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Immune Logics ain't that Immune

Abstract. Da Ré and Szmuc argue that while there is a symmetry between 'infectious' and 'immune' logics, this symmetry fails w.r.t. extending an algebra with an immune or an infectious element. In this paper, I show that the symmetry also fails w.r.t. defining a new logical operation from a given set of primitive (Boolean) operations. I use the case of the material conditional to illustrate this point.

Keywords: defining logical operations; immune logics; infectious logics; material conditionals

1. Introduction

Da Ré and Szmuc (2021) explored a certain family of logics they labelled 'immune logics' (IL). Like its arguably more famous semantic cousin, the infectious semantics for Weak Kleene logic (WK) proposed by Bochvar (1981) and others, IL provides another semantic framework that explains the logical behaviour of 'meaningless' sentences. Unlike in WK, however, meaningless sentences in IL are self-contained and do not contaminate the compound sentences that have them as subsentences.[1]

Despite their differences, IL and WK 'are closely intertwined and can be properly seen as symmetric' (Da Ré and Szmuc, 2021, 34). The distinctive feature of WK is the presence of a value which is a zero element for all the (binary) operations of the algebra, while the characteristic

[1] We can understand 'meaningless' in various ways. For example, it can refer to sentences devoid of cognitive content or to sentences that are not truth-apt. For the purposes of this paper, however, we will not venture into what the right account of meaninglessness is; rather, we will remain neutral about this issue.

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feature of IL is the presence of a value which is an identity element for the binary operations of the algebra. But despite this difference, the logical operations of conjunction and disjunction in IL and WK might be seen as semantic duals: IL-conjunctions correspond to WK-disjunctions while IL-disjunctions correspond to WK-conjunctions.

Da Ré and Szmuc argue, however, that the symmetry between WK and IL fails. For them, while it is straightforward to extend an algebra with an infectious element, it is not so easy to extend an algebra with an immune element since such an extension does not determine one single algebra (Da Ré and Szmuc, 2021, 34).

In this paper, I show that the symmetry also fails w.r.t. defining a new logical operation from a given set of primitive (Boolean) operations. Consider, for example, the material conditional. Using only the set or subset of the Boolean operations of negation, disjunction, and, conjunction, such a logical operation is straightforwardly defined in WK but is not so in IL. Before discussing this issue further, however, let us first rehearse the semantic frameworks of both WK and IL.[2]

2. WK and IL semantics

Both WK and IL are three-valued logics, where each atomic sentence A maps into a trivalent set of semantic values $V = \{1, 0.5, 0\}$. '1' and '0' represent the *classical* truth values: 'true' and 'false', respectively, while '0.5' represents the 'meaningless', non-classical value. Boolean operations are defined in the usual recursive way in both logics. (For our purposes, negation, disjunction, and conjunction are represented as '$\neg$', '$\vee$', and '$\wedge$', respectively.)

In both WK and IL, Boolean operations behave classically if their subsentences only have classical values. This is so since both logics are *sub*-classical. They differ, however, in how they interpret compound sentences that have subsentences with the '0.5' value.

In WK, meaningless sentences are logically *infectious*. They contaminate the compound sentences that have them with their meaninglessness. Let us define this infectious feature of WK as follows:

[2] The discussion of both WK and IL semantics in §2 follows the discussion in (Joaquin, 2022, 2–4).

DEFINITION 2.1 (Infectiousness). For any sentence A, if $v(A) = 0.5$, then any compound sentence that has A as a subsentence would likewise have the value 0.5.

Definition 2.1 implies that meaninglessness begets meaninglessness. And this can be seen in the WK-semantics for Boolean operations represented by the following familiar truth tables (Table 1):

$\neg$	
1	0
0.5	0.5
0	1

$\vee$	1	0.5	0
1	1	0.5	1
0.5	0.5	0.5	0.5
0	1	0.5	0

$\wedge$	1	0.5	0
1	1	0.5	0
0.5	0.5	0.5	0.5
0	0	0.5	0

Table 1. WK truth tables for Boolean operations

The WK tables do imply a rather infectious logic. Conjoining or disjoining any sentence with a sentence that has value 0.5 would result in a meaningless compound sentence. For example, the true sentence '$2 + 2 = 4$' conjoined or disjoined with a meaningless sentence would result in a meaningless compound sentence. Likewise, the false sentence '$2+2 = 5$' conjoined or disjoined with a meaningless sentence would also result in a meaningless compound sentence. Of course, if a meaningless sentence is conjoined or disjoined with another meaningless sentence, the resulting compound would be meaningless as well.

On the other hand, in IL, sentences that have a '0.5' value are logically self-contained and non-contagious. Following Da Ré and Szmuc, let us call this feature of meaningless sentences 'universal idempotent immunity' or 'immunity' for short, and let us define it as follows:

DEFINITION 2.2 (Immunity). For any A, if $v(A) = 0.5$ then any compound sentence that has A as a subsentence will have the value of the *other* constituent subsentence.

