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An Expressivist Strategy to Understand Logical Forms

Abstract. This paper discusses a generalization of logical expressivism. It is shown that, in the wide sense defined here, the expressivist approach is neutral with respect to different theories of inference and offers a natural framework for understanding logical forms and their function. An expressivist strategy for explaining the development of logical forms is then applied to the analysis of Frege's *Begriffsschrift*, Gentzen's sequent calculus and Belnap's display logic.

Keywords: logical expressivism; logical form; Begriffsschrift; sequent calculus; display logic

1. Introduction

Andrea Iacona (2018) notices that logical forms are commonly conceived as servants of two masters, so to speak: on the one hand, they are ascribed with the logical function of explaining why certain inferences are to be considered as logically valid; whilst on the other hand, they are ascribed the semantic function of explaining why the meaning of complex linguistic expressions depends on the meaning their components. Iacona further argues that these functions are incompatible, so that the standard understanding of logical forms is incoherent and should be revised. It is the purpose of this paper to suggest that, in fact, it is possible to characterize a minimal, but unitary notion of logical form. I intend to do that by discussing the way in which logical forms can be understood from the point of view of logical expressivism.

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I shall defend two claims:

1. Logical expressivism offers a natural framework for understanding logical forms and their function.
2. The core expressivist thesis is neutral with respect to different and possibly incompatible theories of inference.

The first of these claims should be clear enough in its formulation: by "natural", here, I simply mean that the framework offered by logical expressivism is intuitive, effective and in accord with the traditional conception of logical forms. The second claim, on the other hand, requires some qualification. Let us start by acknowledging that logical forms are essential to the account of logical consequence. It is also important to notice, however, that logical forms only allow us to say *that* a formula A logically follows from other formulas Γ, while it is the job of a theory of inference to explain *why* the inference from Γ to A is valid. It is crucial to distinguish these two roles. I will argue that logical forms have the *expressive* role of making explicit inferential properties, whereas a theory of inference has the *explanatory* role of accounting for those inferential properties. There are, of course, different theories of inference that one could endorse; my claim is that the use of logical forms as expressive resources does not commit one specifically to any of them.

I shall proceed as follows. First, I define logical expressivism and characterize an expressivist strategy to understand logical forms. Then I apply such a strategy to the analysis of three cases: Frege's *Begriffsschrift*, Gentzen's sequent calculus and Belnap's display logic. Finally, in the light of this analysis, I discuss some of the consequences of the expressivist account of logical forms.

2. The expressivist point of view

Logical expressivism was originally defined in the context of normative inferentialism (Brandom, 1994, 2008; Peregrin, 2014). The core idea, however, can be more generally expressed in the following way:

(LE) Logical vocabulary is characterized by the expressive role of making explicit, in the language, properties of the inferences between the contents of non-logical vocabulary.

The most paradigmatic example that is used to illustrate logical expressivism is the case of conditionals. So, suppose one's theory of inference says that B follows from A. Then if one has the expressive resources of conditionals in one's vocabulary, one can use the formula "$A \to B$" to say in the language that the inference from A to B is a valid one.[1]

It is important to emphasize that a conditional says *in the language* that an inference is valid. Clearly, the formula "$A \to B$" does not have the same content as the metalinguistic expression "$A \vdash B$": where the deduction metatheorem is valid, the correct equivalence is between "$A \vdash B$" and "$\vdash A \to B$" *in the metalanguage*.[2] Notice also that treating an inference as a valid one is something that is only done implicitly in a logical system in which inferring from A to B is allowed. Obviously, one does not need conditionals to say that the inference from A to B is valid. One can say it in a pragmatic metalanguage – e.g. by saying that it is *correct* to infer from A to B – or in a semantic one – e.g. in the standard model-theoretic account of consequence relations. The point, however, is that it is not possible to say that B follows from A *in the language* unless the language contains the expressive resources of conditionals to make that content explicit as a formula. The reason why it is important to say it in the language is that once inferential properties are made explicit as the content of a formula, then it is possible to apply rules to that formula and draw inferences from it. Indeed, the value of the expressive role of logical vocabulary lies not just in what contents are expressed, but especially in the fact that once those contents are made explicit as formulas in the language new things can be done implicitly.

