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ZF-Class Nominalism and the Küng-Armstrong Trilemma. A Plea for Moderate Ineffabilism

Abstract. This paper will examine the Küng-Armstrong trilemma against Class Nominalism. We will see that combining Class Nominalism and Zermelo-Fraenkel set theory (ZF) can provide us with a sophisticated version of Class Nominalism, namely ZF-Class Nominalism, which successfully addresses the objection and leads to a moderate version of ineffabilism about the putative set-membership relation.

Keywords: class nominalism; ineffabilism; properties; set theory

1. Introduction

Class Nominalism reinterprets the essence of medieval nominalism in a unique manner, revitalizing it through the lens of set theory. According to the canonical formulation of the Class Nominalism, property-talk (and relation-talk) must be analyzed in terms of set-membership. So, given a particular, *a*,

> *a*'s having a property, F, should be analyzed as *a*'s being a member of a certain class of things, the class of Fs. (Armstrong, 1978a, p. 28)

Or, equivalently,

> for x to be F is for x to belong to the class of Fs.
> (Garrett, 2003, p. 44)

> [P]roperties are classes (or sets) of things, and so the property of *being red* is the class of all and only red things.
> (Guigon and Rodriguez-Pereyra, 2015, p. 7)

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An “occasional assumption” in vogue among anti-realists and “pervasive tendency” (Armstrong, 1978a, p. 15) in first-order logic, Class Nominalism has rarely been an explicitly defended theory throughout the history of philosophy. However, *pauci sed boni*, it has been brilliantly put forward by W. V. O. Quine (who advocated its classical version), and by D. K. Lewis (who combined Class Nominalism and Modal Realism), as the following passages attest:

> A language adequate to science in general can presumably be formed on this nucleus by annexing an indefinite number of empirical predicates. For this entire language the only ontology required — the only range of values for the variables of quantification — consists of concrete individuals of some sort or other, plus all classes of such entities, plus all classes formed from the thus supplemented totality of entities, and so on. (Quine, 1966, p. 201)

> I do not say that they [viz. universals] are indispensable. The services they render could be matched using resources that are Nominalistic in letter, if perhaps not in spirit. [...] [M]y own usual ontology [...] consists of *possibilia* — particular, individual things, some of which comprise our actual world and others of which are unactualised — together with the iterative hierarchy of classes built up from them. Thus I already have at my disposal a theory of properties as classes of *possibilia*. (Lewis, 1983, p. 343)

More recently Busse (2016) has expressed sympathy with this general perspective by proposing a defense of Class Nominalism from Wolterstorff (1970)’s argument to the effect that properties cannot be identical to classes, while Blumson (2019) has put forward a novel proposal, Convex Class Nominalism, which brings together Resemblance Nominalism and Gärdenfors (2000)’s conception of natural properties as convex subsets of a conceptual space.

In this paper, the Class Nominalist’s theory of properties will be reconstructed (§§ 2–4), and the the Küng-Armstrong trilemma will be presented, according to which the class analysis is either incomplete, or circular, or vicious (§ 5). Drawing on the relationship between Class Nominalism and Zermelo-Fraenkel set theory (ZF) I will set out an answer to the trilemma (§ 6). Finally, we will see that although Class Nominalism does not seem to have any particular momentum in contemporary debate about the nature of properties, a ZF-based Class Nominalism allows us to shed some light on the nature of set-membership, and can provide the ground for a moderate form of ineffabilism (§§ 7–10).

2. Preliminary remarks

For one thing, what we will call here the "Küng-Armstrong trilemma" is an argument explicitly formulated by Armstrong (1978a) exploiting and enhancing an insight found in (Küng, 1967). Although Küng did not provide an explicit case against Class Nominalism, we will still call the objection the "Küng-Armstrong trilemma" to emphasize Küng's indirect contribution to Armstrong's argument. This explains why the following analysis will be mainly concerned with the reconstruction of Armstrong's argument.

The Küng-Armstrong trilemma moves from the Class Nominalist's main thesis, which in turn pivots on three fundamental notions: *class*, *set-membership* ($\in$), and *properties*. Some preliminary remarks about them are in order.

