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Alternative Semantics for Normative Reasoning with an Application to Regret and Responsibility

Abstract. We provide a fine-grained analysis of notions of regret and responsibility (such as agent-regret and individual responsibility) in terms of a language of multimodal logic. This language undergoes a detailed semantic analysis via two sorts of models: (i) relating models, which are equipped with a relation of propositional pertinence, and (ii) synonymy models, which are equipped with a relation of propositional synonymy. We specify a class of strictly relating models and show that each synonymy model can be transformed into an equivalent strictly relating model. Moreover, we define an axiomatic system that captures the notion of validity in the class of all strictly relating models.

Keywords: deontic logic; normative reasoning; propositional synonymy; regret; relating semantics; responsibility

1. Introduction

Recent contributions to the semantics of modal logic have increasingly paid attention to the problem of providing adequate structures to interpret systems where replacement of provable equivalents (RPE) fails.[1] Informally, RPE can be described as follows: if ϕ and ψ are provable equivalents in a system S, then so are any two formulas χ and χ^* s.t. χ^* is obtained from χ by replacing some occurrences of ϕ with ψ. The approaches that have been proposed generally share the view that RPE should be abandoned in favour of notions of replacement that are not

[1] The literature is very vast; for examples of approaches which abstract from a specific context of reasoning see [Pietruszczak, 2009; Jarmużek, 2020; Sedlár, 2021], as well as an earlier unifying framework in [Rantala, 1982].

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based (or, not uniquely based) on provable equivalence. However, ideas on how to handle this issue largely differ. In this article we will analyse a solution that is inspired by the framework of "relating semantics" [Jarmużek, 2020]. More specifically, we will see how a variation of relating semantics can be employed in the context of normative reasoning represented via modal logic.

Normative concepts, such as 'obligation', 'permission', 'prohibition', 'duty', 'right', etc., have some peculiar features. First, these concepts are always relative to a certain source. Current regulations in Slovakia require that passengers wear masks in public transport, while a parallel norm does not hold under the UK regulations right now. A bus passenger in Bratislava is thus legally obliged to wear a face covering, while a bus passenger in Bristol is not. Also, a moral system that grants animals moral standing would view the admissibility of eating meat differently than a moral system that does not.

Second, a normative concept that qualifies a given proposition need not qualify other propositions that are logically equivalent to it. For instance, an obligation 'to feed Thomas (the cat) or to meet and not meet with a friend' does not automatically follow from an obligation 'to feed Thomas', even though a proposition having the structure $p \vee (q \wedge \neg q)$, like the former, is logically equivalent (in a classical setting) to a proposition having the structure p, like the latter. Therefore, replacement of propositions within norms should be carefully performed.

Third, a normative source (or, sometimes, a legal or moral theory integrating its content) establishes which propositions fall within the scope of the norms it includes.[2] For instance, the fact that an employer committed a tort is relevant to the norms on vicarious liability in the German Civil Code, but is not relevant to the corresponding norms in the French Civil Code (at least under its current interpretation [Giliker, 2010]).

A system of formal logic designed for normative reasoning should take these features into account. However, many deontic logics interpreted via possible worlds semantics, following the paradigmatic case of *Standard Deontic Logic* (*SDL*), fail to do so. As Jarmużek and Klonowski [2020] observe, *SDL* and various systems sharing its semantics are logics of

[2] By a normative source (or normative system), we here mean any static or dynamic collection of norms, in particular, *deontic* norms. For instance, this can be a portion of a legal code, but also a set of norms taken from two or more distinct legal codes.

alethic-deontic modalities — dealing primarily with what is the case in a class of possible worlds — rather than proper deontic logics — dealing primarily with what ought to be the case. This observation leads them to formulating a new approach to normative reasoning based on the framework of *relating semantics*.

Relating semantics provides an interpretation for modal languages that extends possible worlds semantics with a relation R which applies to pairs of formulas, saying that the content of such formulas is somehow connected with respect to a given set of parameters (e.g., a world in a model, a source, etc.). Usually, in order to emphasize the role it plays, this relation is called a *relating relation*. If we adopt a broad notion of proposition, according to which the proposition expressed by a formula is the thought expressed by that formula (or even the class of thoughts expressed by that formula), rather than a set of possible worlds, then we can regard the original component of relating semantics as a relation of *propositional pertinence*.

Deontic logic based on relating semantics can be formulated as a linguistic extension of traditional deontic logic. In this case it involves two sorts of deontic modalities: *quasi-deontic modalities* (or alethic-deontic modalities) and *strictly deontic modalities* (or purely deontic modalities). The former are equated with the modalities captured in *SDL* and receive an *intensional* reading (being closed under RPE), whereas the latter constitute a new set of modalities and receive a *hyperintensional* reading (being not closed under RPE). Another way of looking at this difference is saying that quasi-deontic modalities are characterized by a balance between an alethic and a deontic component; for instance, a proposition is *quasi-obligatory* iff it is true in all deontically optimal circumstances. By contrast, in strictly deontic modalities the deontic component is dominant; for instance, a proposition is *strictly-obligatory* iff it is true in all deontically optimal circumstances *and* it is related to some proposition in a normative system.

Relating semantics is, *per se*, a general framework to interpret modal languages, neither committed to a specific context of reasoning nor to a specific interpretation of the relation of propositional pertinence. Therefore, one of the main advantages of this framework, if compared with others available in the literature, is that it allows for introducing relations of a different kind among propositions and, possibly, additional parameters. In turn, any such relation can give rise to new technical devices.

In the present setting, the relation of propositional pertinence gives rise to two functions: *obligation assignment* and *causal contribution assignment*. The whole conceptual construction is the following. First, propositional pertinence is technically defined here as a *quaternary relation*; it specifies whether a proposition (expressed by a formula) ϕ is related to a proposition ψ according to a certain source s — which will be, in our case, a *normative source* — at a state w of a model. From this quaternary relation one can infer a simpler, *ternary relation* of propositional pertinence, that is, whether a proposition ϕ — *on its own* — is relevant to a source s at a state w of a model. In fact, intuitively, a source s may establish a relation between two propositions ϕ and ψ only if both of these are relevant to s. Finally, the meaning of the ternary relation is used to explain the two functions of obligation assignment and causal contribution assignment. The obligation assignment says what is obligatory for an agent a with reference to a source s at a state w. The causal contribution assignment says what causal contributions of an agent are relevant to a source s at a state w. Thus, we have the conceptual dependence represented in Figure 1, where, given two nodes x and y, an arrow from x to y means that x will be used to explain y.

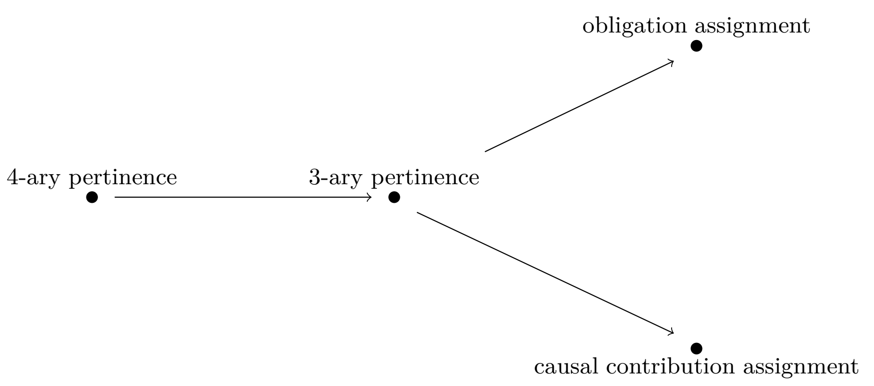

Figure 1. Conceptual dependence among key relations

We have already clarified the connection between normative concepts and normative systems. The connection between the concept of cause (or, as we prefer, causal contribution) and normative systems can be defended as well [see, e.g., Faroldi, 2014].[3] As the following example shows, being the cause is not an absolute issue, but a relative one:

> what counts as the cause of an event depends on the purpose of the inquiry. Someone is killed in a road accident. Was the cause of his death internal bleeding, the driver's mistake, the ice on the road with which the driver failed to cope, or the shortage of funds that led to the local authority's not removing the ice in time? [Honoré, 1999, p. 3]

In line with this observation, we will say that a normative source determines what causal contributions of an agent at a given state are relevant for normative discourse.

