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Presuppositions and the Content Implication

A logical theory may be tested by its capacity for dealing with puzzles B. Russell (1905)

Abstract. In the 1950s Peter Strawson analyzed the works of Bertrand Russell regarding fundamental definitions of meaning, sentences and truth value. The debate between them uncovered many issues that Fregean, truth-functional logics have when defining concepts from natural language. To reconcile the Fregean paradigm with the reality of language use, Strawson proposed the concept of presuppositions — necessary preconditions for the truth of other sentences. We believe that his proposition stemmed primarily from the problem caused by the fact that Fregean, truth-functional logics are not sensitive to the contents of sentences and reduce them to their logical values. This is bound to produce a mismatch between the way logic models reasoning and the way language users reason since real-life reasoning is performed on the contents of sentences and not their logical values. Inspired by the ideas of Strawson and Roman Suszko, who initiated the paradigm of non-Fregean logics, we propose a new solution to the debate between Strawson and Russell. In our solution, the content implication connective is used to express content relations between sentences. We move away from truth and falsehood as the sole two semantic correlates of sentences and instead work in a system where the contents of sentences are their semantic correlates.

Keywords: non-Fregean logic; truth-functional logic; content implication; presupposition; semantic correlates; logic of content

1. Introduction

The principles of classical logic are the bedrock upon which the castle of modern logic was built. However, despite being one of the foundations

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of contemporary science and philosophy, it suffers from some underlying theoretical problems. In particular, it has always been a struggle for logicians to define some of the most fundamental concepts in logic. What is a sentence? What is the meaning of a sentence? What is a true sentence? The answers to these questions are far from trivial and some of the most brilliant logicians of the 20th century vigorously argued about them. For example, Bertrand Russell (1872–1970) and Peter Frederick Strawson (1919–2006) in the 1950s debated these very problems with each other through published works. They argued so brilliantly in fact, that they uncovered some of the fundamental truths about classical logic. Namely, the truth-functionality of classical logic and how it operationalizes meaning makes it insensitive to the contents of sentences. However, buried neck-deep in the formalisms of classical logic, neither Russell nor Strawson realized that fully at the time. This is unfortunate because carefully revising the classical notions of denoting, meaning and truth determination opens logic up to many new applications, which were previously rendered impossible by their conservative definitions. The purpose of this article is to clarify the ideas that Strawson conceived when addressing Russell's works and expand on them. To do that, we will summarize the debate between Strawson and Russell and demonstrate that Strawson had the intuition that something is fundamentally wrong with Russell's way of defining sentences and meaning. However, we will argue that the error did not lie with Russell himself, but with the concept of truth-functionality and using truth values as semantic correlates of sentences. We will build on that and demonstrate that the problems that Strawson and Russell encountered in classical logic do not exist when using a new type of connective: the content implication connective. It is a connective sensitive to the contents of sentences by requiring their semantic correlates to be thoughts expressed by the sentence. As a result, we show how classical logicians inadvertently expressed the need for content-sensitive logic in the past and how their needs can be addressed today.

2. Russell vs. Strawson on denoting and referring

Since the appearance of sentential logic in its mature form logicians were aware of some dire problems within it. For example, it was always a puzzle that the liar paradox breaks the dichotomy of the classes of true and false sentences. This is a big problem if the semantic correlate of a sentence is supposed to be its truth value. After all, people seem to understand the meaning expressed with the liar sentence, but logic fails at denoting that meaning. Awareness of this and other problems related to classical logic did not cause it to be abandoned as useless, but instead, a pragmatic stance was taken. Logicians continued to study it because it was interesting, research-fertile and provided tools for describing non-classical logics. As a result, the problems were not hidden, but everyone counted on a future positive solution. Such (or similar) beliefs underpinned the construction of a paradigm for logical research throughout the 20th century. We use the word paradigm here in the strict sense because we believe that it was an actual paradigm in the sense that T. Kuhn pointed out (1962)[1]. The leading representatives working within this paradigm, and its creators, were B. Russell and W. V. O. Quine.

Within that paradigm, logicians needed at least some working definitions of meaning, even though the existence of the liar paradox would render most of them questionable. Russell did not formulate his definitions explicitly in his article "On denoting" (Russell, 1905). However, Strawson in his article "On referring" (Strawson, 1950) reconstructed Russel's working definitions in the following way:

- If the sentence: 'The king of France is wise' is meaningful (significant) then it is true or false.

This sentence is very important because it forms an equivalence with its inverse implication:

- If the sentence 'The king of France is wise' is true or false, then it is meaningful.[2]

By applying the same principle to any other sentence of natural language we discover that sensibility/meaningfulness is equivalent to having one of two logical values: truth or falsehood. Russell agreed with such a summary of his understanding of meaning and in the article entitled "Mr. Strawson on referring" (Russell, 1957, p. 288) he wrote: "For my part, I find it more convenient to define this word 'false' so that every significant sentence is either true or false." This equivalence then became the basis for defining a sentence in the logical sense as an expression of a certain language to which a logical value of true or false can be assigned.

[1] "Universally recognized scientific achievements that, for a time, provide model problems and solutions for a community of practitioners." (Kuhn, 1962, p. 10)

[2] For the sake of precision, let us recall that the famous sentence in Russell's was: 'The present king of France is bald', while in Strawson's 'The king of France is wise.' Perhaps the reason was one of politeness.

This position is quite common even today in many textbooks on logic, and it seems to be derived from Russell's views above. The difficulties in clearly understanding Russell's article resulted in the following principle being adopted in twentieth-century philosophy of logic:

- (Russell's Principle – *RP*) A sentence (proposition) is significant (meaningful) if and only if it is true or false.

If we apply Russell's Principle to the liar sentence L, which reads: "This sentence is false" then because we are unable to assign it either of the two logical values, it follows that L is not meaningful. Unfortunately for Russell, this is at odds with the common-sense view of L. Presumably he could not deal with L similarly to how he dealt with the sentence S = "The king of France is wise", which, in his view also had no logical value. To solve that problem, he claimed that the correct logical sense of the sentence S is different from its grammatical sense. In other words, the correct logical structure of the sentence S is different from its apparent grammatical form. Unfortunately, the term logical sense as opposed to grammatical sense appears in Russell's work ad hoc. This troubled Strawson when he was reconstructing Russell's arguments in 1950. He warned against the dangers of placing logic over grammar in a footnote: "[a]nd this in spite of the danger-signal of that phrase, 'misleading grammatical form' " (Strawson, 1950, p. 334). Russell's attempts at paraphrasing the sentence S and splitting its logical sense from the grammatical sense resulted in its famous reconstruction into a sentence that is a conjunction of three members: (R_1) "There is a king of France" and (R_2) "There is not more than one king of France" and (R_3) "There is nothing which is king of France and is not wise." (reconstruction after Strawson, 1950, p. 324), of which the conjunction is easily shown to be false, since one of its members is false.

Russel's Principle has important implications for what is considered a valid sentence in classical logic. In particular, its consequences are visible after coupling it with the Frege's Axiom:

- (Frege's Axiom – *FA*) All true sentences have one common reference (denotation), the truth, and all false ones also have one reference (denotation), different from truth, understood as falsehood.

If we couple *RP* and *FA* we arrive at the conclusion that all the meaningful sentences (or propositions) have only one of two references: truth or falsehood. It is important, because *FA* on its own did not claim to cover all the meaningful sentences. Only the true ones and the false ones. *FA* does not strip other sentences from the ability of having some meaning (denotation), but *FA* together with *RP* does. This is an extremely reductive view of meaning, bound to produce problems and friction with the reality of natural language use.

This issue was intuitively picked up by Strawson and led him to distinguish between sentences and their uses. He proposed that truth or falsity belongs to the uses of sentences while meaning belongs to sentences themselves. As a result, he postulated that Russell's equivalence is false. This is so because a sentence does not have a logical value, and therefore it does not have a denotation. Strawson developed his idea in such a way as to retain the meaning of the sentence about the king of France (S), but in order to preserve its bivalence, he forced the introduction of a new, although hitherto unknown linguistic object — a presupposition, coined by him in 1964. The presupposition of the sentence S was to be the use of a sentence which is itself true and then guarantees that the sentence S has a logical value. In the discussed example, the sentence S: "The King of France is wise" presupposes the sentence R_1: "There is a King of France" among many others. It is important to note that when what is presupposed by the specific instance of the use of sentence S does not hold, then the sentence S is neither true nor false in that situation. It fails to make a true or false assertion.