2.1. Classes and sets

What exactly is a Class Nominalist's *class*? The very designation "Class Nominalism" comes from (Armstrong, 1978a)'s influential taxonomy of the varieties of Nominalism, within which "class" and "set" were used interchangeably, as outlined by Armstrong in a later work:

> Nowadays mathematicians and logicians distinguish between sets and classes. All sets are classes, but not all classes are sets. The classes that are not sets behave in a special, disorderly, way. However, the classes that we will be concerned with are quite properly behaved. So why do I not speak of Set Nominalism? The answer is simple enough. The work done on the problem of Universals to which I shall be referring largely antedates the introduction of the class/set distinction and uses the word "class". It has therefore seemed convenient to continue to use the older word. (Armstrong, 1989, pp. 8–9)

For the time being, in referring to the Class Nominalism illustrated by Armstrong, it will not be essential to distinguish the notions of *class* and *set*, but when we move on to illustrate the solution to the trilemma considered here (§§ 6–7), it will be crucial to distinguish these notions clearly.

2.2. Set-membership

As Armstrong noted:

> Class Nominalism must employ one two-place predicate "___$\in$___", that is "being a member of". (Armstrong, 1978a, p. 42)

Thus, concerning the notion introduced by the set-theoretic expression "$\in$", the Class Nominalist uses it in its usual standard meaning, in line with what can be learned from the first pages of any good set theory textbook, as illustrated by the following examples:

> We write "$t \in A$" to say that t is a member of A, and we write "$t \notin A$" to say that t is not member of A. (Enderton, 1977, p. 1)

> Apart from the equality predicate $=$, the language of set theory consists of the binary predicate $\in$, the *membership relation*. (Jech, 2002, p. 4)

> The language of the theory of collections has in it as a non-logical primitive a binary relation symbol "$\in$". The formula "$x \in y$" is read "x belongs to y". (Potter, 2004, p. 23)

It is worth noting at the outset something we will have to return to later (§ 7): as much as these three remarks on "$\in$" may seem equivalent, they differ in the way they spell out its meaning. In the first passage, the meaning of "$\in$" is illustrated by exhibiting an object-language formula that uses it as a dyadic predicative constant ("$t \in A$") and by translating that formula into ordinary English using a two-place predicate in place of "$\in$" ("t is a member of A"). The second passage indicates the meaning of the object-language symbol "$\in$" by naming, in the metalanguage, a relation, namely the *set-membership relation*. Finally, in the third passage it is specified that "$\in$" is a symbol that refers to a relation, the same relation to which the predicate "is a member of" refers. The three extracts thus propose two different styles for fixing the meaning of "$\in$": in the first we have a fixing by elucidation, while in the second and third we have peculiar cases of fixing by ostension. This crucial distinction will be recalled below.

2.3. Properties

When it comes to the question about the notion of *property*, cutting a long story short, Armstrong typically insists that Class Nominalist must accept that when a predicate is true of something, it attributes a property to its subject. But this answer raises more questions than it answers: Why is it necessary to admit properties as entities attributed by predicates? What is the nature of properties? Why should a Class Nominalist admit the existence of properties? Does this not sound like a *petitio principii* in favor of Realism? To answer these questions, we must focus on Armstrong's theory of properties. We shall do this in the next section.

3. The Correspondence-rule

In Armstrong's theory of properties the truth of a subject-predicate sentence requires that its predicate attributes a property to its subject. The following analysis schema, called the "Correspondence-rule" (Armstrong, 1978b, p. 21), highlights precisely this: given a particular, a, and a predicate, "is F"

CORRESPONDENCE-RULE a is $F \Leftrightarrow a$ instantiates F-ness (or the property of *being* F).

