that there is a variety of negations, not equivalent to each other, classical negation being the most famous one. The second most famous one is intuitionistic negation.

The law of excluded middle, that can be expressed as $p \vee \neg p$, is not valid in intuitionistic logic, i.e. $p \vee \neg p$ is not a tautology in this logic. Intuitionistic logic is not the only logic in which this law is not valid. The expression "paracomplete logic" has been coined by Quesada and da Costa to call this family of logics, as well as the expression "paracomplete negation" to denote a negation not obeying the law of excluded middle. Considering the general theory of bivaluations developed by Newton da Costa [see 28, 37], a paracomplete negation is such that there is a proposition p which can be false together with its paracomplete negation $\neg p$.

The paracomplete framework is dual to the paraconsistent framework, the paraconsistent terminology having also been introduced also by Quesada and da Costa [see 41]. A paraconsistent negation is such that there is a proposition p which can be true together with its paraconsistent negation $\neg p$. A paraconsistent logic is then defined as a logic where there is a paraconsistent negation.

Furthermore, Quesada and da Costa introduced the notion of *non-alethic negation*, a negation that is at the same time paraconsistent and paracomplete. And a *non-alethic logic* is a logic in which there is such a negation.⁴

³ This theory provides a general framework to define paraconsistent and paracomplete negations from an intuitive semantical point of view. It is quite general and applies to many-valued logics, considering their corresponding bivalent semantics, for example Łukasiewicz's three-valued logic [see 5].

⁴ The terminology "paranormal" was proposed by the author of the present paper as an alternative to "non-alethic", on the one hand to uniform the terminology using the same prefix "para", on the other hand because "alethic" is used in the context of

One can wonder if it still makes sense to speak of a negation in this case. One of the reasons to defend this position is the most famous non-alethic negation, which is De Morgan negation, for which hold all De Morgan laws as well as the law of double negation [see, e.g., 10]:

- $\neg (p \land q) \dashv \vdash \neg p \lor \neg q$
- $\neg (p \lor q) \dashv \vdash \neg p \land \neg q$
- \bullet $\neg \neg p \dashv \vdash p$

The fact that the same symbol, " \neg ", and the same name, "negation", is used for all these negations is in some sense similar to the fact that the same symbol "0" is used for a variety of numbers which are different. But all these zeros have at least one common property, they nullify. A similar phenomenon takes place with the relation of order, denoted with the same sign "<", despite the fact that there are contradictory theories of order, for example partial order vs. total order.⁵

In the case of negation, the situation is different because there is not a common property valid for all these negations, an essential feature. The two basic properties of classical negation, non-contradiction and excluded middle, are both rejected by some members of the family of negations. But the core of negation cannot be based on De Morgan laws, because some operators not obeying all of them are rightly considered as negations. One may want to use negative properties [see, e.g., 1]:

- $p \nvdash \neg p$
- $\neg p \nvdash p$

But, on the one hand, a negative property is more negative than a positive one, and on the other hand, the second negative property is against the famous Freudian notion of denegation [34], describing a common phenomenon: someone saying "I don't like pizzas", as a disguised way to say: "I adore pizzas".

modal logic with a different meaning. This terminology was first introduced in [29], presenting a paranormal logic called "Overclassical logic".

⁵ We are talking here about strict order, but the same can be said about "equal order". Another way to present things would be to say that the relation of order (defined by the axioms of transitivity and antisymmetry) has different contradictory extensions.

⁶ This was already pointed out in the author's paper "What is paraconsistent logic?" [6], corresponding to his talk at the *First World Congress on Paraconsistency* which took place in Ghent, Belgium in 1997. Also, in this paper, the author emphasized that it does make sense to have a purely negative definition of paraconsistent negation, for example the rejection of the so-called principle of explosion. For more recent work about that, see [4].

Nevertheless, it makes sense to call all these negations "negations", and to use the same symbol for all the members of this family, "¬", as it is currently done, due to a certain strength of all these connectives, which still has to be characterized.

However, to still call "contradiction" the pair p and $\neg p$, even when the two can be true is a different story, another kettle of fish, a whole different ballgame. And there are good reasons not to do so.

5. Square contradiction

We have quoted in section 1, Slater saying:

If we called what is now 'red', 'blue', and vice versa, would that show that pillar boxes are blue, and the sea is red? Surely the facts wouldn't change, only the mode of expression of them. [42]

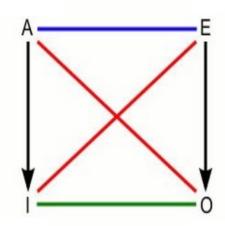
This was as an appetizer, the rest of the quote is:

Likewise, if we called 'subcontraries', 'contradictories', would that show that 'it's not red' and 'it's not blue' were contradictories? Surely the same point holds. And that point shows that there is no 'paraconsistent' logic. [42]

Even if we can criticize Slater considering that what is happening here is not just a swap, and that his argument is not a fatal blow to paraconsistency [see 9], it remains that "contradictory" is used in the theory of oppositions as follows: "Two propositions are *contradictory* iff they can neither be false, nor true together."