The distinction between a sentence and its use caused a heated dispute between the two English philosophers. Russell, in his reply to Strawson's article in the 1957 issue of *Mind*, wrote:

> I must say, to begin with, that I am totally unable to see any validity whatever in any of Mr. Strawson's arguments. Whether this inability is due to senility on my part or to some other cause, I must leave readers to judge.[3] (Russell, 1957, p. 385)

Encouraged by Russell with this statement, we take the liberty of making this independent judgement that Strawson was right in several places, despite that fact that majority of philosophers sided with Russell on that

[3] Speaking of the number of occurrences, one can only say that Russell in the work 'On denoting' mainly uses the term 'proposition' (76 times), while 'sentence' appears only once and 'statement' 9 times. Whereas the important cited passage in 'Mr. Strawson on referring' uses the term 'sentence' and uses it there (9 times); while 'proposition' 1 time. And in Strawson's article 'On referring' the term 'sentence' appears most of the time (152 times), while the term 'proposition' is used sparingly (13 times), just as 'statement' (22 times). All this only adds to the confusion and raises the question of whether both authors were talking about the same thing at all.

matter (e.g., Sainsbury, 1979; Davies, 1981; Evans, 1982; Bach, 1987; Neale, 1990; Reimer and Bezuidenhout, 2004). However, we believe that the correct approach is not to keep tweaking the classical formalisms to fit them to instances of natural language use like most Strawsonian approaches have attempted so far (Ramachandran, 2008). Classical logic does not have to be adjusted to correspond to natural language because it cannot. Human language production and perception have not been found to follow the principles of classical logic (Johnson-Laird, 2010). As a result, we will expand in a non-classical way on the intuitions of Strawson that meaning belongs to the uses of sentences.

3. Strawsonian presupposition and content implication

The following two definitions correspond to Strawson's intuition about presupposition (Beaver and Geurts, 2011):

- *Definition 1 (Strawsonian presupposition).* One sentence presupposes another iff whenever the first is true or false, the second is true.

Or by means of denial:

- *Definition 2 (Presupposition via negation).* One sentence presupposes another iff whenever the first sentence is true, the second is true, and whenever the negation of the first sentence is true, the second sentence is true.

It is appropriate to add a correction here, in line with the position taken by Strawson, who refused to give logical value to sentences. Thus, in each of these two definitions, the expression "sentence" should be replaced by "use of a sentence" or "utterance of a sentence" or just by "proposition" — according to Strawson, since it is not the sentence that is true or false, but its use, utterance or proposition. Thus, more precisely following Strawson's intuition, presupposition can be defined by referring to the concept of a necessary condition:[4]

- The utterance of sentence q is the presupposition for the utterance of sentence p if it is a necessary precondition for both the utterance of p and the utterance of $\neg p$.

It is not revealing to say that the implying/entailment which is meant to exist in the context of presupposition cannot be truth-functional. Oth-

[4] Strawson (1964) himself defined the presupposition of an utterance of a sentence as the necessary condition for the veracity of it.

erwise, for example, any true sentence, and therefore any logical truth, such as: "The present King of France exists or does not exist" would be the presupposition of any other sentence, such as: "Fish live in water" — since the former is clasically implied by the latter, as well as by the negation of the latter. Additionally, every true sentence, and therefore every logical truth, would be a presupposition for itself — for a true A, both the classical implications of $A \to A$ and $\neg A \to A$ would be true. So, if we are going to determine whether a given sentence is a presupposition of another sentence, we must use some other type of implication. Otherwise, the decision about the presupposition would depend only on the logical values of the sentence and its possible presupposition — neither the content of the sentence nor the content of its possible presupposition would matter. This would, however, be incompatible with the very idea of presupposition. It is the content of both sentences that determines whether one sentence is the presupposition of another. Therefore, for the considerations to make sense we must either impose strict limitations on what can be a presupposition (e.g., as relevance logics would) or understand implying/entailment in a "contentual" way. Staying within the truth-functional paradigm means that presuppositions are still vulnerable to problems analogous to paradoxes of strict implication. Thus, we believe that to avoid the situation where sentences that have the same semantic correlates but are content-unrelated (which should never happen when presupposing is considered) we have to employ the non-Fregean paradigm.

In Strawson's opinion, the falsity of the presupposition q of the sentence p deprived the sentence p and the sentence $\neg p$ of meaning, understood there as a logical value. Strawson later began to doubt the validity of such a radical stance and leaned towards Russell's view in this regard, although he never revoked his position.[5] This view, traditionally associated with Strawson, that sentences having a false presupposition

[5] Based on the Stanford text quoted here, it can be concluded that Strawson began to lean towards Russell's position: "What happens when a presupposition is false? The textbook story goes as follows. According to Frege (1948, 214, 221), if an expression A suffers from presupposition failure, then any sentence containing A will lack a truth value; Russell (1905) famously denied this, holding that such a sentence will always be true or false; and then Strawson (1950) reaffirmed Frege's position, more or less. What is less well known is that in subsequent work, Strawson partly recanted his initial view and came to doubt that presupposition failure invariably entails lack of truth value" (Beaver and Geurts, 2011).

have no logical value, is not, however, shared by all philosophers, and is by no means a case of Strawson's opponent, Russell. Centuries before the formulation of the theory of presuppositions, Aristotle perfectly understood the problem represented by the sentence "The present king of France is bald" and without any difficulty attributed logical values to the sentences: "Socrates is sick", "Socrates is healthy" and "It is not true that Socrates is sick", depending on whether the sentence: "Socrates exists" is true or false.[6]

Strawson's view was likely the consequence of his disbelief in the possibility of applying his contemporary logic — the truth-valued one — to solve problems generated by natural language. Strawson manifested his disbelief openly, which exacerbated his conflict with Russell. It seems that Strawson's relative intuitions were correct in this. In fact, truth-valued logics, and thus reducing sentences to their logical values, cannot be useful tools for analyzing issues closely related to the content of sentences. The next part of the paper will present an approach based on *CCL* content-based sentential calculus, whose content implication naturally solves this problem. This approach is characterized by the fact that, on the one hand, it easily solves the problem of assigning logical values to propositions p and $\neg p$, on the other hand, it avoids the need to use, artificially in Strawson's opinion, a logic with quantifiers, which Russell used to create logical paraphrases for p and $\neg p$ to the known quantified conjunction:

- of the sentence p: $\exists x\big(\text{King}(x) \wedge \neg\exists y(y \neq x \wedge \text{King}(y)) \wedge \text{Bald}(x)\big)$,
- of the sentence $\neg p$: $\exists x\big(\text{King}(x) \wedge \neg\exists y(y \neq x \wedge \text{King}(y)) \wedge \neg\text{Bald}(x)\big)$.

In this way, Russell solved the problem of giving to both above sentences p and $-p$ the logical value of falsehood, contrary to the law of the excluded middle. This problem results from confusing *de re* negation with *de dicto* negation. Indeed, since

$$p = \exists x\big(\text{King}(x) \wedge \neg\exists y(y \neq x \wedge \text{King}(y)) \wedge \text{Bald}(x)\big),$$

then there should be

$$-p = \neg\exists x\big(\text{King}(x) \wedge \neg\exists y(y \neq x \wedge \text{King}(y)) \wedge \text{Bald}(x)\big),$$

which simplifies to

$$\forall x\big(\neg\text{King}(x) \vee \exists y(y \neq x \wedge \text{King}(y)) \vee \neg\text{Bald}(x)\big).$$

[6] We will return to these three sentences from Aristotle's Categories at the end of the article.

Thus, one of the key problems that emerges from the starting point for considering the truthfulness of sentences deciding about non-existent objects is the identification of *de dicto* negation with *de re* negation of a given sentence. Let us recall that in "On denoting", Russell clearly states that the sentences p and "The present king of France is not bald" constitute the pair appearing in one substitution of the law of the excluded middle $p \vee \neg p$. It should be recalled that the sentence p Russell presents as "A is B" and the second, that is $\neg p$, as "A is not B",[7] Thus, $\neg(A \text{ is } B) = (A \text{ is not } B)$. "The present king of France is not bald" is, therefore, for Russell, the *de dicto* negation of the sentence "A is B", that is, the sentence $\neg(A \text{ is } B)$. However, in the case of the just mentioned conjunctive reconstruction of p and $\neg p$, as well as, when we compare the two sentences "The present king of France is not bald" and "It is not true that the present king of France is bald", we no longer have this certainty which is the "proper" negation of the sentence p. The first one $p^- = (A \text{ is not-} B)$ is a negation *de re*, the second one $\neg p = \neg(A \text{ is } B)$, a negation *de dicto*. It seems that the solution to this problem depends on what name we consider A to be — individual, empty or general. Regardless of the understanding of the name "A", "B" seems to be undoubtedly a general name.