The expressivist reading of conditionals described here can be generalized to all the expressive resources that allow one to make explicit properties that contribute to determine the validity of inferences. These properties are established by a theory of inference. Different logics can be defined by making explicit inferential properties in different theories of inference. Yet, as is affirmed in Claim (1), the expressivist reading

[1] Here, the symbol "$\to$" generically indicates a conditional. Depending on the inferential theory that is endorsed, obviously, different kinds of conditionals can be characterized by applying this expressive strategy.

[2] Brandom (2008), for instance, defines logical vocabularies as pragmatic metavocabularies of a special kind. On the one hand, they are universal: by making explicit the game of giving and asking for reasons, they allow one to specify what one must be able to do to deploy any vocabulary at all. On the other hand, the practices required to deploy logical vocabularies can be elaborated just from the basic game of giving and asking for reasons. Unlike the semantic case, therefore, both an object vocabulary and its pragmatic metavocabulary may belong to the same language.

of logical vocabulary will clearly remain the same for any of them. In the following sections, this expressivist point of view will be adopted to look at three cases in which new logical forms were developed to prove significant proof-theoretic results. This review will allow us to vindicate Claim (2) as well.

3. Frege's *Begriffsschrift*

The definition of propositional connectives by Gottlob Frege is the motivating example in the original presentation of logical expressivism by Brandom himself. Indeed, there are explicit indications in Frege's work that the development of a "concept script" in symbolic logic can be interpreted as an expressive enterprise on his part. For instance, in the article "Boole's Logical Calculus and the Concept-script," written shortly after the *Begriffsschrift*, he explains: «I sought as far as possible to translate into formulae everything that could also be expressed verbally as a rule of inference» (Frege, 1979, 37).

The problem with applying the expressivist strategy to Frege is that he defines no explicit theory of inference. Frege is characteristically more interested in making inferences as explicit as possible than defining what inferences consist in (see Ricketts & Levine, 1996). However, he certainly has a solid, implicit understanding of the validity of inferences. The most obvious way to extrapolate it is to look at how he explains the rules of inference that he himself uses in his calculus. As is well known, the only inferential rule contained in the *Begriffsschrift* is *modus ponens*. In order to explain why *modus ponens* is a valid rule of inference, Frege says that it is not possible to judge a conditional and its antecedent without also judging the consequent (Frege, 1879, 15–16).[3] When he talks about possibilities in this case, he has in mind the possible permutations of the assertion and the denial of two contents. In more general terms, his idea could be put by saying that an inference is valid if there is no possible case – i.e. no permutation – in which the premisses hold and the conclusion does not.

[3] Frege will repeat the same idea in the definition of his "methods of inference" in the *Grundgesetze* (Frege, 1893/1903, 57–64). Here, the formulation is different because he does not talk about assertions and denials of contents of possible judgments anymore and applies instead the distinction between sense and reference: therefore, he says that his rules are correct because it is not possible for their premisses to be the True and the conclusion to be the False.

With respect to this theory of inference, at the level of propositional contents Frege has three essential expressive needs: saying that a content holds, saying that a content must hold if another does, and saying that a content cannot hold if another does. Clearly, these are the expressive roles of the judgment-stroke, the conditional-stroke, and the negation-stroke, respectively. By introducing this logical vocabulary, Frege provides himself with the resources to express logical truths in his formal language, i.e. contents that hold in virtue of the inferential properties expressed by logical constants.[4] Let us review them one by one.

A judgment-stroke placed before the sign for a content *A* expresses the judgment of *A*. A judgment, Frege explains, is «inwardly to recognize something as true» (Frege, 1979, 2). The judgment-stroke, then, is required to make explicit that a content holds. A content is something that can hold, or, as he will later come to say, a thought. The fact that a content holds has a crucial significance in Frege's theory of inference: it is only when a content holds that it makes a difference with respect to the validity of inferences, because it is only when a content holds that it excludes possible cases. It is for this reason that inferences can be drawn only from judged contents in his view. And yet, Frege notices that there are some cases in which it must be possible to express a content without asserting it. These cases are precisely those in which the other two strokes are applied. We will return to this distinction after the discussion of the two latter notations.