Building on top of this conceptual ground, we will develop a detailed technical analysis of two semantics for normative reasoning that are alternative to each other. In fact, the core of our contribution is a formal comparison between models of a certain version of relating semantics, here simply called *relating models*, and models endowed with a relation of propositional synonymy, here simply called *synonymy models*. The latter are taken from [Glavaničová and Pascucci, 2019], where they are used to interpret a multimodal language and reason on various notions of responsibility. The discussion of these two semantics occupies Sections 2–5. In particular, we show that synonymy models can be transformed into equivalent relating models of a particular kind, here called *strictly relating models*. Furthermore, we provide an axiomatization of the set of formulas that are valid in the class of all strictly relating models.

In Section 6, inheriting the aims of the formal analysis of responsibility carried out in [Glavaničová and Pascucci, 2019], we employ our logical apparatus involving fine-grained notions of obligation and causal contribution to capture new normative notions. In particular, we define notions of:

- *avoiding* (understood as the claim that a proposition ϕ never happens throughout time);
- *agent-avoiding* (understood as the claim that an agent never contributes to ϕ throughout time);

[3] 'Being *the* cause of something' is a stronger condition than 'causally contributing to something'. In cases of causal overdetermination, such as when two gunmen shoot at a single victim, we still want to ascribe responsibility to both of them, even though neither of them is *the* cause.

- *regret* (understood, with a slight simplification, as a situation in which an agent regrets a certain fact, and based on the notion of 'avoiding');
- *agent-regret* (understood, again with a slight simplification, as a situation in which an agent regrets their causal participation in ϕ, and based on the notion of 'agent-avoiding');
- *individual responsibility* (understood as individual blameworthiness, that is, a causal participation in a prohibited proposition that could have been avoided).

We conclude the paper by summarising its contents and pointing out interesting directions for future research.

2. Formal language

We will use a formal language $\mathcal{L}$ that is described by a set of primitive symbols and a grammar to build well-formed expressions. This will turn out to be a variation of the multimodal language employed for a formal analysis of responsibility in [Glavaničová and Pascucci, 2019].

Definition 2.1 (Primitive symbols). The list of primitive symbols in $\mathcal{L}$ is the following:

- a countable set of propositional variables $\texttt{Var}$, denoted by p_1, p_2, p_3, etc. (also p, q, r, etc.);
- a finite set of (names for) agents $\texttt{Agt}$, denoted by a_1, a_2, a_3, etc. (also a, b, c, etc.);
- a finite set of (names for) normative sources $\texttt{Src}$, denoted by s_1, s_2, s_3, etc. (also s, s', s'', etc. and $s(a)$, where $a \in \texttt{Agt}$);
- the unary logical operators $\neg$, H, G, C_a^s and O_a^s (for $a \in \texttt{Agt}$ and $s \in \texttt{Src}$);
- the binary logical operators $\rightarrow$ and $\sim_s$ (for $s \in \texttt{Src}$).

A normative source s represents a set of norms, such as the set of all norms included in a legal code. Our approach will rely on a notion of normative source that is arguably simplistic if compared with real-life norms. For instance, we will work under the assumption that sources are *pairwise disjoint*: every norm belongs to at most one normative source $s \in \texttt{Src}$. However, this simplification will serve the purpose of illustrating the theoretical foundations of the formalism. In principle, it is possible to enrich the framework provided here by taking into account a set of normative sources without such a restriction. In that case, one would have to encode a mechanism for source aggregation. A remark on the label $s(a)$, and similar ones used for normative sources: this can be a formal representation of agent a's personal set of norms, or a system of normative beliefs, that is, beliefs about what should be the case, or simply agent a's preferences about what should and should not happen to the world. To give an example, a cat person has a preference (believes that it should be the case) that no violence is done to cats; in comparison, a person who hates cats might not have such a preference. More generally, a cat person's set of norms will likely take into consideration the cats' well-being, whereas a cat hater's set of norms will probably not.

The intended reading of primitive operators in $\mathcal{L}$ will be clarified below, after the definition of well-formed formulas. We will say that $\neg$ and $\rightarrow$ are Boolean operators, $\sim_s$ is an operator for propositional pertinence (or propositional relatedness) and H, G, C_a^s and O_a^s are modal operators, for past reference, future reference, causal contribution and obligation, respectively. The notion of a proposition plays a central role in the present framework. We stress that a proposition is here not to be equated with a set of possible worlds; it can be rather conceived of as a Fregean thought (or even a set of Fregean thoughts) expressed by a sentence [see Frege, 1892]. We define the set of well-formed formulas (*wffs*) over $\mathcal{L}$ via a two-step procedure. First, we specify formulas over the 'pertinence-free' fragment of $\mathcal{L}$, namely the fragment where no operator $\sim_s$, for $s \in \texttt{Src}$, is involved; this fragment will be labelled as $\mathcal{L}^h$ (*hypo*-$\mathcal{L}$).

DEFINITION 2.2 (Grammar of $\mathcal{L}^h$-formulas). The set of $\mathcal{L}^h$-formulas is defined by the following grammar, where $p \in \texttt{Var}$, $a \in \texttt{Agt}$ and $s \in \texttt{Src}$:

$$\phi ::= p \,|\, \neg\phi \,|\, \phi \rightarrow \phi \,|\, H\phi \,|\, G\phi \,|\, C_a^s\phi \,|\, O_a^s\phi$$

DEFINITION 2.3 (Grammar of $\mathcal{L}$-formulas). The set of $\mathcal{L}$-formulas is defined by the following grammar, where ϕ is an $\mathcal{L}^h$-formula and $s \in \texttt{Src}$:

$$\psi ::= \phi \,|\, \phi \sim_s \phi$$

Thus, arguments of wffs whose main operator is $\sim_s$ *always* belong to $\mathcal{L}^h$. The notion of *main operator* and *argument(s) of an operator* in a formula are understood as usual. The meaning of formulas whose main operator is a Boolean one is also standard. Formulas whose main operator is a modal one read as follows: $H\phi$ says "in all possible past states it is the case that ϕ", $G\phi$ "in all possible future states it is the case that ϕ", $C_a^s\phi$ "it is a result of agent a's causal participation and relevant to source s that ϕ" and $O_a^s\phi$ "it is obligatory for agent a and relevant to source s that ϕ". A formula of the form $\phi \sim_s \psi$ means that ϕ and ψ express propositions that are related according to source s. Since, as we will see below, formulas are evaluated at a state w of a model, from a semantic perspective $\phi \sim_s \psi$ encodes the idea of propositional pertinence as a *quaternary relation*: two formulas (first two arguments of the relation) are said to be related according to a normative source (third argument of the relation) and with reference to a state (fourth argument of the relation).

The remarks on the meaning of logical operators that we have provided so far already point out some important features:

- it might be that an agent's causal contribution to an outcome is relevant to a normative source s_1 and not relevant to a normative source s_2;
- it might be that an agent is obliged to bring about something that is relevant to a normative source s_1 but not relevant to a normative source s_2;
- there might be a relation of propositional pertinence between ϕ and ψ according to a normative source s_1, but not according to a normative source s_2;
- the formal encoding of obligation and causal contribution *always* makes reference to a normative source, as it is witnessed by the superscript s in operators of kind C_a^s and O_a^s, and conveys a notion of relevance; in this regard, the approach presented here differs from the one in [Jarmużek and Klonowski, 2020], since it does *not* employ operators for quasi-deontic modalities.

Usual definitions can be adopted for Boolean operators of conjunction ($\wedge$), disjunction ($\vee$) and material equivalence ($\equiv$), as well as for the modal operators meaning "in some possible past state it is the case that ..." (P) and "in some possible future state it is the case that ..." (F). For instance, $P\phi =_{\text{def}} \neg H \neg \phi$ and $F\phi =_{\text{def}} \neg G \neg \phi$. The logical symbols $\top$ and $\bot$, for arbitrary tautologies and contradictions, are also defined in a standard way.