According to this use of the word, p and $\neg p$ are not contradictory, do not form a contradiction, when \neg is a paraconsistent negation. The theory of oppositions however does not exclude paraconsistent negations; on the contrary, it presents an opposition, subcontariety, which corresponds to one type of paraconsistent negation, and subcontariety is placed within the family of oppositions, in the same way that paraconsistent negation is placed within the family of negations. This is a good reason not to speak of contradiction for p and $\neg p$ in the case of a paraconsistent negation. It is better to speak of subcontrariety (when it is the case).

In the theory of oppositions, we have three notions of oppositions and to these three notions correspond classical, paracomplete, paraconsistent negations [see 8]. Let us remember here the general figure of the square of opposition, the corresponding concepts and definitions:



- In red, contradictories: cannot both be true and cannot both be false.
- In blue, contraries: can both be false but cannot both be true.
- In green, subcontraries: can both be true but cannot both be false.

In black we have subalternation, which is not an opposition, but which helps to structure the relations between these three oppositions. Note that these three notions of opposition correspond to *relations*. A contradiction can be defined as a pair of contradictory propositions. The theory of oppositions applied to proposition and also concepts, we can in particular embed a concept in a proposition.

From this point of view, it is clear that one of the most cited examples of contradiction, a round circle, is indeed not a contradiction, but a contrariety, because, besides circles and squares, there are other animals, like triangles ... [13]. This simply shows that a distinction which was made 2,500 years ago by Aristotle is not implemented in ordinary thought and also not in ordinary language where "contraries" is most often used as synonymous with "contradictories" [14].

The claim that p and $\neg p$ form a contradiction in paraconsistent logic is, according to the theory of oppositions, a similar (or better, a dual) mistake as claiming that a round square is a contradiction.⁷

⁷ Graham Priest [40] has called p and $\neg p$ a true contradiction in case of paraconsistent negation, but from the point of view of the theory of oppositions, funnily enough, we can say that when we have a paraconsistent negation, this is a false contradiction [see 15].

It is important to point out that none of these definitions involve negation, but they can be applied to the theory of negation.⁸ These definitions are generated by the dichotomy truth and falsity, one dichotomy generating three oppositions.⁹

Heloise can ask: why sticking to the archaic terminology of the theory of oppositions? To which Abelard can reply: why changing this terminology? The theory of oppositions has not been rejected or challenged in modern logic. It is good to remember that Frege, considered as one of the fathers of modern logic, presented the square of opposition at the end of his *Begriffsschrift* in 1879 [32], to justify his theory of quantification.

The theory has been expanded and improved in particular with the hexagon of opposition, promoted by Robert Blanché ([27], and also see [11]), but without changing the definitions of the three oppositions (for recent works concerning this theory, see [12, 19–26]).

6. Suggested terminology

Considering what has been explained up to now, it is clear that to say

p and $\neg p$ is a contradiction

is not necessarily a good way of speaking.

Here is the general perspective (facts and definitions) we can have about contradiction, related notions and the corresponding terminology:

- An *antilogy*: a proposition which is always¹⁰ false (a tautology: a proposition which is always true).
- A contradiction: a pair of propositions that can neither be true, nor false together.

⁸ It is possible for example to define the principle of explosion, using the theory of oppositions, without explicitly introducing a negation, see [2].

⁹ One may want to argue that truth and falsity do not form a dichotomy. And this was indeed the position of Aristotle, if we consider the question of future contingents and the way it was formalized by Łukasiewicz [38] using a three-valued logic. But, funnily enough, the theory of oppositions can be applied to the three-valued approach, considering that in this case the truth values form a triangle of subcontrariety. Note also that the theory of oppositions can be formulated avoiding to use the notions of truth and falsity, using concepts such exhaustivity and exclusion.

[&]quot;Always" is a way of speaking, we can alternatively say: according to all truth-possibilities (original terminology in the Tractatus), all valuations, all possible worlds, all models.

- The symbolic example of contradiction is the pair p and $\neg p$ when " \neg " is a classical negation.
- When we have a negation such that p and $\neg p$ can both be true, case of paraconsistent negation, this pair does not form a contradiction, at best a subcontariety.
- The pair p and $\neg p$, for any negation, is a pair of opposites.

It is important to emphasize that what is said here is not to defend any ideology or philosophical position but to avoid linguistic illusion, to develop clarity and promote understanding.

Acknowledgements. I would like to thank Alexandre Costa-Leite, Llyod Humberstone and Jens Lemanski, as well as two anonymous referees, for their useful comments. I presented ideas of this paper in many lectures in different countries around the world, and the feedback of the participants was important.

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