Let us consider the definition of the conditional-stroke first. Frege introduces it in §5 of the *Begriffsschrift*. Here is where he lists all of the four permutations of assertions and denials of two possible contents of judgments *A* and *B*. He then establishes that the judgment of a conditional-stroke corresponds to the denial of the case in which the antecedent *A* is affirmed and the consequent *B* is denied. In this sense, the conditional-stroke makes it explicit that the content *B* must hold if the content *A* does. As he admits, there is no particular reason for choosing the denial of that specific permutation as far as the calculus is concerned, given that there are other ways to exclude possible cases. More precisely, as we know, there are sixteen Boolean functions for two contents that may either hold or not hold, and any of them would do to

[4] There is, in fact, another expressive resource that Frege introduces in his *Begriffsschrift*, namely the concavity notation to express generality. Danielle Macbeth (2005) gives an excellent analysis of the expressive needs that Frege intended to satisfy with a notation for generality in the wider context of his logicist enterprise.

contribute to the definition of a functionally complete set of connectives in the calculus. Yet not all of them have an expressive role.

When it comes to justifying his choice, Frege often says that it is the easiest to use "in inference". This means that the choice is not significant so much for the calculus as for the practice of drawing inferences, but it is still quite vague. In "Boole's Logical Calculus and the Concept-script", however, he adds an important clue: he says that he adopted this interpretation of the conditional-stroke «because its content has a close affinity with the important relation of ground [*Grund*] and consequent [*Folge*]» (Frege, 1979, 37). It is easy to see what the affinity is: the denial of the case where the antecedent A is affirmed and the consequent B is denied is precisely the condition for the inference from A to B to be valid in Frege's theory of inference. This clue explains how he thinks the conditional-stroke to be expressively related to inference: the conditional-stroke makes it explicit as the content of a possible judgment that an inference is valid.

The negation-stroke is the third and last of the expressive resources of the *Begriffsschrift* that we consider here. Negation is required because it must be possible to deny contents. More precisely, it must be possible to say that a content cannot hold if another does: this is essential to make it explicit in the language that an inference is not valid, i.e. that the premisses may hold together with a content that cannot hold together with the conclusion.

There is, however, an interesting complication at this point, because there are in fact two strategies for making explicit that a content cannot hold if another does: this can be done either by admitting negative judgments or by admitting judgments of negative contents. Frege, however, thinks that the former is nonsense. Once the distinction between sense and reference will be firmly under his belt, he will explain that it is because a judgment is already «a choice between opposite thoughts» (Frege, 1979, 189). According to this idea a negative judgment is the judgment of a negative content and the opposition between judgments is explained on the basis of the opposition between judged contents. However, that this is the only sensible approach is not yet obvious in the framework of the *Begriffsschrift*, where a content is only the content of a possible judgment: why could not it be also the content of a possible negative judgment? Although Frege never mentions it,[5] there is an expressivist

[5] Frege (1919) officially insists that judgments cannot be iterated and therefore

reason for that. If the fact that a content cannot hold if another does is made explicit by means of a negative judgment, then the opposition between positive and negative judgments themselves will remain implicit and not expressed in the language.

This last remark highlights an important difference between the expressive roles of the judgment-stroke and the other strokes. Conditionals and negations make explicit inferential properties as contents in the language that can be judged and used to draw inferences. Judgment expresses nothing of the sort. In fact, a judgment-stroke cannot be preceded by other strokes. In particular, it cannot be preceded by another judgment-stroke, meaning that it does not express a content in the language. In the *Begriffsschrift*, therefore, the expressive need to say that a content holds is not addressed on the same level as the others. From the expressivist point of view, the distinction that Frege draws between a sign for content and a sign for judgment can be read as a distinction between two kinds of expressive resources that belong respectively to the language and the pragmatic metalanguage of the calculus.

