BOOK REVIEW

Robert Trueman's Defence of Higher-Order Logic

The book *Properties and Propositions: The Metaphysics of Higher-Order Logic* by Robert Trueman may be considered an extended reply to Willard Van Orman Quine's argument against higher-order logic. According to Quine, the meaning of the first-order existential quantifier is simply the concept of existence. Like Aristotle but unlike Quine, Trueman proposes that terms like "to be" and "to exist" (and consequently "being" and "existence") are ambiguous; in other words, there are several different and mutually irreducible modes (senses) of existence. In particular, existential quantifiers binding variables of different orders form expressions of different concepts of existence. First-order quantifiers refer to objects whereas those of second-order refer to properties (or relations). Consequently, objects and properties are said to exist in different ways. It follows that properties neither are objects nor exist in the sense objects do. Trueman's stance is labelled "Fregean realism". Trueman's main goals are to explicate the way properties are to exist and to defend higher-order logic this way.

It should be immediately noted that the main thesis of the book lacks precision – quite surprisingly for a work involving advanced knowledge of logic. Trueman interchangeably speaks of *ways*, *kinds* and *senses* of existence. It should be, however, clear that what he means is that there are different meanings of the term "existence" in use, not only that there are different kinds of existence. The latter thesis is quite innocent and commonly accepted, e.g., by Quine.

The book consists of fourteen chapters. In chapters 1–5 Trueman argues that the sematic role of terms and predicates differ to such a degree that it is not possible for a term and a predicate to co-refer. For example, the term "the set of all horses" or "the property of being a horse" on one side, and the predicate "is a horse" on the other, cannot have the same reference. Consequently, using predicates does not commit one to the references of the related terms. In chapters 6 and 7, however, Trueman argues that predicates nevertheless do refer and that they refer to properties. He insists that there is nothing paradoxical in this as there are different concepts of reference in play, namely term-reference and predicate-reference. Chapters 8 and 9 are dedicated to discuss difficulties of Fregean realism as well as alternative approaches to the problem in question. Chapter 10 offers Trueman's position on some metaphysical questions. Since properties are not objects and do not exist in the way objects do, Fregean realism is said to easily reconcile with nominalism. Trueman argues that many controversies related in one way or another to problems of universals vanish within the confines of Fregean realism. They turn out to be mere effects of the misuse of language; in particular, mere effects of speaking of properties as if they were objects. Interestingly, in chapters 11-13 Fregean realism gets extended to cover sentences, propositions and states of affairs, to be considered predicates or properties of arity zero. Although to count sentences as zero-place predicates is a rather common and quite convenient simplification, the other idea, that states of affairs are simply zero-place properties, seems to me bold and certainly deserves further discussion. In chapter 14 Trueman makes further use of his account of propositions and delivers an identity theory of truth: true propositions turn out to simply be facts rather than to refer to facts.

Trueman's book is quite comprehensive and well informed. It offers interesting and up-to-date discussions of a wide range of problems in ontology, philosophy of language and philosophy of logic, focused on the criticism of the first-order thesis, i.e., on the defence of higher-order logic. A majority of the discussions is well written and certainly recommendable to interested scholars and even more advanced students, particularly to those searching for a good topic for a research thesis. However, it seems to me that Trueman's theory in its present shape is untenable. Here is why.

Trueman's ontology is language-based. Objects and properties are defined by means of terms, predicates and reference. An object is "anything which can be referred to with a singular term" (p. 1) and a property

is "anything which can be referred to with a predicate" (p. 2). It needs to be mentioned, however, that elsewhere terms get defined by means of objects (p. 25). Sadly, Trueman takes the liberty of offering circular definitions or reasonings, and commits similar logical sins a little too often. Personally, I consider it a major fault of the otherwise interesting and valuable study.

It seems to me that Trueman should accept the term "object" as primitive. Then — to use his words — he could try to define "term" as "an expression which picks out an object", and (first-level unary) predicate as "an expression which says something of an object" (cf. pp. 23, 33). The phrase "to term-refer to" should be treated as a technical synonym of "to pick out the object" applied to terms. Finally, Trueman would find himself in a position to define "property" in the manner given above. More precisely, a property would be anything which can be predicate-referred to with a predicate. At that point the key concept of predicate-reference is needed.

The very concept of predicate-reference is — I think — the engine of Trueman's theory. The other, more familiar concept of term-reference is given by the schema:

$$\ldots \text{ designates } \ldots, \tag{D}$$

with an arbitrary singular term being used in the place of the ellipsis occurring on the right-hand side, and the very same term being mentioned in the place of the ellipsis occurring on the left-hand side. For example, "Rome" designates Rome, "7" designates 7. Trueman proposes that the concept of predicate-reference be given exclusively by another schema:

$$\forall x \, (x \text{ satisfies} \ldots \leftrightarrow \ldots (x)), \tag{S}$$

with an arbitrary unary predicate being used in the place of the ellipsis occurring on the right-hand side, and the very same predicate being mentioned in the place of the ellipsis occurring on the left-hand side. For example, let the sentence "Quine is wrong" consist of the singular term "Quine" and the first-order one-place predicate "is wrong". According to (S), x satisfies "is wrong" if and only if x is wrong, for every x. Predicates are said to refer merely in the sense of (S), i.e. by means of satisfiability conditions. That concept is distinct from the one of term-reference as nothing is postulated to be designated by predicates. In particular, (D) does not apply to predicates. A property can be

exemplified but cannot be designated. In such a sense, (S) constitutes the criterion of existence for properties. Consequently, properties are not objects but exist in another sense, established solely by (S). In the proposed example, the existence of the property of being wrong consists exactly in being exemplified by those being wrong.

