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Vít Punčochář

Logical Forms, Substitutions and Information Types

Abstract. This paper explores the relation between the philosophical idea that logic is a science studying logical forms, and a mathematical feature of logical systems called *the principle of uniform substitution*, which is often regarded as a technical counterpart of the philosophical idea. We argue that at least in one interesting sense the principle of uniform substitution does not capture adequately the requirement that logic is a matter of form and that logical truths are formal truths. We show that some specific logical expressions can produce propositions of different kinds and the resulting diversity of informational types can lead to a justified failure of uniform substitution without undermining the view that logic is a purely formal discipline.

Keywords: logical form; uniform substitution; information types; non-classical logics

1. Introduction

The common practice of modern logic is to employ artificial languages containing expressions which do not represent concrete sentences of natural languages but rather their forms, and are accordingly called formulas. The semantics of a particular logical theory provides a space of possible interpretations, i.e. possible ways in which formulas can be filled in with content. Some formulas are true throughout all interpretations and thus, in a clear sense, true independently of any particular content. Such formulas represent logically valid forms, and concrete fully-fledged sentences that we subsume under these forms can be called logically true (at least from the perspective of the given logical theory). This I believe is the received view of logic.

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There is one principle that is often interpreted as a technical counterpart of the claim that logic is a matter of form. It is called *the principle of uniform substitution* and it says that all substitutional instances of logically valid formulas (or arguments) are also logically valid. The following thesis is usually accepted:

> Since logic is not concerned with content of sentences but only with their forms, any system that deserves to be called logical must respect the principle of uniform substitution.

Indeed, the most common logical systems respect this principle. However, there are significant exceptions: for instance, Carnap's modal logic C (1947), Veltman's data logic DL (1985), public announcement logic PAL (Plaza, 1989), and inquisitive logic InqL (Ciardelli et al., 2019). These examples illustrate that at least in some interesting sense the thesis is mistaken: one can have a system that deserves to be called logical although it does not satisfy the principle of uniform substitution. The goal of the paper is to clarify this point and argue that a failure of uniform substitution does not undermine the view that logic is concerned only with forms and that logical laws are formal laws.

The paper is structured as follows. Section 2 explains the common view of the status of the principle of uniform substitution in logic. Section 3 presents four examples of logics in which uniform substitution fails and explains the reasons behind these failures. Section 4 discusses common features of these examples and compares two alternative interpretations of the role of atomic formulas in logical theories. Only one of these interpretations allows for the failure of uniform substitution. It is argued that this perspective has some significant advantages and that it is not in conflict with the view that logic is concerned with forms but not with content. Section 5 briefly discusses some weaker alternatives to the principle of uniform substitution and specifies a minimal requirement that any logic should satisfy.

2. The principle of uniform substitution

Even though there are interesting problems concerning substitution that are specific to first-order languages (see Schurz, 1995), we will restrict ourselves, for the sake of simplicity, just to the propositional languages. Consider any propositional formal language built up from a collection of atomic formulas by a set of propositional connectives. The central notion of our discussion is the notion of *substitution*. A substitution is a function that assigns a formula to each atomic formula. Such a function can be homomorphically extended to the whole language in accordance with the following constraint:

$$s(C(\varphi_1, \ldots, \varphi_n)) = C(s(\varphi_1), \ldots, s(\varphi_n)), \text{for each connective } C.$$

One can simply say that $s(\varphi)$ is obtained from φ by simultaneous replacement of all occurrences of each atomic formula p with $s(p)$. Now the *principle of uniform substitution* applied to a particular logic in a given propositional language states that the logic is closed under all substitutions. More precisely, if the logic is represented by its notion of logical truth, the principle of uniform substitution says the following:

> For every formula φ and every substitution s, if φ is logically true then $s(\varphi)$ is logically true.

If the logic in question is represented by its notion of a logically valid argument, we can use this formulation:

> For all formulas $\varphi_1, \ldots, \varphi_n, \psi$ and substitution s, if $\varphi_1, \ldots, \varphi_n/\psi$ is a logically valid argument then $s(\varphi_1), \ldots, s(\varphi_n)/s(\psi)$ is also a logically valid argument.

