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Intensional Semantics for Syllogistics: what Leibniz and Vasiliev Have in Common

Abstract. This article deals with an alternative interpretation of syllogistics, different from the classical (extensional) one: an intensional one, in which subject and predicate are not associated with a set of individuals (the extension of the concept) but a set of attributes (the content of the concept).

The authors of the paper draw attention to the fact that this approach was first proposed by Leibniz in works on logical calculus, which for a long time remained in the shadow of his other philosophical works. Currently, the intensional approach is gaining more and more popularity due to the development of non-classical logics, and the article will present several existing intensional formal syllogistic semantics.

The paper will also consider another historical approach to syllogistics, associated with the name of the Russian logician Nikolai Vasiliev, who is not only one of the founders of non-classical (non-Aristotelian logic) but also of a different intensional interpretation of such logic. The authors, along with the already known formalizations of Vasiliev's ideas, present two new systems. One of them is a reconstruction of one type of imaginary logic with statements of three qualities: affirmative and two types of negative statements (with absolute and ordinary negation). The second system is the one that is adequate to semantics, in which instead of the four classical ones, only three types of statements are presented (two particular statements are replaced by one — accidental), and their significance is determined through the relation of the classical logical entailment. Both of them are interpreted intensionally.

The intensional approach in logic and, in particular, in syllogistics allows us to expand the class of accepted principles (which occurs due to the expansion of the class of correct moods of syllogisms).

Keywords: syllogistics; intensional semantics; Vasiliev; Leibniz; multivalued logic; non-classical logic; imaginary logic

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1. An Introduction to the Intensional Leibnizian Semantics

Traditionally, the relations between terms of a categorical statement within syllogistic systems are treated in an extensional way, that is as the relations between extensions of concepts. This interpretation comes from Aristotle. In his "Categories" he writes about the concept which is common to various things as denoting these objects; that is, he characterizes the concept from the point of view of its scope:

> Thus, for example, both a man and an ox are animals. Each of these is called by a common name, 'animal', and the definition of being is also the same. [Aristotle, 1967, 1a12]

Let us recall that a concept is a thought that, by pointing to a certain attribute, distinguishes from the rest of the universe and collects into a class (generalizes) objects that possess this attribute. The extension of a concept is a class of objects that are distinguished from the rest of the universe and united under this concept Aristotle gives the following example:

> Whenever one thing is predicated of another as of a subject, all things said of what is predicated will be said of the subject also. For example, man is predicated of the individual man, and animal of man; so animal will be predicated of the individual man also-for the individual man is both a man and an animal. [Aristotle, 1967, 1b10]

This implies an extensional interpretation of the relationship between subjects and predicates in attributive statements.

In this approach, syllogistic constants are considered as signs of relations between two sets (extensions of concepts): for example, in the fundamental syllogistic constant a represents the relation of set-theoretic inclusion one class to another (extension of the subject in the extension of the predicate), and the constant i represents the presence of at least one common element from two classes: the extensions of subject and predicate, and so on.

In the history of logic, there was an alternative approach to the interpretation of categorical statements. It consisted in the interpretation of the subject and predicate of the proposition as conceptual constructions, but from the point of view of content characteristics. In the modern doctrine of the concept, the content is called the attribute by which the objects in the concept are distinguished and unified (this attribute can be simple or complex). In traditional logic, the content was usually understood as a set of attributes — positive (indicating the presence of properties) and negative (indicating their absence). Recall that between the extensions and contents of concepts (if they have one common universal term), the law of reciprocity applies: if one concept is wider than the other in extension, then it is poorer in content, and vice versa.

In this intensional approach, syllogistic constants are considered as signs of relations between concepts in terms of content. The idea of such an interpretation of categorical statements belongs to Leibniz, who directly contrasted his "content-related" interpretation with the "extensional" one.

Consider two works by Leibniz, where he develops his approach. The first of them is his "Elements of Calculus". In it, he writes:

> The schools speak otherwise, because they are considering not concepts but instances subsumed under universal concepts. Thus they say that metal is wider than gold, since it contains more species than does gold. If we were to count the individuals made of gold on the one hand, and those made of metal on the other, there would certainly be more of the latter than of the former, and hence the former would be contained in the latter as part in a whole.... I prefer to consider universal concepts or ideas and their composition, for these do not depend on the existence of individuals. So I say that gold is greater than metal, because more constituents are required for the concept of gold than for that of metal, and more is needed to produce gold than to produce just a metal.
> [Leibniz, 1989, pp. 237–238]

As we can see, Leibniz shows by example the law of reciprocity and gives preference to the ideas of concepts, rather than their existence.