In this article, we will propose a solution to the problems encountered by Strawson. We believe that he wrestled with the inability of truth-functional logics to model relations between the contents of sentences but was buried too deep in the classical paradigm to notice the underlying reason for his struggles.

4. The underlying reason for Strawson's intuitions

Despite its known and unquestionable advantages, formal logic has some important weaknesses and limitations, which often manifest as incompatibility of its tautologies and rules with the principles of everyday thinking. This means that consistently illustrating inferences and logical truths in natural language becomes difficult and usually leads to

[7] Of course, in the usual sense, the negation of the phrase "The present king of France is bald" is the phrase "The present king of France is not bald," which is negation *de dicto*, but also in the case of a definite description, is negation *de re*. Furthermore, this identification is indicated by the aforementioned elaboration of the phrase p and its negation for triple conjunction (cf. Russell, 1905, p. 485).

paradoxes. Examples include the liar paradox, paradoxes of material implication and deontic paradoxes. Many of these problems are a consequence of the principles that made Strawson hesitant about the Russelian view on denoting: *FA* and *RP*. The core incompatibility between these principles and natural language is the fact that they reduce the semantic interpretation of sentences to their logical values. As a result, they abandon the contents of sentences (i.e., utterances as Strawson would write or situations expressed in the sentences as Roman Suszko would write). In classical logic, the semantic correlate of the sentence "The week has seven days" and "The capital of China is Beijing" is the same. Because of that, logical analysis of reasoning involving these two sentences will treat them identically. Unfortunately, that is not how humans process natural language. When producing language, comprehending it and determining its veracity, humans primarily process the contents of sentences and not their truth values (Brennan, 2016). As a result, classical formal logic is ill-equipped to model natural language processing.

This mismatch between the principles of natural language processing and classical formal logic is easily seen in paradoxes of material implication. In classical logic the sentence: "If the week has seven days, then the capital of China is Beijing" is valid and true. In contrast, for a natural language user that sentence constitutes a semantic error, because the two elements of the implication are not contentually related (Rabovsky and McRae, 2014). The classical sentential logic is a simple calculus on the two numbers, 1 and 0, and not a logic expressing the result of content inference of one sentence from another. This is the reason why Russell and Strawson struggled to define meaning. It was because there was no room for meaning in the paradigm in which they were working since it was reduced to binary truth values. Strawson was forced to make room for meaning by separating the truthfulness of a sentence from the truthfulness of *uttering* a sentence, i.e., from the use of a sentence in a situation. While that solves some problems, there are still so many other incompatibilities between classical formal logic and natural language processing, that it is crucial to develop new systems which would properly model natural language. We call such systems *content logics* — those formal systems in which the semantic interpretation of a sentence, and therefore its meaning, is not a logical value, but content, understood as the thought expressed in a sentence.

This definition of the content of a sentence closely follows Frege's *Sinn* — the sense of a sentence. Thus, we strictly refer to Frege's idea of distinguishing *Bededutung*, the reference of a sentence (i.e., truth or falsehood) from *Sinn*, the sense of a sentence. According to Frege, two sentences with different senses can have the same reference:

> We now inquire concerning the sense and referent of an entire declarative sentence. Such a sentence contains a thought. [footnote 5: By a thought I understand not the subjective performance of thinking but its objective content, which is capable of being the common property of several thinkers.] Is this thought, now, to be regarded as its sense or its referent? Let us assume for the time being that the sentence has a referent! If we now replace one word of the sentence by another having the same referent, but a different sense, this can have no influence upon the referent of the sentence. Yet we can see that in such a case the thought changes; since, e.g., the thought of the sentence *"The morning star is a body illuminated by the sun"* differs from that of the sentence *"The evening star is a body illuminated by the sun."* Anybody who did not know that the evening star is the morning star might hold the one thought to be true, the other false. The thought, accordingly, cannot be the referent of the sentence, but must rather be considered as the sense. (Frege, 1948, pp. 214–215)

Moreover, following Łukowski (2019), we assume that the content of a sentence which is a conjunction: 'A and B', consists of the contents of sentences 'A' and 'B'. At the same time, we assume that the contents of sentences 'A' and 'B' are not components of the content of sentences: 'A implies B' or 'A or B'. These assumptions seem to be in line with Frege's proposal (for the first assumption cf. Frege, 1948, p. 221; for the second cf. Frege, 1948, pp. 224–225 with footnote 10). They mean that the sentence 'A and B' says 'A' and says 'B'. However, neither 'A implies B', nor 'A or B' say 'A' or say 'B'. 'A implies B' and 'A or B' only say about some relationship between the content of 'A' and the content of 'B'.

The assumption that a (indicative) sentence contains a thought and an assertion, which sometimes also acts on the feelings or the mood of the hearer (e.g., Frege, 1948, 1956) means that the thought can be true or false. Max Black, in his Introductory Note to English translation of Frege's Sense and Reference, states that thought can also be understood as a proposition (Frege, 1948, p. 208). Taking all this into account, we will continue to understand the sentence's content in the Fregean sense, which is the thought expressed by the sentence, which can also be called: a statement or utterance or proposition. The content of a sentence in this view is objective, not subjective, so no one is the bearer of the content, but everyone can recognize it to be true or false (e.g., Frege, 1956, p. 301).

5. Content implication – a new tool that Russell and Strawson were missing

The purpose of this work is to investigate whether Classical Contentual Logic (*CCL*) (as defined in Łukowski, 1997, 2011, 2019, 2020, 2022, and slightly amended in this article) is able to deal with the problems that divided Russell and Strawson. *CCL* is a formal system that treats contents of sentences as their semantic correlates and strengthens truth-functional logic with the connective of content implication that forces semantics to have as many semantic correlates as there are sentences in language. [8] We want to follow the advice of Russell who said that the best test for logical theories is their capacity for solving puzzles (Russell, 1905) and see whether content implication connective helps with solving the puzzles that troubled Russell and Strawson. In short, we confront our solution with both Russell's and Strawson's views, without granting ourselves the role of a judge who would point out the winner in the dispute over the meaning of the *S*-sentence. On the contrary, our aim is to show that both approaches were special cases of a solution based on content logic. Moreover, we suppose that adoption of content logic in place of truth-functional logic could bring this long-standing dispute to an end.

Our proposed solution uses the content implication connective ':'), making *CCL* a non-Fregean logic. This new connective makes it possible to express the fact that the content of the successor of an implication is contained in the content of its predecessor. Thus, the truth of the sentence '*A* says that *B*' means that the content of the sentence *B* is a part or the whole of the content of the sentence *A*.[9] This connective refers to the last of the four understandings of the conditional considered by the Stoics, as mentioned by Sextus Empiricus.[10]

[8] Czelakowski (2019, p. 209) rightly links this problem of the true-valued nature of formal logics with the structurality of the logical consequence.

[9] The case of the whole occurs when, in addition, the sentence '*B* says that *A*' is true.

[10] Cf. "[4] And those who judge by 'suggestion' declare that a conditional is true if its consequent is in effect included in its antecedent. According to these, 'If it is day, then it is day,' and every repeated conditional will probably be false, for it is

5.1. Fregean and non-Fregean approach to logic

To solve the puzzles of the liar paradox and the sentence *S*, we must first describe our approach. It originated with Roman Suszko's need to construct logic other than truth-functional ones, which would be closer to human thinking. He proposed non-Fregean logics, which he envisioned rather as "logics of situations" than logics of content, which shows a striking similarity to the ideas of Strawson. However, they satisfied the most important postulate on the road to logics of content and abandoned logical values as the only possible and actual meanings of sentences. Suszko was the first logician to postulate the rejection of the classical paradigm of understanding formal logic (Bloom and Suszko, 1972). Under the influence of Wittgenstein's *Tractatus Logico-Philosophicus*, he decided to change the paradigm of our understanding of logic by freeing it from Frege's Axiom. To achieve that, Suszko constructed propositional logic where semantic correlates of sentences were situations, not logical values of truth and falsehood. In Wittgenstein's opinion, the reality is all facts, i.e., the situations that come true. The logic that models thinking should express the dependencies between these situations in accordance with the structure of language. However, this goal cannot be achieved when the only possible semantic correlates of all sentences are two objects, traditionally understood as truth and falsehood. In the Fregean paradigm logic expresses dependencies not between what the propositions say, but only between the logical values of the propositions. Thus, Suszko considered non-Fregean logic to be one that does not satisfy Frege's Axiom, which in turn means that this logic forces the existence of models with as many semantic correlates as there are sentences in language. This feature is characteristic of non-Fregean formal systems — the mere recognition of semantic correlates as situations or the content of sentences is not enough to make the logic non-Fregean. Its real essence is that the semantic correlate can no longer be a logical value of truth or falsehood or any other. It also means that equivalent sentences can have different semantic correlates — situations or content. This postulate seems to be in line with what Strawson wanted. Since each sentence has its own semantic correlate different from the correlate of another sen-

impossible for a thing itself to be included in itself." We quote after: (Mates, 1961, p. 48), which he quotes from the standard edition of Sextus' works: *Sextus Empiricus. Opera.* Ed. H. Mutschmann, with emendations, additions and corrections by Dr. J. Mau; Leipzig, Teubner, 1954 (vol. 3), and 1958 (vol. 1). *Hyp. Pyrrh*, II, 112.

tence, it means that it is no longer possible to identify sentences that are not related to each other in terms of content. As a result, correlate can be understood in Strawsonian terminology as either: use or statement. In such a model, the sentence has content, while content could have logical value. Thus, the sentence no longer has a logical value.