3. Relating semantics

We describe below the models inspired by relating semantics for modal logic that will be used in this article to interpret the formal language $\mathcal{L}$. These will be simply called *relating models*. A comparison with the original relating models for normative reasoning in [Jarmużek and Klonowski, 2020], as well as with the models used in [Glavaničová and Pascucci, 2019] to interpret a variation of $\mathcal{L}$ and here called *synonymy models*, is offered below.

Definition 3.1 (Relating model). A relating model for language $\mathcal{L}$ is a tuple $\mathfrak{M} = \langle W, \prec, R, c, o, V\rangle$ where:

- W is a set of states (denoted by w_1, w_2, w_3, w, w', w'', etc.);
- $\prec \subseteq W \times W$ is a relation that will be called *temporal precedence*;
- $R \subseteq \mathcal{L}^h \times \mathcal{L}^h \times \mathtt{Src} \times W$ is a relation that will be called *propositional pertinence*;
- $c\colon \mathtt{Agt} \longrightarrow \wp(R)$ is a function that will be called *contribution assignment*;
- $o\colon \mathtt{Agt} \longrightarrow \wp(R)$ is a function that will be called *obligation assignment*;
- $V\colon \mathtt{Var} \longrightarrow \wp(W)$ is a valuation function.

An example can be useful to illustrate the ideas behind Definition 3.1. Given $p, q \in \mathtt{Var}$, the fact that the ordered 4-tuple (p, q, s, w) belongs to R means that the proposition expressed by p and the proposition expressed by q are related by normative source s at state w. As we said above, this is the semantic encoding of the idea of a *quaternary relation* of propositional pertinence (the leftmost node in Figure 1). For instance, in the current state w, pandemic restrictions (s) say that the proposition that one stays home (p) is pertinent to the proposition that one has been exposed to COVID-19 (q). Now, since under this reading it is a normative source, s, that establishes that formulas p and q express related propositions, we can also say that p and q express propositions that are *both* relevant (or pertinent) to s at w. Accordingly, for any formula ϕ, if there is a formula ψ s.t. $(\phi, \psi, s, w) \in R$, then we can say that the proposition expressed by ϕ is relevant to s at w. This is the idea of propositional pertinence as a *ternary relation* (the central node of Figure 1), whose arguments are: a formula, a normative source and a state.

In other words, the relation R plays two roles in relating models:

- specifying whether two propositions ϕ and ψ are *pertinent to each other according to a normative source s* at a state w;

- specifying whether a proposition ϕ is *relevant to a normative source* s at a state w.

The first role is directly conveyed by an expression of kind $(\phi, \psi, s, w) \in R$, for some $\psi \in \mathcal{L}^h$; the second role is not directly expressed in the formal semantics, but can be inferred from the first one (and, in the second role, R ultimately stands for a ternary relation: between ϕ, s and w). This justifies the idea, illustrated in Figure 1, that in the present framework ternary pertinence is *explained* in terms of quaternary pertinence. Such a conceptual dependence is a result of our formal and semantic setting, and a different solution establishing a converse dependence between ternary and quaternary propositional pertinence could be adopted as well.

DEFINITION 3.2 (Truth-conditions). The truth of a wff of $\mathcal{L}$ at a state w of a relating model $\mathfrak{M}$ is recursively defined below:

- $\mathfrak{M}, w \vDash p$ iff $w \in V(p)$, for any $p \in \mathtt{Var}$;
- $\mathfrak{M}, w \vDash \neg\phi$ iff $\mathfrak{M}, w \nvDash \phi$;
- $\mathfrak{M}, w \vDash \phi \rightarrow \psi$ iff $\mathfrak{M}, w \nvDash \phi$ or $\mathfrak{M}, w \vDash \psi$;
- $\mathfrak{M}, w \vDash \phi \sim_s \psi$ iff $(\phi, \psi, s, w) \in R$;
- $\mathfrak{M}, w \vDash H\phi$ iff $\mathfrak{M}, v \vDash \phi$ for all $v \in W$ such that $v \prec w$;
- $\mathfrak{M}, w \vDash G\phi$ iff $\mathfrak{M}, v \vDash \phi$ for all $v \in W$ such that $w \prec v$;
- $\mathfrak{M}, w \vDash O_a^s\phi$ iff $(\phi, \phi, s, w) \in o(a)$;
- $\mathfrak{M}, w \vDash C_a^s\phi$ iff $(\phi, \phi, s, w) \in c(a)$.

While we have already discussed the clause relative to $\sim_s$ to a sufficient extent, and many other clauses are standard ones in the semantics of multimodal logic, a few comments on the last two clauses are needed. We recall that, according to the second of the aforementioned roles played by the relation R, the expression $(\phi, \phi, s, w) \in R$ can be read as saying that (the proposition expressed by) ϕ is relevant to normative source s at state w. Therefore, we get the following explanation of the truth-conditions associated with operators for obligation and causal contribution. $O_a^s\phi$ holds at a state w if and only if the propositional pertinence relation R establishes that ϕ is relevant to normative source s at world w *and* ϕ represents an obligation for agent a. $C_a^s\phi$ holds at a state w if and only if the propositional pertinence relation R establishes that ϕ is relevant to normative source s at world w *and* agent a contributed to ϕ.

The notions of validity of a formula in a model or in a class of models are defined in the usual way. For the sake of simplicity, we will sometimes make reference to the domain of a model $\mathfrak{M}$ and the relations and functions in $\mathfrak{M}$ without a previous thorough description of $\mathfrak{M}$.

From a formal point of view, the relation of propositional pertinence R in a relating model can be specified in an arbitrary way. However, if we want to get closer to the intended meaning of this notion, it is convenient to stipulate some restrictions. Therefore, we will consider a restricted class of relating models that will be called *strictly relating models*. Clearly, our choice of this class targets the specific context of reasoning we are dealing with; thus, it should not be taken as a definition of strictly relating models *tout court*. Given a relation of propositional pertinence R, we will denote by R^{id} (the *identity fragment* of R) the subset of R including all and only those 4-tuples in R that have the form (ϕ, ϕ, s, w).

Definition 3.3 (Strictly relating model). A relating model is a *strictly* relating model iff it satisfies the properties below, for every $\phi, \psi, \chi \in \mathcal{L}^h$, $w \in W$ and $s \in \mathtt{Src}$:

1. if there is $\psi \in \mathcal{L}^h$ such that $(\phi, \psi, s, w) \in R$, then $(\phi, \phi, s, w) \in R$;
2. if $(\phi, \psi, s, w) \in R$, then $(\psi, \phi, s, w) \in R$;
3. if $(\phi, \psi, s, w), (\psi, \chi, s, w) \in R$, then $(\phi, \chi, s, w) \in R$;
4. $R^{id} = \bigcup_{a \in \mathtt{Agt}}(c(a) \, \cup \, o(a))$;
5. if $(\phi, \psi, s, w) \in R$, then: for every $a \in \mathtt{Agt}$, $(\phi, \phi, s, w) \in c(a)$ entails $(\psi, \psi, s, w) \in c(a)$;
6. if $(\phi, \psi, s, w) \in R$, then: for every $a \in \mathtt{Agt}$, $(\phi, \phi, s, w) \in o(a)$ entails $(\psi, \psi, s, w) \in o(a)$.

Thus, according to the first two properties listed in Def. 3.3, at any world of a strictly relating model, whenever a proposition (expressed by) ϕ is related to a possibly different proposition (expressed by) ψ according to a source s, we can infer that (i) ϕ is related to itself according to s, and (ii) ψ is related to ϕ according to s. Furthermore, according to the third property in Def. 3.3, at any world of a strictly relating model, whenever ϕ is related to ψ and ψ is related to χ according to a normative source s, we can infer that ϕ is related to χ according to s. In addition, according to the fourth property in Def. 3.3, in strictly relating models the identity fragment of R corresponds to a set of tuples (ϕ, ϕ, s, w) such that ϕ expresses either something that source s prescribes to some agent a (at w) or a causal contribution of some agent a (at w) that is relevant to source s. Finally, according to the last two properties in Def. 3.3, at any world, if two propositions ϕ and ψ are pertinent to each other according to a normative source s, then, for each agent a, either both of them or none of them is an obligation (causal contribution) relevant to s.