4. Gentzen's Sequent Calculus

It is well known that Gentzen develops sequent calculus to prove his *Hauptsatz*, by overcoming what he thinks to be shortcomings in the logical form of his natural deduction calculus for classical logic (Gentzen, 1934/35, 69). Indeed, the way in which the rules of the calculus are defined allows for a more elegant formulation of the problem of establishing the subformula principle as the problem of cut elimination. Yet, subsequent results of normalization will show that the logical forms of natural deduction can make explicit all the inferential properties required to establish the same principle (Prawitz, 1965; Negri and von Plato, 2001). And in any case, there is an even more obvious expressive feature of sequent calculus that is always universally recognized: sequents make explicit inferences in natural deduction. Prawitz, paradigmatically, notices that «[t]he calculi of sequents can be understood as meta-calculi for deducibility relation in the corresponding systems of natural deduction» (Prawitz, 1965, 90). Here, he uses the notion of "meta-calculus" in a very specific sense. He intends to point out that if a sequent is understood as

rejects the account of negation in terms of negative judgments on the grounds that it could not make sense of embedded negations.

expressing an inference in natural deduction, then the rules of sequent calculus can be seen as instructions to construct proofs in natural deduction. More specifically, he notices that the right rules of sequent calculus govern how a proof in natural deduction can be transformed “on the bottom” by applying introduction rules, while the left rules of sequent calculus govern how a proof in natural deduction can be obtained as a transformation “on the top” by applying elimination rules.

However, the expressive role of sequent calculus is not to be measured only with respect to natural deduction. From the point of view of logical expressivism, in more general terms, sequents are an expressive device that allows one to say explicitly what is implicitly done by treating a sequence of formulas as a proof. The full extent of this achievement might not be obvious at first sight, especially if one only looks at what sequents express explicitly and only sees that a sequent “$\Gamma \Rightarrow A$” can be informally interpreted as “$\Gamma \vdash A$”, but it gets clear as soon as one also considers how sequents are used implicitly in the calculus.

In this sense, it is interesting to compare sequents with conditionals. A conditional “$A \rightarrow B$” makes it explicit that B follows from A. A sequent “$A \Rightarrow B$” makes it explicit that the inference from A to B is a proof of B on the grounds of A. This makes a decisive difference *in the language*: the rules for propositional connectives transform contents into contents, the rules for sequents transform proofs into proofs.[6]

Thanks to this expressive advantage, sequent calculus allows for a proof theory where proofs themselves are the objects of study in a specific, explicit sense (Kreisel, 1971). This can easily be seen by comparing cut elimination in sequent calculus with normalization in natural deduction (Prawitz, 1965, 1971). The two results are usually considered as having the same proof-theoretic significance, which is often encapsulated in the subformula principle. Besides, both results are proved by showing that certain derivations are equivalent to one another. It must not be forgotten, however, that in fact they say different things in completely different ways.

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[6] As is well known, Gentzen originally declared that the “informal meaning” of a sequent “$A_1, \dots, A_n \Rightarrow A$” is the same as that of the conditional “$A_1 \wedge \dots \wedge A_n \rightarrow A$” (Gentzen, 1934/35, 82). In a similar way, Hertz had suggested to consider his *Sätze* “$a \rightarrow b$” as «formal “implications” in the sense of Russell» (Hertz, 1922, 12, n. 1). In fact, however, neither Gentzen’s sequents nor Hertz’s sentences behave like implications in their calculi. While both can be intuitively understood as expressing a conditional relation, they have a different expressive role than logical connectives.

The rules of natural deduction establish that certain formulas follow from other formulas. Normalization proceeds by showing that a given procedure for deriving a certain conclusion from certain premisses by applying certain rules can be transformed into another procedure for deriving the same conclusion from the same premisses by applying different rules: it shows that in *doing* the former one is also *doing* the latter.

The rules of sequent calculus already establish that certain proofs can be transformed into other proofs. Cut elimination proceeds then by showing that these transformations can always be performed without losing content because of the cut rule: it shows that in *saying* that the sequences transformed with the cut rule are proofs then one is also *saying* that sequences transformed without the cut rule are proofs. From an expressivist point of view, this is precisely what the admissibility of the cut rule reveals.

5. Belnap's Display Logic

Display logic is a direct expressive development of sequent calculus. In display logic, sequents $X \Rightarrow Y$ are composed of structures X, Y that are built out of formulas by using structural connectives. The way in which structures are constructed determines the inferential properties that are made explicit by logical operators. In fact, substructural logics show how the meaning of logical operators varies depending precisely on how sequences are handled. By making structures explicit in the language, display logic allows to establish rules to operate on and transform them in the calculus.

As is well known, Gentzen's sequent calculus already distinguishes between logical rules applied to complex formulas and structural rules applied to sequences of formulas. So, display logic can be seen as a generalization of the expressive resources required to make structures explicit.

