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What Should the Logic Formalizing Human Cognition Look Like? Psychologism as Applying Logic in Cognitive Science

Abstract. Contemporary logicians have expanded upon the old notions of psychologism in logic and proposed new, weakened versions of it. Those weakened versions postulate that psychologistic logic does not have to inform about the ontology or metaphysics of reasoning. Instead, logic applied in cognitive science could serve as one of many paradigms for making empirical predictions about the observable process of human reasoning. The purpose of this article is to entertain this notion and answer the question: what properties should a logical system formally representing actual human reasoning have? Based on the existing evidence from cognitive science and neuroscience we identified three potential candidates: context-sensitivity (satisfied for example by adaptive logics), content-sensitivity (satisfied by non-Fregean logics) and probabilism (satisfied for example by fuzzy logics).

Keywords: psychologism; applied logic; cognitive science; cognitivism

For those who have eyes to see, logic and the empirical cognitive sciences interface today in many interesting ways, and that to mutual benefit. J. van Benthem, 2008

1. Introduction

Psychologism as a stance with regard to the status of logic has received many definitions throughout the last 200 years. At its conception, F. E. Beneke (1832) categorized logic as a sub-field within psychology, while J. S. Mill (1843) proposed that the laws of logic are derived from

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psychological phenomena. As such, they reasoned that the structure of logic ought to represent the structure of human reasoning.

> Logic is not a science, separate from and coordinate to psychology. To the extent that it is a science at all, it is a part or branch of psychology, distinguished from it on [the] one hand as the part is from the whole, and on the other hand as the art is from the science.
>
> (Mill, 1865, p. 388)

This perspective was subject to intense critique by Frege (1884) and Husserl (1900), who contended that the laws of logic are obtained *a priori* and remain valid irrespective of the reality of human reasoning. Conversely, the laws of human reasoning are empirically derived, which imbues them with an unacceptable degree of vagueness and a lack of universality that is unsuitable for logic. Consequently, logic and psychology diverged as scientific disciplines and drifted apart in the second half of the 20th century (see Stenning and van Lambalgen, 2008).

Fortunately, logicians in the 21st century have revisited these old notions of psychologism and found that they halted a potentially fruitful collaboration between logic and psychology (see Gabbay and Woods, 2005; Urbański, 2011; Wheeler, 2008; Pelletier et al., 2008; Chater and Oaksford, 2002; Johnson-Laird et al., 2015). In particular, it appears that both psychologism and Fregean anti-psychologism were unnecessarily strong stances. According to them, the domain of logic was either located entirely within the psychological field or entirely outside of it. In contrast to this duality, contemporary advances at the crossroads of logic and cognitive science have convincingly argued that *psychologizing* can be one of many applications of logic. After all, logic lies at the foundation of many applied sciences — computer science, artificial intelligence and economics. The common denominator for them all is their interest in the formal representation of information processing Stenning and van Lambalgen (2008). Given that *thinking* and *reasoning* in modern cognitive science are defined for the most part as information processing, it is only natural that logic can also be used to formally represent them.

The purpose of this article is to take a closer look at the contemporary understanding of *psychologism* in logic. In particular, we will discuss select applied logics that attempted to formalize some aspects of human reasoning. These logics designed their formalisms specifically to mimic the discrepancies between human reasoning and classical logic. As such, psychologism will present itself as a paradigm that defines specific logical properties that are expected from a formal system intended to model some instances of human thinking. Finally, we will identify and discuss three such properties which emerge from the psychological literature — context-sensitivity, content sensitivity and probabilism/fuzziness. We believe that these three properties are necessary but not exhaustive for a system that accurately models human reasoning.

2. The many definitions of psychologism

Psychologism in logic has been defined in a vast variety of ways. These various definitions are sometimes so different from each other that they describe almost completely different ideas. For example, *epistemological psychologism* holds that the laws of logic exist only as instances of their use by agents who perform reasoning (Sober, 1978). This idea was developed in part by the neo-Kantian F. E. Beneke who posited that the sole condition for veracity of logical form was its utility for human reasoning. As such, logic is said to *emerge* from the actuality of human reasoning, which is studied by psychology. This view positioned logic within the realm of psychology as a discipline, which was not received well by logical positivists. In fact, this claim outraged them enough to cause a rift between logic and psychology which lasts to this day (Sober, 1978).

Already in 1978, E. Sober argued that a more sensible definition of psychologism could be adopted — bringing together logicians and psychologists again.

> The psychologistic position that I will defend likewise assumes a view of what a psychological theory is. I will assume that an adequate account of memory, perception, problem-solving, learning, and cognition in general can be provided by an information-processing model. Providing such a model must involve selecting some logical system (i.e., some set of rules of inference characterized within a formal system) and claiming that it has psychological reality. (Sober, 1978, p. 167)

It appears that Sober (1978) already argued for what are also the central theses of this paper: a) that logic can be applied to formalize human reasoning, and b) when logic is applied this way, then mental phenomena *become* the subject of logical inquiry. This view is becoming more and more popular among logicians and cognitive scientists. Pelletier et al. (2008) took a closer look at different types of psychologism and proposed some modifications which could make them more compatible with contemporary logic. They believe that psychologism is not a faulty interpretation of the subject of logic — but rather a *possible* interpretation — and not the only one.

> From this perspective, psychologism accepts that humans reason, perceive, and think about "the external world" in particular ways, and thereby psychologism could be a general doctrine that can be applied (methodologically or otherwise) in any realm of study. [...] In the context of psychologism, one might choose to be psychologistic about some [realms] and not about others.
>
> (Pelletier et al., 2008, pp. 6 and 14)

The idea that psychologism can be *conditionally adopted* instead of being a metaphysical view of logic as a whole is the key to unlocking collaboration between logicians and cognitive scientists. The purpose of such collaboration would be to develop applied logic(s) which accurately symbolically represent the process of human reasoning. Both logicians and cognitive scientists typically agree that classical first-order logic is ill-equipped for that task (see Johnson-Laird et al., 2015; Pelletier et al., 2008). To address this issue logicians developed many non-classical logics; some explicitly stated a psychologistic rationale for their work (see, e.g., Batens, 1999, 2000), while others kept their motivations vague and left the *psychologising* to those who would want to apply their logic (see, e.g., Da Costa et al., 1995). In turn, when cognitive scientists wanted to symbolically represent the reasoning processes, they employed tools provided by other disciplines (eg., statistics or Bayesian probabilistics). However, we believe that logic may provide a paradigm which is at least *just as good* for modeling actual human reasoning as the ones that were used to date.

3. The rifts between logic and cognitive science

In their pursuit to dissociate logic[1] from psychologism, logical positivists implicitly endorsed classical first-order logic and classical probabilistics as normative frameworks of *rationality* — the standards that people should strive to achieve in their reasoning and decision-making

[1] In this article we employ the broadest definition of *logic* as the study of reasoning, which has three essential components: premises, conclusions, and rules of reasoning (see Jaakko and Gabriel, 2007).

processes (Russell, 1914). However, contemporary cognitive science has challenged this belief. Contrary to the assumptions underlying the normative/descriptive gap, it is increasingly recognized that classical logic, in fact, does not offer an adequate normative account of rational behavior (see, e.g., Gigerenzer and Selten, 2001; Johnson-Laird et al., 2015; Kahneman, 2011).