Strictly relating models are those that we will use to simulate the semantic approach to the formal analysis of responsibility ascription carried out in [Glavaničová and Pascucci, 2019]. Before proceeding to this, a further clarifying note is needed. As a matter of fact, in the present article we defined the notion of a relating model for normative reasoning (see Def. 3.1) in a way that diverges from the definition in [Jarmużek and Klonowski, 2020]. The main differences can be summarized as below:

- Jarmużek and Klonowski formally encode a ternary — rather than quaternary — relation of propositional pertinence, since they do not make explicit reference to normative sources;
- the ternary pertinence relation for a propositional variable p is defined by Jarmużek and Klonowski in a way that holds either for each state $w \in W$ or for no state $w \in W$ (see the first clause of Definition 3.3 and Fact 3.4 in [Jarmużek and Klonowski, 2020]), whereas here it can change across states, since we want to capture the idea that the content of a normative source (whence, the relations it establishes on propositional variables) may vary over time;
- the pertinence relation is here a component of the formal language too and is defined in terms of formulas, rather than in terms of their internal components (see the *demodalization* procedure illustrated in Section 3.2 of [Jarmużek and Klonowski, 2020] and discussed below);
- since we do not employ quasi-deontic modalities, we do not use a set of accessible worlds to interpret deontic modalities; we rather directly use the two functions called 'obligation assignment' and 'contribution assignment' to interpret fine-grained modalities, and these functions are specified, for each agent a, as subsets of the relation R.

For a better assessment of these differences, we briefly explain how demodalization, a key procedure to define propositional pertinence in [Jarmużek and Klonowski, 2020], could be implemented in our setting (with straightforward adaptations to the current notation).

DEFINITION 3.4 (Demodalization). The demodalizer function $\mathbf{d} : \mathcal{L}^h \longrightarrow \mathcal{L}^h$ is recursively defined as follows:

- for any $p \in \mathtt{Var}$, $\mathbf{d}(p) = p$;
- $\mathbf{d}(*\phi) = \mathbf{d}(\phi)$, for $*$ a unary logical operator;
- $\mathbf{d}(\phi * \psi) = \mathbf{d}(\phi) * \mathbf{d}(\psi)$, for $*$ a binary logical operator.

Thus, when it is applied to the language $\mathcal{L}^h$, demodalization erases every occurrence of a logical operator in the set $\{H, G, O^s_a, C^s_a, \neg\}$ from an input formula ϕ. It is noteworthy that negation is treated by this procedure as a sort of modality. Jarmużek and Klonowski assume that "relations between sentences *modulo* a given normative system do not pertain to their modal nature, but only to the factual content they bear" [Jarmużek and Klonowski, 2020, p. 346]. A way of subscribing to this view in our framework would be adding the following properties to any relating model, which would force the pertinence relation to be closed under the addition or deletion of modal operators (and negation) in formulas, as well as under Boolean operations:

- for any $p \in \mathtt{Var}$, $(p, p, s, w) \in R$ iff for each $u \in W$ there is $\phi_u \in \mathcal{L}^h$ such that $(p, \mathbf{d}(\phi_u), s, u) \in R$ or $(\mathbf{d}(\phi_u), p, s, u) \in R$;
- for any binary Boolean operator $*$,
  $(\phi * \psi, \phi * \psi, s, w) \in R$ iff $(\mathbf{d}(\phi), \mathbf{d}(\psi), s, w) \in R$;
- for any unary logical operator $*$,
  $(*\phi, *\phi, s, w) \in R$ iff $(\mathbf{d}(\phi), \mathbf{d}(\phi), s, w) \in R$.

Alternatively, Jarmużek and Klonowski assume that demodalization concerns only deontic operators and negation, and, *mutatis mutandis*, analogous adjustments could be performed in our framework to meet this requirement.

4. From synonymy models to relating models

We describe below the alternative semantics in terms of *synonymy models* that was used in [Glavaničová and Pascucci, 2019] to interpret a language closely related (although not identical) to $\mathcal{L}$. The aim of this section is showing that synonymy models can be seen as relating models of a particular kind, namely as strictly relating models. The semantics in terms of synonymy models originates from the idea that in hyperintensional contexts (such as in reasoning about obligations or causal contributions) the proposition expressed by a formula does not correspond to a set of possible worlds. These models share some features with the hyperintensional models that are proposed by [Sedlár, 2021].

Definition 4.1 (Synonymy model). A *synonymy model* for language $\mathcal{L}$ is a tuple $\mathfrak{M} = \langle W, \prec, \mathsf{Cnt}, f, c, o, V \rangle$ where:

- W is a set of states denoted by w_1, w_2, w_3, etc. (also w, w', w'', etc.);

- Cnt is a set of *semantic contents* (or *propositions*) denoted by k_1, k_2, k_3, etc. (also k, k', k'', etc.);
- $\prec \subseteq (W \times W)$ is a relation called *temporal precedence*;
- $f\colon (\mathcal{L}^h \times W) \longrightarrow \mathsf{Cnt}$ is a relation called *content assignment*;
- $c\colon (\mathtt{Agt} \times \mathtt{Src} \times W) \longrightarrow \wp(\mathsf{Cnt})$ is a function called *contribution assignment*;
- $o\colon (\mathtt{Agt} \times \mathtt{Src} \times W) \longrightarrow \wp(\mathsf{Cnt})$ is a function called *obligation assignment*;
- $V\colon \mathtt{Var} \longrightarrow \wp(W)$ is a valuation function.

Thus, the crucial difference between *relating models* and *synonymy models* is the fact that the former include a relation of propositional pertinence R, whereas the latter include a set of semantic contents Cnt and a content assignment function f. Furthermore, the functions c and o are defined in different ways in the two models.

The definition of the truth-conditions in synonymy models needs to take into account the following syntactic differences between the language $\mathcal{L}$ adopted here and the original language in [Glavaničová and Pascucci, 2019]:

- for any $s \in \mathtt{Src}$, we now have an operator $\sim_s$ which encodes the relation of propositional pertinence with reference to s and replaces the single operator $\sim$ originally used;
- the operators for causal contribution used in the present setting make reference to a normative source (having the shape C^s_a), since their meaning is explained in terms of causal relevance, which, in turn, depends on propositional pertinence, and the latter is a source-relative notion.

Moreover, we add the assumption that the relation of propositional synonymy in accordance to a normative source s concerns only propositions that are either normatively relevant or causally relevant to s. This is required by the specific procedure that we will use to translate synonymy models into relating models, as we will see below, and it will have as a consequence that $\phi \sim_s \phi$ might not be valid in a model (since ϕ might be neither normatively nor causally relevant with reference to s). The truth-conditions for $\sim_s$ will be adapted accordingly.

DEFINITION 4.2 (Truth-conditions). The truth-conditions of a wff of $\mathcal{L}$ at a state w of a synonymy model $\mathfrak{M}$ are as in a relating model, except for the clauses below:

- $\mathfrak{M}, w \vDash \phi \sim_s \psi$ iff
  - $f(\phi, w) = f(\psi, w)$;
  - $f(\phi, w) \in c(a, s, w) \cup o(a, s, w)$, for some $a \in \mathtt{Agt}$;
- $\mathfrak{M}, w \vDash C^s_a \phi$ iff $f(\phi, w) \in c(a, s, w)$;
- $\mathfrak{M}, w \vDash O^s_a \phi$ iff $f(\phi, w) \in o(a, s, w)$.

The notions of validity of a formula in a synonymy model and in a class of synonymy models are defined in the usual way.

The following procedure, called a REL-*translation*, shows how one can transform a synonymy model into a relating model.