Belnap (1982) notices three main aspects of the use of sequences in sequent calculus that have an inferential significance. First, sequences can occur as antecedents or as consequents in a sequent: formulas occurring in sequences in the antecedents are treated as having a positive polarity, while formulas occurring in sequences in the consequents are treated as having a negative polarity. Second, sequences can group formulas together by means of ",": sequences of formulas grouped together indicate a single content and are treated as conjunctions or disjunctions depending on their polarity. Third, sequences can be empty: empty sequences indicate null contents and again are treated as truth or falsity depending on their polarity.

In light of this, Belnap introduces three structural connectives to make these significances explicit: a unary connective "$*$" to reverse the polarity of a substructure, a binary connective "$\circ$" to fuse formulas together in conjunctive and disjunctive substructures depending on the polarity, and a zero-place item "$\boldsymbol{I}$" to express truth and falsity, again depending on the polarity.

The original purpose of display logic was to provide a strategy to prove cut-elimination for relevant and other substructural logics. In fact, the cut rule that is normally used in cut elimination is a form of mix. The problem is that mix is often invalid in substructural logics (Restall, 1998).

Belnap solves the problem in two steps. First, he associates connectives with structural operators by means of introduction rules like the following ones:

$$\frac{X \circ A \Rightarrow B}{X \Rightarrow A \rightarrow B} \qquad \frac{X \Rightarrow A \qquad B \Rightarrow Y}{A \rightarrow B \Rightarrow *X \circ Y}$$

Second, he defines reversible "display rules" for structural operators such that the cut formula can always be isolated either in the antecedent or the consequent and the simple cut rule can be applied:

$$\begin{array}{c} X \circ Y \Rightarrow Z \\ \hline\hline X \Rightarrow *Y \circ Z \end{array} \qquad \begin{array}{c} X \Rightarrow Y \circ Z \\ \hline\hline *Y \circ X \Rightarrow Z \\ \hline\hline X \Rightarrow Z \circ Y \end{array} \qquad \begin{array}{c} X \Rightarrow Y \\ \hline\hline *Y \Rightarrow *X \\ \hline\hline **X \Rightarrow Y \end{array}$$

The possibility to always reduce sequences and display single (but possibly complex) formulas on the left-hand side and right-hand side of a sequent is often already conceived as an expressive result that improves on Gentzen's inferential characterization of logical vocabulary. As Belnap puts it:

> The nub is this. If a rule for $\rightarrow$ only shows how $A \rightarrow B$ behaves *in context*, then that rule is not *merely* explaining the meaning of $\rightarrow$. It is also and inextricably explaining the meaning of the context.
>
> (Belnap, 1996, 81)

The idea is that if inferential rules are specified for sequences, then the inferential roles that they establish are the result of the meaning of the formulas *and* the way in which they occur in sequences. By allowing formulas to stand as antecedents and consequents in isolation, display logic allows a separated and more precise characterization of their inferential roles.

Once again, however, the expressive significance of display logic is far more general. From the expressivist point of view, the really interesting result achieved by display logic is the possibility of making explicit the structure with which formulas occur in an inference, so that rules can be established to govern how such structures can be transformed.

Since the inferential properties of logical operators are grounded in these transformations, the framework is suitable to accommodate various non-classical logics. As a consequence, of course, Belnap's proof of cut-elimination can be extended to several of these. The strategy is to define rules for the introduction of connectives in which *enough* structural operators are explicit and then establish rules for these structural operators to enforce the inferential behavior of the connectives in different logics.[7]

6. Some consequences of the expressivist approach

There are a few points worth highlighting in the aftermath of the applications of the expressivist strategy that we have considered.

First, logical forms have been discussed here with respect not so much to their impact on proof theory, as to their expressive role: this might suggest that the former depends on the latter. Such an implication, however, is not defended in this paper. In all the examples that we have reviewed, a proof-theoretic problem is solved by improving the expressive resources of logical languages: the problem of defining rigorous derivations for Frege, the problem of proving the coherence of a system for Gentzen, and the problem of proving cut elimination in relevance logic for Belnap. However, in none of these cases are logical forms the solution