The second work to which we will turn is "New Essays on Human Understanding". Here, Leibniz proposes a program for the interpretation of the syllogism as intensional theory:

> For in saying, every man is an animal, I mean to say that all men are included in all animals; but I mean at the same time that the idea of animal is included in the idea of man. Animal includes more individuals than man, but man includes more ideas or more formalities; the one has more examples, the other more degrees of reality ; the one more extension, the other more intension. It may also be truly said that the entire syllogistic doctrine may be demonstrated by that *de continente et contento*, the containing and the contained,which is different from that of the whole and the part; for the whole always exceeds the part.
> [Leibniz, 1916, p. 569]

There are many papers devoted to this problem. In the past 20–30 years, attempts have been made to formalize this Leibnizian approach in syllogistics [Glashoff, 2010; Lenzen, 1983; Van Rooij, 2014, see, e.g.,], but these works have been more focused on the arithmetic side of the formalization and directly follow his approach. In this article, we present formal syllogistic variants of the actualization of Leibniz's idea and the authors' research within the framework of the presented systems.

One of the variants of a semantics for such a system was constructed by Russian logician Vladimir Markin [2001]. He also showed that the system for which the Leibnizian approach to the interpretation of categorical statements is adequate is Łukasiewicz's syllogistic, known as traditional syllogistics.

He considered the set of literals L, which represent the attributes $L = \{p_1, \sim p_1, p_2, \sim p_2, \dots\}$. An attribute can be positive (p_i) and negative ($\sim p_i$).

Leibniz interprets the concept in intensional way, that is, as a set of attributes. He does not mention that this set is non-empty, since this fact seems obvious to him. A concept is an arbitrary non-empty subset of L. Only noncontradictory concepts are used to construct the semantics for traditional syllogistic. The noncontradictory concept $\alpha \subseteq L$ satisfies the following conditions:

1. $\alpha \neq \varnothing$;
2. $\neg \dot{\exists}\, p_i (p_i \in \alpha \mathbin{\dot{\wedge}} \sim p_i \in \alpha)$.

Let $\boldsymbol{H}$ be the set of all noncontradictory concepts. Over this set is defined the operation $^{\#}$, which compares every concept of α to its contrary, denoted by $\alpha^{\#}$:

$$(p_i \in \alpha^{\#} \Leftrightarrow \sim p_i \in \alpha) \mathbin{\dot{\wedge}} (\sim p_i \in \alpha^{\#} \Leftrightarrow p_i \in \alpha).$$

We now further define the interpretation function π that compares each common term for a noncontradictory concept (interpreted in intensional way). The conditions of the truth of formulas of the language of syllogistic in the interpretation of π are defined like this:

$$|SaP|_\pi = 1 \iff \pi(P) \subseteq \pi(S), \tag{D1}$$

$$|SeP|_\pi = 1 \iff \pi(P)^{\#} \cap \pi(S) \neq \varnothing, \tag{D2}$$

$$|SiP|_\pi = 1 \iff \pi(P)^{\#} \cap \pi(S) = \varnothing, \tag{D3}$$

$$|SoP|_\pi = 1 \iff \pi(P) \setminus \pi(S) \neq \varnothing. \tag{D4}$$

The syllogistic formula $\boldsymbol{A}$ is generally valid, i.e. $|\boldsymbol{A}|_\pi = 1$ under any interpretation of π.

This semantics adequately explicates Leibniz's approach.

Leibniz's interpretation of statements like a (All S are P) fully corresponds to the formulated semantics. He writes:

> So we can learn in this way whether any universal affirmative proposition is true. For in such a proposition the concept of the subject, taken absolutely and indefinitely and in general viewed in itself, always contains the concept of the predicate. For example, all gold is a metal, that is, the concept of metal is contained in the concept of gold generally and viewed in itself, so that whatever is assumed to be gold is by this fact assumed to be metal. [Leibniz, 1989, p. 239]

Regarding the interpretation of particular statements, Leibniz writes:

> But in the particular affirmative proposition it is not necessary for the predicate to be contained in the subject per se and viewed absolutely, or for the concept of the subject per se to contain the concept of the predicate. It suffices that the predicate be contained in some species of the subject or that the concept of some instance or species of the subject contain the concept of the predicate. [Leibniz, 1989, p. 239]

Species, according to Leibniz, is a broader concept in terms of content:

> Considered as concepts or component terms as I am here viewing them, these differ as part and whole, so that the concept of genus is part, the concept of species the whole. [Leibniz, 1989, p. 237]

> Since nothing more is required in a particular affirmative proposition than that a species of the subject contains the predicate, the subject will itself be related to the predicate either as species to genus, or as species to something coinciding with it or a reciprocal attribute, or as a genus to a species. [Leibniz, 1989, p. 240]

Thus, a partial statement is true, according to Leibniz, if the subject and the predicate have a common genus. In terms of the semantics constructed above it can be expressed as:

$$|SiP|_\pi = 1 \iff \dot{\exists}\, \alpha \in \boldsymbol{H}(\pi(S) \subseteq \alpha \mathbin{\dot{\wedge}} \pi(P) \subseteq \alpha).$$

Markin showed the equivalence of this statement to (D2).