Definition 4.3 (REL-translation). Consider a synonymy model $\mathfrak{M}_1 = \langle W_1, \prec_1, \mathsf{Cnt}_1, f_1, c_1, o_1, V_1 \rangle$. Then, the REL-translation of $\mathfrak{M}_1$, denoted by $\mathtt{REL}(\mathfrak{M}_1)$, is the relating model $\mathfrak{M}_2 = \langle W_2, \prec_2, R_2, c_2, o_2, V_2 \rangle$ specified below:

- $W_2 = W_1$;
- $\prec_2 = \prec_1$;
- $(\phi, \psi, s, w) \in R_2$ iff
  - $f_1(\phi, w) = f_1(\psi, w)$;
  - $f_1(\phi, w) \in c_1(a, s, w) \cup o_1(a, s, w)$, for some $a \in \mathtt{Agt}$;
- $c_2(a) = \{(\phi, \phi, s, w) : f_1(\phi, w) \in c_1(a, s, w)\}$;
- $o_2(a) = \{(\phi, \phi, s, w) : f_1(\phi, w) \in o_1(a, s, w)$;
- $V_2 = V_1$.

Notice the following result:

Proposition 4.1 (REL-strictness). *The* REL-*translation of a synonymy model* $\mathfrak{M}$ *is a strictly relating model.*

Proof. By inspection of the properties of strictly relating models. Consider the first three properties of Def. 3.3. According to Def. 4.3, if we have $(\phi, \psi, s, w) \in R_2$, then we must also have $(\psi, \phi, s, w) \in R_2$ and $(\phi, \phi, s, w) \in R_2$. Furthermore, if we have $(\phi, \psi, s, w), (\psi, \chi, s, w) \in R_2$, then we must also have $(\phi, \chi, s, w) \in R_2$. In fact, all these properties — which have the shape of conditionals — follow from the fact that if their antecedents hold for some formulas ϕ. ψ and χ, then we have $f_1(\phi) = f_1(\psi) = f_1(\chi)$, and their consequents must hold as well, due to the properties of $=$. Consider the fourth property in Definition 3.3. According to Definition 4.3 we must have that $R^{id}_2 = \bigcup_{a \in \mathtt{Agt}} (c_2(a) \cup o_2(a))$. In fact — taking into account also the definition of R^{id}_2 in terms of R_2 — we have that $(\phi, \phi, s, w) \in R_2$ iff $f_1(\phi, w) \in c_1(a, s, w) \cup o_1(a, s, w)$

for some $a \in \mathtt{Agt}$ iff $(\phi, \phi, s, w) \in c_2(a) \cup o_2(a)$ for some $a \in \mathtt{Agt}$ iff $(\phi, \phi, s, w) \in \bigcup_{a \in \mathtt{Agt}}(c_2(a) \cup o_2(a))$. Consider the fifth and the sixth property in Definition 3.3. According to Definition 4.3, if we have $(\phi, \psi, s, w) \in R_2$ and, for some $a \in \mathtt{Agt}$, $(\phi, \phi, s, w) \in c_2(a)$, then we must also have $(\psi, \psi, s, w) \in c_2(a)$. In fact, the result follows from the fact that $f_1(\phi) = f_1(\psi)$. The same conclusion can be reached when we replace $c_2(a)$ with $o_2(a)$ in the argument. ⊣

We prove below that REL-translations are invariant with respect to the truth of formulas at a state. This property will be called REL-*invariance*.

PROPOSITION 4.2 (REL-invariance). *For every formula $\theta \in \mathcal{L}$, every synonymy model $\mathfrak{M}_1$ and every state w in its domain, it holds that $\mathfrak{M}_1, w \vDash \theta$ iff* $\mathtt{REL}(\mathfrak{M}_1), w \vDash \theta$.

PROOF. By induction on the syntactical complexity of θ. We assume that $\mathtt{REL}(\mathfrak{M}_1) = \mathfrak{M}_2$ in order to have an easier notation when we describe its components. The basic case in which θ is a propositional variable and the inductive step in which it is a formula whose main operator is in the set $\{\neg, \rightarrow, H, G\}$ can be easily dealt with as immediate consequences of the definition of the components W_2, $\prec_2$ and V_2 in $\mathfrak{M}_2$.

Consider the case $\theta = \phi \sim_s \psi$. For the left-to-right direction, assume $\mathfrak{M}_1, w \vDash \phi \sim_s \psi$. Then (by Def. 4.2), $f_1(\phi, w) = f_1(\psi, w)$ and there is some $a \in \mathtt{Agt}$ such that $f_1(\phi, w) \in c_1(a, s, w) \cup o_1(a, s, w)$. From this (by Def. 4.3), we can infer that $(\phi, \psi, s, w) \in R_2$. The last step leads (by Def. 3.2) to the claim that $\mathfrak{M}_2, w \vDash \phi \sim_s \psi$. For the right-to-left direction, assume that $\mathfrak{M}_1, w \nvDash \phi \sim_s \psi$. Then (by Def. 4.2), either $f_1(\phi, w) \neq f_1(\psi, w)$ or there is no $a \in \mathtt{Agt}$ such that $f_1(\phi, w) \in c_1(a, s, w) \cup o_1(a, s, w)$. From this (by Def. 4.3), we can infer that $(\phi, \psi, s, w) \notin R_2$. The last step allows one to infer (by Def. 3.2) that $\mathfrak{M}_2, w \nvDash \phi \sim_s \psi$.

Consider the case $\theta = C_a^s\phi$. For the left-to-right direction, assume that $\mathfrak{M}_1, w \vDash C_a^s\phi$. Then (by Def. 4.2), we can infer that $f_1(\phi, w) \in c_1(a, s, w)$. This entails (by Def. 4.3) that $(\phi, \phi, s, w) \in c_2(a)$. Therefore (by Def. 3.2), we can conclude that $\mathfrak{M}_2, w \vDash C_a^s\phi$. For the right-to-left direction, assume that $\mathfrak{M}_1, w \nvDash C_a^s\phi$. Then (by Def. 4.2), we can infer that $f_1(\phi, w) \notin c_1(a, s, w)$. This entails (by Def. 4.3) that $(\phi, \phi, s, w) \notin c_2(a)$. Thus (by Def. 3.2), we can conclude that $\mathfrak{M}_2, w \nvDash C_a^s\phi$.

Consider the case $\theta = O^s_a\phi$. For the left-to-right direction, assume that $\mathfrak{M}_1, w \vDash O^s_a\phi$. Then (by Def. 4.2), we can infer that $f_1(\phi, w) \in o_1(a, s, w)$. This entails (by Def. 4.3) that $(\phi, \phi, s, w) \in o_2(a)$. Therefore (by Def. 3.2), we can conclude that $\mathfrak{M}_2, w \vDash O^s_a\phi$. For the right-to-left direction, assume that $\mathfrak{M}_1, w \nvDash O^s_a\phi$. Then (by Def. 4.2), we can infer that $f_1(\phi, w) \notin o_1(a, s, w)$. This entails (by Def. 4.3) that $(\phi, \phi, s, w) \notin o_2(a)$. Thus (by Def. 3.2), we can conclude that $\mathfrak{M}_2, w \nvDash O^s_a\phi$. ⊣

5. Axiomatic system

In the present section we provide an axiomatization for the set of formulas that are valid in the class of all strictly relating models. Hereafter we will denote by $\mathcal{C}_{\mathrm{SRM}}$ the class of all strictly relating models, and the fact that a formula $\phi \in \mathcal{L}$ is valid in such class by $\vDash_{\mathcal{C}_{\mathrm{SRM}}} \phi$.

The following list of deductive principles constitutes a logical system that we will simply call $\mathbb{S}$. The fact that a formula $\phi \in \mathcal{L}$ is provable in $\mathbb{S}$ will be denoted by $\vdash_{\mathbb{S}} \phi$.