This property is often endorsed only implicitly. It is often hidden behind schematic representation of logical principles, and behind the talk about instantiations of schemata. However, such a presentation is easily translatable into formulations that explicitly involve the notion of substitution.

In fact, the best-known logical systems satisfy the principle of uniform substitution. It holds for example for classical logic, intuitionistic logic, C. I. Lewis' modal logics S1–S5, the relevant logic R, Łukasiewicz fuzzy logic, Lambek calculus and so on. Of course, this is not a coincidence. This property is regarded as an essential feature of these logics, which is preserved even in the most abstract generalizations of the notion of a logical system.

For example, a set $\mathcal{L}$ of formulas (of the usual language of intuitionistic logic) is called an *intermediate logic* if it satisfies the following properties: (a) it contains all intuitionistically valid formulas but only classically valid formulas, i.e. $\mathsf{IL} \subseteq \mathcal{L} \subseteq \mathsf{CL}$; (b) $\mathcal{L}$ is closed under modus ponens; (c) $\mathcal{L}$ is closed under uniform substitution. There are uncountably many such "logics". (Chargov and Zakharyaschev, 1997)

Similarly, a set $\mathcal{L}$ of formulas (of the usual language of modal logic) is called a *normal modal logic* if it satisfies the following properties: (a) $\mathcal{L}$ contains all classically valid formulas; (b) $\mathcal{L}$ contains the formula $\Box(p \to q) \to (\Box p \to \Box q)$; (c) $\mathcal{L}$ is closed under the rules of modus ponens and necessitation; (d) $\mathcal{L}$ is closed under uniform substitution. Again, there are uncountably many such "logics" (see, e.g., Chargov and Zakharyaschev, 1997).

Uniform substitution is also regarded as a cornerstone of abstract algebraic logic where we encounter the following abstract definition of a logic viewed as a consequence relation (Font, 2016). A *(sentential) logic* of type L is a pair $\mathcal{L} = \langle L, \vdash \rangle$, where L is an algebraic language and $\vdash$ is a subset of $\mathcal{P}(\mathrm{Fm}_L) \times \mathrm{Fm}_L$ (i.e., a relation between sets of formulas and formulas) that satisfies the following properties, for all $\Gamma, \Delta \in \mathcal{P}(\mathrm{Fm}_L)$ and $\varphi \in \mathrm{Fm}_L$:

(a) If $\varphi \in \Gamma$, then $\Gamma \vdash \varphi$.
(b) If $\Gamma \vdash \varphi$ and $\Gamma \subseteq \Delta$ then $\Delta \vdash \varphi$.
(c) If $\Gamma \vdash \varphi$ and $\Delta \vdash \psi$ for all $\psi \in \Gamma$ then $\Delta \vdash \varphi$.[1]
(d) If $\Gamma \vdash \varphi$, then $s(\Gamma) \vdash s(\varphi)$, for every substitution s.

Why is the principle of uniform substitution so essential for logic? Because it is supposed to correspond somehow to the essential idea that logic is concerned with form and not with content. Let us spell out this connection more carefully. It seems reasonable to claim that if a natural language sentence is an instance of the form $s(\varphi)$, then it is also an instance of the form φ. For example, the sentence

(A) *Peter is a lawyer and Berta is a politician.*

is a natural language instance of the form $\varphi = p \wedge q$. In comparison, the sentence

(B) *If Paul is now at home, he is watching TV and if he is not at home, he is running.*

is of the form $s(\varphi) = (t \to r) \wedge (\neg t \to s)$, assuming that $s(p) = t \to r$ and $s(q) = \neg t \to s$. But since (B) is also a conjunction of two sentences it can alternatively be formalized in a less fine-grained way by φ. Then it seems

[1] The condition (b), even though included by Font, is actually redundant because it can be obtained from the conditions (a) and (c).

that the principle of uniform substitution must hold. In general, if a formula $s(\psi)$ is not logically valid then there should be a natural language instance of this form that is false. But then this natural language sentence is also an instance of the form ψ. Hence, the formula ψ should not be regarded as valid, too, because it has false natural language instances.