Armstrong pointed out that the Correspondence-rule is appealing in that it is part and parcel of our ordinary inferential practice:

> There is one sense in which everybody agrees that particulars have properties and stand in relations to other particulars. The piece of paper before me is a particular. It is white, so it has a property. It rests upon a table, so it is related to another particular. Such gross facts are not, or should not be, in dispute between Nominalists and Realists. (Armstrong, 1978a, p. 11)

With reference to Armstrong's example, if this paper is white, then, by the Correspondence-rule, it has the property of *being white*, so it has a property. As Armstrong puts it, this is something anyone should agree to. And indeed all versions of Nominalism that Armstrong listed accept these "gross facts", but one: Ostrich Nominalism, the weakest type of Nominalism that, for this very reason, according to Armstrong, does not deserve special attention.[1] Rather, disagreement arises among non-ostrich-like theories when it comes to providing an analysis of what a property is and what it means to have, or instantiate, a property:

> What is in dispute [...] is the account or analysis to be given of the gross facts. This appears to be the situation in the dispute between Nominalism and Realism. (Armstrong, 1978a, p. 11)

Notice that this implies that recognizing the existence of properties does not automatically commit us to an ontology of universals. In fact, only Realists hold that properties (or at least some of them) are universals. But for many Nominalists the acknowledgment of the existence of properties as ontological *correlata* of predicates is perfectly consistent

[1] In (Calemi, 2016) I suggested that the particular view embraced by the Ostrich Nominalist turns out, upon careful analysis, to be far less weak than Armstrong believed.

with construing them as non-universal entities. That is why, in principle, demanding prior acceptance of the Correspondence-rule does not mean to beg the question against Nominalism.

In any case, as intuitive as it may seem, the Correspondence-rule rests upon some assumptions that must be examined to best understand the role it plays in the Küng-Armstrong trilemma.

At the core of the Correspondence-rule lies an assumption regarding the nominalizability of predicates (or general terms), along with an assumption about the semantics of nominalized predicates (or general terms). Usually, natural languages allow predicate nominalization via appropriate morphological modifications. In English, these are related to the use of suffixes such as "-ness", "-ity", "-hood", or to the use of a gerundive construction such as "the property of being ___" or simply "being ___": for example, the predicate "is red" could be nominalized in the forms

- "redness"
- "the property of *being red*"
- "*being red*"

In what follows, we shall use the following notation: if "P^n" is an n-place predicate, "$Nom(P^n)$" is a placeholder for its nominalization.

Correlatively, every nominalized predicate is an *abstract singular term*. An abstract singular term is a linguistic device that behaves like a name (or individual constant), and that purports to refer to a property. Moreover, the Correspondence-rule assumes that when a nominalized predicate involves a genuine reference, its reference is identical to the property attributed (or expressed) by its non-nominalized counterpart. Defining "$[\![\alpha]\!]$" as follows

$$[\![\alpha]\!] = \begin{cases} \text{the property attributed by } \alpha \text{ if } \alpha = P^n \\ \text{the property named by } \alpha \text{ if } \alpha = Nom(P^n) \end{cases}$$

we can formulate the Property identity principle, that expresses what is stated above:

PROPERTY IDENTITY $[\![P^n]\!] = [\![Nom(P^n)]\!]$.

So, according to the theory of properties we are examining, the truth of the subject-predicate discourse requires to posit properties along the following analysis schema:

ANALYSIS SCHEMA s is $P \Leftrightarrow \ulcorner s$ is $P\urcorner$ is true $\Leftrightarrow$ there is a particular, u, named by $\ulcorner s \urcorner$, there is the property $[\![Nom(P)]\!]$, and u instantiates $[\![Nom(P)]\!]$.

This of course applies not only to monadic predicates. To generalize the point, let "$\langle s\rangle_n$" be a finite sequence of individual constants, "$s_1, \ldots, s_n$"; let "P^n" be a n-adic predicate; let "*Ins*" stand for "instantiate":

$$P^n(\langle s\rangle_n) \iff \ulcorner P^n(\langle s\rangle_n)\urcorner \text{ is true}$$
$$\ulcorner P^n(\langle s\rangle_n)\urcorner \text{ is true} \iff Ins([\![\langle s\rangle_n]\!], [\![Nom(P^n)]\!])$$

From what we have seen so far, the assumptions underlying the Correspondence-rule are multiple, and not all can be regarded as "gross facts", at least not without further clarification. For example, according to (Schiffer, 1996), Correspondence-rule-like principles are "something-from-nothing" schemes in that they obtain ontologically-loaded sentences through mere linguistic transformations from ontologically neutral sentences. On the other side, Schnieder (2006) pointed out that on the sole basis that natural languages have processes that transform predicates into abstract singular terms "it should not be suggested that properties are *dependent* on language" (Schnieder, 2006, p. 329).[2]

In spite of possible disagreements on these particular issues, for the sake of the argument we will suppose that Armstrong's version of the Correspondence-rule is tenable.