Acting according to the conclusions derived with first-order classical logic and rationality are now separate concepts in cognitive science. That rift happened because of two reasons: 1. always adhering to classical logic sometimes produces *paradoxes*, 2. always adhering to classical logic generates maladaptive behaviors from both an evolutionary perspective and a psychological perspective.

Classical logic produces many paradoxes, but some of them are especially problematic with respect to the *rationality* of an agent that would employ them in everyday reasoning. The most well-known are the paradoxes of material implication. Material implication in classical logic ($p \rightarrow q$) can be false only when the antecedent (p) is true, while the consequent (q) is false. As a result, we may produce sentences that are true according to truth valuation in classical logic but are not accepted as true in natural language. For example, 'If Poland is an apple then apples are a fruit'. Importantly, such sentences do not only appear invalid/false at first glance, but even after longer deliberation.

Another example of a mismatch between everyday human reasoning and classical logic (or classical probability theory) is Hempel's paradox (1945). Sentence A: 'If something is a raven then it is black' (i.e., 'All ravens are black') is equivalent via contraposition to: A': 'If something is not black then it is not a raven'. For such sentences which make claims about a class of objects we can collect evidence in the form of observable instances that either confirm or disconfirm the claim. For example, the observation expressed with: B: 'My pet raven is black' is evidence in support of sentence A. Similarly, we can search for observations in support of sentence A'. For instance, C: 'My pet dog is white and it is not a raven' is evidence in support of A'. However, given that A' is equivalent to A, sentence C about my white dog is also evidence for A that all ravens are black.

It is not controversial to say that a rational agent should avoid making inferences that involve paradoxes of material implication or a Hempel paradox. From an evolutionary perspective, it would be an enormous waste of time and resources to process information in such a manner. As a result, organisms that employ classical material implication or look for evidence in the manner described above would handicap themselves and face extinction. This issue highlights that *rationality* in an environment which exerts survival pressure on individuals is different from hypothetical rationality in a world containing infinite time and resources. As a result, cognitive scientists developed the paradigm of *bounded rationality* — the idea that the limitations of our environment and our processing capacity dictate what "*logic*" are we going to use when making inferences — and that it is not going to be classical first-order logic (see Gigerenzer and Selten, 2001; Johnson-Laird et al., 2015; Kahneman, 2011; Simon, 2010).

The notion that *rational* and *logical* can be vastly different gained traction in the beginning of the 21st century. Hertwig and Gigerenzer (1999)) entitled an article: "The 'conjunction fallacy' revisited: How intelligent inferences look like reasoning errors" and their work spelt an advent of research on the deep rationality behind heuristic information processing performed by humans. What were once called *cognitive errors*, *fallacies* or *biases* were now revisited to uncover their utility and adaptability (see e.g., Dosi et al., 2020; Hertwig and Herzog, 2009; Hahn and Oaksford, 2007; Doody, 2020; Hutzler, 2014). At the same time, logicality or classically understood rationality was not rejected altogether as inadequate for modeling human reasoning. Instead, it is now understood as one of many cognitive tools available to reasoning agents (Sturm, 2012).

Pelletier et al. (2008) in their work on the taxonomy of psychologistic arguments in logic devote much space to describing this type of framework. They call it the *Teleological argument* and the *Cognitive Architecture argument*. These two arguments come together to form the basis for *bounded rationality* because they capture both types of *boundaries* for human cognition. First, those that come from the person (their cognitive architecture) and those that come from the environment (the teleological/evolutionary explanation).

> The cognitive architecture element of this position holds that there is some specifiable mechanism according to which the human mind works. The mechanism has access to certain information and abilities at various times during the execution of a task, and its resulting actions (including its decisions) are to be understood as conditioned by the ways in which the architecture can operate. [...]

> The teleological element comes in if we suppose that aspects of this architecture are aimed at the same goal, or are serving the same purpose — for instance, social dominance or mating success or individual survival or species survival. In the theoretical biology literature, such view is called a teleological view, meaning that there is some common end that this mental apparatus serves to further.
>
> (Pelletier et al., 2008, p. 42)

If one takes these stances as arguments in favor of *metaphysical* or *epistemological psychologism*, they fall short. The argument there would be that our cognitive architecture is the source of logical laws, and that we are able to communicate about them effectively because we all share the same underlying structure of that architecture. The reason why we share that structure is evolutionary (adaptive) — it reflects the evolutionary pressures exerted on our ancestors. (Pelletier et al., 2008) astutely observe that such an explanation does not sufficiently distinguish between the inherent truth of logic and the circumstantial truth derived from recurring empirical observations. In other words, a phenomenon that necessarily occurs exerts the same evolutionary pressure as one that has always occurred, even if it was not a necessity. Therefore, human cognitive architecture can, in itself, only give rise to a logic where true statements are qualified with: 'It appears to humans that ...' That is not enough to substantiate metaphysical or epistemological psychologism. However, it is ample to endorse a modern, attenuated version of psychologism, positioning itself as a paradigm for logic as employed in cognitive science.

4. Psychologism as a paradigm for applying logic in cognitive science

J. Corcoran (1994) postulated that logic can be divided into formal ontology and formal epistemology. Through that lens, the idea of applying logic to studying human cognition is contained entirely within formal epistemology, leaving formal ontology intact. Instead, it would merely utilize the achievements of formal ontology — the rigorous proof theory and semantics — in order to model human reasoning. Logicians are becoming more and more aware of this possibility. Urbański (2011) argues strongly in favor of embracing this contemporary understanding of psychologism — which he calls *cognitivism*.

The way of applying logic to cognitive science is relatively straightforward since it would follow the same scientific criteria as any other paradigm. van Benthem (2008) summarizes it in a simple way:

> Logical theories should then be quite welcome here, as a means of deriving predictions, even if they turn out refuted. Indeed, the abovementioned logical theories of inference, update, and interaction all suggest interesting testable hypotheses about human behaviour, and one could easily imagine a world where a logician who has created a new logical system does two things instead of one: like now, submit to a logic conference, usually far abroad, but also: telephone the psychologist next door to see if some new nice experiment can be done.
>
> (van Benthem, 2008, p. 77)

Incremental empirical falsification of predictions made by logical theories with regard to human reasoning could potentially contribute to the search for something of immense value — a logic that describes human information processing (cognition and emotion). However, creating such a logic prompts the question — what properties should it have?