Having this picture in mind, the principle of uniform substitution seems to have a different status than other logical laws like, for instance, the law of excluded middle, the law of non-contradiction, transitivity of implication, contraposition and distributivity. It is quite common to motivate a non-classical logic by formulating natural language counterexamples to classical logical laws. The counterexamples usually illustrate a sense in which a principle does not hold, and it is legitimate just to take this sense seriously and explore whether it can motivate a system with appealing mathematical properties. For example, one can ask whether the sentence

(C) *Peter's wife is a lawyer or she is not a lawyer.*
(But Peter has no wife!)

is a counterexample to the principle of excluded middle. In the usual manner, one can explain its problematic features away to preserve classical logic. Alternatively, one can take the problems seriously and design a perhaps many-valued logic that aims at reflecting such phenomena as presuppositions in a way that invalidates the principle of excluded middle.

Is it possible to provide in an analogous way a natural language counterexample to the principle of uniform substitution? This looks a priori implausible. Such a counterexample would consist in providing a natural language sentences (A) of a form φ that would be intuitively regarded as logically valid, and another sentence (B) of a form $s(\varphi)$ (for some substitution s) that would be intuitively invalid. However, the fact that (B) is intuitively invalid together with the fact that it can be formalized as a substitutional instance of the form of (A) seems to commit us already at the level of natural language to regard (A) as invalid.

Note that this picture, according to which the principle of uniform substitution must hold, presupposes a particular interpretation of the role atomic formulas play in the formalization. Atomic formulas here cannot be viewed as placeholders for some elementary sentences that do not have any further analysable logical structure, for example in the sense of (Wittgenstein, 1922), but rather they must be viewed as placeholders for arbitrary sentences, even the complex ones. This will be important in what follows.

3. Logics violating uniform substitution

Despite the prima facie plausibility of the principle of uniform substitution, there are some unusual logical systems that violate this principle. Some motivation for its violation comes from the area of nonmonotonic logic because it employs supraclassical consequence relations. For example, David Makinson says:

> The moral of this story is that the supraclassical closure relations that we shall be offering as bridges between classical consequence and nonmonotonic consequence relations are *not closed under substitution*. Nor, for that matter, are the nonmonotonic relations that issue from them. This runs against ingrained habit. Students of logic are brought up with the idea that any decent consequence relation should be purely formal, or structural, and hence satisfy substitution. Indeed, those terms are often used in the texts as synonyms for closure under substitution. To understand nonmonotonic logic, this is a habit to suspend.
> (Makinson, 2003, p. 74)

This seems to indicate that we might also drop the idea that logic is a discipline concerned purely with logical forms. I do not want to go that far. In this section, we focus on several other examples of logics in which the principle of uniform substitution fails and in the next section I will argue that this is not in conflict with viewing logic as a purely formal science.

Our first example of a logic in which uniform substitution fails is Carnap's modal logic C (Carnap, 1946, 1947). Carnap, when developing one of the first versions of a possible world semantics, proceeded as follows: First, he introduced a notion of truth relative to a given state description (a syntactic analogue of the notion of a possible world). Then he defined the notion of L-truth (logical truth) as truth in all state descriptions. Finally, he incorporated this metalinguistic notion directly into the object language by introducing a modal operator N and defining the following truth condition:

$N\varphi$ is true (in a state description w) if and only if φ is L-true.

One can easily check that all the principles of the logic $\mathsf{S5}$ are valid in this semantics. However, the resulting logic is not identical with $\mathsf{S5}$. If we incorporate logical validity directly inside the object language, we can express also satisfiability ($\Diamond\varphi =_{\text{def}} \neg N \neg\varphi$) but, unlike logical validity, satisfiability is not closed under uniform substitution. For instance, $\Diamond q$ is L-true but $\Diamond(q \wedge \neg q)$ is not. See (Schurz, 2001), for further discussion.