As for the requirement of consistency of concepts, in the work "Elements of calculus", where the intensional semantics is formulated, negative signs are not mentioned. However, in another work, "Calculi universalis investigatione" Leibniz considers concepts containing negative signs, and even suggests an analog of the $^{\#}$ operation. He writes:

> If, in turn, this term is denied — 'the scientist is not-smart, not-fair' then, obviously, it turns out 'fair, smart non-scientist'.
>
> [Leibniz, 1903, p. 70]

We see that, as in the case of the operation $^{\#}$, positive signs are replaced by negative ones, and negative ones are replaced by positive ones.

Thus, the representation of the statement i in Leibniz's interpretation is equivalent to that proposed in the semantics. But it should be noted that the concept should be interpreted as a set of negative and positive attributes with the adoption of the principle of their consistency.

In [Markin, 2001], the consistency and completeness of the **C4** system with respect to this intensional semantics are proved. Here are the axiomatics for **C4**:

(A1) $(MaP \wedge SaM) \supset SaP$
(A2) $(MeP \wedge SaM) \supset SeP$
(A3) $SeP \supset PeS$
(A4) $SaP \supset SiP$
(A5) $SiP \supset SaS$
(A6) $SeP \equiv \neg SiP$
(A7) $SoP \equiv \neg SaP$
(A8) SiS

The only deduction rule is *modus ponens*.

2. Introduction to Imaginary Logic and the Intensional Ideas of Nikolai Vasiliev

In the Russian logical heritage, the ideas of Nikolai Vasiliev are the most interesting. The fate of the ideas of the Russian logician is similar to the fate of the ideas of Leibniz. Vasiliev's works, like Leibniz's ones, were an impetus for the development of various scientific spheres, including mathematics, logics and philosophy. We will try to show that they are connected not only by the historical underestimation of ideas, but also by the fact that the ideas themselves have a common foundation.

Vasiliev's works are undoubtedly of great importance for the further development of non-classical logics, in particular, paraconsistent and multi-valued logic. But, unfortunately, his own system remained in the shadow for a long time, and many more aspects of his logic remain not fully known. The reconstruction of his ideas by means of modern logic is of historical and practical importance for the advancement of logic and science.

Vasiliev saw the possibility of constructing not one single system, but a multitude of imaginary logics, each of which is different from the Aristotelian system. Imaginary logic is the principle of constructing a new logic, and this principle should be looked upon as a tool in the study of the foundations of logic.

> The method of constructing the imaginary logic should be used for addressing the issues of laws of thought. This method allows to take apart the complicated and messy fabric of the "logical" where all the threads are linked and intertwine, criss-cross and entangle each other. This method allows to take different layers of the "logical" apart and trace the most important ways — the basis of the fabric — and their relations.[1] [Vasiliev, 1989, p. 92]

Looking back to 1910, Vasiliev was definitely far-sighted in realizing that the method of constructing an imaginary logic could help in solving problems related to the laws of our thinking.

Vasiliev is widely known for his revolutionary work in the field of "Imaginary (non-Aristotelian) Logic" [1989, pp. 53–93] across the world. He sets out the logic of the syllogistic type in it. But this logic differs from the classical one. Aristotle's logic undoubtedly is a tool for cognition of the real world around us. We can also think of other worlds, different from ours, in which the logical laws will be different. The logic proposed by Vasiliev is the logic of a contradictory reality, in which properties can simultaneously be inherent and not inherent in some objects. So his logic expands the class of accepted judgments. Along with the generally accepted affirmative and negative ideas, it accepts ones which combine negation and affirmation (contradiction). Vasiliev calls them — indifferent.

Nowadays, when non-classical logics are honorable and firm, it is not necessary to explain that the attempts to create different from Aristotelian logics opened new possibilities in that ancient science. However, back in 1910, the creation of such a logic was viewed as a radical move and its consistency had to be defended in scientific spheres.

[1] Here and further translated from Russian by authors.