Until now, when logicians created a non-classical logic, two kinds of justifications were used:

1. Arbitrary — when the purpose of the system is to satisfy some purely formal criteria. For example, Da Costa et al. (1995) when they justified the creation of paraconsistent logic:

> [...] we wish to point out that, from our viewpoint, when presenting a formal system, one does not need to be concerned with the formulation of philosophical rationales for the mathematical constraints introduced. [...] Moreover, such systems are not thought of as capturing the true nature of the world, nor of logic, of logicality or whatever. In the first instance, they were just devised with the aim of putting forward a particular logical system meeting certain theoretical constraints.
>
> (Da Costa et al., 1995, pp. 115–116)

2. Bounded — when the purpose of the non-classical logic is to symbolically represent an aspect of reality. For example, Makinson and Gärdenfors (1991) when they explained the purpose of non-monotonic logic:

> For example, the logic of conditional propositions has always reflected a desire to understand the import of "if ... then" locutions of ordinary language. Nonmonotonic reasoning has been studied with hopes for use in artificial intelligence, with consequent attention to the requirements of finitude and eventual practical computability.
>
> (Makinson and Gärdenfors, 1991, p. 186)

Thus, formal systems designed to symbolically represent human information processing are naturally bounded. Their properties ought to be dictated by the human cognitive architecture as described by Pelletier et al. (2008). This encompasses psychological data on human reasoning (i.e., what humans say and do) as well as neuroscientific data on the underlying processes (i.e., what is the underlying physiology of human information processing). This means that the process of determining properties that we desire in our applied logic(s) is an empirical one. This is reminiscent of *eliminative psychologism* — an old view that postulated a replacement of logic with the psychology of human reasoning, originated by J. Locke who suggested that philosophers should "observe what people do when they ordinarily reason" (George, 2003, p. 33). However, here we do not postulate the replacement of logic at all. On the contrary, the modern advancements of formal mathematical logic may prove to be an invaluable tool for creating an accurate logic of human reasoning.

5. Properties of psychologistic applied logic

The 20th century gave birth to a myriad of different non-classical logics. These give us an unprecedented opportunity to empirically test which of their properties we should expect from a formal system designed specifically for modeling human reasoning. For example, Rudnicki and Łukowski (2021) wanted to determine whether the *virtual entailment principle* is a property of human cognitive architecture. That principle, devised by J. Buridan in the 14th century, states that every sentence in natural language implicitly asserts its own veracity. There are many consequences of adopting such a principle — for example, the liar paradox ('This sentence is false') stops being a paradox and becomes a false sentence. By analyzing brain activity in response to true sentences, false sentences and paradoxical sentences, Rudnicki and Łukowski (2021) provided evidence in support of applying the *virtual entailment principle* in the logic of human reasoning. The results showed that *it seems to humans that* the liar sentence is false. As such, this experiment exemplifies the approach that can be taken for testing whether some properties of logical systems correspond to the properties of human reasoning.

In this section, we will analyze several properties that a logic modeling human reasoning needs to have and look for already existing nonclassical logics that could address that need.

5.1. Modularity, a.k.a. context-sensitivity

So far, when referring to human reasoning and the potential applied logic used to symbolically represent it, we treated them as singular, unified constructs. Yet, contemporary cognitive science provides compelling evidence suggesting that one system of logic might not be sufficient to embody all aspects of our reasoning processes. Depending on the modality of information[2] humans may process it differently. This idea is also referred to as the *modularity* of the human mind.

Modularity refers to the idea that the human mind is made up of a number of distinct specialized systems or modules that are dedicated to performing specific functions. This idea suggests that different aspects of cognition and behavior are mediated by separate neural systems that are specialized for particular tasks. James et al. (1890) developed the groundwork for this idea at the end of the 19th century. He proposed that there were two modes of reasoning: *associative reasoning* — used for analyzing past experiences to derive new knowledge, as well as *true reasoning* — where new, unfamiliar information is analyzed with pre-established rules of thought. These ideas were picked up in 1975 by M. Posner and C. Snyder who created the dual process model of the human mind and by Kahneman (2011) who polished and popularized it (see Posner and Snyder, 1975).[3]

In its contemporary version, this theory identifies at least two systems of information processing: System 1 and System 2. System 1 is described as fast, effortless, and intuitive. It is responsible for automatic and mostly unconscious processing of information, such as recognizing faces or making snap judgments. System 2 is described as slower, more effortful, and more "logical". It is responsible for more complex and

[2] In cognitive science, *modality* refers to the type of processed information. For example, it may mean a distinction between auditory, visual or somatosensory information or a distinction between information represented in language or non-linguistic symbols.

[3] While popular in cognitive science, the concept of the modular mind has been criticised for its hyperintellectualism — excessive focus on the mind as a series of computational systems engaged in processing informational input. For example, radical enactivists instead posit a more integrative, active, and embodied interaction with our environment, where cognition is not confined to discrete, abstract modules but is a product of our direct and dynamic engagement with the world around us (Hutto and Myin, 2013). We believe that there is no fundamental disagreement between modularity and enactivism and that emphasis on embodied and extended cognition could be a welcome property of psychologistic applied logic.

mostly conscious thought processes, such as solving math problems or analyzing arguments. Interestingly, logicians have devoted some work to analysing this dual processing idea (Frankish, 2010). However, in this work, we would like to distance ourselves from this particular understanding of modularity.

In the discourse of the dual processing paradigm, researchers often describe System 2 as the "logical" one in the sense that it adheres more tightly to the principles of classical logic. As a result, researchers sometimes relegated the role of logic to describing some limited aspect of human reasoning — "the logic module". When Pelletier et al. (2008) summarized the works of (Gigerenzer et al., 1996; Cosmides and Tooby, 1992; Cummins, 2002) on that matter they identified the postulated existence of such a construct:

> A natural interpretation of how a "logic module" should work within a cognitive architecture is that the module (and its place within the architecture) determines what is the field under consideration and what should count as correct and incorrect in that realm.
>
> (Pelletier et al., 2008, p. 47)

This understanding of modularity drastically underestimates the potential applicability of logic. All information processing performed by humans can potentially be symbolically represented with some formal logic. That concerns the supposedly "logical" cognitive processes, as well as the more "heuristic" ones (vide System 1). Information processing is an umbrella term that encompasses conscious reasoning, but also emotions, attitudes, beliefs or even basic psychophysiological processes like hunger. The "logic module" is not a separate entity — THE ARCHITECTURE ITSELF IS THE LOGIC encoded with neurons and neurochemical signals (for an example of modeling cellular neuronal activity as logic gates, see Goldental et al., 2014). The task of psychologistic applied logic is now to translate that neuronal logic into a comprehensible system of rules. Naturally, there still can potentially be a "classical logic module" (vide System 2) embedded within a larger picture of many logics that humans use.

This is important because cognitive science teaches us that humans may employ very different rules of inference depending on modality and context. A famous example of the way in which context alters human reasoning is the "Asian Disease" problem originally studied by Tversky and Kahneman (1985). In an experiment, participants were presented with a scenario of a deadly disease outbreak among 600 people and a dilemma. They could choose one out of two strategies for dealing with the disease outbreak. The strategies had certain, predictable outcomes and participants had to choose a strategy based on them. The outcomes presented to the participants were:

1. 200 individuals are saved
2. with a probability of $\frac{1}{3}$, 600 individuals are saved and with a probability of $\frac{2}{3}$, 600 individuals are not saved.

When the dilemma is phrased like this, participants overwhelmingly prefer option 1. However, participants' choices are very different when equivalent options are presented, but framed in terms of losses:

1. 400 individuals die
2. with a probability of $\frac{1}{3}$ 600 individuals will not die and with a probability of $\frac{2}{3}$ 600 individuals will die.