A second example of a logic in which substitution fails is Veltman's data logic (Veltman, 1985). Let us denote it as DL. Instead of the notions of truth and falsity simpliciter, DL is based on the notions of truth and falsity *on the basis of available evidence.* Its semantics is very much like Kripke semantics for intuitionistic logic with some subtle differences. As in Kripke semantics, models of DL are some partially ordered sets. The points in such a model are interpreted as bodies of available evidence and the ordering represents growing of evidence. There are two relations between these points and formulas: truth and falsity relative to available evidence.

"$s \vDash \varphi$" means: φ is true on the basis of s,
"$s \dashv \varphi$" means: φ is false on the basis of s.

A formula is called *T-stable* (*F-stable*) if it is always upward persistent with respect to $\vDash$ ($\dashv$). The idea is that growing evidence means the growth of the body of established basic truths and falsities. Atomic formulas are both T-stable and F-stable in this sense. However, the language contains operators *may* and *must* which reflect some non-persistent features of the current state. They are characterized by the following semantic clauses generating unstable sentences:

$s \vDash \mathit{may}\ \varphi$ iff for some $t \geqslant s$, $t \vDash \varphi$,
$s \dashv \mathit{may}\ \varphi$ iff for no $t \geqslant s$, $t \vDash \varphi$,
$s \vDash \mathit{must}\ \varphi$ iff for no $t \geqslant s$, $t \dashv \varphi$,
$s \dashv \mathit{must}\ \varphi$ iff for some $t \geqslant s$, $t \dashv \varphi$.

While q is both T-stable and F-stable, *may* q is not T-stable and *must* q is not F-stable. This causes failure of uniform substitution on the level of logic. Entailment is defined as preservation of $\vDash$, and so, for instance, q entails *must* q but *may* q does not entail *must may* q.

Our third example is dynamic epistemic logic, or more specifically, public announcement logic PAL (Plaza, 1989; van Ditmarsch et al., 2007). PAL characterizes basic logical features of the following epistemic operators indexed by agents: $K_a\varphi$ represents the form: *the agent a knows that* φ; and $\langle\varphi\rangle\psi$ represents the form: *if all agents receive the true information that* φ*,* ψ *will be true.*

The formula $q \to \langle q\rangle q$ is valid in PAL. It expresses that if q is true then after it is announced, it remains true. This sounds plausible if we assume that q stands for a factual sentence. For example, after it is announced that Earth is heavier than Venus, this fact remains to be true. However, this is not so, if we announce an epistemic fact. In this case the announcement can change its truth value. For example, assuming that Paul does not know that Earth is heavier than Venus, after this fact is announced, Paul's epistemic state is changed by the announcement and it ceases to be true that Paul does not know that Earth is heavier than Venus. So, while $q \rightarrow \langle q \rangle q$ is logically true in PAL, its substitutional instance $(q \wedge \neg K_a q) \rightarrow \langle q \wedge \neg K_a q \rangle (q \wedge \neg K_a q)$ is not.

Our final example is Inquisitive Logic InqL (Ciardelli et al., 2019). Inquisitive logic is a logic of questions. Its language contains a question generating operator $\mathbin{\vee\!\!\!\vee}$ called *inquisitive disjunction*. $p \mathbin{\vee\!\!\!\vee} q$ represents the form of a disjunctive question: *whether p or q*. A polar (yes/no) question *whether p* can be defined as a special case of a disjunctive question, namely as the question *whether p or not p*, and it can be formalized as $?p =_{\mathrm{def}} p \mathbin{\vee\!\!\!\vee} \neg p$.

The syntax of inquisitive logic allows one to embed $\mathbin{\vee\!\!\!\vee}$ under other connectives. One can form in this way conjunctions of two questions (the question *whether p and whether q*, i.e. $?p \wedge ?q$) or conditional questions (the question *whether q if p*, i.e. $p \rightarrow ?q$). One can also negate questions: $\neg(p \mathbin{\vee\!\!\!\vee} q)$. The meaning of this construction is a claim that the presupposition of the question $p \mathbin{\vee\!\!\!\vee} q$ is not true. But the presupposition of the question *whether p or q* is the claim *that p or q*. So, $\neg(p \mathbin{\vee\!\!\!\vee} q)$ is equivalent to $\neg(p \vee q)$, and thus $\neg\neg(p \mathbin{\vee\!\!\!\vee} q)$ is equivalent to $\neg\neg(p \vee q)$.