- A0 All $\mathcal{L}$-instances of tautologies of the Propositional Calculus;
- A1 $((\bigvee_{a\in\mathtt{Agt}} C^s_a\phi) \vee (\bigvee_{a'\in\mathtt{Agt}} O^s_{a'}\phi)) \equiv (\phi \sim_s \phi)$;
- A2 $(\phi \sim_s \psi) \to (\phi \sim_s \phi)$;
- A3 $(\phi \sim_s \psi) \to (\psi \sim_s \phi)$;
- A4 $(\phi \sim_s \psi) \to ((\psi \sim_s \chi) \to (\phi \sim_s \chi))$;
- A5 $H(\phi \to \psi) \to (H\phi \to H\psi)$;
- A6 $G(\phi \to \psi) \to (G\phi \to G\psi)$;
- A7 $\phi \to HF\phi$;
- A8 $\phi \to GP\phi$;
- A9 $(\phi \sim_s \psi) \to (C^s_a\phi \to C^s_a\psi)$;
- A10 $(\phi \sim_s \psi) \to (O^s_a\phi \to O^s_a\psi)$;
- R0 from the set of assumptions $\{\phi, \phi \to \psi\}$, infer ψ;
- R1 if $\vdash_{\mathbb{S}} \phi$, then $\vdash_{\mathbb{S}} H\phi \wedge G\phi$.

Notice that the formulation of R0 is intended to capture deductive reasoning under assumptions, whereas the formulation of R1 allows only for reasoning on provable formulas. The result that will be proven below will point out that system $\mathbb{S}$ constitutes an axiomatization of validity in the class of models $\mathcal{C}_{\mathrm{SRM}}$.

PROPOSITION 5.1 (Soundness). *For any $\theta \in \mathcal{L}$, if $\vdash_{\mathbb{S}} \theta$ then $\vDash_{\mathcal{C}_{\mathrm{SRM}}} \theta$.*

PROOF. An induction on the length of derivations in $\mathbb{S}$. The result for A0 and R0 is straightforward. In the case of A5–A8 and R1 the result follows from the correspondence theory for systems of tense logic. Therefore, we discuss only A1–A4 and A9–A10.

In the case of A1, for the left-to-right direction of the biconditional, suppose that there is a state w of a model $\mathfrak{M}$ such that either (i) $\mathfrak{M}, w \vDash C^s_a\phi$ or (ii) $\mathfrak{M}, w \vDash O^s_{a'}\phi$, for some $a, a' \in \mathtt{Agt}$. In the sub-case (i) by Def. 3.2 we get that $(\phi, \phi, s, w) \in c(a)$. In the light of Def. 3.3, $R^{id} = \bigcup_{a\in\mathtt{Agt}}(c(a) \cup o(a))$, whence $(\phi, \phi, s, w) \in R^{id}$ and $(\phi, \phi, s, w) \in R$. Thus, by Def. 3.2, $\mathfrak{M}, w \vDash \phi \sim_s \phi$. In the sub-case (ii) we get the same result by replacing $c(a)$ with $o(a')$ in the previous argument. For the right-to-left direction of the biconditional, assume that there is no $a \in \mathtt{Agt}$ such that $\mathfrak{M}, w \vDash C^s_a\phi$ and that there is no $a' \in \mathtt{Agt}$ such that $\mathfrak{M}, w \vDash O^s_{a'}\phi$. Then, by Def. 3.2, we have that $(\phi, \phi, s, w) \notin \bigcup_{a\in\mathtt{Agt}}(c(a) \cup o(a))$. In the light of Def. 3.3 we can infer that $(\phi, \phi, s, w) \notin R^{id}$, whence that $(\phi, \phi, s, w) \notin R$. Finally, by Def. 3.2, we can reach the conclusion that $\mathfrak{M}, w \nvDash \phi \sim_s \phi$.

In the case of A2, assume that $\mathfrak{M}, w \vDash \phi \sim_s \psi$. Then, due to Def. 3.2 and Def. 3.3, $(\phi, \psi, s, w), (\phi, \phi, s, w) \in R$ and, by Def. 3.2, one can infer $\mathfrak{M}, w \vDash \phi \sim_s \phi$.

In the case of A3, assume $\mathfrak{M}, w \vDash \phi \sim_s \psi$. Then, due to Def. 3.2 and Def. 3.3, $(\phi, \psi, s, w), (\psi, \phi, s, w) \in R$ and, by Def. 3.2, one can infer $\mathfrak{M}, w \vDash \psi \sim_s \phi$.

In the case of A4, assume $\mathfrak{M}, w \vDash \phi \sim_s \psi$ and $\mathfrak{M}, w \vDash \psi \sim_s \chi$. Then, due to Def. 3.2 and Def. 3.3, $(\phi, \psi, s, w), (\psi, \chi, s, w), (\phi, \chi, s, w) \in R$ and, by Def. 3.2, one can infer $\mathfrak{M}, w \vDash \phi \sim_s \chi$.

In the case of A9, assume that $\mathfrak{M}, w \vDash \phi \sim_s \psi$ and $\mathfrak{M}, w \vDash C^s_a\phi$. Then, by Def. 3.2, $(\phi, \psi, s, w) \in R$ and $(\phi, \phi, s, w) \in c(a)$. Due to Def. 3.3, we also have $(\psi, \psi, s, w) \in c(a)$, whence we can conclude $\mathfrak{M}, w \vDash C^s_a\psi$. The case of A10 is analogous. ⊣

PROPOSITION 5.2 (Completeness). *For any $\theta \in \mathcal{L}$, if $\vDash_{\mathcal{C}_{\mathrm{SRM}}} \theta$ then $\vdash_{\mathbb{S}} \theta$.*

PROOF. We rely on the method of canonical models. The canonical model $\mathfrak{M}$ for system $\mathbb{S}$ is defined as follows:

- W is the set of all maximal $\mathbb{S}$-consistent sets of wffs of $\mathcal{L}$;
- for all $w, w' \in W$, $w \prec w'$ iff $\{\phi : G\phi \in w\} \subseteq w'$ iff $\{\phi : H\phi \in w'\} \subseteq w$;[4]

[4] The equivalence of these two definitions of $w \prec w'$ is granted by classical

- $R = \{(\phi, \psi, s, w) : \phi \sim_s \psi \in w\}$;
- for every $a \in \mathtt{Agt}$, $c(a) = \{(\phi, \phi, s, w) : C_a^s\phi \in w\}$;
- for every $a \in \mathtt{Agt}$, $o(a) = \{(\phi, \phi, s, w) : O_a^s\phi \in w\}$;
- for every $p \in \mathtt{Var}$, $V(p) = \{w : p \in w\}$.

Usual techniques for completeness theory in multimodal languages allow one to prove that every maximal $\mathbb{S}$-consistent set is closed under deduction in $\mathbb{S}$: if $\Gamma \subseteq w$ and ϕ can be inferred from Γ in $\mathbb{S}$, then $\phi \in w$ as well. The same applies to the truth-lemma for the canonical model $\mathfrak{M}$, namely the fact that for every $\phi \in \mathcal{L}$ and $w \in W$, $\mathfrak{M}, w \vDash \phi$ iff $\phi \in w$. What remains to be proven is that $\mathfrak{M}$ satisfies all the properties of a strictly relating model.

Consider the first property of Def. 3.3. Assume that $(\phi, \psi, s, w) \in R$. Then, by Def. 3.2, we can infer that $\mathfrak{M}, w \vDash \phi \sim_s \psi$. Thus, due to the truth-lemma, it follows that $\phi \sim_s \psi \in w$ and, due to the fact that every maximal $\mathbb{S}$-consistent set of wffs includes all instances of A2, A0 and is closed under deduction, it follows that $\phi \sim_s \phi \in w$, whence that $(\phi, \phi, s, w) \in R$ as well.

The second and the third property of Def. 3.3 can be dealt with in a similar way relying on A3 and A4.