Perhaps I overlook some key details but I cannot help to think that it is (S) itself which is untenable in its present form and role. Its objective is to reduce the existence of properties to satisfying predicates. However, an unavoidable side-effect of it is an ontology based entirely upon language. In particular, the existence of a predicate turns out to be the criterion (a necessary and sufficient condition) of the existence of a property. Therefore, (S) — analogically to the infamous axiom of comprehension — is open to diagonalization. And it comes as no surprise that — taken literally — Trueman's theory is simply inconsistent. For on the one hand, in higher-order logic, for some unary predicate, say "Φ", it is provable that

$$\forall x\,(\Phi(x) \leftrightarrow \neg(x \text{ satisfies } x)).$$

On the other hand, the following formula is an instance of (S):

$$\forall x\,(x \text{ satisfies "}\Phi\text{"} \leftrightarrow \Phi(x)).$$

As predicates are objects, even if properties are not, it follows respectively, by elimination of the universal quantifier, that

$$\Phi(\text{"}\Phi\text{"}) \leftrightarrow \neg(\text{"}\Phi\text{" satisfies "}\Phi\text{"}),$$
$$(\text{"}\Phi\text{" satisfies "}\Phi\text{"}) \leftrightarrow \Phi(\text{"}\Phi\text{"}).$$

And by propositional logic those formulas entail a well known absurdity:

$$(\text{"}\Phi\text{" satisfies "}\Phi\text{"}) \leftrightarrow \neg(\text{"}\Phi\text{" satisfies "}\Phi\text{"}).$$

In set theory antinomies arise because of the assumption that sets are simply denotations of any expressions. In Trueman's theory, analogically, a contradiction arises by assuming that properties are simply satisfiability conditions of predicates. To avoid such inconsistencies, the question of which properties there are must be decided by some other, non-linguistic criteria. However, it makes understanding properties simply as satisfiability conditions impossible.

An old-school approach to realism represents an alternative to Trueman's theory. As ancient Greek philosophers taught, something exists,

then some of that what exists may be conceived of, and finally some of what has been conceived of may be expressed in words. Some linguistic constructions simply fail to correspond with properties. Language turns out to not mirror the being accurately. In such an account (S) is not needed. And if it were nevertheless adopted in a restricted way, it would not have the status of the criterion of existence.

For example, according to Aristotle the term "being" ("existence") is ambiguous. Primary substances simply exist, whereas beings of all other categories exist in something else. The difference of meaning between the sentences "x exists" and "x exists in y" gives the ground of the thesis that there are different senses of being, yet it does not serve as a criterion of existence. An ontology of second-order logic — involving the ambiguity of existence but based on Aristotle's version of realism – was offered by Innocent Joseph Bocheński decades ago. Bocheński does not attempt to offer linguistic criteria of existence. Instead, he contents himself with offering criteria of identity of objects and of properties. Objects are said to be identical if and only if they exemplify exactly the same properties, whereas properties are said to be identical if and only if they are exemplified by exactly the same objects. Bocheński's ontology is not as wide and elaborated as Trueman's, but seems to be free from antinomies.

Furthermore, even if Trueman's theory were repaired and were turned into a consistent one, it would hardly be apt to decide the controversy over higher-order logic. It would certainly be an interesting piece of ontology but no threat to Quine's first-order thesis whatsoever. In my opinion distinguishing senses of existence does not actually bear importantly on the issue of higher-order logic either regarding the calculus itself or its relation to the natural language.

It should be clear that whether there is only one sense of existence or more makes no difference for logic whatsoever. There are well-examined models of second-order logic involving different senses of existence (Henkin models) as well as models with a single concept of existence, treating second-order logic as if it was a many-sorted first-order logic. In Trueman's conceptual framework it seems legitimate to say that properties are objects in first-order models of second-order logic but not in Henkin models. And yet, Henkin models and first-order models of second-order logic are characteristic of exactly the same operation of consequence. There is a one-one mapping from Henkin models onto first-order models such that any Henkin model returns exactly the same collection of true sentences as its first-order counterpart. And in both

kinds of models satisfiability conditions for all logical constants are exactly analogical.

The distinction of different modes of existence does not seem also to be particularly helpful in defending higher-order logic on the basis of natural language. Quine's objection — as far as I can understand it — does not have much to do with senses of being. It is more deeply rooted in linguistic analyses than in ontology. According to Quine variables are pronouns. For example, the sentence "x is a horse" means the same as the sentence "this is a horse". Therefore, the reference of variables is essentially the same as the reference of terms. Using Trueman's words, Quine could say that variables by their own nature pick out objects. The only difference between terms and variables consists in the way the reference gets fixed. The reference of a term is integrated into its meaning whereas the reference of a variable comes next to its meaning as an addendum, e.g., as a demonstrative gesture or a function of valuation. Therefore, using variables — either free or bound — in place of predicates is inherently erroneous, whether predicates do or do not enjoy references of any ontological kind. That is, I think, the very main problem with higher-order logic, but it is not properly addressed by Trueman. It is only briefly mentioned in passing and Peter Geach's answer is briefly cited. According to Geach in the vernacular there are pronouns which do not commit to objects, particularly the pronoun "somehow". However, Geach overlooks that in the contexts in which pronouns do not pick up objects they serve simply as abbreviations of default short disjunctions (or respectively conjunctions). For example, the sentence "I will come back home somehow" either does commit to objects of a kind or hides an expression of the metalanguage or serves as an abbreviation of a short (certainly finite) disjunction like "I will come back home either by train or by plane". Unfortunately, Trueman passes over that crucial problem of second-order logic in silence. Therefore, it seems to me, Quine's objection against higher-order logic is immune to the claims Trueman's book contains. This is not to say that Quine is right. It is only to say that Trueman has not addressed his objections successfully.

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