The declarative part of InqL is just classical logic. For example, $\neg\neg q \leftrightarrow q$ and $\neg\neg(p \vee q) \leftrightarrow (p \vee q)$ are logically valid. However, in the light of what was said, the formula $\neg\neg(p \mathbin{\vee\!\!\!\vee} q) \leftrightarrow (p \mathbin{\vee\!\!\!\vee} q)$, as a substitutional instance of $\neg\neg q \leftrightarrow q$, cannot be valid. Its validity would imply the validity of $(p \vee q) \leftrightarrow (p \mathbin{\vee\!\!\!\vee} q)$, which would lead to a collapse of the inquisitive disjunction into the declarative one.

4. General factors leading to a failure of substitution

Let us generalize and reflect on some common features of these examples: if substitution fails in a given logic $\mathcal{L}$, it is so typically because

(a) the language of $\mathcal{L}$ involves different types of sentences;
(b) atomic formulas in $\mathcal{L}$ are related to some specific basic type;

(c) the other types in $\mathcal{L}$ are generated by some specific logical operators of the language of $\mathcal{L}$;
(d) the language of $\mathcal{L}$ involves operators that are polymorphic, i.e. they can be applied to sentences of various types.

As Carnap (1946) himself indicates the logic C is committed to the Tractarian theory of mutually independent atomic facts and atomic sentences representing (truly or falsely) these facts. These sentences form a specific type and atomic formulas of the propositional formal language are placeholders for sentences of this type. Nevertheless, the formal logical language involves logical operators that generate from this basic type other types of sentences, for instance the type of logical truths. These logical operators are polymorphic. For example, negation can be applied to an atomic sentence as well as to a logical truth.

The logic DL relates atomic formulas to T- and F-stable sentences and contains operators like *may* and *must* that generate unstable sentences. Other logical operators can be applied without restrictions.

The logic PAL distinguishes between factual and epistemic sentences. Atomic formulas are related to the factual sentences, and the public announcement operator generates epistemic sentences from the factual ones. Again the other operators are polymorphic.

Finally, InqL distinguishes between declarative and inquisitive sentences. Atomic formulas are related to the former type and inquisitive disjunction creates the latter from the former. All operators of the propositional language can be applied to statements as well as to questions.

Let us discuss the difference between the two basic views of atomic formulas that we have encountered:

A1. atomic formulas as placeholders for arbitrary sentences;
A2. atomic formulas as placeholders for "elementary sentences" or sentences of a particular kind that provides the basic building blocks.

The view A1 is directly related to the possibility of substituting any formula for any atomic formula and thus it implies that the principle of uniform substitution should hold. So, only the second view A2 provides space for a failure of substitution. If a logical system is built in accordance with A2 and it relates atomic formulas to a specific type of sentence, it might very well happen that there are particular logical laws valid for this type but not valid generally. This was illustrated in the previous section with the four examples.

Both these alternative views of atomic formulas, A1 and A2, are legitimate and even the first perspective A1 could be, in a sense clarified below, applied to the logical systems discussed in the previous section. For any logic $\mathcal{L}$, even if it is not closed under uniform substitution, we can meaningfully ask what its *schematic fragment* is. The schematic fragment can be defined as the set

$$\mathrm{Sch}(\mathcal{L}) = \{\varphi \mid s(\varphi) \in \mathcal{L}, \text{ for every substitution } s\}.$$

If $\mathcal{L}$ is based on A2, like the four logics from the previous section, the schematic fragment of $\mathcal{L}$ is the system of universal invariants, that is, of the laws that the logic $\mathcal{L}$ generates when we reinterpret atomic formulas and switch from A2 to A1.