4. From the Correspondence-rule to the Class analysis

Taking the Correspondence-rule at face value, it introduces properties into the discourse domain, for its right-hand side includes a property designator. But, as we have seen earlier, this does not mean that every application of the Correspondence-rule commits us to an ontology of universals. When given a predicate, "is F", the Realist will typically hold that it expresses a property, *being* F, and since all properties are universals, every true application of the predicate "is F" commits us to the universal F-ness. But the Class Nominalist does not share this conception of properties.

[2] In (Calemi, 2014) I argued that the realist can make explicit the ontological commitment of terms occupying the predicative position within sentences of the form "a is F" without the aid of morphological transformations.

Following the Class Nominalist's *credo*, one is not obliged to admit universals in order to explain what a property is. Indeed, for a Class Nominalist properties are sets and, therefore, instantiating a property is nothing but being a member of a set. This means that given a predicate, "is F", the following holds

PROPERTY IDENTITY* $[\![Nom(\text{is } F)]\!] = \{x \mid Fx\}$

that is, a nominalized predicate is nothing more than a device for naming a set. Accordingly, the two-place predicate "instantiates" is to be construed as a misleading linguistic expression of set-membership. So, according to the Class Nominalist, a's instantiating F-ness boils down to a's being a member of the Fs-set:

CLASS ANALYSIS a instantiates $[\![Nom(\text{is } F)]\!] \iff a \in \{x \mid Fx\}$.

In the end, the standard version of Class Nominalism incorporating the Correspondence-rule and the Class analysis, seems to account for the truth of the subject-predicate discourse without appealing to universals, but only to particulars and sets.

5. The Küng-Armstrong trilemma

The Küng-Armstrong trilemma moves from the Class Nominalist's tenets and aims to show that the Class analysis is either incomplete, or vicious, or infinitely regressive. Let us briefly recall the passages from Küng and Armstrong from which the objection can be reconstructed:

> Once the introduction of classes is allowed, relations can be defined. A diadic relation, for example, can then be defined as a class of ordered pairs, and an ordered pair $\langle a, b\rangle$ can be defined as a class of two classes: $\langle a, b\rangle := \{\{a, b\}, \{a\}\}$ [...] But even in such a system some undefined signs for relational functions remain, e.g., "$\in$", the sign for the membership function. (Küng, 1967, p. 28, n. 16)

> Although properties can be treated as classes of particulars, and relations as certain classes of classes of particulars, Class Nominalism must employ one two-place predicate "___$\in$___", that is "being a member of". But what corresponds to this predicate is a certain type of relation whose tokens are all those ordered pairs consisting of, first, a particular or particulars and second, all those classes of which these particulars are members. The Class Nominalist, however, is committed to giving a

> reductive analysis of all types in terms of particulars. Hence the Class Nominalist is forced to attempt a Class analysis of the class membership relation. (Armstrong, 1978a, p. 42)

The underlying assumption in Küng's passage seems to be that the non-analyzabilty of the predicate "$\in$" is a shortcoming of the Class analysis, that apparently should apply to any subject-predicate sentence. Armstrong, instead, emphasizes the other side of the coin: "$\in$" must somehow be analyzed. But in doing so, the Class analysis becomes either circular or vicious:

> If *being a member of* must also be analysed, it will be a matter of the ordered pair consisting of a and the class of Fs *being a member of* the class of all those ordered pairs which "stand in the relation of class-membership". But then a type-notion of *being a member of* has reappeared unanalysed in the analysis. Whether this is declared to be a new type (higher-order class-membership) or the very same type as before, it is clear that the attempt to remove unanalysed type-notions from the right-hand side of the analysis has failed.
> (Armstrong, 1978a, p. 42)

Let us consider more closely Armstrong's argument. Let us assume that a is F, and let us see what happens when the Class Nominalist tries to deliver her class analysis:

(1) a is F (assumption)
(2) a instantiates $[\![Nom(\text{is } F)]\!]$ (from (1) by Correspondence-rule)
(3) $a \in \{x \mid Fx\}$ (from (2) by Class analysis)