Consider the fourth property of Def. 3.3. Assume, for the sake of contradiction, that $R^{id} \neq \bigcup_{a\in\mathtt{Agt}}(c(a) \cup o(a))$. Then, due to the definition of R^{id}, either (i) there is $(\phi, \phi, s, w) \in R$ such that $(\phi, \phi, s, w) \notin \bigcup_{a\in\mathtt{Agt}}(c(a) \cup o(a))$, or (ii) there is $(\phi, \phi, s, w) \in \bigcup_{a\in\mathtt{Agt}}(c(a) \cup o(a))$ such that $(\phi, \phi, s, w) \notin R$. In the sub-case (i), due to Def. 3.2, we get that $\mathfrak{M}, w \vDash \phi \sim_s \phi$ and, by the truth-lemma, $\phi \sim_s \phi \in w$. Since every maximal $\mathbb{S}$-consistent set includes all instances of A1, then $(\bigvee_{a\in\mathtt{Agt}} C_a^s\phi) \vee (\bigvee_{a'\in\mathtt{Agt}} O_{a'}^s\phi) \in w$ and, by the truth-lemma, $\mathfrak{M}, w \vDash (\bigvee_{a\in\mathtt{Agt}} C_a^s\phi) \vee (\bigvee_{a'\in\mathtt{Agt}} O_{a'}^s\phi)$. This entails that either there is $a \in \mathtt{Agt}$ such that $\mathfrak{M}, w \vDash C_a^s\phi$ or that there is $a' \in \mathtt{Agt}$ such that $\mathfrak{M}, w \vDash O_{a'}^s\phi$. In both cases, by Def. 3.2, we get that $(\phi, \phi, s, w) \in \bigcup_{a\in\mathtt{Agt}}(c(a) \cup o(a))$: contradiction. In the sub-case (ii), we must have that for some $a \in \mathtt{Agt}$, either $\mathfrak{M}, w \vDash C_a^s\phi$ or $\mathfrak{M}, w \vDash O_a^s\phi$. Therefore, $\mathfrak{M}, w \vDash (\bigvee_{a\in\mathtt{Agt}} C_a^s\phi) \vee (\bigvee_{a'\in\mathtt{Agt}} O_{a'}^s\phi)$ and $(\bigvee_{a\in\mathtt{Agt}} C_a^s\phi) \vee (\bigvee_{a'\in\mathtt{Agt}} O_{a'}^s\phi \in w)$, by the truth-lemma. Then, due to the properties of maximal $\mathbb{S}$-consistent sets, $\phi \sim_s \phi \in w$ and from this we can conclude $(\phi, \phi, s, w) \in R$: contradiction.

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results in completeness theory for tense logic [see van Benthem, 1983], and relies on the bridge-axioms A7–A8.

Consider the fifth property of Def. 3.3. Assume that $(\phi, \psi, s, w) \in R$ and that for some $a \in \mathtt{Agt}$, $(\phi, \phi, s, w) \in c(a)$. Then, by Def. 3.2, $\mathfrak{M}, w \vDash \phi \sim_s \psi$ and $\mathfrak{M}, w \vDash C^s_a\phi$. By the truth-lemma, $\phi \sim_s \psi, C^s_a\phi \in w$, whence, by A9 and the properties of maximal $\mathbb{S}$-consistent sets, $C^s_a\psi \in w$ as well. From this we can infer, by the truth-lemma, that $\mathfrak{M}, w \vDash C^s_a\psi$, whence, by Definition 3.2, that $(\psi, \psi, s, w) \in c(a)$.

The sixth property of Definition 3.3 can be dealt with in an analogous way. $\dashv$

As a consequence of Propositions 5.1 and 5.2, system $\mathbb{S}$ is not closed under replacement of provable equivalents (RPE). In particular, even if $\vdash_{\mathbb{S}} \phi \equiv \psi$, it may be the case that $\nvdash_{\mathbb{S}} C^s_a\phi \equiv C^s_a\psi$ or that $\nvdash_{\mathbb{S}} O^s_a\phi \equiv O^s_a\psi$, for some $a \in \mathtt{Agt}$ and $s \in \mathtt{Src}$. For the sake of example, consider the formulas p and $p \wedge \top$. Due to the fact that $\mathbb{S}$ includes all theorems of the Propositional Calculus, $\vdash_{\mathbb{S}} p \equiv (p \wedge \top)$. However, there are some maximal $\mathbb{S}$-consistent sets of formulas s.t., for every $\phi, \psi \in \mathcal{L}^h$ and $s \in \mathtt{Src}$, $\phi \sim_s \psi$ is not included in those sets (the simplest way of justifying this is by observing that no axiom of $\mathbb{S}$ has the form $\phi \sim_s \psi$ and that neither the rule R0 nor the rule R1 allows one to derive formulas of that form without further assumptions); some of those sets will include $C^s_a p$ but not $C^s_a(p \wedge \top)$ (or $O^s_a p$ but not $O^s_a(p \wedge \top)$) for some $s \in \mathtt{Src}$ and $a \in \mathtt{Agt}$, since A9 (A10) is the only axiom of $\mathbb{S}$ establishing a logical connection between pairs of formulas of the form $C^s_a\phi$ and $C^s_a\psi$ ($O^s_a\phi$ and $O^s_a\psi$), but this connection relies on an assumption of the form $\phi \sim_s \psi$. Moreover, any logical connection between pairs of formulas of the form $C^s_a\phi$ and $O^s_a\phi$ always depends on whether one assumes $\phi \sim_s \phi$ to hold or not (the only relevant principle in this regard is A1).

6. Responsibility and regret

The present section applies the proposed framework to the analysis of various notions related to responsibility. While in the previous sections we aimed at general semantic results, making use of the broad class of relating models $\mathcal{C}_{\mathrm{SRM}}$ and the weak formal system $\mathbb{S}$, throughout this section we will assume principles of deductive reasoning that extend the axiomatic basis of $\mathbb{S}$, as well as additional properties of models, in order to more closely adhere to the formal definitions of responsibility provided in [Glavaničová and Pascucci, 2019]. In particular, we take the following additional axioms:

A11 $H\phi \to HH\phi$;
A12 $G\phi \to GG\phi$;
A13 $(H\phi \wedge G\phi \wedge \phi) \to GH\phi$.

These principles force the relation $\prec$ to be transitive and backward linear (thus, only forward branching is admissible). The resulting models are tree-like and can be used to draw a comparison with other frameworks for non-deterministic time where notions of responsibility have been analysed; for instance, see the recent systematic analysis based on STIT-logic that is carried out by Canavotto [2020].

We will analyse notions of avoiding (a certain outcome or a participation in it), regret, (a simple version of) agent-regret, and individual (contributory) responsibility. The last notion is crucial for the debate on moral and legal responsibility (individual responsibility as we understand it can be either legal or moral, and thus it is a notion applicable in both law and morality). The two notions of regret were introduced by Williams [1981] in his analysis of moral luck and Raz [2011] debated them in the context of responsibility and broadened their understanding. We will start our discussion with two notions of avoiding, since these notions will feature in our definitions of (agent-)regret as well as in individual responsibility (on the importance of the possibility of avoiding a certain outcome, or participation in it see [Watson, 2004; Braham and Van Hees, 2012]).

The notion of avoiding employed here is rather simple. It is understood as a property of a state w from whose perspective ϕ never happens throughout time, which can be equated with the conjunction of three claims made at w: 'ϕ is not the case now', 'ϕ has never been the case' and 'ϕ will never be the case'. Let us now express this notion formally in the present framework (all notions will be defined relying on purely syntactic tools available in our language $\mathcal{L}$ and on the logical principles of system $\mathbb{S}$, together with the additional axioms A11–A13):[5]

DEFINITION 6.1 (Avoiding). The claim 'ϕ is avoided throughout time' corresponds to the following formula:

$$A\phi =_{\text{def}} \neg\phi \wedge H\neg\phi \wedge G\neg\phi$$

[5] Glavaničová and Pascucci [2019] introduce a notion of *avoidability* which is related to the present notion of agent-avoiding (later in this section): a causal participation of the agent in something that was avoidable at some moment in the past.

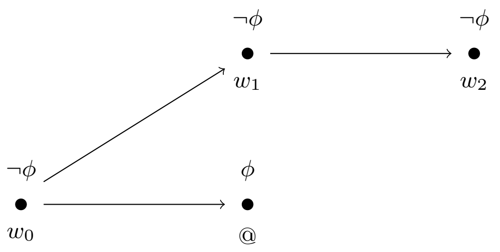

Figure 2. Avoidability

The above definition reads as follows: ϕ never happens, that is, it is false at the time of evaluation, and it always has been and always will be false. The *possibility of avoiding* ϕ (avoidability) can be expressed via $PFA\phi$ (that is, there is a past state w and a state w' in its future from whose perspective ϕ is avoided throughout time), as Figure 2 illustrates.