The difference between a logic and its schematic fragment reflects the difference between the two alternative views of atomic formulas. The question what is the schematic fragment of a given logic is often quite interesting and non-trivial. For Carnap's modal logic C we obtain $\mathrm{Sch}(\mathsf{C}) = \mathsf{S5}$. This was proved already by Carnap (1946). I am not aware of any results concerning the schematic fragment of Veltman's data logic. The schematic fragment of PAL was characterized in (Holliday et al., 2012). The schematic fragment of inquisitive logic corresponds to the Medvedev logic of finite problems ML (Ciardelli and Roelofsen, 2011).

The relation between a logic and its schematic fragment can be very tricky, as the case of inquisitive logic shows. The system of InqL is quite simple and well-behaving but its schematic fragment ML is a rather mysterious logic. Some deep facts about this logic are known, for example that it is not finitely axiomatizable (Maksimova et al., 1979), but in spite of a serious investigation many basic questions remain open. Most importantly, it has been a long standing open problem in the area of intermediate logics whether ML is decidable and recursively axiomatizable.

One may prefer the first interpretation of atomic formulas A1 and then regard the schematic fragment as the real target of logical study. However, it should be taken into account that the relation between a logic and its schematic fragment is asymmetric. Clearly, every logic determines uniquely its schematic fragment. However, in general, it is not possible to recover the logic $\mathcal{L}$ from its schematic fragment $\mathrm{Sch}(\mathcal{L})$. We can have different logics with the same schematic fragment. For example, there are uncountably many different non-classical inquisitive logics that all have ML as their schematic fragment (Punčochář, 2016).

This means that when we focus directly on the schematic fragment we may lose some structure and thus some interesting information. This is a strong argument in favour of the restricted interpretation of atomic formulas A2 that allows for the failure of uniform substitution.

Moreover, I want to argue that the view of atomic formulas A2 is not in conflict with the claim that logical truths are true purely due to their form. In each example presented in the previous section uniform substitution failed because the logic in question contained some basic type of sentences and operators generating from this basic type some other types (not always substitutable for the basic type). This can be viewed as reflecting a formal feature of natural language expressions: it is the logical form of a sentence, and not its particular content, that determines the type of the sentence.

Take, for an illustration, the case of modus tollens. Imagine a context in which one marble is randomly selected from a box that contains marbles classified as big or small and as blue or red. One can produce natural language instances of modus tollens like the following one:

$$\frac{\textit{If the drawn marble is big, then it is blue} \qquad \textit{The drawn marble is not blue}}{\textit{The drawn marble is not big}}$$

This seems to be a perfectly valid argument. Is it valid on the basis of its form or on the basis of its content? There seems to be nothing specific about the content of the premises and conclusion that would be responsible for its validity. One can completely change the content without loosing validity, for example by producing this argument:

$$\frac{\textit{If Paul is a minister, then he has a good salary} \qquad \textit{Paul does not have a good salary}}{\textit{Paul is not a minister}}$$

Nevertheless, there are counterexamples to modus tollens like the one discussed in (Yalcin, 2012). Assume that the marbles in the box are distributed according to the following table:

	blue	red
big	10	30
small	50	10

Moreover, consider this argument:

$$\frac{\begin{array}{c}\textit{If the marble is big, then it is probably red}\\ \textit{The marble is not probably red}\end{array}}{\textit{The marble is not big}}$$

It seems that in the specified context the premises are true though the drawn marble may very well be big. This means that the following argument form is not valid:

F1 $\quad p \to \textit{probably } q, \neg \textit{probably } q \ / \ \neg p$

However, it is crucial for the counterexample that the consequent of the implication in the first premise is of a particular *form*, namely of the form *probably p*. "Probably" can surely be viewed as an operator that generates a specific type of sentence. So, the failure of F1 does not imply that also the argument form

F2 $\quad p \to r, \neg r / \neg p$

is invalid, even though F1 is a substitutional instance of F2. This example illustrates that we can reasonably have logically valid arguments, i.e. arguments that are valid due to their form rather than content, and at the same time we can have substitutional instances of these arguments that are invalid. So, the view A2 is not in conflict with the usual interpretation of formulas as capturing logical forms and the usual understanding of logical validity as purely formal validity.