Fregean logic does not have this feature, because it allows to assign the same semantic correlate to sentences with different content. Only in a non-Fregean logical reality, do we have the realization of the postulate important for Strawson: distinguishing *sentence* from *statement/proposition.* The problem of assigning a logical value to the latter is solved in a simple way in non-Fregean semantics — not by special, additional evaluation, but simply by belonging of the statement/proposition/ utterance to the set of designated correlates or to the complement of that set. The designated correlates are those statements/propositions/ utterances that we consider true. The other correlates are false. As can be seen, in semantics for non-Fregean logic, a sentence does not have a logical value, but rather its use/statement does. In cognitive science, we would say that the sentence contains information that the sender wants to convey to the recipient.

Naturally, just as Strawson did not limit himself to putting forward the above-mentioned postulate, so Suszko did not limit himself to simple rejecting the Fregean Axiom. He constructed a propositional logic which rejects this axiom. Suszko defined his logic on the classical sentential language extended by the new binary connective $\equiv$, called by Suszko "identity", and the logic is known as the *Sentential Calculus with Identity* (*SCI*). *SCI* is the *Classical Sentential Calculus* (*CL*) strengthened by the following axioms:

$$\alpha \equiv \alpha \tag{$\mathrm{A}_{1\equiv}$}$$

$$(\alpha \equiv \beta) \to (\neg\alpha \equiv \neg\beta) \tag{$\mathrm{A}_{2\equiv}$}$$

$$(\alpha \equiv \beta) \land (\gamma \equiv \delta) \to ((\alpha \,\S\, \gamma) \equiv (\beta \,\S\, \delta)), \text{for } \S \in \{\land, \lor, \to, \leftrightarrow, \equiv\} \tag{$\mathrm{A}_{3\equiv}$}$$

$$(\alpha \equiv \beta) \to (\alpha \to \beta) \tag{$\mathrm{A}_{4\equiv}$}$$

Modus Ponens stays the only prime rule of inference. It is not difficult to see that an axiomatic strengthening of *SCI* by a reverse of ($\mathrm{A}_{4\equiv}$), i.e.:

$$(\alpha \to \beta) \to (\alpha \equiv \beta)$$

trivializes the connective of identity, identifying it with the classical equivalence. Then, non-Fregean *SCI* becomes Fregean *CL*.

The semantics adequate for SCI are created by the class of, so called, SCI-models, i.e., such matrices $\mathcal{M}_{SCI} = (\mathcal{A}_{SCI}, D)$, that $\mathcal{A}_{SCI} = (A_{SCI}, \neg, \cap, \cup, \Rightarrow, \Leftrightarrow, \approx)$ is an algebra similar to the SCI-language $\mathcal{L}_{SCI} = (\mathrm{For}_{SCI}, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, \equiv)$. D is a non-empty subset of A_{SCI}, and for any $a, b \in A_{SCI}$,

1. $-a \in D$ iff $a \notin D$,
2. $a \cap b \in D$ iff $a \in D$ *and* $b \in D$,
3. $a \cup b \in D$ iff $a \in D$ *or* $b \in D$,
4. $a \Rightarrow b \in D$ iff $a \notin D$ *or* $b \in D$,
5. $a \Leftrightarrow b \in D$ iff $a, b \in D$ *or* $a, b \notin D$,
6. $a \approx b \in D$ iff $a = b$.

The semantic inference is defined in a standard way: $X \models_{SCI} \alpha$ *iff* for any SCI-model $\mathcal{M}_{SCI} = (\mathcal{A}_{SCI}, D)$ and $\nu \in \mathrm{Hom}(\mathcal{L}_{SCI}, \mathcal{A}_{SCI})$: $\nu(\alpha) \in D$, if only for any $\beta \in X$, $\nu(\beta) \in D$.

Non-Fregean logic SCI is not extensional due to equivalence of sentences. On the other hand, it is extensional due to the identity of sentences, i.e., identical sentences are mutually substitutable in all contexts, and equivalent sentences are not. As can be seen, there is a clear distinction between the equivalence of sentences and their identity. If the sentence $p \equiv q$ is true, and so sentences p and q are identified as expressing the same situation/content, then they have the same semantic correlate assigned to that situation/content: $\nu(p) = \nu(q)$. Then, by necessity (i.e., by virtue of $\mathrm{A}_{4\equiv}$) p and q are also equivalent, i.e., the situations/contents expressed by them have the same logical value belonging either to the set D or to its complement $A_{SCI} - D$. However, the equivalence of p and q does not mean that they are identical. Each of these sentences can express a real situation, i.e., a fact, which means that both have true content, and yet the facts expressed by them can be different, e.g., p = "A cat is a mammal", q = "A week has seven days". It is similar with sentences with false contents — they are equivalent, but do not have to express the same situation/content.

As already mentioned, in creating his non-Fregean logic, Suszko drew inspiration from Wittgenstein's (1921). Therefore, he maintained that the semantic correlates of sentences are situations. In this work we assume that the semantic correlate of a sentence is its content understood in a Fregean way, as the sense of the sentence, i.e., the thought expressed by that sentence (Łukowski, 1997, 2011). From this point of view, every indicative sentence has a sense, or in the terminology of the present:

meaning. This may seem to be a major departure from the assumptions of (Wittgenstein, 1921). Indeed, the postulate 4.461 states:

> The proposition shows what it says, the tautology and the contradiction that they say nothing.
> The tautology has no truth-conditions, for it is unconditionally true; and the contradiction is on no condition true.
> Tautology and contradiction are without sense.
> (Like the point from which two arrows go out in opposite directions.)
> (I know, e.g. nothing about the weather, when I know that it rains or does not rain.) (Wittgenstein, 1921)

It seems obvious that neither tautology nor contradiction says anything about the world — tautology because it is true in every possible world, and contradiction because it is false in every possible world. However, even though they do not say anything about the world (situations), they are still comprehended/understood, which means they still have content. Even Wittgenstein phrases it like that: *„I know, e.g. nothing about the weather, when I know that it rains or does not rain.”* When he says that he knows that it rains or does not rain, it means that he comprehended the thought/content expressed with that tautological sentence. That is the difference between understanding meaningfulness as teaching an agent about something (in psychology called *“uncertainty reduction”*) and meaningfulness as being understandable, albeit sometimes useless. In contrast, a sentence: *“Rain under for a pencil beneath the alphabet,”* without some other specific sentences that set up a context for it, is incomprehensible and thus does not have any content. In contrast, we do understand what thought the sentence: *“it rains and does not rain at the same time”* conveys. In Wittgenstein's sense, it is meaningless because it is informationally empty/useless but in the *CCL* sense it still has content, which is its semantic correlate.

From this point of view, it should be also noted that “non-Fregean logic” can be a misleading name. This is because it suggests that Frege only considered references for sentences and not senses. Meanwhile, as we well know, Frege considered both. Thus, one can venture to say that non-Fregean logics are systems that do not so much break with Frege's paradigm, but rather fully embrace it.