When the dilemma is phrased like this, participants select the riskier option 2. As such, researchers show that what is equivalent in terms of classical logic or probabilistics, can be processed very differently by the human cognitive architecture. They explain it in terms of *modules* by pointing out that humans treat gains and losses as separate domains and showing that the phenomenon disappears when framing takes the form of a statistical problem, instead of gains vs. losses (Bless et al., 1998).

Cognitive science has not only focused on the influence of external context-sensitivity but has also extensively explored internal variables such as individual differences. It's widely recognized that the interpretation of the same piece of information can vary significantly from one person to another. This variance isn't solely limited to differences in normative judgements, which pertain to individuals' unique viewpoints on how things ought to be. Beyond these normative judgements, the differences extend to factual judgements — how people perceive things as they are, and the logic that underlies their reasoning process.

Spychalska et al. (2016) investigated the time course of brain activity when people judge the truthfulness of statements. In particular, participants judged statements that take the form: 'Some A's are B's' while it is otherwise known to the participants that actually 'All A's are B's' is true. The results showed similar brain activity in everyone who perceives the sentence 'All A's are B's'. However, two distinct groups of people emerge when they are asked about the truth of the sentence 'Some A's are B's'. Some reject them as false (the so-called *pragmatic responders*), while others accept them as true (the so-called *logical responders*). When analyzing the brain activity of these two groups, researchers found that they process the same information very differently — they use different verification strategies. As such, it is likely that at least some reflexively employed rules of inference are fundamentally different between these two groups.

Logicians have already recognized the importance of individual differences in the past. For example, the first paraconsistent logic created by S. Jaśkowski (1948) was intended to capture the dynamics of discourse between people with differing opinions. Hence, its name: *discursive logic*. When debating, each person provides some information — their beliefs or opinions. As such, premises provided by different participants of the discourse are considered true by them, but do not have to be considered so by others. As a result, what is true in the discourse as a whole (the sum of all participants' assertions) is most often inconsistent — which is why Jaśkowski made his logic paraconsistent.

The creation and development of paraconsistent logics provide invaluable foundations for representing the modularity of the human mind. That is because inconsistent beliefs surface not only when two different people debate, but also within one person. Humans are able to live with contradictory beliefs, attitudes and behaviors. That is possible in large part by partitioning those contradictions into different modules and using them independently of each other Rudnicki and Łukowski (2021). That modularity could potentially be represented with inconsistency-adaptive logics (see Batens, 1999, 2000).

Adaptive logics were already created with the weakly psychologistic motivation of applying them to actual human reasoning:

> Adaptive logics are intended to explicate actual forms of reasoning and only their dynamic proofs provide one with such an explication.
>
> (Batens, 2001, p. 47)

> The origin of adaptive logics does not lie in any technical insights, but in an attempt to explicate reasoning processes that occur in actual reasoning, both everyday reasoning and scientific reasoning.
>
> (Batens, 2007, p. 222)

Adaptive logics are defined as systems that dynamically adjust themselves to the premises of different reasonings. From a semantic perspective this means that depending on the presence or absence of *abnormalities* in the premises, different models of these premises are going to be selected. For example, an inconsistency-adaptive logic is going to act differently if there are some premises that take the form of: $(p \wedge \neg p)$

than it would if the set of premises was consistent. In particular, it will prevent the conclusions from *exploding*. From a proof-theoretic perspective, contingent on the same type of *abnormalities* in premises — different rules of inference may hold or be suspended.

The idea of adaptive logics seems to be a perfect fit for symbolically representing the modularity of the human mind. The alternative — creating countless separate logics to account for a plethora of specific phenomena in human reasoning — is not feasible. In fact, the very essence of adaptive logics — the idea that they are a single cohesive system that switches between rules of inference depending on the premises — may find justification in neuroscientific findings. Wainstein et al. (2023) investigated the neurophysiological mechanism behind perceptual switches — a phenomenon where ambiguous stimuli can be interpreted in many ways. A famous example of such a situation is the rabbit-or-duck drawing from 1892 (Figure 1). At any given moment, people are able to see the very same scribbles as either a duck or a rabbit — but not both at the same time. Wainstein et al. (2023) determined what happens when a person switches from one meaning to another and they showed that it involves a large-scale network reconfiguration mediated by noradrenaline. In other words, shifting between different modes of seeing the world is not a small tweak that just changes the meaning of one small stimulus — rabbit to duck. Instead, it is a brain-wide shift in neural communication, with certain brain areas being targeted by excitatory noradrenaline in order to increase their activity. As such, it can be understood as a significant change in the global perspective/framing/perception of the world in a given situation. Noradrenergic communication is known to be one of the key elements of human attention and reasoning (see Aston-Jones and Cohen, 2005; Wainstein et al., 2022). It may be hypothesized that large-scale shifts in brain activity mediated by noradrenaline underlie our ability to employ different rules of inference adaptively, as the premises change (Freeman, 1998). This is why in this article we argue for a coordinated effort to create a single applied logic useful for cognitive science (which may have to be an adaptive logic), instead of separate logics for different cognitive modules.

5.2. Non-Fregeanity, a.k.a. content-sensitivity

In Section 3 we signalled that the paradoxes of material implication are one of the many issues that prevented classical logic from providing an

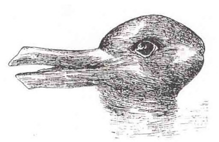

Figure 1. Duck-or-rabbit, an ambiguous stimulus used by L. Wittgenstein to differentiate between *seeing that* and *seeing as*. In modern cognitive science this distinction still has merit, since at first, the brain discerns basic physical properties of the object (*seeing that*) and then integrates them into the whole which assigns them meaning (*seeing as*) (Hart, 2015).

accurate account of human reasoning. They are caused by the fact that in classical logic (as well as in most non-classical logics) the antecedent in an implication can be unrelated to the consequent. When humans process information, the *relevance* of premises to each other is necessary and preserved. In other words, under typical circumstances, humans judge a reasoning as valid only if all the premises used in that reasoning *are related to each other*. What it means exactly and why it is true can be explained with an analogy between neuroscience and linguistics.

In linguistics, when researchers operationalize the *meaning* of symbols (words, phrases, etc.), they turn to a corpus-based analysis of frequencies relevant for those symbols. In other words, they automatically analyze enormous amounts of human-produced writing and determine how often different symbols (e.g., words) occur, where they occur and what other symbols they co-occur with (see, e.g., Bybee and Hopper, 1997; Barlow and Kemmer, 2000; Ellis, 2002; Goldberg, 2006).

> [...] corpus-linguistic analyses are always based on the evaluation of some kind of frequencies, and frequency as well as its supposed mental correlate of cognitive entrenchment is one of several central key explanatory mechanisms within cognitively motivated approaches.
>
> (Gries, 2009, p. 1226)

As a result, a *semantic network* is created where the meaning of symbols can be defined through their relation to every other symbol in their language (Sowa, 2014). As such, the *relevance* of symbols to each other can be expressed by their distance from each other in the semantic network. This is important for the logic of human reasoning because neuroscience identified similar semantic networks within the human cognitive architecture (Marupaka et al., 2012). In fact, researchers are already able to study whether people consider two symbols to be related purely by analyzing their brain activity while perceiving those symbols.