In Figure 2, at the actual state @, ϕ holds. However, there is a past state w_0 such that there is a state w_1 in its future where it holds that ϕ is never true: it is false at w_1, it is always false in its past (in this model there is just one state in its past, namely w_0), and it is always false in its future (in this model, the only state in its future is w_2). That means that while ϕ actually holds, it could have been avoided.

Agent-avoiding is a similar notion but focuses on causal participation rather than on any event whatsoever (that is, agent-avoiding is a specific case of avoiding):

DEFINITION 6.2 (Agent-Avoiding). The claim 'ϕ is avoided throughout time by agent a' corresponds to the following formula:

$$A^s_a\phi =_{\text{def}} \neg C^s_a\phi \wedge H\neg C^s_a\phi \wedge G\neg C^s_a\phi$$

The above definition reads as follows: the agent does not actually participate, never has participated, and never will participate in ϕ (that is, a never causally participates in ϕ throughout time).

Let us now turn to the notions of *regret* and *agent-regret.*

> The constitutive thought of regret in general is something like 'how much better if it had been otherwise', and the feeling can in principle apply to anything of which one can form some conception of how it might have been otherwise, together with consciousness of how things would then have been better. [Williams, 1981, p. 27]

Regret is to be distinguished from a peculiar sub-species of regret, agent-regret:

> a person can feel only towards his own past actions (or, at most, actions in which he regards himself as a participant). In this case, the supposed possible difference is that one might have acted otherwise, and the focus of the regret is on that possibility, the thought being formed in part by first-personal conceptions of how one might have acted otherwise.
>
> [Williams, 1981, *ibid.*]

Williams [1981, 30] also notes that agent-regret involves "a wish on the agent's part that he had not done it."

In addition, while Williams reserves agent-regret only to cases of regret that are integral to an agent's identity and character (and thus, 'life-changing' events), Raz allows it to apply also to a more mundane kind of regret related to our own conduct and its consequences that we normally experience. We will follow this broader understanding of agent-regret. According to Raz [2011, p. 237], agent-regret is important for the debate because it sheds light on the reasons why we are attached to the consequences of our actions, even if they are beyond our control: our actions (and their consequences) are "related to our sense of who we are."

Our formalisation of regret contains three elements: ϕ holds; it could have been avoided; and the agent wishes that it is avoided. The agent's wish is here formalised as the agent's normative preference: that is, what ought to be the case according to the agent's normative system. To this aim, we employ the notation $s(a)$, which explicitly connects a source to an agent (we recall that $s(a)$ is a possible label for an element of the set `Src`). In more detail: ϕ happened to be the case; yet, according to a's preferences ($s(a)$), it should have never been the case ($O_a^{s(a)} H \neg\phi$). For instance, a's friend died climbing (ϕ). Then, agent a deeply regrets this, and thus has it that it should have been otherwise.

Our definition of regret is then as follows:

Definition 6.3 (Regret). The claim 'agent a regrets ϕ' corresponds to the following formula:

$$R_a^{s(a)}\phi =_{\text{def}} \phi \wedge PFA\phi \wedge O_a^{s(a)}(A\phi)$$

The notion of *agent-regret* is similar, but it focuses on an agent's causal participation:[6]

[6] As we indicated above, we take it that causal participation is sufficient for responsibility; in this way we can account for cases of causal overdetermination.

DEFINITION 6.4 (Agent-Regret). The claim 'agent a regrets their contribution to ϕ' corresponds to the following formula:

$$R_a^{s,s(a)}\phi =_{\text{def}} C_a^s\phi \wedge PFA_a^s\phi \wedge O_a^{s(a)}(A_a^s\phi)$$

To illustrate how these definitions work, consider how we can employ a tense operator to talk about regret which concerns past state-of-affairs rather than those that presently obtain. Regret as related to past can be expressed as $P(\phi \wedge PFA\phi) \wedge O_a^{s(a)}(A\phi)$; agent-regret related to the past can in turn be formalised as $P(C_a^s\phi \wedge PFA_a^s\phi) \wedge O_a^{s(a)}(A_a^s\phi)$.

The final bit we want to cover in the present section is *individual responsibility*, which we will understand as a backward-looking notion of responsibility for a prohibited state-of-affairs to which the agent causally participated (thereby, contributory responsibility) while their participation was avoidable (this notion of responsibility can be perhaps best qualified as *individual blameworthiness*). This understanding of responsibility is very useful both in the context of law and in the context of morality. To give an example, White and Baum [2017, p. 66] note that generally, "the law punishes those who have caused harm, particularly the harm that could and should have been avoided." We can formalise such a notion of responsibility as follows:

DEFINITION 6.5 (Individual Responsibility). The claim 'agent a is individually responsible for ϕ' corresponds to the following formula:

$$\mathcal{IR}_a\phi =_{\text{def}} P(C_a^s\phi \wedge PFA_a^s\phi \wedge O_a^s\neg\phi)$$

This definition contains three core elements:

- a fine-grained notion of causal contribution (which avoids problems noted at the beginning of our paper, but also improves the notion of cause);
- the possibility of agent-avoiding, that is, avoidability of the causal participation rather than avoidability of the event itself;
- a fine-grained notion of prohibition.

That is, an individual has causally participated in ϕ ($C_a^s\phi$ holds in a past state w); the participation in ϕ could have been avoided (the past state w is preceded by a state w_1, which in turn precedes a future state w_2 such that, from the perspective of the latter state, the agent never — throughout the entire course of time — participates in ϕ, that is, $\neg C_a^s\phi \wedge H\neg C_a^s\phi \wedge G\neg C_a^s\phi$); and ϕ should have been avoided (i.e., in the past state w, $O_a^s\neg\phi$ holds).

Note that this notion is a notion of historic responsibility (that is, a backward-looking notion) as opposed to prospective responsibility (a forward-looking notion). The latter would be here analysed simply as an obligation that is not past-oriented (in other words, does not contain an operator for past reference in its scope).

All notions that have been syntactically defined in this section, apart from the notion of avoiding (Def. 6.1), correspond to hyperintensional modalities in systems extending $\mathbb{S}$ with A11–A13, since replacement of provable equivalents cannot be safely performed within their scope. In fact, all these notions involve occurrences of operators of the type C^s_a or O^s_a, which are, in turn, hyperintensional modalities (the addition of A11–A13 does not affect what we observed on the deductive behaviour of the latter modalities in $\mathbb{S}$).

7. Concluding remarks and future work

Various semantics can be used to interpret modal languages in normative contexts. Two recent proposals were taken into account and compared in this article: a semantics based on relating models and a semantics based on synonymy models. It turned out that, in the end, the two semantics are not 'rival': synonymy models can be admitted to the family of relating models, since it is possible to provide a translation function which takes a synonymy model as an input and gives a strictly relating model as an output (for instance, the REL-translation employed here). Our envisaged future research in this area includes translations between specific classes of models over the two semantic approaches.

Regardless of the semantics one chooses to interpret it, the multimodal language $\mathcal{L}$ used in the present work allows one to represent deontic and causal modalities in a fine-grained manner. Also, as we pointed out, it allows one to represent interesting notions of regret and responsibility. From the philosophical point of view, our future aim is capturing more complex normative notions, such as: group responsibility (for a simple formal notion of group responsibility [see Glavaničová and Pascucci, 2019]) vicarious responsibility and retroactive responsibility. Finally, we plan to deal with 'positive' notions of responsibility (e.g., praiseworthiness or vicarious praise). In fact, apart from the notion of *prospective responsibility*, which can be easily encoded as a present- or future-oriented obligation in this setting, we focused mainly on responsibility in the *negative sense* (i.e., for something wrong). This is so because such sense seems to be more pressing from a practical point of view. Thus, positive notions of responsibility are left as another interesting topic for future research.

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