Vasiliev repeatedly focuses on the similarity of his logical system to the geometry of Lobachevsky. He does to Aristotelian logic what Lobachevsky did to Euclid's geometry. Lobachevsky rejected Euclid's 5th postulate to build a new geometry and Vasiliev rejected the fundamental law of classical logic (which he called the law of earthly logic) to build a new logic.

Vasiliev emphasizes that there have been attempts to criticize the law of contradiction for a long time. He proposes to consider two formulations of the law of contradiction. In the Kant–Leibnizian tradition, this law prohibits the simultaneous presence of two incompatible features in an object. And it is precisely by abandoning it that we do not violate the basis of logic, which allows us to build non-classical imaginary logics

The second formulation says that one and the same judgment cannot be both true and false. Vasiliev calls it the law of absolute distinction between truth and falsity, or the law of non-self-contradiction. And it is in no way violated in the theory he proposed.

Vasiliev faced the task of not only constructing a new logic, but also finding a real interpretation for it. He believed that if Lobachevsky's geometry had such an interpretation, it would certainly be found for his new logic.

> We can give a real interpretation for non-Euclidean geometry, we can find formations, the geometry of which will be non-Euclidean in our Euclidean space . . . The real interpretation of Lobachevsky's geometry will be geometry on a surface with constant negative curve, the so-called pseudosphere . . . This is exactly how you can find in our world the formations logically similar to the imaginary logic.
>
> [Vasiliev, 1989, p. 81]

In the final part of his main work, Vasiliev expressed the idea of an intensional interpretation of attributive ideas of three different qualities. It turns out to be interesting that Vasiliev never mentions Leibniz's works and does not refer to his ideas. Whether he was familiar with the corresponding works of Leibniz or not is still a puzzle.

In this approach, generally affirmative ideas of the form "All S are P" carry information that the content of the predicate P is a part of the content of the subject S. A generally negative thought means that the contents of S and P have conflicting features in this approach. It is not at all necessary that for every feature in the predicate of the subject there would be a contradictory one. It would be enough to find that in one the feature of the predicate:

> While denying that Columbus was the first European to sail to America, we do not deny that he was a European and that he sailed to America.
> [Vasiliev, 1989, p. 87]

Vasiliev goes further and speaks about the possibility of such a concept, where there is absolute falsity and the concept of absolute negation.

> But you can create a concept [...] of absolute denial. One can imagine a non A that would not have any of the attributes A. If the concept A consists of attributes $\{p, q, r, s, \}$ [...] then the concept of non-A should consist of attributes non-p, non-q etc. Along with this, one can think of our negation as preserved, which denies not all attributes of A, but only some of them. [Vasiliev, 1989, pp. 87–88]

Thus, Vasiliev introduces two negations instead of one: the idea of an absolute falsity and of simple falsity. The distinction between absolute and weak negation is clearly expressed in the article by Zaitsev and Markin [1999, p. 135]:

> [...] The interpretation of general statement of the other two qualities is based on the distinction between absolute and weak negation: a proposition of the type "All S is not (in the absolute sense) P" expresses the idea that in the concept S all attributes of the concept P are denied, and a proposition like "All S is not (in the weak sense) P"—that in the concept S only some features of P are denied (i.e., some features of P are denied in S, and some are asserted).
> [Zaitsev and Markin, 1999, p. 135]

With such an intensional approach to imaginary logic, conclusions that were impossible both in classical logic and the main version of the imaginary version now become possible. For example, Vasiliev gives one mode of intensional imaginary logic, which was previously impossible. In fact, the class of correct inferences in this logic is significantly wider. Here, for now, we give just one example of a new syllogism: its premises are: A is non-P, S is non-A; and conclusion is: S is P. Let the term P be associated with a set of features $\{p_1, p_2, \ldots, p_n\}$. According to the major premise, A contains features that contradict those indicated, that is, negative features $\{\text{non-}p_1, \text{non-}p_2, \ldots, \text{non-}p_n\}$. And according to the minor premise, S contains features that contradict those included in A, with features that contradict non-p_1, non-p_2, dots, non-p_n. These are positive signs p_1, p_2, …, p_n. Therefore, each of the features included in P is also contained in S.

3. Vasiliev's Syllogistics with Two Types of Negation

As has already been stated above, the imaginary logical theory remained in a half-page rudimentary form for a long time.