When a sentence begins[4] and the words become perceived the neural semantic network encoding their meaning becomes activated. However, to perceive the word 'grandma' does not mean that some specific 'grandma neurons' become active (Galus, 2022). Instead, a whole network encoding related information becomes active. The degree of activation for any given part of that network is a function of its *relevance* to the meaning of 'grandma'. This is explained in large part by the past experiences of co-occurrences of the word 'grandma' with other words (hence the analogy with linguistic semantic networks). Thus, if a sentence starts by saying: 'My grandma went to the Christmas market ... ' a large neuronal semantic network becomes active encoding the meaning of these individual words. Due to that activity, comprehending the sentence 'My grandma went to the Christmas market to buy presents' would be easy — the word 'presents' would already have been activated due to its presence in the semantic network encoding 'grandma' and 'Christmas'. As a result, not much additional neuronal excitation would be required to complete the meaning of the whole sentence. However, if instead the sentence would unfold in an *unexpected* manner: 'My grandma went to the Christmas market to buy a rocket' — the word 'rocket' would not have been pre-activated and would require a relatively separate semantic network to be active for comprehension. This can be detected with contemporary neuroimaging techniques (Kutas and Federmeier, 2011).

This phenomenon is robust enough and our analytic techniques are reliable enough that it already found application in marketing (Gorin et al., 2022). Researchers and corporations are able to determine what associations customers have with certain commercial brands. For example, Camarrone and Van Hulle (2019) investigated the overlap between semantic networks encoding the brands: "Netflix" and "Rex & Rio" (an online video streaming platform and a detective TV-series). Analyzing brain activity of the participants was able to demonstrate that "Netflix"

[4] The *beginning* of a sentence can be understood normally within any modality in which people could perceive it: reading written language, hearing spoken language, etc. However, in cognitive science experiments, sentences are typically presented word-by-word — displayed on a screen or played as a recording. Thus, researchers control exactly the timing of words in order to be able to match brain activity to the exact word it is supposed to represent.

is associated strongly with "fun" and "theater", but not with "luxury" or "cool".

This research strongly suggests that a logic applied to human cognition ought to take *relevance* into account. That very requirement as well as the problems caused by using extensional implication for modeling human cognition motivated logicians to develop relevance logics (Mares, 2006). Some of them introduced the idea of *variable sharing* to avoid the paradoxes of material implication (see, e.g., Anderson and Belnap, 1975; Kielkopf, 1977; Read, 1988). The weak version of variable sharing postulates that the antecedent and consequent in an implication must *share a variable* (i.e., some information/content must appear in both). Such an implication becomes intensional (necessarily true in all possible worlds), which unfortunately brought some new problems of its own. For example, if a knowledge operator K is introduced then the standard postulates for knowledge are not satisfied because neither: (if $\models A \to B$, then $\models KA \to KB$), nor (if $\models A$, then $\models KA$), or ($\models \neg(KA \wedge K\neg A)$. This would mean that if something is a logical truth then every user of language would necessarily have to know it, which is definitely not a desirable property. Another issue with intensionality was elucidated by M. Cresswell:

> It is well known that it seems possible to have a situation in which there are two propositions p and q which are logically equivalent and yet are such that a person may believe the one but not the other. If we regard a proposition as a set of possible worlds then two logically equivalent propositions will be identical, and so if 'x believes that' is a genuine sentential functor, the situation described in the opening sentence could not arise. I call this the paradox of hyperintensional contexts. (Cresswell, 1975, p. 25)

The strong version of variable sharing takes it one step further and requires that consequent may contain only variables which occur in the antecedent (for an overview of different types of variable sharing, see Estrada-González and Tanús-Pimentel, 2021), but it still does not eliminate all counter-intuitive implications. For example, the formula: $[\neg p \wedge (p \vee q)] \to q$, is classically valid and fulfills even the strong version of variable sharing known as the proscriptive Parry principle (Parry, 1968), but is not accepted by users of natural language (see also Dunn, 2015).

These issues prompted the emergence of other relevance logics which employ the possible worlds approach (see Routley, 1982; Hintikka, 1975;

Rantala, 1982) and hyperintensional logics (hyperintensionality is the notion that two sentences can have different semantic correlates not only when they have the same logical value, but even when they are logically equivalent) — P. Tichý's Transparent Intensional Logic, where meaning/concept are considered to be abstract procedures (see Tichý, 1969; Duží et al., 2015); H. Leitgeb's (2019) system of hyperintensional logic HYPE; I. Sedlár's (2021) hyperintensional modal logics with an equivalence connective representing identity of content.

As such, we build on the work of many relevance logicians who invented various techniques for modeling implication used in natural language, especially if we include the later works that embraced hyperintensionality. We believe that a psychologistic goal is not foreign to relevance logics, given their history of trying to establish what *relevance* as a general construct is (Dunn, 2015). Consider how Wilson and Sperber (2004) defined *relevance*, a definition we happily accept, since it encompasses the user of language, their prior knowledge and how they process information:

> Any input (a sight, a sound, an utterance, a memory) is relevant to an individual when it connects with previous information they have to yield conclusions that matter to them: say, by answering a question they had in mind, improving their knowledge on a certain topic, settling a doubt, confirming a suspicion, or correcting a wrong belief. In terms of our theory, an input is relevant to a person when its processing in the context of a set of previously available assumptions produces a POSITIVE COGNITIVE EFFECT. A positive cognitive effect involves a significant difference to a subject's mental representation of the world: a true conclusion, for example.
>
> (Wilson and Sperber, 2004, pp. 239–240)

Relevance represents the first facet of content-sensitivity that a logic intended for application to human reasoning should respect. The second facet involves the aspects of truth-functionality and *Fregeanity*. Fundamentally, most logics — including relevance logics — are truth-functional and *Fregean*. Truth-functionality is a characteristic indicating that the truth value of compound propositions is calculated as a function of the truth values of its constituent propositions. Fregeanity, conversely, means that sentences with equal truth-values have identical denotation. This essentially means that, under most logics, if the actual content of all propositions is replaced with their truth values (typically 1 for truth and 0 for falsehood, with many-valued logics providing additional options), the system will function identically. Consequently, logic is transformed into a formal system representing a restricted set of information (most of the time truth and falsehood, but many-valued logics may employ more). From a perspective aspiring to construct an applied logic informed by neuroscience and cognitive science, this could be viewed as a limitation, since there is no evidence suggesting that humans evaluate the truth values of compound statements by deconstructing them into their elements and analyzing the interrelations of their truth-values.

This logical emphasis on truth-values becomes less problematic when we consider another body of work in formal linguistics related to a principle that G. Frege formulated: *the principle of semantic compositionality* (Pelletier, 1994). This is a fundamental concept in linguistics, asserting that the meaning of a complex expression is determined by the meanings of its individual components and the manner in which they are combined. In simpler terms, this principle suggests that one can understand the meaning of an entire phrase or sentence by understanding the meanings of its parts and the rules used to combine them. This foundational idea plays a pivotal role in linguistics, serving as a guideline for analyzing and constructing meaning in language. As such, much work in formal linguistics has been devoted to understanding how exactly meaning can be constructed and how this process can be symbolically represented (Hodges, 2001), without the overemphasis on truth-valuations typical in logic.