5.2. Content implication – a new content sensitive connective

A connective of sentential identity that guarantees non-Fregeanity of logic is not an implication but some kind of equivalence. It turns out, however, that it is possible to define a non-truth-functional implication, referred to as the content implication. The content implication is a binary conjunction ':' given by the following axioms (see Łukowski, 1997, 2011):[11]

$$((\alpha : \beta) \wedge (\beta : \delta)) \rightarrow (\alpha : \delta) \tag{A$_{1:}$}$$

$$(\alpha \wedge \beta) : \alpha \tag{A$_{2:}$}$$

$$(\alpha \wedge \beta) : (\beta \wedge \alpha) \tag{A$_{3:}$}$$

$$\alpha : (\alpha \wedge \alpha) \tag{A$_{4:}$}$$

$$((\alpha : \beta) \wedge (\beta : \alpha)) \rightarrow ((\neg\alpha : \neg\beta) \wedge (\neg\beta : \neg\alpha)) \tag{A$_{5:}$}$$

$$\begin{gathered}((\alpha : \beta) \wedge (\beta : \alpha) \wedge (\delta : \gamma) \wedge (\gamma : \delta)) \rightarrow \\ (((\alpha \mathbin{\S} \delta) : (\beta \mathbin{\S} \gamma)) \wedge ((\beta \mathbin{\S} \gamma) : (\alpha \mathbin{\S} \delta))), \quad \text{for } \S \in \{\vee, \rightarrow, \leftrightarrow, :\}\end{gathered} \tag{A$_{6:}$}$$

$$((\alpha : \beta) \wedge (\delta : \gamma)) \rightarrow ((\alpha \wedge \delta) : (\beta \wedge \gamma)) \tag{A$_{7:}$}$$

$$(\alpha : \beta) \rightarrow (\alpha \rightarrow \beta) \tag{A$_{8:}$}$$

Replacing Suszko's axioms for identity $A_{1\equiv}$–$A_{4\equiv}$ with the axioms for content implication $A_{1:}$–$A_{8:}$, leads to the new non-truth-functional calculus hereinafter referred to as the *Classical Contentual Logic (*CCL*)*. One of the most important theses of the *CCL* is the seemingly trivial content implication:

$$\alpha : \alpha$$

which follows from the axioms $A_{1:}$, $A_{2:}$, $A_{4:}$.[12] It expresses what is known as Buridan's thesis, also known as *the virtual entailment principle*, which states that every sentence implicitly asserts its own truth, which means that we think with the logic of truth, that is, logic whose distinguishing value is truth — even when we lie.

SCI-models are replaced by *CCL*-models, being matrices $\mathcal{M}_{CCL} = (\mathcal{A}_{CCL}, D)$ such that $\mathcal{A}_{CCL} = (A_{CCL}, \neg, \cap, \cup, \Rightarrow, \Leftrightarrow, \supset)$ is an algebra

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[11] Contrary to the correct forms in this paper, in Łukowski (1997, 2011, 2019, 2020); Rudnicki and Łukowski (2021), the axioms $A_{6:}$ and $A_{7:}$ have the following forms: $A_{6:}((\alpha : \beta) \wedge (\beta : \alpha) \wedge (\delta : \gamma) \wedge (\gamma : \delta)) \rightarrow (((\alpha \mathbin{\S} \delta) : (\beta \mathbin{\S} \gamma)) \wedge ((\beta \mathbin{\S} \gamma) : (\alpha \mathbin{\S} \delta)))$, for $\S \in \{\rightarrow, \leftrightarrow, :, \vee\}$; and $A_{7:}((\alpha : \beta) \wedge (\delta : \gamma)) \rightarrow ((\alpha \S \delta) : (\beta \S \gamma))$, for $\S \in \{\wedge, \vee\}$.

[12] Indeed, we have $((\alpha \wedge \alpha) : \alpha)$ by $A_{2:}$, and so $(\alpha : (\alpha \wedge \alpha)) \wedge ((\alpha \wedge \alpha) : \alpha)$ by $A_{4:}$. Thus, $\alpha : \alpha$ by $A_{1:}$.

similar to CCL-language $\mathcal{L}_{CCL} = (\mathrm{For}_{CCL}, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, :)$, D is a nonempty subset of A_{CCL}, and for any $a, b, c \in A_{CCL}$,

1. $a = a \cap a$
2. $a \cap b = b \cap a$
3. $a \cap (b \cap c) = (a \cap b) \cap c$
4. $-a \in D$ *iff* $a \notin D$
5. $a \cap b \in D$ *iff* $a \in D$ *and* $b \in D$
6. $a \cup b \in D$ *iff* $a \in D$ *or* $b \in D$
7. $a \Rightarrow b \in D$ *iff* $a \notin D$ *or* $b \in D$
8. $a \supset b \in D$ *iff* $a = b \cap c$, for some $c \in A_{CCL}$

The last 8th condition can be replaced by the equivalent one given by Grzegorczyk (2011):

8′. $a \supset b \in D$ iff $a = a \cap b$.

However, from a philosophical point of view, as well as because of our attempt to formally simulate human everyday thinking, condition 8 seems to be more natural and useful than 8'.[13]

The semantic inference is defined in a standard way: $X \models_{CCL} \alpha$ *iff* for any CCL-model $\mathcal{M}_{CCL} = (\mathcal{A}_{CCL}, D)$ and $\nu \in \mathrm{Hom}(\mathcal{L}_{CCL}, \mathcal{A}_{CCL})$: $\nu(\alpha) \in D$, if only for any $\beta \in X$, $\nu(\beta) \in D$.

Completeness Theorem.[14] *For any* $\alpha \in \mathrm{For}_{CCL}$, $X \subseteq \mathrm{For}_{CCL}$:

$$\alpha \in C_{CCL}(X) \quad \textit{iff} \quad X \models_{CCL} \alpha.$$

Sketch of the proof. The proof bases on the fact that the relation $\alpha \sim \beta$ *iff* $(\alpha : \beta) \wedge (\beta : \alpha) \in T$, where T is a CCL-theory,[15] is a congruence of the matrix $(\mathcal{L}_{CCL}, T)$. A desired Lindenbaum CCL-model is the matrix $(\mathcal{L}_{CCL}/_{\sim}, T/_{\sim}$, where T is a maximal CCL-theory, and the valuation $\nu \in \mathrm{Hom}(\mathcal{L}_{CCL}, \mathcal{L}_{CCL}/_{\sim})$. The proof that $(\mathcal{L}_{CCL}/_{\sim}, T/_{\sim}$ satisfies the first three conditions of the CCL-model is based on the fact that, $(\alpha \wedge \alpha) : \alpha, (\alpha \wedge \beta) : (\beta \wedge \alpha), (\alpha \wedge (\beta \wedge \gamma)) : ((\alpha \wedge \beta) \wedge \gamma), ((\alpha \wedge \beta) \wedge \gamma) :$

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[13] A similar relationship connects the inclusion and identity of sets: $A \subseteq B$ *iff* $A = A \cap B$. In many cases, the inclusion of sets is a much more convenient relation than identity. It is the same with content implication. The use of the sentential identity to express everyday inferences seems unnatural.

[14] $C_{CCL}(X)$ is understood in a standard way, i.e. $\alpha \in C_{CCL}(X)$ *iff* there is a sequence $\beta_1, \ldots, \beta_k$ being a proof of α on the ground of CCL. It means that $\alpha = \beta_k$, and every β_i belongs to X or is an axiom of CCL or is a result of Modus Ponens applied to earlier formulas.

[15] X is a theory of the consequence operation C, so called C-theory, if and only if $X = C(X)$.

$(\alpha \wedge (\beta \wedge \gamma))$ are theses of *CCL*. The first one comes from $A_{2:}$, the second is simply $A_{3:}$, and the third and the forth is derivable from $A_{1:}, A_{2:}, A_{3:}, A_{4:}, A_{7:}$.[16] Verification of the conditions 4–7 is the same as in classical propositional logic. Thus, let us check if $(\mathcal{L}_{CCL}/_{\sim}, T/_{\sim}$ satisfies the last (i.e. the eighth) condition. Let us assume that $\nu(\alpha) \supset \nu(\beta) \in T/_{\sim}$. Thus, $[\alpha]/_{\sim} : [\beta]/_{\sim} \in T/_{\sim}$. It means that $(\alpha : \beta) \in T$, and so, $(\alpha : (\alpha \wedge \beta)) \in T$, by $(\alpha : \alpha)$, $A_{1:}, A_{4:}, A_{7:}$.[17] Since $A_{2:} \in T, \alpha \sim (\alpha \wedge \beta)$. Thus, $[\alpha]/_{\sim} = [\alpha]/_{\sim} \wedge /\_{\sim}[\beta]/_{\sim} = [\beta]/_{\sim} \wedge [\alpha]/_{\sim}$. It means that there is $c = ([\alpha]/_{\sim}) \in T/_{\sim}$ such that, $\nu(\alpha) = \nu(\beta) \wedge /_{\sim}c$. Now, let us assume that $\nu(\alpha) = \nu(\beta) \wedge /_{\sim}c$. Since $c \in \mathrm{For}_{CCL}/\sim$, thus $c = [\gamma]/_{\sim} = \nu(\gamma)$, for some $\gamma \in \mathrm{For}_{CCL}$. It means that $[\alpha]/_{\sim} = [\beta]/_{\sim} \wedge /_{\sim}[\gamma]/_{\sim}$, and so, $\alpha \sim (\beta \wedge \gamma)$. Thus, $(\alpha : (\beta \wedge \gamma)) \wedge ((\beta \wedge \gamma) : \alpha) \in T$. Then, $\alpha : (\beta \wedge \gamma) \in T$, and so, by $A_{1:}$ and $A_{2:}, \alpha : \beta \in T$. $[\alpha]/_{\sim} : /_{\sim}[\beta]/_{\sim} = [\alpha : \beta]/_{\sim} \in T/_{\sim}$.