But "$\in$" is a predicate, a dyadic one in fact. Therefore, (3) has a predicative term, "$\in$". Either "$\in$" is a primitive and not analyzable predicate, or it is an analyzable one. In the first case, the Class analysis is incomplete: indeed on the basis of the Correspondence-rule, every predicate carries an ontological commitment and the task of the analysis is precisely to make this commitment explicit, that is to untangle the ontological contribution that each predicates makes in the statements in which it occurs. For this reason, in this perspective, leaving a predicate unanalyzed means producing an incomplete analysis, that is, an analysis that has not fully spelled out the ontological commitment of the statement to which it applies. In the second case, the Correspondence-rule allows one to infer

(4) a and the set $\{x \mid Fx\}$ instantiate $[\![Nom(\in)]\!]$, namely the $\in$-relation (from (3) by Correspondence-rule)

But the $\in$-relation, *qua* relation, has to be analyzed in class-terms, and since in set theory an n-adic relation is typically construed as a set of ordered n-uples, the dyadic set-membership relation must be looked upon as the set of all ordered pairs such that the first term belongs to the second:

$$\{\langle x, y\rangle \mid x \in y\}$$

thus

(5) $\langle a, \{x \mid Fx\}\rangle \in \{\langle x, y\rangle \mid x \in y\}$ (from (4) by the Class analysis)

Once again, (5) contains the predicative term "$\in$", so it must be analyzed in terms of relation-talk *via* the Correspondence-rule:

(6) $\langle a, \{x \mid Fx\}\rangle$ and $\{\langle x, y\rangle \mid x \in y\}$ instantiate the $\in$-relation$'$ (from (5) by the Correspondence-rule)

Now, either the $\in$-relation = the $\in$-relation$'$, or not. If the $\in$-relation = the $\in$-relation$'$, then the analysis is circular in that its *analysans* contains the *analysandum*. If the $\in$-relation $\neq$ the $\in$-relation$'$, then the analysis becomes vicious. Indeed, being committed to analyzing in class terms all properties and relations, the Class Nominalist must also apply the class analysis to (6):

(7) $\langle\langle a, \{x \mid Fx\}\rangle, \{\langle x, y\rangle \mid x \in y\}\rangle \in \{\langle\langle x, y\rangle, z\rangle \mid \langle x, y\rangle \in z\}$ (from (6) by the Class analysis)

But (7) contains, once again, a predicative terms, "$\in$", therefore, via the Correspondence-rule one can easily infer the following

(8) $\langle\langle\langle a, \{x \mid Fx\}\rangle, \{\langle x, y\rangle \mid x \in y\}\rangle, \{\langle\langle x, y\rangle, z\rangle \mid \langle x, y\rangle \in z\}\rangle$ instantiates the $\in$-relation$''$ (from 7 by the Correspondence-rule)

$\vdots$

In turn, either the $\in$-relation$'$ = the $\in$-relation$''$, or not. In the former case the analysis is circular, in the latter case the (vicious) regress continues. So, the Class analysis is incomplete, or vicious, or infinitely regressive.

In the next sections I will try to show that the Class Nominalist who adopts Zermelo-Fraenkel axiomatic set theory (ZF) can respond to the Küng-Armstrong trilemma. I shall call this kind of Nominalism *ZF-Class Nominalism*.

6. ZF-Class Nominalism

Let us recall (1)–(3):

(1) a is F (assumption)
(2) a instantiates F-ness (from (1) by the Correspondence-rule)
(3) $a \in \{x \mid Fx\}$ (from (2) by the Class analysis)

Is it legitimate to reapply the Correspondence-rule to (3)? The Küng-Armstrong trilemma presupposes that the Correspondence-rule should be applied across the board. Let us assume that this is the case. If it were legitimate to apply the Correspondence-rule to (3), then, as we have seen, there would exist the set, say C, of all the ordered couples such that their first element belongs to their second one:

$$C = \{\langle x, y\rangle \mid x \in y\}$$

Let us hypothetically assume that C exists. By two application of the Union axiom, according to which