5. Weaker forms of substitution

Is there a weaker version of the principle of substitution that logics should satisfy? It is clear that some form of substitution should be present in a logical system. However, it is difficult to specify generally what the restrictions should look like. They depend on the goal of a particular logical inquiry. There may be different reasons for the failure of substitution and these different reasons lead to different weakenings of the principle of uniform substitution. Let me illustrate this with an example.

In (Punčochář, 2022) I introduced a logic in which substitution also fails, but for a very specific reason. The logic is based on a stratification of formulas into degrees. Every formula φ is of a particular degree $d(\varphi)$. We say that a substitution is *normal* if it assigns only formulas of the same degree, that is, if there is a k such that $d(s(p)) = k$, for each atomic formula p. Hence, normal substitutions are those that do not assign formulas of different kinds. The proposed logic is not in general closed under non-normal substitutions but it is closed under all normal substitutions, i.e. it has the following property:

> If $\varphi_1, \ldots, \varphi_n/\psi$ is a valid argument form then for every normal substitution s, the argument form $s(\varphi_1), \ldots, s(\varphi_n)/s(\psi)$ is also valid.

This is a desirable result given the motivation with which the logic was introduced but it can be hardly regarded as a general requirement that would be applicable to all logics. For instance, all the examples of a failure of uniform substitution that we discussed in Section 3 concerned formulas/arguments with just one atomic formula ($\Diamond q$ in C, $q \vDash \textit{must}\ q$ in DL, $q \to \langle q \rangle q$ in PAL, and $\neg\neg q \leftrightarrow q$ in InqL). In such cases non-normality cannot play any role since it requires at least two atomic formulas to which formulas of different degrees are assigned by the substitution. If we want a generally applicable restriction, we must find something weaker. The following principle might seem to be a suitable candidate:

> *The principle of basic substitutions*: a logic should be closed under the substitutions assigning only formulas that are of the same type as atomic formulas.

Unfortunately, even this principle seems to be too strong. It holds for all logics that we discussed except Carnap's logic C, where $\Diamond(p \wedge \neg q)$ is valid but $\Diamond(p \wedge \neg p)$ is not.

One can find suggestions of minimal principles of substitutions, under which all logics should be closed, in (Schurz, 2001). One such suggestion is based on semantic considerations and I will not reconstruct it here. Instead, I will just mention the following simple and purely syntactic candidate for the weakest principle of substitution:

> *The weakest principle of substitution*: a logic should be closed under injective substitutions of the shape $s\colon \mathrm{At} \to \mathrm{At}$.

Schurz calls such substitutions *syntactically isomorphic*. I believe that this principle is universal. A mere renaming of atomic formulas, respecting the differences between atomic formulas, cannot have any impact on the questions of validity and invalidity. This, however, does not mean that one cannot introduce a logical system that has different categories of atomic formulas for which different laws hold. In such a case one should be able injectively substitute at least atomic formulas of matching categories.

6. Conclusion

Let us summarize the main points made in this paper. In a sense, the principle of uniform substitution is a technical articulation of the formal nature of logic. However, this sense is related to a particular view of the role of atomic formulas in formal languages. There is an alternative understanding of this role that allows for violation of uniform substitution without undermining the view of logic as a discipline purely concerned with forms. In order to obtain such a failure we need: plurality of sentential types, atomic formulas related to a particular type, type-transforming logical operators, and polymorphic logical operators. There might be different sources of the failure of uniform substitution and they are related to different weakenings of the general principle. There is a minimal principle of substitution that should be respected by any logic. Any logic should be closed under injective substitutions of the shape $s\colon \mathrm{At} \to \mathrm{At}$.

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VÍT PUNČOCHÁŘ
Department of Logic, Institute of Philosophy
Czech Academy of Sciences
Prague, Czechia
puncochar@flu.cas.cz
https://orcid.org/0000-0003-2198-4574