Following straight the Vasiliev's theory, Zaitsev and Markin offered the next semantics. A concepts is a subset (α) of the set $\{p_1, \sim p_1, \ldots, p_n, \sim p_n\}$, which includes contradictory both positive and negative signs with the same indices p_i and $\sim p_i$. Besides, all concepts must satisfy several conditions: (1) Each concept is not empty ($\alpha \neq \varnothing$); (2) there can be no contradicting attributes $\neg \dot{\exists} p_i (p_i \in \alpha \mathbin{\dot{\wedge}} \sim p_i \in \alpha)$ in one and the same concept. Next a function * was defined on the set of all concepts (M). It matches each attribute with an attribute that contradicts it. We pay attention to the idea offered by Vasiliev: it is similar to the semantics for **C4** described earlier. The semantics are described in detail in [Zaitsev and Markin, 1999].

The interpreting function δ assigns common meanings to the terms: $\delta(P) \in M$, i.e. it assigns a concepts to each term. The evaluation of formulas $|\cdot|^\delta$ is associated with δ. The possible values for true interpretation are **1** (true) and **0** (false).

A syllogistic constant A with subscripts $_1$, $_2$, $_3$ was offered for writing down general statements [Zaitsev and Markin, 1999]. The subscript $_1$ indicates affirmative statements, $_2$ – absolute negation statements, $_3$ – weak negation statements. The article [Konkova and Markin, 2020] draws attention to the correlation of the properties of statements this type of the imaginary logic with its original version. Let us recall that Vasiliev defines semantic definitions for general statements only in his version of this imaginary logic. In the main version of imaginary logic, Vasiliev still uses indefinite-partial premises. This method serves to expand the system by introducing accidental statements. So, these authors introduced additional syllogistic constants I with subscripts $_1$, $_2$, $_3$, where information conveyed by the subscripts is the same as in general statements.

Zaitsev and Markin of the reconstruction showed the importance of formulas for general and accidental statements. General statements (A_1) have the value 1 only if each feature from the concept P is included in the concept S. Statements with strong negation (A_2) take the value 1 only if for each feature from concept P there is a contradicting feature in the concept S. Statements with weak negation (A_3) have the value 1 only if there is such a feature from the concept P, which is a feature from the concept S.

Accidental statements are significant under following conditions: affirmative accidental statements I_1 take the value 1 with the interpretation δ iff there are no properties in the concept associated with P that contradict those ones which are the part of the concept associated with S. Accidental statements I_2 are significant iff there are no identical properties in the concepts associated with S and P. Accidental statements I_3 are significant iff the concept associated with P contains at least one property that is absent in the concept associated with S, and there is a property that contradicts the second concept.

The axiomatic calculus **IL2** (expressed using the same language as above) may now be presented. The axiomatization of the calculus is the spirit of Łukasiewicz and is based on the classical propositional calculus:

(A1) $(A_1MP \wedge A_1SM) \supset A_1SP$
(A2) $(A_1MP \wedge A_2SM) \supset A_2SP$
(A3) $(A_2MP \wedge A_1SM) \supset A_2SP$
(A4) $(A_2MP \wedge A_2SM) \supset A_1SP$
(A5) $(A_1MP \wedge I_1SM) \supset I_1SP$
(A6) $(A_1MP \wedge I_2SM) \supset I_2SP$
(A7) $(A_2MP \wedge I_1SM) \supset I_2SP$
(A8) $(A_2MP \wedge I_2SM) \supset I_1SP$
(A9) A_1SS
(A10) $\neg(A_1SP \wedge I_2SP)$
(A11) $\neg(A_2SP \wedge I_1SP)$
(A12) $I_1SP \supset I_1PS$
(A13) $I_2SP \supset I_2PS$
(A14) $A_1SP \supset I_1SP$
(A15) $A_2SP \supset I_2SP$
(A16) $A_3SP \equiv \neg I_1SP \wedge \neg I_2SP$
(A17) $I_3SP \equiv \neg A_1SP \wedge \neg A_2SP$

The only deduction rule is *modus ponens*.

Metatheorems of the semantic consistency and completeness of **IL2** have been proved Zaitsev and Markin [1999]. Thus, the class of formulas provable in **IL2** coincides with the class of formulas that are generally valid in the above semantics.

It is interesting that while preserving the laws of traditional syllogistics such as the laws of syllogistic identity A_1SS and I_1SS or the laws of subordination, the intensional approach opens up new laws that differ not only from the traditional approach, but also from the main version of imaginary logic. So, for example, in the basic version, the laws of inversion for general negative (strong negation) and indifferent (weak negation) statements are possible only as quasi-inversions $A_2SP \supset \neg I_2PS$, proposed by Kostiouk and Markin [1998]. According to this interpretation, the laws of conversion are adopted for ideas with strong negation and the constraint $A_2SP \supset I_2PS$. Weak negation statements fully convert $A_3SP \supset A_3PS$. The laws of opposites of the new logic state that: any general judgment is incompatible with either a general or a particular judgment of a different quality, and any particular judgment is incompatible only with general judgments of a different quality. All the accepted laws in **IL2** were proved by Konkova [2019].