Naturally, like the *SCI*-model, the *CCL*-model fulfills the Strawson's postulate: the meaning of a sentence is proposition/statement/ utterance, which may turn out to be true or false, but the sentence itself is neither true nor false. True or false could be the thought expressed in the sentence, i.e., the content of the sentence. It is easy to see that the entire *CCL*-model is the context for understanding and interpreting sentences in a language, mainly because of the eighth condition.

The content implication $p : q$ is a sentence with true content in a given *CCL*-model, colloquially the sentence p contentually implies the sentence q, i.e. $\nu(p) \supset \nu(q) \in D$, if and only if, $\nu(p) = \nu(q) \cap \nu(r)$, for some sentence $r \in \mathcal{L}_{CCL}$, in other words, when $\nu(q)$, the content of the sentence q is a part (in the sense of the conjunction) of $\nu(p)$, the content of the sentence p. We will then say that the sentence p says what the sentence q says, or in short: p says q. Naturally, the fact that p says q does not mean that q says p. For example, the sentence $A =$ "Tomorrow is Christmas Eve", among others, says that: $B =$ "The day after tomorrow is a holiday" — the content of the sentence $A : B$ is true. At the same time, it is clear that the content of the sentence $B : A$ is not

[16] (1) $(\alpha \wedge (\beta \wedge \gamma)) : \alpha$, by $A_{2:}$. (2) $(\alpha \wedge (\beta \wedge \gamma)) : (\beta \wedge \gamma)$, by $A_{3:}, A_{2:}$. Similarly, (3) $(\beta \wedge \gamma) : \beta$ and (4) $(\beta \wedge \gamma) : \gamma$. (5) $(\alpha \wedge (\beta \wedge \gamma)) : \beta$, by $A_{1:}$, (2), (3). (6) $(\alpha \wedge (\beta \wedge \gamma)) : \gamma$, by $A_{1:}$, (2), (4). (7) $(\alpha \wedge (\beta \wedge \gamma)) : (\alpha \wedge \beta)$, by $A_{1:}$, (1), (5). (8) $((\alpha : \beta) \wedge (\alpha : \gamma)) \rightarrow ((\alpha \wedge \alpha) : (\beta \wedge \gamma))$, by $A_{7:}$. (9) $(((\alpha \wedge (\beta \wedge \gamma)) \wedge ((\alpha \wedge (\beta \wedge \gamma))) : (\alpha \wedge \beta)$, by (1), (5), (8). (10) $(\alpha \wedge (\beta \wedge \gamma)) : ((\alpha \wedge \beta) \wedge (\alpha \wedge \gamma))$, by (9), $A_{4:}$. (11) $(\alpha \wedge (\beta \wedge \gamma)) : (\alpha \wedge \beta)$, by (9), (10), $A_{1:}$. Similarly, $(\alpha \wedge (\beta \wedge \gamma)) : ((\alpha \wedge \beta) \wedge \gamma)$. The proof for $(\alpha \wedge \beta) \wedge \gamma : \alpha \wedge (\beta \wedge \gamma)$ is analogous.

[17] Like in the footnote 16.

true, because the content of the sentence A is not part of the content of the sentence B — the sentence B says nothing about Christmas Eve, but only about a day off from work. Moreover, in addition to the content of sentence B, sentence A says much more, for example $C =$ "The day after tomorrow is a non-working day" as well as $D =$ "The ninth day from today is New Year's Day." This fact agrees with the presence of a certain c in the eighth condition, which is the content of a sentence that is also mentioned in it with the content of a. Returning to our example, it should be noted that not only the sentences $A : B$, $A : C$, $A : D$ express true thoughts. Unlike $C : A$, the content of the sentence $D : A$ is also true. Then, since $A : D$ and $D : A$, so $A \equiv_c D$, that is sentence A says exactly the same as D — everything that can be read from the content of A, you can also read from the content of D and vice versa. Content implication can then be viewed as being an ingredient in Leśniewski (1916) sense: the content of one sentence is part or the whole of the content of another sentence. Naturally, the sentential identity '$\equiv_c$' is defined as follows:

$$((\alpha : \beta) \wedge (\beta : \alpha)) \leftrightarrow (\alpha \equiv_c \beta)$$

It is not the Suszko's identity '$\equiv$', but it is an axiomatic strengthening of '$\equiv$' by the formulas:

$$(\alpha \wedge \alpha) \equiv \alpha$$
$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$$
$$(\alpha \wedge \beta) \wedge \gamma \equiv \alpha \wedge (\beta \wedge \gamma).$$

Naturally, Suszko's identity is the most trivial one. For example, the formula $p \wedge q \equiv q \wedge p$ is not an *SCI*-tautology, which means that the content of $p \wedge q$ is not identical with the content of $q \wedge p$. To distinguish from the connective of Suszko's identity, our $\equiv_c$ is called the connective of the *content identity*, and hence the index 'c'. [18]

It is also worth noting how the content implication implements Wittgenstein's postulates from (1921), which inspired Suszko. Let us quote three theses that not only refer to the analysis presented above, but are also its perfect summary:

[18] The idea behind the construction of the content implication was to "cut up" Suszko's identity into two mutually opposite implications. It seems, however, that such a construction is not possible in logics that satisfy the condition of a structural operation/consequence relation.

> 5.122 If p follows from q, the sense of "p" is contained in that of "q".
> 5.14 If a proposition follows from another, then the latter says more than the former, the former less than the latter.
> 5.141 If p follows from q and q from p, then they are one and the same proposition. (Wittgenstein, 1921)

Certainly, these theses do not refer to the truth-functional implication, but to the content implication — although the sentence "A cat is a mammal" is truth-functionally derived from the sentence "A fish is a mammal," there can be no question of any concluding the senses of these sentences. It may seem that the fundamental thesis of *CCL*, which says that every sentence says what it says, that is $A : A$ is in contradiction with postulate 5.14. However, it is not. It is enough to confront 5.14 with 5.141 and we will see that the *CCL*-tautology $A : A$ expresses the postulate 5.141 and not 5.14, which somehow assumes that since we are talking about two sentences then they must be different.[19] Equipped with the axiomatic definitions of our new tool — the content implication, we can now embark on a journey to solve the problems that divided Russell and Strawson.

6. The presupposition of a sentence as a part of its content

Let us assume the same symbols for Russell's sentence, its negation, and presupposition as in the previous sections:

$p =$ "The present king of France is bald"
$p^{-} =$ "The present king of France is not bald"
$\neg p =$ "It is not true that the present king of France is bald"
$q =$ "The present king of France exists"

Therefore,

$$p : q.$$

This sentence expresses our intuitive belief that the sentence p says that q. It is also consistent with the above proposed understanding of

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[19] Unfortunately, a very common error in the utterances is the statement: *"two objects a and b are equal if and only if ..."*. However, since they are two, they are not equal because they would be one and the same object. The correct wording of an exemplary fragment of a sentence should therefore have the form: *"a"* and *"b"* are names of the same object if and only if ... or *"a"* and *"b"* mean the same if and only if ... ". Wittgenstein did not make that mistake. In 5.14 he wrote *"If a proposition follows from another"*, meanwhile, in 5.141 *"p follows from q and q from p"*, which means that *"p"* and *"q"* are two names of the same proposition.

the connective of content implication. Naturally, the sentence p also says more than q, but at this point in time, this remainder is not important. So,

$$\nu(p) = \nu(q) \cap a,$$

where a is the one that goes beyond the q sense of the sentence p. Thus, among others, the sense a contains that the king is bald, that there can be only one king, that France is no longer a republic, etc.