Many attempts have been made to discredit semantic compositionality, but with little success (Werning et al., 2012). Some arguments against it are somewhat psychologistic in nature, noting that humans tend to “fill in the gaps” in understanding messages with interpretations not explicitly contained within them. These arguments are often countered by highlighting that the principle asserts that meaning is derived from the symbols and *the way they are combined*. Thus, if different agents interpret the message differently, they may attribute slightly different meanings to it. However, such objections to semantic compositionality can be addressed by adopting a physicalist understanding of what meaning is (or more precisely: the act of comprehending meaning). Specifically, the *meaning* of a symbol (or a composed set of symbols) is the act of it being processed by an agent (either the sender or the receiver of the message).[5] Consider the example sentence: ‘My grandma went to the Christmas market to buy presents’. We know that the activation

[5] This physicalist understanding of meaning can still accommodate the intersub-

of neuronal ensembles encoding the meanings of *grandma* and *Christmas* necessarily involves partial activation of ensembles encoding related concepts. Therefore, within the very act of a person comprehending the meaning of the word *grandma*, the associated notion of *presents* is implicitly present. This provides scope for the human tendency to "fill in the gaps" evident in metaphors, sarcasm, idioms, and other related linguistic phenomena.[6]

In neuroscience, we understand that the processes of sentence comprehension and truth-determination are invariably tied to the content, or information, of those sentences. In this context, individuals can comprehend sentences without necessarily assigning them any truth value. If required to attribute a truth value to a statement, they might employ diverse rules depending on the nature of the content being evaluated. For instance, cognitive neuroscientists have pinpointed specific neural networks that become active when determining truth values for counterfactual versus factual statements. Counterfactual statements address a hypothetical scenario (e.g., 'If N.A.S.A. had not developed its Apollo Project, the first country to land on the moon would have been Russia/America'), as opposed to factual statements that reference actual events (e.g., 'Because N.A.S.A. developed its Apollo Project, the first country to land on the moon was America/Russia'). Nieuwland and Martin (2012) examined brain activity in response to both counterfactual and factual statements and found that prior knowledge did not impede the assignment of truth values to counterfactual statements. Essentially,

jectivity of meaning by operationalizing it as the shared patterns in which multiple agents process the message/symbol.

[6] This interpretation of cognitive compositionality assumes that the context in which symbols are perceived (e.g., the body language of the message sender) forms an integral part of the message itself and can be symbolically represented. However, when many researchers formalize meaning, they distinguish symbols from their context. If symbols and their context are treated separately, cognitive neuroscience encounters challenges with compositionality. A comprehensive discussion of this issue is provided by Baggio (2021), who argues that compositionality, as commonly understood, is not a universally applicable principle of language comprehension, but rather a specific capability of the human language system. "[The principle of semantic compositionality] is an abstract, high-level, general statement on a specific aspect of semantic competence: the capacity to assign, to (some) complex expressions, meanings that are a function only of the meanings of the parts and of the expression's syntactic form [...] It is no longer a principle applying to language or to linguistic theory as a whole, but a computational constraint on one processing phase of four, in one processing stream of two, in the brain's language system" (Baggio, 2021, pp. 20–21).

when individuals assessed the truth-value of a statement about a hypothetical scenario, their judgement was uninfluenced by their existing knowledge of real-world events. Consequently, Nieuwland and Martin (2012) presented preliminary evidence suggesting the engagement of separate neural networks when attributing truth values to sentences, contingent upon the complexities of their content. This reinforces the notion, introduced in Section 5.1, that modules processing varied sentence types could potentially operate under distinct inferential rules (adaptivity).

This hegemony of content and information in cognitive science stands in stark contrast to *Fregeanity* of most logical systems.[7] Logicians began their attempts at moving beyond that paradigm for the first time only a century after Frege's anti-psychologistic publications. They were hesitant to develop non-Fregean systems because they believed that they could not be properly algebraized, but that view was about to change (Béziau, 1997).

R. Suszko, the progenitor of that new movement, rejected Frege's axiom ("all true sentences have the same common referent, and similarly all false sentences also have the one common referent" (Frege, 1892)) when creating his own logic — *The Sentential Calculus with Identity* (*SCI*) (see Bloom and Suszko, 1972, 1975; Suszko, 1968, 1971). In his logic Suszko chose *situations* as semantic correlates of sentences, a notion that paralleled developments in the realm of artificial intelligence around the same time. McCarthy and Hayes (1969), in their seminal work on situation calculus, also introduced the concept of *situations* to represent distinct states or configurations of the world. Their use of situation variables is analogous to time-instants in A. Prior's (1968) U-T calculi and borrowed inspiration from Kripke's (1963) modal logic and his revolutionary introduction of possible worlds semantics. The idea of considering *situations* or *worlds* as distinct instances or snapshots of reality allowed for a deeper examination of how states change over time, offering a nuanced alternative to traditional truth-functional approaches. It was further developed by Perry and Barwise (1983) who described situations as providing only partial information about the world, making them better at capturing the context-dependent nature of meaning in natural language. However, Suszko's approach takes a step further in the direction of hyperinten-

[7] It is important to note that psychologistic thought has become deeply unpopular after Frege's work. Before it was developed by Mill (1843), Lipps (1893), Heymans (1905), Wundt (1883), Jerusalem (1905), Sigwart (1904), Elsenhans (1897), Erdmann (1892).

sionality (or content-sensitivity) since situations are elevated to the role of semantic correlates, not only domains over which truth-values are evaluated.

Thanks to Suszko there is already some work on representing information in logic with semantic correlates that move beyond simple truth values. His work was picked up and developed further by other logicians. For example, Łukowski (1997, 2019) developed the *Contentual Classical Logic* (*CCL*) by strengthening classical logic with formulas defining a non-Fregean implication connective. However, Łukowski's non-Fregean implication connective was recently separated entirely from classical logic to achieve three key properties (see Łukowski, 2022):

1. To be defined semantically and not syntactically.
2. To be contentual and not truth-functional.
3. To be defined by singular models and singular mappings instead of classes of models and classes of mappings.

Independently, Grzegorczyk (2011) inspired by *SCI* created the non-Fregean *Logic of Descriptions*, which is now being further developed by J. Golińska-Pilarek and T. Huuskonen (see Golińska-Pilarek, 2016; Golińska-Pilarek and Huuskonen, 2017).

These non-Fregean logics may form the foundation for logic applied in cognitive science, since they are more flexible at representing content-driven variation in human reasoning. However, this strong emphasis on connecting semantic correlates to contents of sentences also has its pitfalls. Namely, the potential for *maximal hyperintensionality.* In a maximally hyperintensional non-Fregean logic, no two sentences have different transcriptions but the same semantic correlate. Suszko's *SCI* is an example of such a system. In *SCI* there are no two sentences with the same semantic correlate, for example, α is not identical to $(\alpha \wedge \alpha)$, and $(\alpha \wedge \beta)$ is not identical to $(\beta \wedge \alpha)$. As such, maximally hyperintensional logics would be almost impotent in making generalized statements, which are necessary for formulating empirical predictions and testing them. Berto and Nolan (2021) warn against such situations:

> Propositions may be more fine-grained than sets of possible worlds, but they had better not be mappable 1:1 to the sentences expressing them. (Berto and Nolan, 2021, p. 1)

Fortunately, hyperintensional logics do not have to be maximally hyperintensional. F. Berto (2019) created a system of *Hyperintensional Belief Revision.* In that system, he also clearly defined the boundaries that prevent hyperintensionality from trivializing its logic. Thus, the task of psychologising non-classical logicians is to define the conditions when two instances of cognitively processed information should be formalized as equivalent and when not. This process must be dictated by the desired utility of the logical system for predicting human behavior — not its pretense at grasping some kind of *absolute truth*.