But the main interest that an intensional approach to imaginary logics opens up is the possibility of expanding the class of correct judgments. Since imaginary logic is presented as a syllogistic theory, the main task of this theory is to identify correct two-premise syllogisms.

The concepts of a syllogism and a greater and a lesser term, a greater and lesser premise are defined in [Konkova and Markin, 2020]. They are introduced in a standard way, with the only difference from traditional syllogistics being that each premise and conclusion is a statement of one of the six types we described earlier. According to this system, as with any other logical theory, we are not dealing with the statements themselves, but with their formulas, meta-statements $B_1, B_2 \vdash C$, where B_1, B_2 and C are atomic formulas of the language of imaginary logic. Then the syllogism is called **IL2**-correct only if the formula is provable in the calculus **IL2**.

On the basis of the **IL2** system, Konkova [2019] managed to consider all possible 864 moods of syllogisms for all four figures. All correct moods of the syllogism were proved and the countermodels were proposed for all incorrect moods. Let us now give a general scheme of proofs and refutations for all figures. In each figure of the syllogism, all possible combinations of premises were sequentially considered (thirty six combinations for each figure) with all possible conclusions (for each combination of premises, six possible conclusions were considered). The initial consideration was carried out using the above truth values for judgments by the method of selecting counter-models. For all those combinations of premises with possible conclusions that could not be refuted by the countermodel, a proof was made of their truth in the **IL2** system. To make the article more self-contained, we repeat the proof of syllogisms using the example of the 4th figure:

$$(A_2PM \wedge A_3MS) \supset A_3SP$$

Proof:

1.	$(A_2PM \wedge I_1SP) \supset I_2SM$	(A7)
2.	$(A_2PM \wedge I_2SP) \supset I_1SM$	(A8)
3.	$I_2SM \supset I_2MS$	(A13)
4.	$I_1SM \supset I_1MS$	(A12)
5.	$(A_2PM \wedge I_1SP) \supset I_2MS$	1, 3
6.	$(A_2PM \wedge I_2SP) \supset I_1MS$	2, 4
7.	$(A_2PM \wedge \neg I_1MS) \supset \neg I_1SP$	6, PL
8.	$(A_2PM \wedge \neg I_2MS) \supset \neg I_2SP$	5, PL
9.	$(A_2PM \wedge \neg I_1MS \wedge \neg I_2MS) \supset (\neg I_1SP \wedge \neg I_2SP)$	7, 8, PL)
10.	$A_3SP \equiv (\neg I_1SP \wedge \neg I_2SP)$	(A16)
11.	$A_3PM \equiv (\neg I_1MS \wedge \neg I_2MS)$	(A16)
12.	$(A_2PM \wedge A_3MS) \supset A_3SP$	9, 10, 11, PL

Thus, a completely new intensional approach in logic made it possible to single out sixty-four **IL2**-correct syllogisms in the **IL2** system, of which fifty-two are perfect (the conclusion in them is the most powerful consequence of these premises), and twelve are imperfect. The 1st figure contains eighteen correct syllogisms, twelve of them are perfect ($A_1A_1A_1$, $A_1A_2A_2$, $A_2A_1A_2$, $A_2A_2A_1$, $A_3A_1A_3$, $A_3A_2A_3$, $A_1I_1I_1$, $A_1I_2I_2$, $A_2I_1I_2$, $A_2I_2I_1$, $A_3I_1I_3$, $A_3I_2I_3$) and six more imperfect ones. The most interesting of them are $A_2A_2A_1$, in this syllogism, from two premises with strong negation, an affirmative conclusion is deduced, and $A_3A_2A_3$, in which a large premise with a weak negation, and a smaller one with a strong one. In the 2nd Figure there are twelve correct syllogisms, eight perfect ones ($A_3A_1A_3$, $A_3A_2A_3$, $A_1A_3A_3$, $A_2A_3A_3$, $A_3I_1I_3$, $A_3I_2I_3$,$A_1I_3I_3$, $A_2I_3I_3$) and 4 more imperfect ones. Figure 3 contains eighteen correct syllogisms ($A_1A_1I_1$, $A_1A_2I_2$, $A_2A_1I_2$, $A_2A_2I_1$, $A_3A_1I_3$, $A_3A_2I_3$, $A_1I_1I_1$, $A_1I_2I_2$, $A_2I_1I_2$, $A_2I_2I_1$, $A_3I_1I_3$, $A_3I_2I_3$, $I_1A_1I_1$, $I_1A_2I_2$, $I_2A_1I_2$, $I_2A_2I_1$, $I_3A_1I_3$, $I_3A_2I_3$). And finally, in the 4th Figure, the number of correct syllogisms is sixteen, fourteen of them are perfect $A_1A_1I_1$, $A_1A_2I_2$,