At this point, it seems necessary to recall what was previously announced. Namely, that establishing a relationship between the content of sentences p and q is possible assuming:

1. a specific understanding of these sentences expressed by the mapping ν; and
2. a specific model $\mathcal{M}_{CCL}$ of reality (i.e., a fragment of reality) establishing this relationship.

All the following considerations are carried out with the understanding of sentences given by ν and with the image of reality expressed in the model $\mathcal{M}_{CCL}$.

Because all semantic correlates of sentences, i.e., the values of the mapping ν on sentences, belong either to the set of designated values D, or to its complement $A_{CCL} - D$, it should be assumed that all correlates are treated either as true or false. In this way, non-Fregean by nature, the logic of the content retains its bivalence. Therefore, if France does not currently have a king, then the sentence q is false, $\nu(q) \notin D$, and so $\nu(q) \in A_{CCL} - D$. This means that $\nu(p) \notin D$, among other things, $\nu(\neg p) = \neg\nu(p) \in D$ (CCL-model condition 4). Thus, if there is no king of France, the sentence $p =$ "*The present king of France is bald*" has false content and its *de dicto* negation $\neg p =$ "*It is not true that the present king of France is bald*", is true.

In a similar way, it is possible to establish the logical value of the sentence $p^- =$ "The present king of France is not bald", which is a *de re* negation of the sentence p. Naturally,

$$p^- : q,$$

so

$$\nu(p^-) = \nu(q) \cap b,$$

where b is that sense of p^- which goes beyond the sense of q. Thus, among others, the sense b contains that the king is not bald, that there can be only one king, that France is no longer a republic, etc. As in the previous case, the non-existence of the king of France means falsity of q, and thus falsity of p^{-}. To sum up, if France does not have a king, then both p and p^{-} have false contents. Each of them, among other things, says that q. As we can see, the logical analysis of the content using *CCL*-model, solves the problem not only of the meaning, but also of the logical values of sentences p and p^{-} in the model without referring to the concept of presupposition. Naturally, the sentence q should be considered as a presupposition of both sentences p and p^{-}. It is true that each of these two sentences contentually implies a sentence q and both sentences $p : q$ and $p^{-} : q$ have true contents. The sentence p as well as its *de re* negation p^{-} says what q says. The fact that these three sentences meet the definition of presupposition does not affect the analysis of the truthfulness of the content of these sentences — everything depends on the mapping and the model. Both relationships $p : q$ and $p^{-} : q$ result only from a simple analysis of the content of sentences p and p^{-}.

Strawson was forced to invent presuppositions because he was only equipped with Fregean logic, which was not suitable to examine content-based relations. However, in a non-Fregean logic we no longer have to use the concept of presuppositions at all. They are replaced with the natural, content-based form of the relations that take place between the utterances of natural language. If the present king of France does not exist, it is false to claim both that he is bald and that he is not bald, and so, it is true to say that both these statements are false. If the present king of France exists, it is enough to confront what they say with reality in order to recognize the logical value of the sentences p, p^{-}, $\neg p$, $\neg(p^{-})$.

Content implication allows not only to account for Strawsonian distinction between a sentence and its proposition. It also makes it possible to reconcile both seemingly mutually contradictory Russell approaches. Let us recall that according to the first approach $p = (A \in B)$ and its negation $\neg p = \neg(A \in B) = (A \notin B)$. Thus, $p \vee \neg p$, the law of the excluded middle holds. A graphical presentation of both cases on figures 1 and 2. Of course, $\neg p = p^{-}$ here.

However, we have a different situation when $\neg p \neq p^{-}$, e.g., in the case of Russell's well-known conjunctive reconstruction of sentences p and $\neg p$ (see figures 3–5).

In the first case, the sentence p^{-} has false content, because there is no "dot A" in the field "not bald." In the second case, the sentence p has false content, because there is no "dot A" in the field "bald." In the

1	0
p	$\neg p, p^{-}$

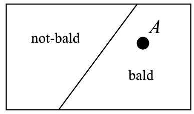

Figure 1.

1	0
$\neg p, p^{-}$	p

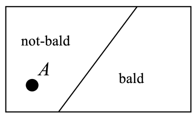

Figure 2.

1	0
$p, \neg(p^{-}), q, s$	$p^{-}, \neg p, \neg s$

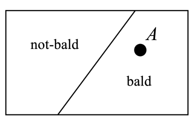

Figure 3.

1	0
$p^{-}, \neg p, q, \neg s$	$p, \neg(p^{-}), s$

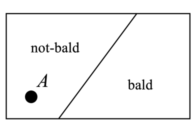

Figure 4.

third case, both p and p^{-} have false contents, because there is not "dot A" either in the field "not bald" or "bald." That is why $\neg p$ and $\neg(p^{-})$ have true contents.

1	0
$\neg p, \neg(p^-)$	$p, p^-, q, s, \neg s$

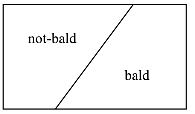

Figure 5.

7. Just as Aristotle wanted

Aristotle himself had spoken on the issue discussed by Russell and Strawson centuries later. In *Categories* (Cat. 13b 17–35), he posed the question: does the logical value of sentences "*Socrates is sick*", "*Socrates is healthy*", "It *is not true that Socrates is sick*" depend on whether Socrates exists or not? The answer he gave was "*yes.*" In fact, it turns out that the way non-Fregean logic assigns utterances logical values is exactly the same way as Aristotle did.

Let us take the following indications:

$$s = \text{"Socrates is sick"}$$
$$h = \text{"Socrates is healthy"}$$
$$t = \text{"Socrates is not sick"} = \text{"It is not true that Socrates is sick"}$$

Aristotle sets the values of these three sentences according to whether Socrates exists or not. If Socrates exists, then each of these sentences can be both true and false, with the additional assumption that the logical value of sentence s is different from that of sentences h and t. If Socrates does not exist, then sentences s and h are false and t is true. These assignments Laurence R. Horn collected in the following table (Horn, 2001, pp. 9, 14–16).

	Socrates exists	**Socrates does not exist**
$s =$ "Socrates is sick"	1 0	0
$h =$ "Socrates is healthy"	0 1	0
$t =$ "It is not true that Socrates is sick"	0 1	1

Let us introduce an important sentence $e =$ "Socrates exists". A simple reconstruction of the content of sentences s and h shows that the contents of sentences $(s : e)$, $(h : e)$, $(s \equiv_c h^-)$, $(h \equiv_c s^-)$ are true. Obviously, s^- and h^- are negations *de re*, respectively, of s and h. Thus, let us take the mapping $\nu \in \mathrm{Hom}(\mathcal{L}_{CCL}, \mathcal{A}_{CCL})$ and a CCL-model $\mathcal{M}_{CCL} = (\mathcal{A}_{CCL}, D)$ such that:

$$\nu(s) = \nu(e) \cap a, \quad \nu(h) = \nu(e) \cap b, \quad \nu(\neg s) = \nu(t),$$

where a says about Socrates's lack of health, and b says about Socrates's lack of sickness. Reading Aristotle's distribution expressed in the columns of the Horn table we receive:

1. If Socrates exists and is sick, then $\nu(e) \in D$ and $a \in D$, and so $\nu(s) \in D$ and $\nu(t) \notin D$. Moreover, since $b \notin D$, so $\nu(h) \notin D$.
2. If Socrates exists and is healthy, then $\nu(e) \in D$ and $b \in D$, and so $\nu(h) \in D$. Moreover, since $a \notin D$, so $\nu(s) \notin D$ and $\nu(t) \in D$.
3. If Socrates does not exist, then $\nu(e) \notin D$, and so $\nu(s) \notin D$ and $\nu(h) \notin D$ and $\nu(t) \in D$.

As we can see, the above distribution of logical values proposed by Aristotle is fully consistent with the way we define our content implication.