UNION AXIOM For every set, X, there exists a set, denoted by $\bigcup X$ and called the union of X, whose elements are all the elements of the elements of X,

we can build from C

$$\bigcup\bigcup C$$

Since every set, X, is a member of its power set (whose elements are all the subsets of X), $\mathcal{P}(X)$, it follows that

$$\bigcup\bigcup C \in \mathcal{P}(\bigcup\bigcup C)$$

Taking into consideration that in ZF an ordered couple could be defined through Kuratowski's schema

$$\langle x, y\rangle := \{\{x\}, \{x, y\}\}$$

we have

$$\langle \bigcup\bigcup C, \mathcal{P}(\bigcup\bigcup C)\rangle = \{\{\bigcup\bigcup C\}, \{\bigcup\bigcup C, \mathcal{P}(\bigcup\bigcup C)\}\}$$

so, by definition of C

$$\{\{\bigcup\bigcup C\}, \{\bigcup\bigcup C, \mathcal{P}(\bigcup\bigcup C)\}\} \in C$$

Lastly, by two consecutive applications of the Union axiom we have, respectively,

$$\{\bigcup\bigcup C\} \in \bigcup C$$
$$\bigcup\bigcup C \in \bigcup\bigcup C$$

That is, if C existed, then there would exist a set, $\bigcup\bigcup C$, which belongs to itself. But ZF rules out the existence of self-contained sets because of the Foundation axiom,

FOUNDATION AXIOM Every non-empty set X contains an element such that no element of X belongs to it,

Therefore, C does not exist. The next section will highlight the consequences of this result for Class Nominalism.

7. Plea for a moderate ineffabilism

A serious Class Nominalist should take at face value the result of the previous section. Indeed, if by Property identity and Property identity*

$$[\![\in]\!] = [\![Nom(\in)]\!] = \{\langle x, y\rangle \mid x \in y\} = C$$

and if C does not exist, then "$\in$" is not a property-predicate; that is, it does not attribute the set-membership relation to its terms. But if C does not exist, and "$\in$" is not a property-predicate, the Correspondence-rule cannot be applied across the board; that is, an unrestricted version of the Correspondence-rule leads to postulating the existence of sets that cannot exist. This is the reason why we should take (3) as the ontological bedrock of the Class analysis: it cannot proceed further. At the same time, this does not mean that the analysis is incomplete: the analysis would have been incomplete if (i) the predicate "$\in$" had been a property-predicate and (ii) "$\in$" had been left unanalyzed. But since "$\in$" is not a property-predicate, leaving it unanalyzed does not entail the incompleteness of the analysis. Instead, it attests that the analysis has hit rock bottom, making explicit everything there was to make explicit: in (3) there is no hidden ontological commitment that needs to be further specified. This is tantamount to showing that the Correspondence-rule is flawed, at least in its unrestricted version. Finally, since the Class analysis cannot proceed further, it cannot be either circular or regressive.

It often happens in philosophy that some theses are stated in paradoxical form, either because they are such or, quite frequently, to better emphasize their underlying idea. Just to mention a few famous examples we could recall Kant's *dictum* according to which the predicate "exist(s)" is not a predicate, Frege's warning according to which the concept *horse* is not a concept, and proposition 6.522 of Wittgenstein's *Tractatus* which states that the propositions of the *Tractatus* are not propositions (i.e., they are pseudo-propositions):

> [it] *is obviously not a real predicate*, i.e., it is not a concept of anything that can be added to the concept of a thing
>
> (Kant, 1781, A593/B621, italics mine)

> It must indeed be recognized that here we are confronted with an awkwardness of language, which I admit cannot be avoided [...] the concept *horse* is not a concept (Frege, 1952, p. 46)

> My propositions serve as elucidation in the following way: anyone who understands me eventually recognizes them as nonsensical, when he has climbed out through them, on them, over them.
>
> (Wittgenstein, 1961, 6.522)

If by following these illustrious examples, we wanted to express the ZF-Class Nominalist's main thesis in paradoxical form, we would have to say the following: *set-membership cannot be named*.