$A_1A_3A_3$, $A_2A_1I_2$, $A_2A_2I_1$, $A_2A_3A_3$, $A_3A_1I_3$, $A_3A_2I_3$, $A_3I_1I_3$, $A_3I_2I_3$, $I_1A_1I_1$, $I_1A_2I_2$, $I_2A_1I_2$, $I_2A_2I_1$) and two are imperfect.

4. Vasiliev-style Intensional Semantics for Syllogistics with Three Constants

There is another way to construct an syllogistic intensional semantics. We will show how it is possible to present the traditional Łukasiewicz syllogistic using, firstly, Vasiliev's ideas, and, secondly, intensional semantics of a special type.

Shalack [2015] proposed a semantics in which formulas of classical propositional logic are attributed to general terms, and syllogistic constants are interpreted through the relation of classical inferability using the falsity constant **f**. Such a semantics was developed by Markin [2016] by using the inferability relation instead of the relation of the classical logical entailment.

We will construct here such an intensional semantics for Vasiliev-style syllogistics. In his work "On particular statements, triangle of oppositions, the law of excluded fourth" [1989, pp. 12–53], Vasiliev criticizes the interpretation of particular statements "Some S are P" and "Some S are not P" in traditional syllogistics. The quantifier "some" is interpreted in the sense of "there is at least one" and is not opposed to the quantifier "all". In this interpretation, particular judgments do not express, according to Vasiliev, a complete knowledge of the relation of their subject to the predicate, leaving uncertainty whether all S is P (for affirmative) and all S is not P (for negative). Science, according to Vasiliev, "uses 'some' in the sense of 'not all' and otherwise it can not be used" [1989, p. 4].

A truly scientific particular statement should give complete information about the ratio of the extension of S to the extension of P. Such is the accidental statement "Only some S are P". It combines an exposition and a negation: some S are P, and the other S are not P. An accidental statement has the same cognitive value as a general one, because it contains information about the entire extension of the term S.

Vasiliev uses the constant m to denote the type of accidental judgment. He suggests that syllogistics should be based on three types of statements: general propositions (affirmative and negative) and accidental one. The atomic formulas of the language of this syllogistic are

SaP, SeP, and SmP. The original propositions of Vasiliev's syllogistic (with the same subjects and predicates) form the so-called "triangle of opposites", which replaces the logical square of traditional syllogistic. These propositions are pairwise opposite, and all three are incompatible with respect to falsity (one of them is necessarily true).[2]

In a language with the original constants a, e, m an axiomatic calculus of Vasiliev-style syllogistic (called **C4V**) was constructed and it was proved that is definitively equivalent to the traditional Łukasiewicz syllogistic constructed in the standard language [Kostiouk and Markin, 1998].

The **C4B** system axiomatizes the traditional Vasiliev-style syllogistic based on the classical propositional calculus. Axiom schemes for this system are:

(C4V1) $(MaP \wedge SaM) \supset SaP$
(C4V2) $(MeP \wedge SaM) \supset SeP$
(C4V3) $SeP \supset PeS$
(C4V4) $\neg(SaP \wedge SmP)$
(C4V5) $\neg(SeP \wedge SmP)$
(C4V6) $SaP \vee SeP \vee SmP$
(C4V7) SaS
(C4V8) $\neg(SaP \wedge SeP)$

The only deduction rule is *modus ponens*.

The system **C4V**, which formalizes Vasiliev's syllogistic with three initial syllogistic constants, is recursively equivalent to Łukasiewicz's syllogistic **C4** with standard constants. We use the translation of $^+$ from the system language **C4V** to the language of systems **C4** and the back-translation $^\times$ from the standard language to "Vasiliev-style" language:

$$SaP^+ = SaP \qquad SaP^\times = SaP$$
$$SeP^+ = SeP \qquad SeP^\times = SeP$$
$$SmP^+ = SoP \wedge SiP \qquad SiP^\times = \neg SeP$$
$$(\neg \boldsymbol{A})^+ = \neg \boldsymbol{A}^+ \qquad SoP^\times = \neg SaP$$
$$(\boldsymbol{A} \circ \boldsymbol{B})^+ = \boldsymbol{A}^+ \circ \boldsymbol{B}^+ \qquad (\neg \boldsymbol{A})^\times = \neg \boldsymbol{A}^\times$$
$$(\boldsymbol{A} \circ \boldsymbol{B})^\times = \boldsymbol{A}^\times \circ \boldsymbol{B}^\times$$

where $\circ$ is an arbitrary binary connective. Translating $^+$ inserts **C4V** to **C4** (Łukasiewicz syllogistics), and the translation of $^\times$ inserts **C4** in **C4V**.