[7] Things are actually more complicated than this. The development of display calculi for non-classical logics has seen the introduction of an increasing number of structural operators to match the different inferential behavior of different connectives. For a comprehensive review, see (Goré, 1998b,a). However, this is not necessarily an advance on the expressivist front. Following this approach, in fact, the boundaries between logical systems and their algebraic semantics become more and more diaphanous: expressive power and generality are achieved not so much in terms of logical forms that make explicit inferential properties, as in the metalanguage of algebraic semantics in which models are defined to represent those properties.

per se. The solution for Frege is to sanitize proofs against any infection from psychological processes, and the solution for Gentzen is to establish the inversion principle in his proof systems, the solution for Belnap is to allow the elimination of simple cuts. The expressive advance in logical forms that they introduce is essential to make explicit the inferential properties that their solutions address, but does not constitute the solution itself. In the case of Gentzen's inversion principle, the proofs of normalization show that they are not even required.

Second, the three cases considered here may look like three levels of a cumulative development of the expressive resources of logical languages, that improves more and more our understanding of inferences and their structure. The first step in such a progressive path would be to introduce propositional connectives to say that contents are related to one another in ways that are significant for our inferential practices. This achievement, however, would still leave implicit how transitions are possible from one content to another. The second step, then, would be to put forward sequents to say that certain transitions between contents are good derivations. This would still leave implicit what makes a transition a good one. Therefore, the third step would be to introduce structural operators to say that sequents have structures on which good derivations depend. It may even be tempting to extrapolate what the next step would be along that path: presumably, it would feature expressive resources to say explicitly what is implicitly done by taking contents to have structures that determine the inferential properties of the derivations in which they occur. It must be clear, however, that the acknowledgment of the expressive role of logical forms is distinct from the evaluation of the expressivist strategy itself. Depending on the theory of inference that is endorsed, the expressive results delivered by certain logical forms could be considered in different ways. An inferentialist, for instance, might welcome them as allowing a better understanding of inferential relations and, therefore, conceptual contents. A representationalist, on the other hand, might acknowledge their value merely as new techniques in proof analysis that, however, do not really improve our logical knowledge.

Third, the expressivist understanding of logical forms presented here clearly shifts most of the explanatory burden on the theory of inference that defines the reasons why they determine valid inferences. In this sense, logical expressivism offers a rather deflationary account of logical forms. The only thing that can be said about logical forms *per* *se*, from an expressivist point of view, is that they are patterns of logical constants and variables. Yet as logical constants and variables are necessarily defined with respect to valid inferences, any determination of logical forms therefore presupposes a theory of inference. One of the consequences of this approach is that conflicts about the interpretation of logical forms cannot be construed as regarding logical forms themselves, but must be traced back to incompatibilities between different theories of inference. Interestingly, however, such incompatibilities sometimes only emerge thanks to the expressive role of logical forms. In this sense, therefore, logical expressivism could still have a positive role to play in the philosophy of logic.

Fourth, in the expressivist understanding of logical forms their *only* role is to make explicit inferential properties. This idea invites us to rethink a certain standard conception according to which the process of formalization consists in the translation of natural language sentences into formulas in logical languages, that are *then* related to one another by rules of inference. The expressivist reading, instead, starts from inferences and conceives formalization as the development of logical forms to make explicit inferential rather than linguistic properties. In this sense, the fact that the languages of logical systems may share certain structural features with natural languages depends on whether such features are intrinsic to the very possibility to express contents and drawing inferences. Of course, this is a possibility that must be taken seriously into account by both logic and semantics. Still, the notion of logical form, as it is understood by logical expressivism, does not coincide with the notion of syntactic form.

7. Conclusion

From a historical point of view, the standard languages of modern logic were essentially defined in the period roughly between Frege's *Begriffsschrift* (1879) and Gentzen's *Untersuchungen* (1934/35). Afterwards, the problem of developing logical forms could have appeared as essentially solved. New generations of logicians had begun to encounter well-established logical vocabularies and see their most urgent task to be rather the development of a strong theory of inference to ground them. Yet, of course, the development of logical forms has never really stopped, because they are part and parcel of the enterprise of understanding inference in which logic consists. With respect to such a process, the examples considered in this paper are particularly clear in illustrating why new logical forms are defined whenever the expressive need rises not just to talk about the properties of valid inferences, but to infer their consequences in a calculus. Logical expressivism ascribes to logical forms precisely the function of satisfying such a need.

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