8. Summary

Thomas Kuhn (1962) in his *Structure of Scientific Revolutions* writes that a paradigm shift in science is preceded by growing frustration in some researchers at the inadequacy of the old paradigm at explaining reality. He also devotes a lot of attention to the concept of *incommensurability* — the impossibility or difficulty of communication between different scientific paradigms. Sometimes even the same words can mean vastly different things depending on the paradigm. For example, mass in Newtonian physics is very different from mass in general relativity. We believe that Peter Strawson was one of the researchers who voiced their frustration with the inadequacy of Fregean, truth-functional paradigm of logic at properly modeling natural language. However, his frustration was largely incommensurable, because the new paradigms of logic were either just beginning or did not exist at all. Nowadays, we can build upon the achievements of Suszko and his followers to develop a new paradigm in logic — the logic of content. In this article we show that the problem for which the presuppositions were created disappears if we change the way we understand what a sentence, its content and its truth value is. In the classical paradigm, contents of sentences are reduced to logical values, but that is not the only way in which logic can be carried in. In the paradigm we propose there are as many semantic correlates of sentences as there are sentences in language. This allows us to retain the information carried by utterances of natural language while still being able to determine bivalent truth values for them. We base our approach on the dynamically growing field of cognitive science that is interested in describing the logic of human everyday reasoning. One of the pioneers of that field, Philip Johnson-Laird (2010) expresses the dominant sentiment of cognitive scientists regarding the old logical positivism by writing:

> Human reasoning is not simple, neat, and impeccable. It is not akin to a proof in logic. (...) Reasoning is more a simulation of the world fleshed out with all our relevant knowledge than a formal manipulation of the logical skeletons of sentences.
>
> (Johnson-Laird, 2010, p. 18249)

Russell and Strawson got caught in the trap of trying to determine the "real" truth value of the sentence S — "The current King of France is bald." Truth value in their paradigm was absolute and the Strawsonian idea of assigning truth values to instances of sentence use instead of sentences was somehow controversial to logicians. However, that idea is not only non-controversial but natural for cognitive scientists and a system that tries to bring together logic and human cognition must account for that. In the logic of content we propose, truth values are always determined for single utterances, rather than sentences. In logical terms it is expressed by the fact that the content implication connective maps a relation between two sentences only for a specific understanding of these sentences expressed by the mapping ν; and a specific model $\mathcal{M}_{CCL}$ of reality establishing this relationship. In cognitive terms, Johnson-Laird (2010) summarizes it again the best:

> However, intuition is not always enough for rationality: a single mental model may be the wrong one. [...] In reasoning, the heart of human rationality may be the ability to grasp that an inference is no good because a counterexample refutes it.
>
> (Johnson-Laird, 2010, p. 18249)

In other words, humans perform reasonings flexibly, constantly changing their "model" of reality. They reason in counterfactuals ("What if X was true") and empathically take the perspective of others to determine "what is true from their point of view." The ability to do all these things suggests that the logic governing our everyday reasonings does not struggle with presuppositions. It does not require concessions and very special cases to account for them but handles them rather effortlessly. We believe that our solution to the problem of the *S* sentence as it was debated by Russell and Strawson captures the sentiment of both philosophers. In terms of the proposed content implication, sentence *S* says each of the sentences in Russell's analysis: R_1, R_2, R_3 (i.e., *S* says their conjunction R_1 & R_2 & R_3). However, this conjunction does not exhaust everything that the sentence *S* says, otherwise it would be synonymous with sentence *S*. It looks as if Russell inadvertently used our connective in his reconstruction and simply wrote down formally what the sentence *S* says. On the other hand, Strawson's presupposition of sentence *S*, i.e., the sentence R_1, is also what sentence *S* says. Thus, we come to the conclusion that both solutions of the famous philosophers, are in fact instances of use of the content implication, albeit unknown to them at the time. Moreover, from this point of view, the solutions of the two philosophers are not contradictory, but complementary. They were both correct, but because they worked in a Fregean, truth-functional paradigm, they could not have expressed themselves fully.

References

Bach, K., 1987, *Thought and reference*, Oxford: Clarendon Press.

Beaver, D. I., and B. Geurts, 2011, "Presuppositions", *Stanford Encyclopedia of Philosophy* (access: 05.08.2020). https://plato.stanford.edu/entries/presupposition/

Bloom, S. L., and R. Suszko, 1972, "Investigations into the sentential calculus with identity", *Notre Dame Journal of Formal Logic* 13(3): 289–308. DOI: 10.1305/ndjfl/1093890617

Brennan, J., 2016, "Naturalistic sentence comprehension in the brain", *Language and Linguistics Compass* 10(7): 299–313. DOI: 10.1111/lnc3.12198

Czelakowski, J., 2019, "Abstract algebraic logic as a bridge between algebra and logic" [in Polish], *Wiadomości Matematyczne* 52(2): 201–217. DOI: 10.14708/wm.v52i2.3295

Davies, M., 1981, *Meaning, Quantification, Necessity: Themes in Philosophical Logic*, Routledge. DOI: 10.4324/9780367853952

Evans, J., 1982, *The Varieties of Reference*, Oxford: Clarendon Press. DOI: 10.2307/2219077

Frege, G., 1948, "Sense and reference", tr. T. Geach, with Introductory note by M. Black, *The Philosophical Review* 57(3): 207–230. DOI: 10.2307/2181485

Frege, G., 1956, "The thought: A logical inquiry", *Mind, New Series* 65(259): 289–311. DOI: 10.1093/mind/65.1.289

Grzegorczyk, A., 2011, "Filozofia logiki i formalna logika niesymplifikacyjna" [Philosophy of logic and perceptive equivalence], *Zagadnienia Naukoznawstwa* 4(190): 445–450.

Horn, L. R., 2001, *A Natural History of Negation*, CSLI Publications.

Johnson-Laird, P. N., 2010, "Mental models and human reasoning", *Proceedings of the National Academy of Sciences* 107(43), 18243–18250. DOI: 10.1073/pnas.1012933107

Kuhn, T. S., 1962, *The Structure of Scientific Revolutions*, University of Chicago Press: Chicago.

Neale, S., 1990, *Descriptions*, Cambridge, Mass.: MIT Press.

Leśniewski, S., 1916, "Podstawy ogólnej teorii mnogości I" [Foundations of the General Set Theory I] *Prace Polskiego Koła Naukowego w Moskwie. Sekcya matematyczno-przyrodnicza* (2).

Łukowski P., 1997, "An approach to the liar paradox. New aspects in non-classical logics and their Kripke semantics", *RIMS Kôkyûroku* 1010: 68–80.

Łukowski, P., 2011, *Paradoxes*, Dordrecht–Heidelberg–London–New York: Springer.

Łukowski, P., 2019, "A 'distributive' or a 'collective' approach to sentences?", *Logic and Logical Philosophy* 28(2): 331–354. DOI: 10.12775/LLP.2019.011

Łukowski, P., 2020, "Content implication and Yablo's sequence of sentences", *Logic and Logical Philosophy* 29(1): 57–69. DOI: 10.12775/LLP.2019.012

Łukowski, P., 2022, "Propozycja formalizacji myślenia codziennego", in D. Leszczyńska-Jasion, S. Chlebowski, A. Tomczyk and A. Zakosztowicz (eds.), *Język – Struktura – Ontologia. Tom dedykowany pamięci Romana Suszki*, Wydawnictwo Nauk Społecznych i Humanistycznych Uniwersytet im. Adama Mickiewicza w Poznaniu.

Mates, B., (1961), *Stoic Logic*, Berkeley, University of California Press.

Rabovsky, M., and K. McRae, 2014, "Simulating the N400 ERP component as semantic network error: Insights from a feature-based connectionist attractor model of word meaning", *Cognition* 132(1): 68–89. DOI: 10.1016/j.cognition.2014.03.010

Ramachandran, M., 2008, "Descriptions and presuppositions: Strawson vs. Russell", *Suid-Afrikaanse Tydskrif vir Wysbegeerte* 27(3): 64–79. DOI: 10.4314/sajpem.v27i3.31515

Reimer, M., and A. Bezuidenhout (eds.), 2004, *Descriptions and Beyond*, Oxford: Clarendon Press.

Rudnicki, K., and P. Łukowski, 2021, "Psychophysiological approach to the liar paradox: Jean Buridan's virtual entailment principle put to the test", *Synthese* 198(22): 5573–5592. DOI: 10.1007/s11229-019-02107-x

Russell, B., 1905, "On denoting", *Mind, New Series* 14(56): 479–493.

Russell, B., 1957, "Mr. Strawson on referring", *Mind* 66(263): 385–389. DOI: 10.1093/mind/lxvi.263.385

Sainsbury, R. M., 1979, *Russell*, London: Routledge and Kegan Paul.

Strawson, P. F., 1950, "On referring", *Mind, New Series* 59(235): 320–344.

Strawson, P. F., 1952, *Introduction to Logical Theory*, Oxford.

Strawson, P. F., 1964, "Intention and convention in speech acts", *The Philosophical Review* 73(4): 439–460. DOI: 10.2307/2183301

Suszko, R., 1975, "Abolition of the Fregean axiom", *Lecture Notes in Mathematics* 453: 169–239. DOI: 10.1007/BFb0064874

Wittgenstein, L., 1921, *Tractatus Logico-Philosphicus*, Annalen der Naturphilosophie.

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