[2] For more, see [Zaitsev, 2017].

For syllogistics **C4B** it is possible to construct a semantics in which atomic statements are interpreted through the relation of entailment between the formulas of propositional logic.

Let the interpreting function δ associate to each general term some *satisfiable* formula of the language of propositional logic that does not contain any other connectives, except $\wedge$, $\vee$, and $\neg$; and so, the formula $\neg\delta(S)$ is not provable in the propositional calculus.

$$\begin{aligned} V(SaP,\delta) &\iff \delta(S)\vDash\delta(P), \\ V(SeP,\delta) &\iff \delta(S)\vDash\neg\delta(P), \\ V(SmP,\delta) &\iff \delta(S)\nvDash\delta(P)\ \dot{\wedge}\ \delta(S)\nvDash\neg\delta(P), \end{aligned}$$

where V is a predicate of significance.

The significance conditions for complex formulas remain standard.

The formula $\boldsymbol{A}$ is called V*-generally valid* if and only if $\dot{\forall}\,\delta\,V(\boldsymbol{A},\delta)$. To prove the adequacy of this semantics for the calculus **C4V** first, we use the intensional semantics constructed in [Shalack, 2015] and modified by Markin [2016] for **C4**. We give the significance conditions for syllogistic constants:

$$\begin{array}{ll} \Phi(SaP,\delta)\Leftrightarrow\delta(S)\vDash\delta(P); & \Phi(\neg\boldsymbol{A},\delta)\Leftrightarrow\neg\,\Phi(\boldsymbol{A},\delta); \\ \Phi(SeP,\delta)\Leftrightarrow\delta(S)\vDash\neg\delta(P); & \Phi(\boldsymbol{A}\wedge\boldsymbol{B},\delta)\Leftrightarrow\Phi(\boldsymbol{A},\delta)\ \wedge\ \Phi(\boldsymbol{B},\delta); \\ \Phi(SiP,\delta)\Leftrightarrow\delta(S)\nvDash\neg\delta(P); & \Phi(\boldsymbol{A}\vee\boldsymbol{B},\delta)\Leftrightarrow\Phi(\boldsymbol{A},\delta)\ \vee\ \Phi(\boldsymbol{B},\delta); \\ \Phi(SoP,\delta)\Leftrightarrow\delta(S)\nvDash\delta(P); & \Phi(\boldsymbol{A}\supset\boldsymbol{B},\delta)\Leftrightarrow\neg\,\Phi(\boldsymbol{A},\delta)\ \vee\ \Phi(\boldsymbol{B},\delta). \end{array}$$

where δ is a function that matches each general term with a satisfiable formula of the language of classical propositional logic, which does not contain any other propositional connectives except $\neg$, $\wedge$, $\vee$.

Now, to demonstrate adequacy, it is necessary to first prove the following metatheorem:

Theorem 4.1. *An arbitrary syllogistic formula* $\boldsymbol{A}$ *of a language with the original constants* a, e, *and* m *is* V*-generally valid if and only if its translation* $\boldsymbol{A}^{+}$ *is* Φ *generally valid.*

Proof. We first prove the following statement:

$$\dot{\forall}\boldsymbol{A}\dot{\forall}\delta(V(\boldsymbol{A},\delta)\iff\Phi(\boldsymbol{A}^{+},\delta)).$$

We will use course-of-values induction on the number of propositional connectives in $\boldsymbol{A}$. ⊣

Next, we can prove a metatheorem about the adequacy of the proposed semantics to the Vasiliev-style syllogistic **C4V**:

Theorem 4.2. *An arbitrary formula* $\boldsymbol{A}$ *is provable in* **C4V** *if and only if the formula* $\boldsymbol{A}$ *is* V*-generally valid.*

Proof. For the proof we can use:

1. the result of Kostiouk and Markin on the embeddability of **C4V** in Łukasiewicz's syllogistic **C4** by using the translation $^+$;
2. Theorem 4.1;
3. Shalak's result that the set of Φ-generally valid formulas coincides with the set of formulas provable in the system **C4**. ⊣

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