The Fregeanity of most logics was one of the key reasons why logic and cognitive science drifted apart. P. Johnson-Laird (2010), one of the most prominent researchers of human reasoning saw them as very distant from each other. However, a deeper analysis of the arguments against importing logic into cognitive science reveals that their authors argue rather against Fregeanity and using classes of models and mappings, not logic as a whole:

> The mistake is to import logic directly into psychological theory, and to assume that the mental processes of everyday reasoning extract the logical form of premises, and use it to reason. [...] One reason for the difficulty of logical analysis is that everyday reasoning depends on the meanings of propositions, whereas logic does not. Another is that reasoning depends on knowledge of context, whereas logic does not. And yet another difference is that reasoning depends on general knowledge and beliefs, whereas logic does not. In short, everyday reasoning depends on the meanings of words, general knowledge, and beliefs.
> (Johnson-Laird, 2010, p. 196)

To that, we say that logic can account for the meanings of propositions, context of reasoning and individual differences, as long as it does not attempt to reduce all meaning to a handful of truth-values. In fact, in this article, we keep discussing how logicians independently created a plethora of non-classical logics, each of which carefully accounts for one of these boundary conditions. To address the limitations pointed out by Johnson-Laird (2010) we postulate that applied logicians working together with cognitive scientists should develop a non-Fregean system to accommodate the content-sensitivity of human reasoning.

5.3. Probabilism/fuzziness

The environment in which the human brain evolved involves a constant, high degree of uncertainty. We do not know for certain if any information we possess is true, which is exacerbated by the fact that our senses provide limited insight into the world around us. As a result, all living organisms evolved to deal with that uncertainty. In fact, the so-called *uncertainty reduction* is one of the leading paradigms in studying how humans explore their surroundings for information and form beliefs (Kobayashi and Hsu, 2017). As such, it would have been odd if our brain processed information without assigning it some degree of uncertainty. After all, there are big risks waiting for those who act on information that is unlikely to be true. Neuroscientists identified two kinds of ways in which information can be uncertain: ambiguity[8] and risk (Kobayashi and Hsu, 2017). Risk describes situations where we know the properties of our environment well, but that environment produces unpredictable outcomes anyway (e.g., tossing a fair die). Ambiguity describes situations where we do not know the properties of our environment well enough (e.g., tossing a die that might be fair or weighted). Because ambiguity can be reduced by studying the environment with senses, whereas risk cannot, human brains process them relatively separately — to the point of having different neural correlates (Kobayashi and Hsu, 2017).

Because uncertainty is a property assigned both to our environments and to our cognition, a logic that represents the latter ought to take that into account. Most logics operate only on a handful of possible truth values, which would make it difficult to represent concepts like: *probably true*, *almost certainly true*, or the most terrifying: *50/50 to be true*. The necessity of logically representing these concepts is warranted also by the most basic properties of human neurophysiology. At first glance, human neurons may resemble the simplest logic gates that take inputs (0,1) and produce outputs (0,1). That is because of the *all or nothing* principle. This principle states that neural outputs — the action potentials — of most neurons do not code information with their magnitude. Instead, they either fire (1) if the input was large enough or stay silent (0) if the input did not reach the required threshold. As a result, in most cases, properties of the processed information are encoded with the number and the frequency of action potentials — not their size. However, even at this elementary scale, the apparent simplicity crumbles when we realize that action potentials are generated in a probabilistic fashion (Yang and Shadlen, 2007). For each neuron, we can generate a probability density function, which describes the likelihood that the

[8] The word 'ambiguity' in cognitive science is used in a broader sense than in logic. Logical ambiguity typically does not encompass vagueness, under-specification and generality, whereas cognitive ambiguity does.

neuron will fire when given a certain magnitude of input. For most of them, the probability that they will fire follows the logistic growth function — which means that within a certain range of inputs, the probability of action potential rapidly rises (Yang and Shadlen, 2007). This means that probabilistic processing can be identified at multiple levels of brain organisation. First, at the level of cell physiology, but also at the level of whole-brain network activity (Pouget et al., 2013).

Some logics have already been created to accommodate the way in which the human mind handles uncertainty. For example, in 1965, L. Zadeh created *fuzzy set theory* and *fuzzy logic*.

> One of the basic aims of fuzzy logic is to provide a computational framework for knowledge and inference in an environment of uncertainty and imprecision. (Zadeh, 1992, p. 2)

Fuzzy logic is based on the theory of fuzzy sets, which replaces the dichotomy of belonging or not-belonging to a set, with the *degree of belonging*. As such, in logical terms, propositions can be assigned various *degrees* of veracity or falsity. For example, something can be *entirely true* (1), *entirely false* (0), but also *half true, half false* (0.5) or anything else between 0 and 1. These degrees can be defined with various membership functions formalizing the relation between some empirical data and the assigned logical values/set membership. For example, the classical *sorites* problem of the heap is solved with a membership function. If we have a heap of sand and we take one grain of sand away, is it still a heap? A membership function can describe the relation between the number of grains of sand and belonging to the set of *heaps*.

Fuzzy sets theory is commonly applied to theories in neuroscience as a supplement, to account for the uncertainty of empirical data. For example, it was used to formally represent the *cross-talk* between different cognitive modules when high-order cognitive functions are active (Wallace, 2014). Different modules are then thought to apply their own different fuzzy transformations to the processed information, which then later has to be re-integrated to produce a single output (behavior/conclusion). Fuzzy logic was designed to handle exactly that type of problem.

> Fuzzy logic enables the overcoming of the artificial dichotomy between neurology and psychology that has been created by Aristotelian logic, introducing and organization of physical sets and psychological sets working in continuous complementarity [...].
> (Pinelli, 2006, p. 22); translated by Marchese (2008)

Consequently, fuzzy logic was proposed as a method to account for uncertainty in probabilistic models of behavior (Marchese, 2008) since fuzziness can be described as the ambiguity of a situation (see Ivancevic and Ivancevic, 2007; Kobayashi and Hsu, 2017).