This prohibition could be seen as introducing a form of *ineffabilism* into the ontological debate about properties, and in some ways it is but with caveats. Of course the ZF-Class Nominalist's answer to the Küng-Armstrong trilemma does not constitute a strong — and questionable if not self-defeating [3] — version of the *property-ineffability thesis*, that is, the view that there are ineffable/inexpressible properties (or relations): it is not the ZF-Class Nominalist's intention to argue that there is a relation, *namely* set-membership, which cannot be named as this would lead to a form of contradiction. ZF-Class Nominalism embeds rather a much more moderate ineffabilism — if we wish to continue to use this suggestive term —, formulated in the formal mode: if ZF defines the boundaries of the ZF-Class Nominalist's ontology, the predicate "$\in$" can be used in its usual ways but cannot be nominalized. The non-applicability of the Correspondence-rule to "$\in$" attests that there are

[3] See (Hofweber, 2016, 231) and (Jonas, 2016, pp. 60–71). See also (Priest, 2023) for a bite-the-bullet strategy to deal with contradictions arising from strong ineffabilism.

some predicates that, in the apparatus of the ZF-Class Nominalism, cannot have properties (or relations) as ontological *correlata* on pain of set-theoretical inconsistency. So, if a predicate cannot have a property as its ontological *correlatum*, and if the nominalization process implies that its nominalization is a genuine property-designator, then this leads to the affirmation that some predicates cannot be nominalized (on pain of set-theoretical inconsistency).

Coming to this point, three not negligible questions may arise:

(A) Why would a Class Nominalist choose ZF as the core theory and not, for example, another well-defined set theory that might instead admit the existence of collections broader than sets, such as Bernays-Gödel set theory (BG)?

(B) If "$\in$" is not a property-predicate, what does "$x \in y$" could possibly mean?

(C) As a matter of fact we often use nominalizations of "$\in$" and find it extremely convenient and functional to express ourselves in this way: does this view imply that what is written in most of the introductory textbooks about set theory is false?

7.1. Is ZF ontologically preferable over BG?

Concerning (A), it would not be easy to answer a question like this if it were understood in a generalized form; that is, if it were such as to ask why, in general, we should prefer a certain axiomatic set theory over the others available. However, we can make some observations limited to the case we are discussing by translating the original question into the following: *Is ZF ontologically preferable over BG?*

BG is a conservative extension of ZF (sse Mostowski, 1979) that permits the existence of collections larger than sets without encountering Russell's famous paradox. Time has come to distinguish between the notion of *proper class* and that of *set*. In general, a class is a collection of elements, so every set is a class. Moreover, a *proper class* is a collection that is not a set. In BG every set is a class but not every class is a set: e.g., the collection of all sets is a proper class.[4] In particular — which is

[4] In (Lewis, 1991)'s influential work, the distinction between classes and sets is more complex, as, according to Lewis's usage of the term "class," there are classes that are not sets (e.g., the class of all sets), sets that are not classes (e.g., the empty set), and sets that are classes.

what we are interested in — in BG axiomatic set theory, C exists as a proper class. Why one should prefer ZF over BG in providing a tenable account of predication?

It is my contention that, if nothing else, the Class Nominalist has some good reasons to prefer ZF over BG. First, the argument from ontological economy: BG-Class Nominalism is less economical than ZF-Class Nominalism insofar as the former should maintain that there are three fundamental kinds of entities (proper classes, sets, and individuals) while the latter admits only two (sets, and individuals — in ZF the notion of *class* is informal and dispensable).

In second place, ZF-Nominalism could mount an argument from explanatory unification. Indeed, in ZF-Class Nominalism properties are nothing but sets, and so property-attributions of the form "x is F" are explained in terms of x's being a member of Fs. In a BG-Class Nominalism some properties should be identified with sets that are classes, others with sets that are not classes, and (at least following Lewis, 1991) still others with classes that are not sets. If having a unified account is a virtue when it comes to choosing between alternative nominalistic accounts of predication, then the Class Nominalist has a good reason to prefer ZF over BG.