Given Definition 2.2, Table 2 represents the IL-semantics for Boolean operations:

$\neg$	
1	0
0.5	0.5
0	1

$\vee$	1	0.5	0
1	1	1	1
0.5	1	0.5	0
0	1	0	0

$\wedge$	1	0.5	0
1	1	1	0
0.5	1	0.5	0
0	0	0	0

Table 2. IL truth tables for Boolean operations

The IL tables show that some compound sentences are *immune* from the 'meaninglessness' virus only if they have a non-meaningless (classically valued) subsentence.[3] For example, the true sentence '$2+2=4$' conjoined or disjoined with a meaningless sentence would result in a true compound sentence. On the other hand, the false sentence '$2+2=5$' conjoined or disjoined with a meaningless sentence would still be a false compound sentence. In IL, a Boolean operation would only be meaningless if all of its subsentences are meaningless.

Finally, negations behave in the same way in both WK and IL. On the one hand, in both these logics, negations behave classically if their negands have classical values. (That is, the negation of a false sentence is a true sentence, and vice-versa.) On the other hand, if their negands are meaningless, then the negation is meaningless as well.

3. Defining logical operations in WK and IL

After rehearsing the semantic frameworks of both WK and IL, let us now go back to the issue of their symmetry. As was noted in §1, WK and IL can be seen as symmetric in the sense that they are semantic duals: IL-conjunctions correspond to WK-disjunctions and IL-disjunctions correspond to WK-conjunctions. (This is easy to see given Table 1 and Table 2 above.) But while this symmetry is true of these Boolean operations, it might not be true of other logical operations. Let us consider the case of the material conditional '$\supset$'.

It is easy to show that given the infectious semantics of WK, i.e., a semantics with an infectious element for every primitive operation, any definable operation will also be infectious. Consider the case of the material conditional. In WK, '$\supset$' could be simply defined in terms of '$\vee$' and '$\neg$' such that $(A \supset B) =_{\mathrm{df}} (\neg A \vee B)$. The corresponding truth table (Table 3) shows this.[4]

Given Table 1 and Table 3, we can already see that $(A \supset B)$ and $(\neg A \vee B)$ are indeed logically equivalent. In some sense, therefore, we

[3] These IL truth tables are the same with those proposed by C. S. Peirce and the semantics for 'ordinary logic' due to Cooper (1968). For a discussion of Peirce's logic, see (Belikov, 2021) and (Fisch and Turquette, 1966); for Cooper's, see (Humberstone, 2011).

[4] Infectious conditionals can also be defined as $\neg(A \wedge \neg B)$.

$\supset$	1	0.5	0
1	1	0.5	0
0.5	0.5	0.5	0.5
0	1	0.5	1

Table 3. WK truth table for conditionals

can say that infectiousness is independent of the particular primitive vocabulary, so long as the defined logical operation abides by Definition 2.1 and the truth tables of Boolean operations in WK. More generally, this means that, in WK, logical operations are adequately inter-definable.

On the other hand, this does not seem to hold for immune systems like IL since immunity seems to be sensitive to the set of primitive constants involved. Consider again the case of the material conditional. If the material conditional is immune (as per Definition 2.2), then Table 4 will represent immune conditionals:

$\supset_{\mathrm{i}}$	1	0.5	0
1	1	1	0
0.5	1	0.5	0
0	1	0	1

Table 4. IL truth table for immune conditionals I

However, given Table 2 and Table 4, we can show that immune conditionals cannot be adequately defined as $(\neg A \vee B)$ since if $v(A) = 1$ and $v(B) = 0.5$, then $(\neg A \vee B)$ has value 0 but $(A \supset_{\mathrm{i}} B)$ has value 1.[5] This means, more generally, that logical operations in IL are not adequately inter-definable.

If we try to preserve the inter-definability of logical operations, e.g., we want to maintain that the material conditional is defined as $(\neg A \vee B)$, then we will have a different semantics for the material conditional, which may be presented by Table 5:

Given Table 2 and Table 5, it is apparent that $(A \supset B)$ and $(\neg A \vee B)$ are logically equivalent. But then the semantics for the material conditional violates Definition 2.2 since if $v(A) = 0$ and $v(B) = 0.5$, then the value of $(A \supset B)$ is not 0 but 1.[6]

[5] In the same manner, such immune conditionals cannot be defined by $\neg(A \wedge \neg B)$ either.

[6] The same reasoning applies if IL conditionals are defined in terms of negations and conjunctions. I will leave it to the reader to prove this as an exercise.

$\supset$	1	0.5	0
1	1	0	0
0.5	1	0.5	0
0	1	1	1

Table 5. IL truth table for immune conditionals II

4. Conclusion

Given the foregoing, we may now conclude that if material conditionals are immune, then logical operations in IL are not inter-definable. On the other hand, if material conditionals in IL are definable in terms of Boolean operations, then they violate the definition of immunity. In either case, the semantics for immune logics ain't that immune.

The material conditional, however, is just one example of a more general phenomenon. While it is true that every definable operation from a primitive set of infectious operations is infectious, it is not true that every definable operation from a primitive set of immune operations is immune. In this sense, one can see the limit of the intended symmetry between WK and IL.

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