Even though not specifically stated as such, fuzzy sets were present in neuroscience from the time T. Young (1802) described how only three kinds of photoreceptors are able to encode all colors of the visible light spectrum. Since each wavelength value excites each kind of receptor with different strength, they can be formalized as degrees of membership. Similarly classic in the field is the later-developed fuzzy description of neuronal coding for the chemical senses. It describes taste perception in terms of parallel processing where one neuron responds to a broad range of stimuli and the end-product (sensation) is derived from relative activation of the whole cellular ensemble (Erickson et al., 1994). Bearing in mind knowledge about neuronal networks, it might now seem obvious that activity of each particular cognitive module can be formalized as memberships to certain fuzzy sets. For example, a "strongly" active ("strongly" as in the way in which fuzzy sets theory assigns linguistic labels to certain values of set membership) *fight-or-flight* system together with the "strongly" active neural network encoding the meaning of *fire* indicates that the person is probably observing a burning building. In contrast, a "strongly" active parasympathetic nervous system and the oxytocinergic system together with the "strongly" active neural network encoding the meaning of *fire* indicates that the person is probably making smores with friends/family by a campfire.

However, empirical data is required to properly define the structure of these sets. It is said that when the membership to a fuzzy set is not definable by an easily observable numerical value, it is instead taken from the *psychological continuum* — a function that describes the likelihood that a human would describe designate n as belonging to the set A (Ivancevic and Ivancevic, 2007). That continuum can be empirically defined. The idea of an experimental approach to vagueness is not new. In the thirties, M. Black (1937) conducted empirical studies by collecting peoples' judgments about the vagueness of certain expressions. His procedure led to the determination of the so-called *profiles of density* for vague predicates, which are just a different version of fuzzy membership functions. Currently, we can either fall back on these classical ways of operationalizing *degrees of membership* or employ the neuroscientific techniques mentioned in Section 5.2, where we described how neural semantic networks encoding different meanings are detected. These techniques would operationalize meaning as patterns of brain activity and *degrees* of similarity between them.

6. Conclusion

The 21st century has witnessed an increasing interest among logicians in applying logic to cognitive science. Many of them have published works attempting to formalize some aspects of actual human reasoning (see, e.g., Pfeifer, 2013; Batens, 2007). However, there is still a big knowledge gap between logic and cognitive neuroscience. While cognitive scientists have limited exposure to the advancements of symbolic logic, logicians lack access to empirical neuroimaging data. As a result, logic succeeds only at symbolically representing select, isolated aspects of human reasoning. Certain logics are well-equipped to account for the content of reasoning (Łukowski, 1997), while others handle its adaptivity (Batens, 2007) or fuzziness (Zadeh, 1992). We believe that the time has come for the *Human Logic Project.* Inspired by the coordinated interdisciplinary efforts of the *Human Brain Project* (Human Brain Project, 2015), we suggest that logicians and cognitive scientists collaborate to develop a logical system of human reasoning.

A psychologistic logic, designed specifically to model genuine instances of human reasoning, would parallel other theories in cognitive science. It would produce falsifiable predictions about behavior and neurophysiological responses. The unique focus of logic on the form of reasoning, rather than its content (as even content-sensitive logics do), could provide a distinctive perspective on the information processing behind cognitive biases and heuristics. Such an insight might find that various biases observed across different cognitive and emotional domains have a common underlying logical structure. This is useful because as cognitive and social sciences primarily highlight the content of reasonings in their theories, they often introduce unnecessary theoretical constructs.

Consider several biases that are extensively studied due to their influence on politics and broader societies: ethnocentric bias (prioritizing one's own group), in-group bias (viewing one's own group more positively), and hostile-media bias (perceiving media reports as slanted against one's *own side*). Researchers have delved into these heuristics across varied and distinct fields of study, leading to the proposal of numerous unrelated theories explaining their origin. However, a recent study by a team of psychology researchers revealed that these biases, along with many others described in different scientific fields, share a common structure (Oeberst and Imhoff, 2023). This underlying structure is succinctly encapsulated as *confirmation bias*, the tendency to interpret new information in a manner aligned with pre-existing beliefs.

Oeberst and Imhoff (2023) explain that the theory of confirmation bias can predict and explain many seemingly unrelated biases by identifying the core belief being preserved. For instance, the core belief *"I am good"*, combined with confirmation bias, can result in the *self-serving bias* — the tendency to attribute one's failures to external factors and successes to internal ones (Mullen and Riordan, 1988). Similarly, the core belief 'My group is good' alongside confirmation bias helps explain why people might be less receptive to criticism when it emanates from an in-group member, a phenomenon termed the *intergroup sensitivity effect* (Hornsey et al., 2002). By pinpointing this shared structure among many biases, (Oeberst and Imhoff, 2023) present new hypotheses about ways to mitigate their adverse impacts. The Human Logic Project seeks to make parallel strides, aiming to integrate existing data by identifying shared mechanisms across different cognitive domains, all while leveraging the logical focus on form over content without entirely dismissing the latter.

This potential of logic extends beyond just higher-order cognitive processes. For instance, a fundamental belief in the neuropsychology of visual perception is that human unconscious visual processing operates inferentially (Aggelopoulos et al., 2015). This perspective treats sensory inputs and prior knowledge as premises, with the architecture of the neural visual system dictating the rules of inference. Ritchie (2022) delves into the arguments supporting the idea that a visual sensation serves as the *conclusion* of an unconscious inferential process.[9] Two main points support this position: Firstly, sensory inputs are inherently underdetermined (i.e., incomplete), yet humans reliably perceive a consistent image of their surroundings and swiftly attribute meaning to visual experiences. Secondly, visual perception exhibits *invariance*, implying that we perceive a consistent image and ascribe meaning to visual experiences, despite sensory inputs never being exactly replicated. In layman's terms,

[9] The term 'inference' is interpreted differently in logic and cognitive science. In logic, inference is typically defined in a strict sense as a deductive relation. In cognitive science, inference might encompass any form of reasoning, be it deductive, inductive, or abductive. In this context, the latter perspective is adopted.

we can easily recognize an object regardless of variations in lighting, viewing angles, or distances. To explain this consistency, it's hypothesized that an inductive unconscious inference is at work (Ritchie, 2022). Consequently, neuroscience is positioned to classify certain neural activations (like sensory inputs at the receptors) as premises and others (such as activity in integrative sensory areas) as conclusions. The remaining challenge is discerning which inferential processes in the brain are better suited for logical modeling versus, say, Bayesian modeling. This is an area ripe for exploration, especially since there is a deficit of research bridging logic and neuroscience.

The ultimate vision for the Human Logic Project is to engineer an adaptive logic capable of addressing a diverse array of problems — including deductive, inductive, probabilistic, and modal challenges — and producing solutions that reflect genuine human reasoning. Here, the term "*logic*" is understood in its broadest sense: a system of inference encompassing premises, rules, and conclusions. Our ambition is not merely to craft a single, traditional logic. Instead, we aspire to create an adaptive system, a hybrid construct that fuses multiple inferential frameworks, each tailored for specific reasoning tasks. This system would synergistically integrate rule-based logical analysis, non-Fregean content-sensitivity, and the nuanced probabilistic capabilities of fuzzy logic. Through this multifaceted approach, we aim to achieve both structural rigor and adaptability, thus accounting for the intrinsic uncertainties and context dependencies that characterize human cognition. The project's success would be gauged in two primary ways: firstly, through an evaluation of its logical coherence; and, secondly, by its efficacy in predicting human behavior. However, many open questions remain. For instance, should such an adaptive logic have non-monotonic and paraconsistent properties? Such questions can only be answered through a collaboration between logicians and cognitive scientists.

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