Lastly, it seems that BG-Class Nominalism could not address the Küng-Armstrong trilemma. Let us take into consideration Bernays axiomatic system (1937), assuming that small italics refer to sets and capital italics refer to classes, and that

$$a \in b \qquad \text{and} \qquad a \,\eta\, B$$

mean, respectively, "a is in set b" and "a is member of the class B". If the class C exists, then $\langle a, \{x \mid Fx\}\rangle \,\eta\, C$; so, by the Correspondence-rule, $\langle a, \{x \mid Fx\}\rangle$ and C instantiate the η-relation, and this implies that $\langle\langle a, \{x \mid Fx\}\rangle, C\rangle$ is member of the class of all ordered pairs in which the first element is an ordered pair (consisting of an individual and a set), and whose second element is a class; but then we would have another relation, η-relation$'$, and we would need to ask whether the η-relation $=$ the η-relation$'$ or not. If the η-relation $=$ the η-relation$'$, the analysis is circular, while if the η-relation $\neq$ the η-relation$'$ the regress continues in the usual way, following the Küng-Armstrong argument. Since delivering a good account of predication is the main goal of Class Nominalism, once again the Class Nominalist has another good reason to prefer ZF as the core theory in so far as its apparatus allows a regress-stopping strategy.

(Of course, this does not mean that such a preference should be understood in a completely generalized sense. Rather, it applies specifically when the primary objective is to establish a viable class-nominalistic account of predication.)

7.2. Set-membership is not the meaning of "$\in$"

Coming to the concern raised by (B), it should be pointed out that, from a nominalistic standpoint, the fact that a predicate does not express a property does not imply that it has no meaning at all. As Quine rightly indicates

> One may admit that there are red houses, roses, and sunsets, but deny, except as a popular and misleading manner of speaking, that they have anything in common [...] we can use general terms, e.g., predicates, without conceding them to be names of abstract entities.
>
> (Quine, 1948, pp. 29–31)

That is, a theory of properties is not, by itself, a theory of meaning, so from the fact that "$\in$" is an ontologically non-committing predicate does not follow that it is meaningless: it can retain its usual meaning and can be used and understood in its usual sense, without this being in any way affected by its lack of an alleged ontological *correlatum*. This hints at the answer to question (C) that we will address in the next section.

7.3. Ostension *vs.* elucidation

It is true that we can talk about the set-membership relation (as often happens in textbooks on set theory, and as is happening in this very sentence), but it is equally true that the purpose of our language is not always to express a fundamental ontology: many of the uses we make of nouns, predicates, and other items of our everyday language are far more relaxed, and do not aim to perspicuously mirror our ontological preferences. Sometimes, talking about a relation holding between an element, a, and a set, B, is simply a convenient *façon de parler* to say something more complex, that is that certain object-language statements (ZF-statements) are true. Though, the ZF-Class Nominalist will underline that if we want to talk about ZF in an ontologically accurate manner (and that is usually not the main concern of an introductory textbook on set theory), we should respect ZF-Class Nominalism's ontology. This involves that in providing clues about the meaning of "$\in$" in an ontologically accurate language one cannot refer ostensively to an entity that "$\in$" expresses and that its nominalization names: in such a language, definitions of the meaning of "$\in$" cannot be of the fixing-by-ostension type but only of the fixing-by-elucidation type (see § 2.2).

8. Conclusion

We have seen that according to the Küng-Armstrong trilemma the Class Nominalist's analysis of predication is either incomplete, or circular, or regressive. It has been shown that a more sophisticated form of Class Nominalism, namely ZF-Class Nominalism, can provide a Class analysis that is neither incomplete, nor circular, nor regressive. The argument we have presented illustrates that, in ZF-Class Nominalism, the predicate "$\in$" is ontologically non-committal, and precisely for this reason, its nominalizations are not admissible in a language suited to express ontological commitments.

The moderate ineffabilism so far exposed has, moreover, an interesting outcome concerning the metaphysical debates about the nature of the relation/nexus holding between a property and its bearer: since "$\in$" is the set-theoretical counterpart of the predicate "instantiates", from ZF-Class Nominalist's point of view any attempt to investigate into the nature of *instantiation* is futile. Indeed, if for compelling ontological (set-theoretical) reasons "$\in$" is not nominalizable, then "instantiates" must also be considered as such in ZF-Class Nominalism. Therefore, from the ZF-Class Nominalist's point of view, any discourse that uses "$[\![Nom(\text{instantiates})]\!]$" must be looked upon — so to say — as a special kind of attempt to speak of the unspeakable.

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